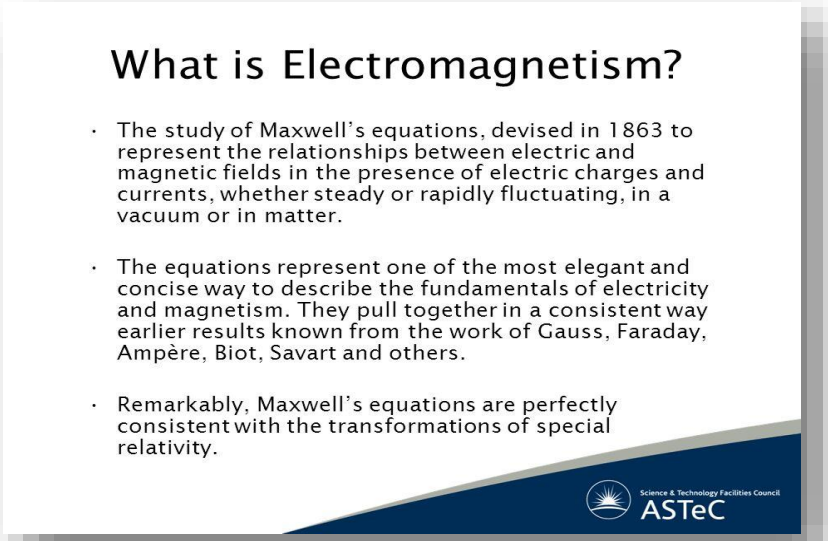
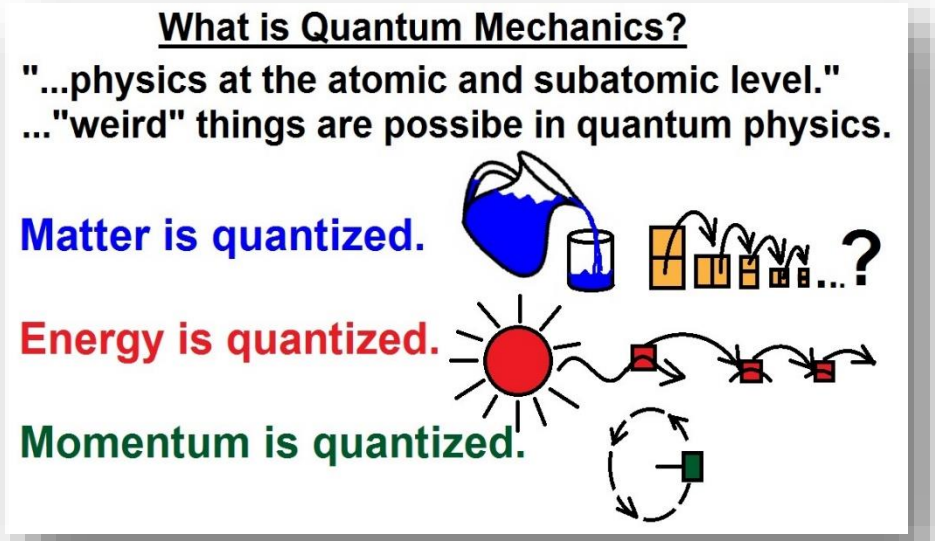
PYL100: EM Waves & Quantum Mechanics (3-0-0)





- **Course Instructor(s): Brajesh K Mani & Amartya Sengupta (Co-Ordinator)**
- **Room: LH 121**
- **Schedule: Monday & Thursday (8:00 – 9:20, Slot A)**
- **Office: VI-415D (BKM) & VI-415B (ASg)**

II Semester 2018-2019

Course Contents

Electrostatics and Magnetostatics in medium, Maxwell's equations, Plane wave solutions, Polarization & Poynting Vector

Minor 1

Phase & Group velocities, Propagation of EM waves in free space and plasma

Introduction to QM, Wave particle duality, de-Broglie waves, QM operators

Minor 2

Superposition, Continuity equation, potential well problems, QM tunnelling, Krönig Penney model and applications

Majors

Contact Points

Faculty Level

- Entire Course Administrative/Grading/other miscellaneous issues: **Amartya Sengupta** (amartya@physics.iitd.ac.in)

