Department of Mathematics
Indian Institute of Technology Delhi

MTL 145: Number Theory

Course Outline For 2025–26, Semester II

December 29, 2025

- Contents

  - **Divisibility.** Basic properties and basic results; GCD & LCM, properties and results; Prime numbers, proofs on the infinitude of primes, theorems of Gandhi and Chebyshev, rate of growth of $\sum_{p\leq x} 1/p$ and $\pi(x)$.
  - **Congruences.** Elementary properties, theorems of Fermat–Euler, Wilson, Wolstenholme & von Staudt–Clausen; Solutions of linear and systems of linear congruences; Chinese Remainder Theorem & Hensel's Lemma; Solutions of quadratic congruences, Gauss's Lemma & Quadratic Reciprocity Law; Existence of primitive root.
  - **Arithmetical functions.** Study of ϕ, μ, σ_k, λ, Λ; the group structure of arithmetical functions; Möbius Inversion Formula; Abel's Summation Formula; average orders of ϕ, μ, d.
  - **Theory of Partitions.** Basic definitions, Ferrar's graphs and consequences; generating functions of partitions and related functions; theorems of Euler & Jacobi; Congruence properties of $p(n)$; Rogers–Ramanujan Identities.
  - **Diophantine Approximations.** Simple continued fractions; representations of real numbers by continued fractions, Equivalent numbers; Approximations by convergents; Measure of approximation of real numbers by rationals & Markoff constants; Orders of Approximation; Theorems of Dirichlet & Liouville; Algebraic & Transcendental numbers; transcendence of e and π.
  - **Binary Quadratic Forms.** Definite & Indefinite Binary Quadratic Forms; Equivalence of Quadratic Forms; Sums of two, three and four squares; Reduction of Indefinite Binary Quadratic Forms; The Pell equations; The Class Group.
  - **Diophantine Equations.** Review of the linear Diophantine equation; Cases $p = 2$ and $p = 3$ of Fermat's "Last" Theorem; Rational points on Curves; Elliptic Curves and the Group Law; Theorems of Mordell–Weil, Lutz–Nagell, Mazur, Taniyama–Shimura and the "abc" conjecture; the equations $y^2 = x^3 + k$; Waring's Problem, bounds for $g(k)$ and $G(k)$.
  - **Distribution of Primes.** Merten's Theorem; the functions π, ψ, ϑ; The Prime Number Theorem and its equivalent results; exact order of $\pi(x)$ and p_n, The Zeta functions of Riemann and Hurwitz, and the L-function of Dirichlet; Multiplicative characters, Dirichlet's Theorem for primes in arithmetic progression; Conjectures on prime numbers.

- References

  - Tom M. Apostol, Introduction to Analytic Number Theory, Narosa Publishing House, New Delhi, Third reprint, 1991.
  - G. H. Hardy & E. M. Wright, An Introduction to the Theory of Numbers, Oxford University Press, Fourth Edition, 1959.
  - Kenneth Ireland & Michael Rosen, A Classical Introduction to Modern Number Theory, Graduate Texts in Mathematics, Springer-Verlag, New York, Second Edition, 1993.
  - Ivan Niven, Herbert S. Zuckerman & Hugh L. Montgomery, An Introduction to the Theory of Numbers, John Wiley & Sons, New York, Fifth edition, 1991.
  - H. E. Rose, A Course in Number Theory, Clarendon Press, Oxford, First Edition, 1988.

- Exam Dates & Syllabus

	Dates	Proposed Syllabus
Quiz 1	5 February	Divisibility & Congruences
Minor	20 – 25 February	Divisibility, Congruences, Arithmetical Functions & Partitions
Quiz 2	24 March	Diophantine Approximations
Quiz 3	16 April	Binary Quadratic Forms & Diophantine Equations
Major	29 April – 5 May	Entire Syllabus, with emphasis on portions covered after Minor

- Grading Policy

  Grades will be assigned based on performance in Minor (30%) and Major (40%), and best two of three quizzes (15% each). An 'A' will be assigned for scores of 80% and over. You need 30% to get a 'D' or 'NP' grade.

- Attendance & Makeup Policy

  Students are expected to regularly attend classes, although no attendance will be taken. No re-Minor or re-Major will be conducted without valid medical certificate.

- Tutorial Policy

  There are no regular tutorials in this course, but we will discuss problems at the end of each chapter. Problem sheets may be downloaded from my homepage.

- Instructors & Lecture Schedule

Name	Venue	E-mail
Amitabha Tripathi	LHC 603	atripath@maths.iitd.ac.in

  Lectures will be held from 2:00 to 2:50 on `Mondays, Wednesdays & Fridays`.

- Web page

  All updated information on this course may be found at: `web.iitd.ac.in/~atripath`.

Mon	Wed	Fri	Mon	Wed	Fri
		Jan 2 Divisibility	**Mar 2** Mid Semester	**Mar 4** Mid Semester	**Mar 6** Mid Semester
Jan 5 Divisibility	**Jan 7** Divisibility	**Jan 9** Divisibility	**Mar 9** Diophantine App.	**Mar 11** Diophantine App.	**Mar 13** Diophantine App.
Jan 12 Congruences	**Jan 14** Makar Sankranti	**Jan 16** Congruences	**Mar 16** Diophantine App.	**Mar 18** Diophantine App.	**Mar 20** Diophantine App.
Jan 19 Congruences	**Jan 21** Congruences	**Jan 23** Congruences	**Mar 23** Binary QFs	**Mar 25** Binary QFs	**Mar 27** Binary QFs
Jan 26 Republic Day	**Jan 28** Congruences	**Jan 30** Congruences	**Mar 30** Diophantine Eqns.	**Apr 1** Diophantine Eqns.	**Apr 3** Good Friday
Feb 2 Arith. Fns.	**Feb 4** Arith. Fns.	**Feb 6** Arith. Fns.	**Apr 6** Diophantine Eqns.	**Apr 8** Diophantine Eqns.	**Apr 10** Diophantine Eqns.
Feb 9 Partitions	**Feb 11** Partitions	**Feb 13** Partitions	**Apr 13** Diophantine Eqns.	**Apr 15** Primes	**Apr 17** Primes
Feb 16 Partitions	**Feb 18** Partitions	**Feb 20** Minor Exam	**Apr 20** Primes	**Apr 22** Primes	**Apr 24** Primes
Feb 23 Minor Exam	**Feb 25** Minor Exam	**Feb 27** No Class	**Apr 27** Primes	**Apr 29** Major Exam	**May 1** Major Exam
			May 3 Major Exam	**May 5** Major Exam	

• **Sat, Jan 10 works as a Wed**: Divisibility

Amitabha Tripathi
(Course Coordinator)
617, Building 99B, Department of Mathematics
atripath@maths.iitd.ac.in
1486 (Internal)