Department of Mathematics
Indian Institute of Technology Delhi

MTL 145 & MTL 735: Number Theory

Course Outline For 2023–24, Semester II

January 11, 2024

- Contents

  - **Divisibility.** Basic properties and basic results; GCD & LCM, properties and results; Prime numbers, proofs on the infinitude of primes, theorems of Gandhi and Chebyshev, rate of growth of $\sum_{p\leq x} 1/p$ and $\pi(x)$.
  - **Congruences.** Elementary properties, theorems of Fermat–Euler, Wilson, Wolstenholme & von Staudt–Clausen; Solutions of linear and systems of linear congruences; Chinese Remainder Theorem & Hensel's Lemma; Solutions of quadratic congruences, Gauss's Lemma & Quadratic Reciprocity Law; Existence of primitive root.
  - **Arithmetical functions.** Study of ϕ, μ, σ_k, λ, Λ; the group structure of arithmetical functions; Möbius Inversion Formula; Abel's Summation Formula; average orders of ϕ, μ, d.
  - **Theory of Partitions.** Basic definitions, Ferrar's graphs and consequences; generating functions of partitions and related functions; theorems of Euler & Jacobi; Congruence properties of $p(n)$; Rogers–Ramanujan Identities.
  - **Diophantine Approximations.** Simple continued fractions; representations of real numbers by continued fractions, Equivalent numbers; Approximations by convergents; Measure of approximation of real numbers by rationals & Markoff constants; Orders of Approximation; Theorems of Dirichlet & Liouville; Algebraic & Transcendental numbers; transcendence of e and π.
  - **Binary Quadratic Forms.** Definite & Indefinite Binary Quadratic Forms; Equivalence of Quadratic Forms; Sums of two, three and four squares; Reduction of Indefinite Binary Quadratic Forms; The Pell equations; The Class Group.
  - **Diophantine Equations.** Review of the linear Diophantine equation; Cases $p = 2$ and $p = 3$ of Fermat's "Last" Theorem; Rational points on Curves; Elliptic Curves and the Group Law; Theorems of Mordell–Weil, Lutz–Nagell, Mazur, Taniyama–Shimura and the "abc" conjecture; the equations $y^2 = x^3 + k$; Waring's Problem, bounds for $g(k)$ and $G(k)$.
  - **Distribution of Primes.** Merten's Theorem; the functions π, ψ, ϑ; The Prime Number Theorem and its equivalent results; exact order of $\pi(x)$ and p_n, The Zeta functions of Riemann and Hurwitz, and the L-function of Dirichlet; Multiplicative characters, Dirichlet's Theorem for primes in arithmetic progression; Conjectures on prime numbers.

- References

  - Tom M. Apostol, Introduction to Analytic Number Theory, Narosa Publishing House, New Delhi, Third reprint, 1991.
  - G. H. Hardy & E. M. Wright, An Introduction to the Theory of Numbers, Oxford University Press, Fourth Edition, 1959.
  - Kenneth Ireland & Michael Rosen, A Classical Introduction to Modern Number Theory, Graduate Texts in Mathematics, Springer-Verlag, New York, Second Edition, 1993.
  - Ivan Niven, Herbert S. Zuckerman & Hugh L. Montgomery, An Introduction to the Theory of Numbers, John Wiley & Sons, New York, Fifth edition, 1991.
  - H. E. Rose, A Course in Number Theory, Clarendon Press, Oxford, First Edition, 1988.

- Exam Dates & Syllabus

	Dates	Proposed Syllabus
Quiz 1	7 February	Divisibility & Congruences
Minor	19 – 24 February	Divisibility, Congruences, Arithmetical Functions & Partitions
Quiz 2	13 March	Diophantine Approximations
Quiz 3	10 April	Binary Quadratic Forms & Diophantine Equations
Major	27 April – 4 May	Entire Syllabus, with emphasis on portions covered after Minor

- Grading Policy

  Grades will be assigned based on performance in Minor (30%) and Major (40%), and best two of three quizzes (15% each). An 'A' will be assigned for scores of 80% and over. You need 30% to get a 'D' or 'NP' grade.

- Attendance & Makeup Policy

  Students are expected to regularly attend classes, although no attendance will be taken. No re-Minor or re-Major will be conducted without valid medical certificate.

- Tutorial Policy

  There are no regular tutorials in this course, but we will discuss problems at the end of each chapter. Problem sheets may be downloaded from my homepage.

- Instructors & Lecture Schedule

Name	Venue	E-mail
Amitabha Tripathi	LHC 520	atripath@maths.iitd.ac.in

  Lectures will be held from 11:00 to 11:50 on `Mondays & Wednesdays` and from 12:00 to 12:50 on `Thursdays`.

- Web page

  All updated information on this course may be found at: `web.iitd.ac.in/~atripath`.

Mon	Wed	Thur	Mon	Wed	Thur
Jan 1 Divisibility	**Jan 3** Divisibility	**Jan 4** Divisibility	**Feb 26** Diophantine App.	**Feb 28** Diophantine App.	**Feb 29** Diophantine App.
Jan 8 Divisibility	**Jan 10** Divisibility	**Jan 11** Congruences	**Mar 4** Friday tt	**Mar 6** Diophantine App.	**Mar 7** Diophantine App.
Jan 15 Congruences	**Jan 17** Congruences	**Jan 18** Congruences	**Mar 11** Diophantine App.	**Mar 13** Binary QFs	**Mar 14** Binary QFs
Jan 22 Congruences	**Jan 24** Congruences	**Jan 25** Congruences	**Mar 18** Binary QFs	**Mar 20** Diophantine Eqns.	**Mar 21** Diophantine Eqns.
Jan 29 Arith. Fns.	**Jan 31** Arith. Fns.	**Feb 1** Arith. Fns.	**Mar 25** Mid Semester	**Mar 27** Mid Semester	**Mar 28** Mid Semester
Feb 5 Partitions	**Feb 7** Partitions	**Feb 8** Partitions	**Apr 1** Diophantine Eqns.	**Apr 3** Diophantine Eqns.	**Apr 4** Diophantine Eqns.
Feb 12 Partitions	**Feb 14** Partitions	**Feb 15** Partitions	**Apr 8** Eid-ul-Fitr	**Apr 10** Diophantine Eqns.	**Apr 11** Primes
Feb 19 Minor Exam	**Feb 21** Minor Exam	**Feb 22** Minor Exam	**Apr 15** Primes	**Apr 17** Ram Navami	**Apr 18** Primes
			Apr 22 Primes	**Apr 24** Primes	**Apr 25** Primes
			Apr 29 Major Exam	**May 1** Major Exam	**May 2** Major Exam

Amitabha Tripathi
(Course Coordinator)
MZ 167, Department of Mathematics
atripath@maths.iitd.ac.in
1486 (Internal)