- Electrodynamics Course Material: **Brajesh Kumar Mani** (bkmani@physics.iitd.ac.in)
  - You can download the PDF copy of the lectures from: http://web.iitd.ac.in/~bkmani/courses.html
  - Office Hours: Tuesday, 04 - 05PM

- Quantum Mechanics Course Material: **Amartya Sengupta** (amartya@physics.iitd.ac.in)

TA Level (Electrodynamics, i.e. upto Mid Semester Break)

- Rajat Dhawan (rajat.dhawan@physics.iitd.ac.in)
- Sheetal Punia (sheetal.punia@physics.iitd.ac.in)
- Rajni Bala (rajni.bala@physics.iitd.ac.in)

For day to day attendance issues, help with tutorial problems, please contact the TAs. Ensure that you cc the faculty in any emails that you send to TAs.

Some book-keeping

- If a student's attendance is less than 75%, the student will be awarded one grade less than the actual grade that he/she has earned, less than 60% will get 2 grades less.
- TIMBLE ATTENDANCE IS MANDATORY AND COMPULSORY FROM DAY 1. PAPER ATTENDANCE IS FOR DATA COLLECTION PURPOSES.
- If you are more than 10 minutes late, you will NOT be allowed entry to the class.

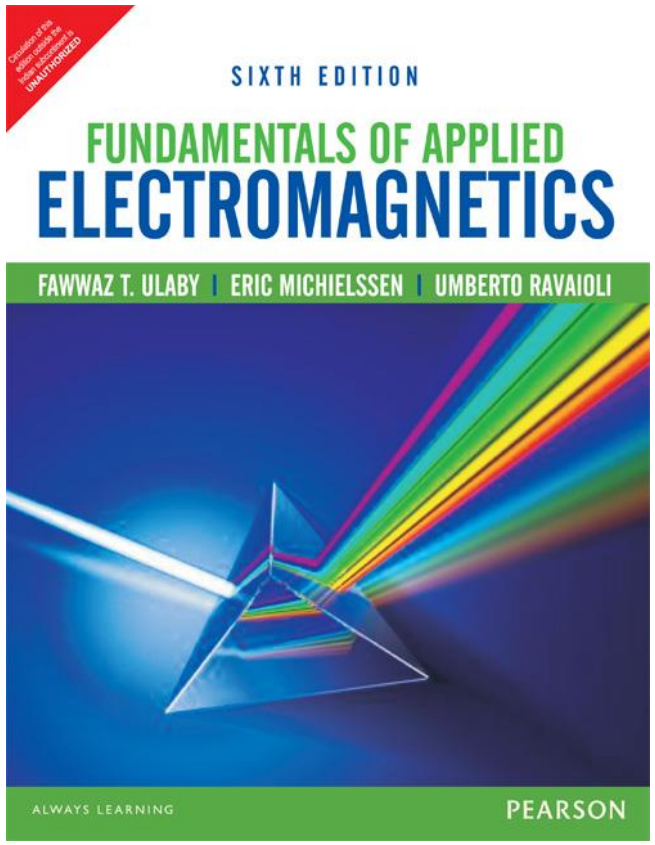
- Minor I: 20+0 marks
- Minor II: 10+10 marks (all topics covered upto Minor II)
- Majors: 10+30 marks (entire course syllabus)
- HW: 10+10 marks

