Space Vector PWM

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Space vectors

• The origin of space vectors lies in rotating mmf in machines.
• The resultant mmf for a three phase system is a rotating mmf having a fixed magnitude and direction at every instant of time.
• Space vector is a mathematical concept which is useful for visualizing the effect of three phase variables in space.
Space vectors

• Resultant space vector for load phase voltage or current are defined as,
  • \( V_R(t) = \frac{2}{3} [v_{An}(t) + v_{Bn}(t)e^{\frac{j2\pi}{3}} + v_{Cn}(t)e^{\frac{j4\pi}{3}}] \)
  • \( I_R(t) = \frac{2}{3} [i_A(t) + i_B(t)e^{\frac{j2\pi}{3}} + i_C(t)e^{\frac{j4\pi}{3}}] \)

• The space vectors \( V_R(t) \) or \( I_R(t) \) have both magnitude and angle. Individual voltages/currents can be balanced or unbalanced and need not be sinusoidal.
Current space Vector

- For the sinusoidal three phase currents, the resultant current space vector is shown.
- The resultant space vector (pink) is rotating at a uniform speed and having a constant radius.
• The pole voltage of one phase of the converter has two switching states: 1 (=$V_D$) and 0(=0).

• The converter has total eight switching states ($2\times2\times2=8$). These are: (000,111,100,110,010,011,001,101).

• There are six active vectors and two zero vectors.

• What is the load phase voltage space vector for 100 combination?
Space vector for 100 combination

- $v_{AO}(t) = VD$, $v_{BO}(t) = 0$, $v_{CO}(t) = 0$
- $v_{An}(t) = \frac{2}{3} v_{AO}(t) - \frac{1}{3} v_{BO}(t) - \frac{1}{3} v_{CO}(t) = \frac{2}{3} V_D$
- $v_{Bn}(t) = \frac{2}{3} v_{BO}(t) - \frac{1}{3} v_{CO}(t) - \frac{1}{3} v_{AO}(t) = -\frac{1}{3} V_D$
- $v_{Cn}(t) = \frac{2}{3} v_{CO}(t) - \frac{1}{3} v_{AO}(t) - \frac{1}{3} v_{BO}(t) = -\frac{1}{3} V_D$

- $V_R(t) = \frac{2}{3} \left[ v_{An}(t) + v_{Bn}(t)e^{j\frac{2\pi}{3}} + v_{Cn}(t)e^{j\frac{4\pi}{3}} \right] = \frac{2}{3} V_D e^{j0}$

- Similarly we can deduce the resultant space vector for other combinations.
# Space vector for all combinations

<table>
<thead>
<tr>
<th>Space Vector</th>
<th>Switching States</th>
<th>Resultant space vector ($\mathbf{v}_R(t)$)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>V0</td>
<td>000</td>
<td>$\mathbf{v}_0 = 0$</td>
<td>Zero Vector</td>
</tr>
<tr>
<td>V1</td>
<td>100</td>
<td>$\mathbf{v}_1 = \frac{2}{3} V_D e^{j0}$</td>
<td></td>
</tr>
<tr>
<td>V2</td>
<td>110</td>
<td>$\mathbf{v}_2 = \frac{2}{3} V_D e^{j\pi/3}$</td>
<td></td>
</tr>
<tr>
<td>V3</td>
<td>010</td>
<td>$\mathbf{v}_3 = \frac{2}{3} V_D e^{j2\pi/3}$</td>
<td></td>
</tr>
<tr>
<td>V4</td>
<td>011</td>
<td>$\mathbf{v}_4 = \frac{2}{3} V_D e^{j3\pi/3}$</td>
<td></td>
</tr>
<tr>
<td>V5</td>
<td>001</td>
<td>$\mathbf{v}_5 = \frac{2}{3} V_D e^{j4\pi/3}$</td>
<td></td>
</tr>
<tr>
<td>V6</td>
<td>101</td>
<td>$\mathbf{v}_6 = \frac{2}{3} V_D e^{j5\pi/3}$</td>
<td></td>
</tr>
<tr>
<td>V7</td>
<td>111</td>
<td>$\mathbf{v}_7 = 0$</td>
<td>Zero Vector</td>
</tr>
</tbody>
</table>
The space vectors can be obtained also from a graphical method.
Eight space vectors