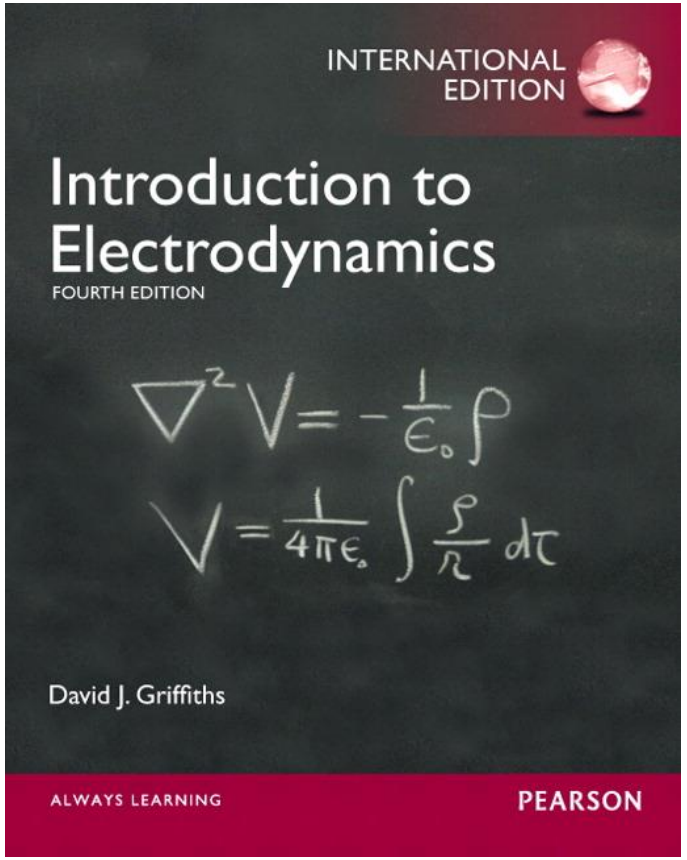
Exams will be closed books and notes. One A4 size handwritten formula sheet will be allowed.
Exams are covered by the Academic Honour Code.

Attendance Policy:

- Exam attendance is mandatory with exemptions of individual emergencies, health conditions etc. Valid documentation will be required.
- Pass marks is 30%.
- No RE-MINORS if attendance is below 75%.

Suggested Books for Electrodynamics





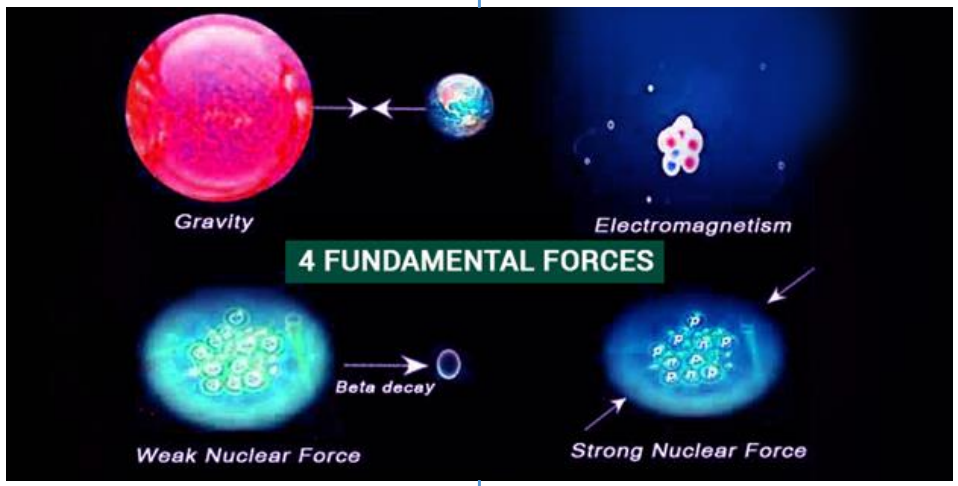
The Fundamental Forces of Nature

Acts in: two pieces of matter in the Universe

Strength: $\mathbf{10^{-41}}$

Range: long ranged, Infinite

Mediator particles: Graviton



Acts in: between charged particles

Strength: $\mathbf{10^{-2}}$

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Acts in: processes like radioactive decay

Strength: 10^{-14}

Range: short ranged, 10^{-18} m (less than the diameter of a proton)

Mediator particles: Bosons

Acts in: between nuclei

Strength: 1

Range: short ranged, 10^{-15} m (diameter of a medium sized nucleus)

Mediator particles: Gluons

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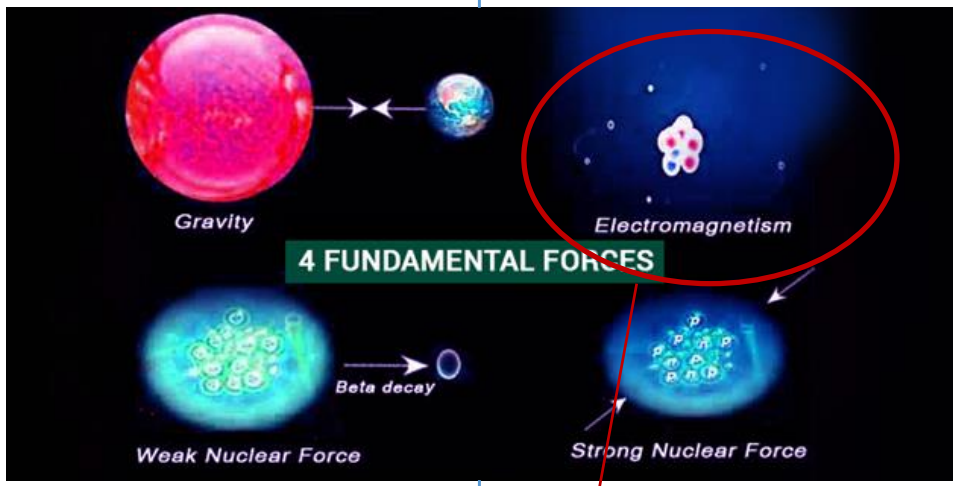

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Mediator particles: Gluons

The Electromagnetic Interaction
(electric and magnetic fields and their interactions/propagation with/in matter

The Electrostatic Force

"charges are stationary"

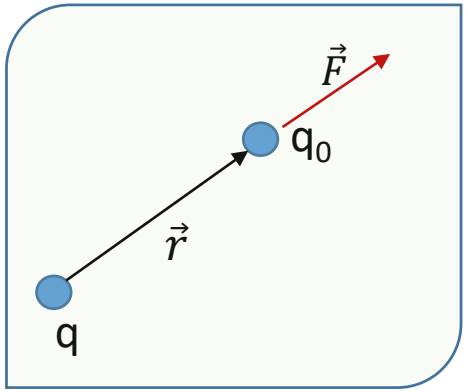
The electrostatic force on a charge q_0 due to a point charge q

$$\vec{F} = \frac{1}{4\,\pi\,\epsilon_0}\,\frac{q\,q_0}{r^2}\,\hat{r}$$

(Coulomb Force)

- Acts along the line joining the two charges
- Attractive if two charges have the same sign
- Repulsive if the two charges have opposite sign
- ϵ_0 is the permitivity of the free space

$$\epsilon_0 = 8.85 \times 10^{-12}\ \frac{C^2}{N\,m^2}$$



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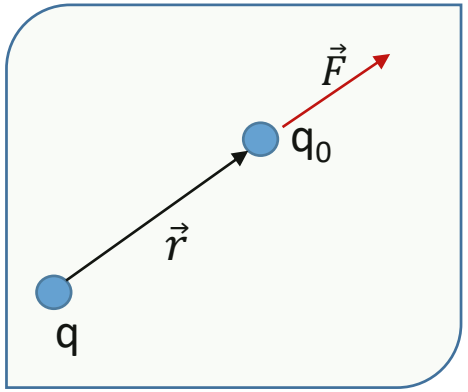