\[ V_3 = \frac{2}{3} V_D e^{j(2\pi/3)(010)} \]
\[ V_2 = \frac{2}{3} V_D e^{j(\pi/3)(110)} \]
\[ V_1 = \frac{2}{3} V_D e^{j0(100)} \]
\[ V_4 = \frac{2}{3} V_D e^{j(\pi)(011)} \]
\[ V_5 = \frac{2}{3} V_D e^{j(4\pi/3)(-+1)} \]
\[ V_6 = \frac{2}{3} V_D e^{j(5\pi/3)(+-+)} \]
Boundary of space vector diagram

\[ V_3 = \frac{2}{3} V_D e^{j(2\pi/3)} (010) \]
\[ V_2 = \frac{2}{3} V_D e^{j(\pi/3)} \]
\[ V_4 = \frac{2}{3} V_D e^{j(\pi)} (011) \]
\[ V_7 = 0 \]
\[ V_5 = \frac{2}{3} V_D e^{j(4\pi/3)} (-+) \]
\[ V_6 = \frac{2}{3} V_D e^{j(5\pi/3)} \]
Sectors in space vector diagram

- **V₀(000)**
- **V₁(100)**
- **V₂(110)**
- **V₃(010)**
- **V₄(011)**
- **V₅(001)**
- **V₆(101)**

- **Sector**
- **Switching States**
- **Space Vector**
- **Imaginary hexagon**
Space Vector PWM

• How to switch the eight vectors so that the correct voltage is impressed on the load?
• Space vector PWM is an extension of sine triangle PWM. Here the PWM is done by using space vectors.
The space vectors are switched for certain duration of time in a cycle so as to produce the resultant vector.

\[ V_R T_S = V_1 T_1 + V_2 T_2 + V_0 T_0 = V_1 T_1 + V_2 T_2 + V_0 T_{01} + V_0 T_{07} \]

\[ T_S = T_1 + T_2 + T_0 \]

In space vector PWM, \( T_{01} = T_{07} = T_0/2 \)
Mathematical expression of timings

\[
\frac{OA}{\sin\left(\frac{\pi}{3} - \theta\right)} = \frac{OB}{\sin(\theta)} = \frac{OC}{\sin\left(\frac{2\pi}{3}\right)}
\]

\[
\frac{V_1 T_1}{\sin\left(\frac{\pi}{3} - \theta\right)} = \frac{V_2 T_2}{\sin(\theta)} = \frac{V_R T_S}{\sin\left(\frac{2\pi}{3}\right)}
\]

\[
T_1 = \sin\left(\frac{\pi}{3} - \theta\right) \frac{V_R}{V_1} \frac{2}{\sqrt{3}} T_S = \sin\left(\frac{\pi}{3} - \theta\right) \frac{V_R}{V_D} \sqrt{3} T_S
\]

\[
T_2 = \sin \theta \frac{V_R}{V_1} \frac{2}{\sqrt{3}} T_S = \sin \theta \frac{V_R}{V_D} \sqrt{3} T_S
\]

\[
T_0 = T_S - T_1 - T_2
\]

What happens at $\theta = 0$, and $V_R = 2/3 \ V_D$?
Zero vector

• Usually the zero vectors are kept equal. This gives the best harmonic performance.

• $T_0 = T_S - T_1 - T_2$ is divided into equal parts of $T_0/2$ at the beginning and end of the cycle i.e. $T_{01} = T_{07} = \frac{T_0}{2}$

• For special switching sequences (e.g. discontinuous PWM), the division is made not equal.
Example of switching

• For example, we can switch in a switching cycle $T_s$: 111 for $T_{01}$ time period, 110 for $T_1$ time period, 100 for $T_2$ time period and 000 for $T_{07}$ time period. This will realize the reference vector ($V_R$) in the switching cycle $T_s$.