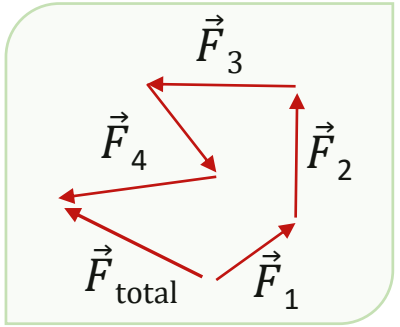
Superposition of the Forces

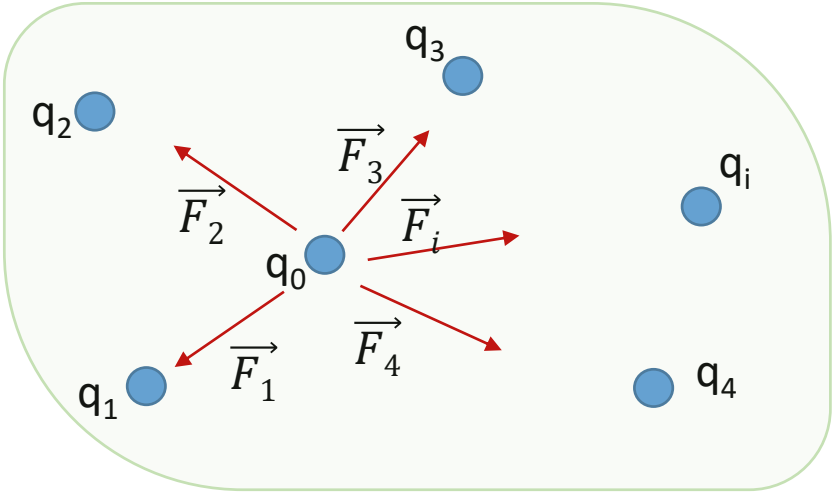
The net force will be the vector sum of all the forces

$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_i + \ldots + \vec{F}_n$$

Using above relation

$$\vec{F}_{total} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_0\,qi}{r_i^2}\,\hat{r}_i$$



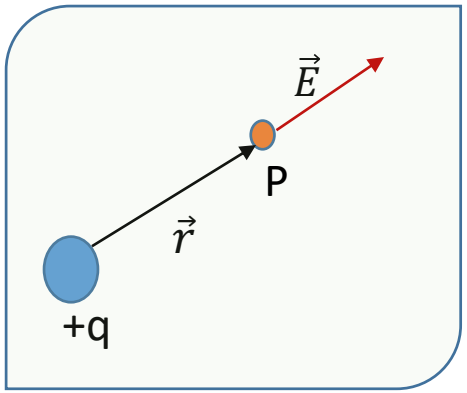


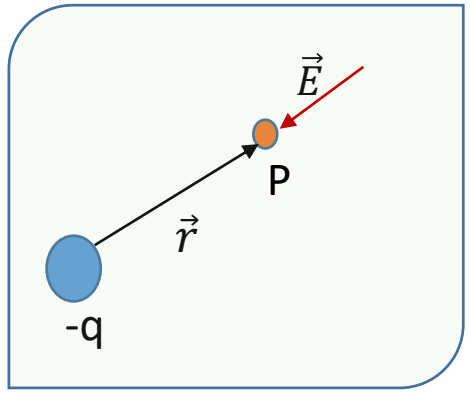
The Electric Field

The electric field of charge a source q at a point P

$$\vec{E}\,(r) = \frac{1}{4\,\pi\,\epsilon_0}\,\frac{q}{r^2}\,\hat{r}$$

- Field is independent of test charge
- Depends on the source charge
- Points away from positive charge
- Points toward negative charge





Using the expression for electrostatic force, field can also be written as

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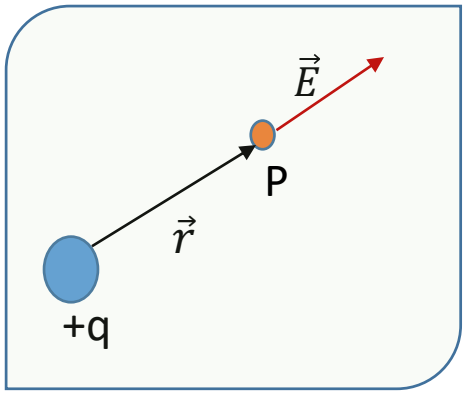

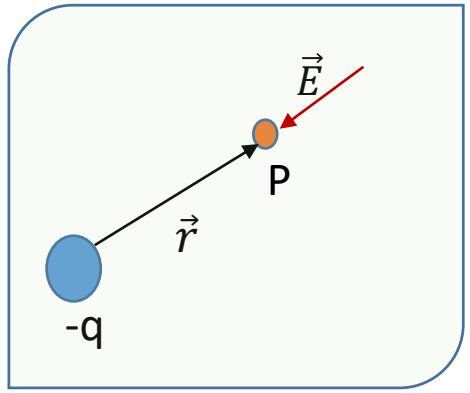

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Superposition of Fields

The net electric field of a collection of source charges

$$\vec{E}_{\,total} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots + \vec{E}_i + \ldots \vec{E}_n$$

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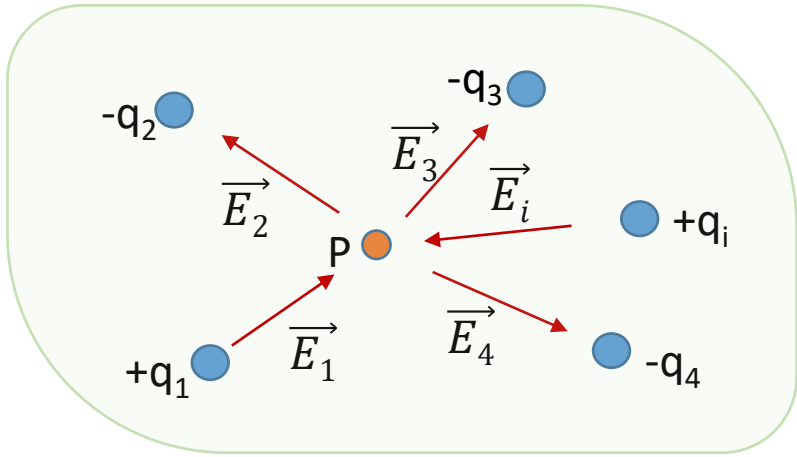
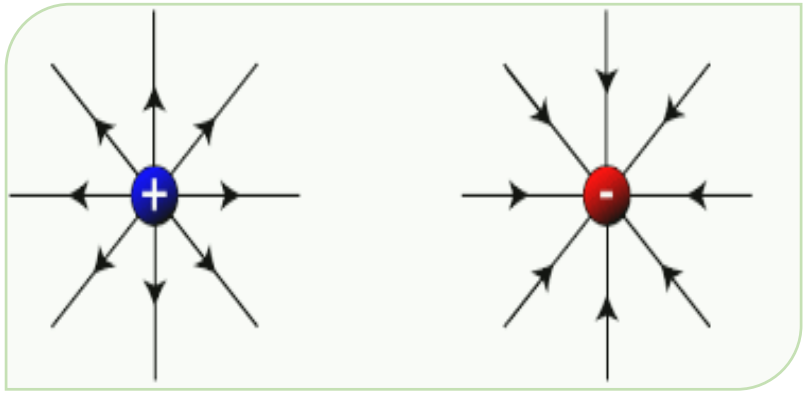
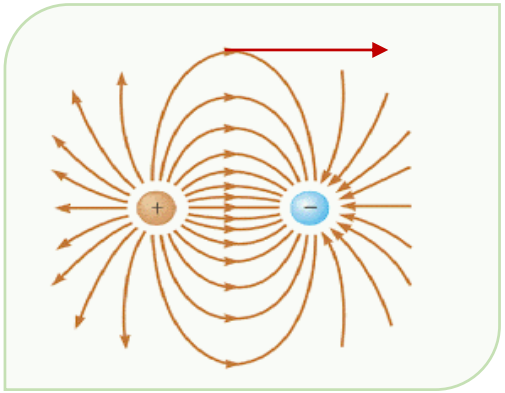


The Electric Field Lines

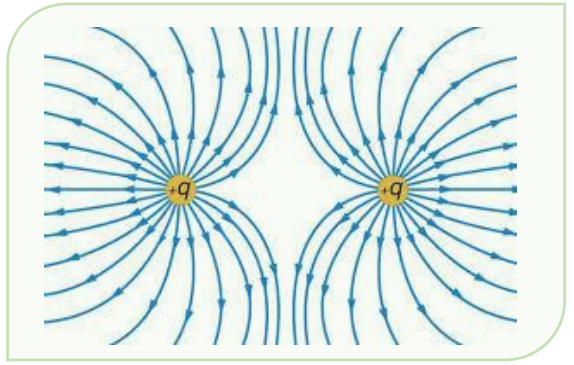
- Electric field lines start at positive charge and end on negative charge
- Electric field vector points along the tangent to a field line
- Magnitude of the electric field at a point is proportional to the density of the field lines at that point.
- The number of field lines starting/ending from/on a charge depends on the magnitude of the charge.
- Electric field lines never cross each other