• $V_R T_s = V_1 T_1 + V_2 T_2 + V_0 T_0 = V_1 T_1 + V_2 T_2 + V_0 T_{01} + V_0 T_{07}$
Example of switching

- The instantaneous pole voltages can be seen from the diagram. The switching sequence is 111-110-100-000-100-110-111 and so on in sector 1.
- The sequence ensures minimum switching.
Example of switching

- Similarly, 111-110-010-000-010-110-111 and so on in sector 2.
What is the maximum voltage?

- The maximum voltage is obtained in linear modulation when the inscribed circle touches the hexagon.
  \[ V_{\text{Rmax}} = \frac{2}{3} V_D \cos \frac{\pi}{6} = 0.577 \ V_D \]
- In sine-PWM the peak AC voltage that was obtained was 0.5 \ V_D.
How to realize using carriers?

• The SVPWM technique discussed so far involves substantial calculation, sector identification etc.

• It can be done very easily using carriers where no calculation, sector identification or switching sequence design is required.

• In order to realize SVPWM through carriers, we can observe the sine PWM more in details.
Sine PWM for 3 phases

Zoomed later
• What is the pattern of switching?
Zoomed view in sine PWM

- 111-110-100-000-100-110-111 and so on in sector 1.
- 111-110-010-000-010-110-111 and so on in sector 2.
Zoomed view in one carrier in sine PWM

- Let us zoom further into one carrier.
- We observe that the two zero vector periods are not equal.
- In Sine PWM, $T_{01}$ time period and $T_{07}$ time period are not always equal.
Switching sequence in space vector PWM

- The switching sequence here in one carrier period is 111-110-100-000-100-110-111.
- This is same as sine PWM, however the two zero periods are equal.
Switching sequence in space vector PWM

- Minimum switching is ensured.
- Switching frequency is same as carrier frequency.
- Thus space vector PWM is an extension of sine PWM, and can also be realized using carriers.
The space vector PWM is an extension of sine PWM by addition of a common mode voltage.

\[ v_a = V_m \cos \theta, v_b = V_m \cos(\theta - \frac{2\pi}{3}), \]
\[ v_c = V_m \cos(\theta - \frac{4\pi}{3}) \]

What are the line voltages \( v_{ab} \) and \( v_{bc} \)?

\[ v_{ab} = \sqrt{3} V_m \sin\left(\frac{\pi}{3} - \theta\right) \]
\[ v_{bc} = \sqrt{3} V_m \sin \theta \]
The line voltage expressions follow the $T_1$ and $T_2$ expressions.

- $v_{ab} = \sqrt{3} V_m \sin\left(\frac{\pi}{3} - \theta\right)$
- $T_1 = \sqrt{3} \frac{V_R}{V_D} T_S \sin\left(\frac{\pi}{3} - \theta\right)$
- $v_{bc} = \sqrt{3} V_m \sin \theta$
- $T_2 = \sqrt{3} \frac{V_R}{V_D} T_S \sin \theta$

The active vectors are represented by the line voltages.

What about the zero vectors?
• To make the two zero vectors equal we have to add a common mode voltage.

• What should be its value?

  \[ 1 - (v_a + v_{cm}) = 1 - v_c + v_{cm} \]

  \[ v_{cm} = -\frac{v_a + v_c}{2} \]

• In general, \( v_{cm} = -\frac{v_{max} + v_{min}}{2} \)
Waveforms

$m=1$

$m=1.15$

Resultant Waveform
Simulation Waveforms

Pole Voltage

- $V_{DC} = 600V$, the fundamental pole voltage is \((1.154 \times 600) \times 0.5 \times 0.98 = 339.27 V\)
- $mf = 21$, harmonics reside around $mf$, $2mf$, $3mf$ ...

**Triplen Harmonics**

Fundamental (50Hz)(peak) = 339.1
Simulation Waveforms

Line voltage

- \( V_{DC} = 600V \), the fundamental line voltage is \((1.154 \times 0.98 \times 600) \times 0.5 \times 1.732 = 587.62 \text{ V}\)
- \( mf = 21 \), harmonics reside around \( mf \), \( 2mf \), \( 3mf \) ...

![Simulation Waveforms Graph]

Fundamental (50Hz) (peak) = 588

Harmonic order
Simulation Waveforms

Pole voltage

- The phase voltage does not contain any triplen harmonic, so the phase current will be absent from it.