Single charge



Two opposite charge



Two same charge

Field due to a continuous charge distribution

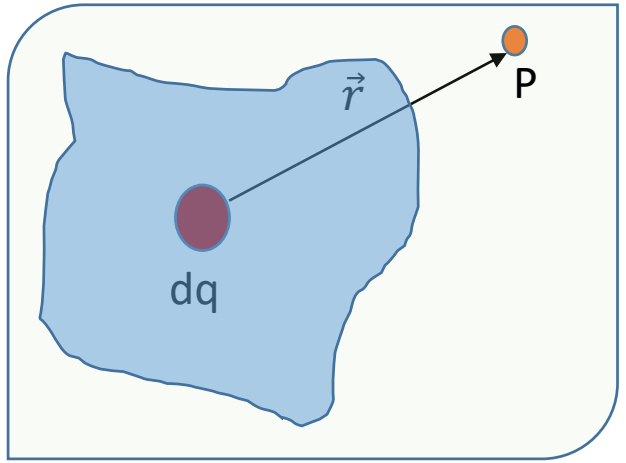
The electric field of a continuous charge distribution *dq* at a point P

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\int \frac{1}{r^2}\,\hat{r}\,dq$$

Integrate over the space of charge configuration

Discrete → Continuous

Σ → $\int$



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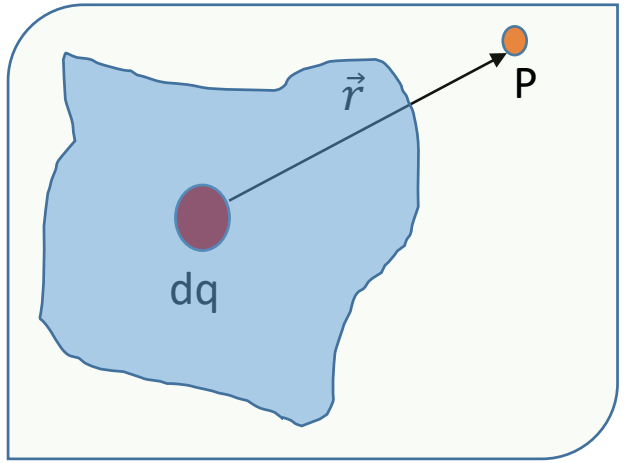


$dq = \lambda(r')\,dl'$ for line charge distribution

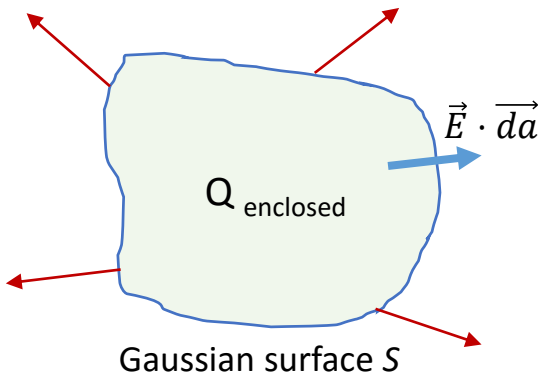
$dq = \sigma(r')\,da'$ for surface charge distribution

$dq = \rho(r')\,dv'$ for volume charge distribution

Divergence of Electric Field (Gauss's Law)

"The total electric flux through any closed surface is a measure of the total charge enclosed in the surface". Mathematically,

$$\oint_S \vec{E} \cdot \overrightarrow{da} = \frac{Q_{enclosed}}{\epsilon_0}$$



Gaussian surface *S*

* Gives you the value of electric field of a charge configuration very quickly.
* However, the charge configuration should be symmetrical (Spherical symmetry, Cylindrical symmetry, etc.)
* No charge inside, no field lines

Let us dig into the past…

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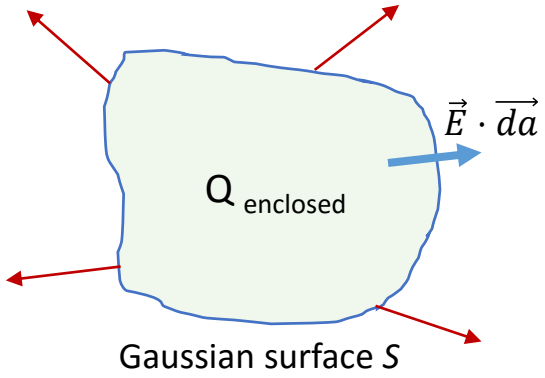

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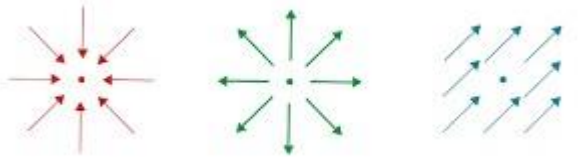
Divergence of a Vector

"measure of spreads of a vector"

$$\nabla \cdot \vec{A} = \left[\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] \cdot \left[\hat{\imath}A_x + \hat{\jmath}A_y + \hat{k}A_z\right]$$

$$= \left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right]$$

$\nabla \cdot \vec{A} < 0$ $\nabla \cdot \vec{A} > 0$ $\nabla \cdot \vec{A} = 0$



Divergence theorem

$$\oint_S \vec{A} \cdot \overrightarrow{da} = \int_V (\nabla \cdot \vec{A}) \mathrm{d}\tau$$

Flux of a vector field through a closed surface is equal to the volume integral of the divergence over the region inside the surface.

Let us dig into the past…

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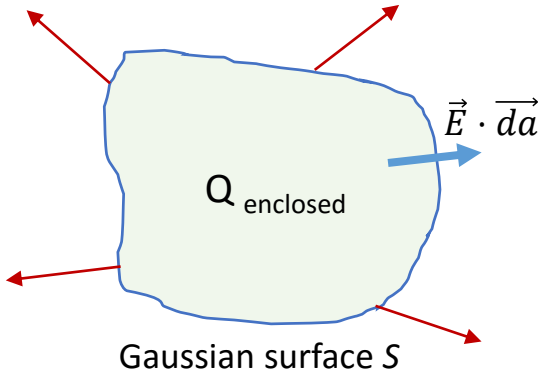

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* Gives you the value of electric field of a charge configuration very quickly.
* However, the charge configuration should be symmetrical (Spherical symmetry, Cylindrical symmetry, etc.)
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Using the divergence theorem

$$\int_V \left(\nabla \cdot \vec{E}\right) d\tau = \int_V \left(\frac{\rho}{\epsilon_0}\right) d\tau$$

ρ is the volume charge density

$$\Longrightarrow \quad \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

(Differential form of Gauss's law)

Divergence of a Vector

"measure of spreads of a vector"

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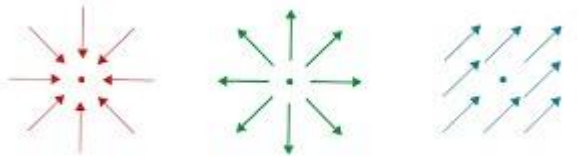

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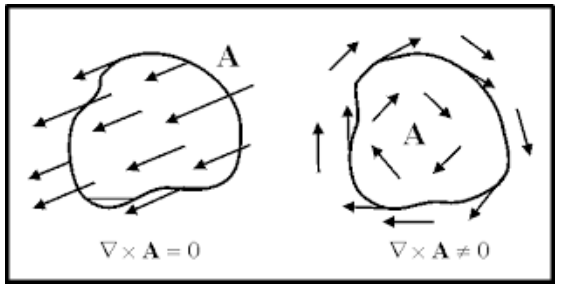
Let us dig into the past…

Curl of Electric Field

Curl of a Vector

"measure of how much a vector curls around a point"

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Ax & A_y & A_z \end{vmatrix}$$



Stokes' Theorem

$$\int_S (\nabla \times \vec{A}) \cdot \overrightarrow{da} = \oint_P \vec{A} \cdot \overrightarrow{dl}$$

The integral of a derivative of a vector over a region is equal to the value of the function at the boundary.

Let us dig into the past…

Curl of Electric Field

"The electric field lines do not form closed curves. The field diverges or converges from/to a charge body"

$$\nabla \times \vec{E} = 0$$

Curl of a Vector

"measure of how much a vector curls around a point"

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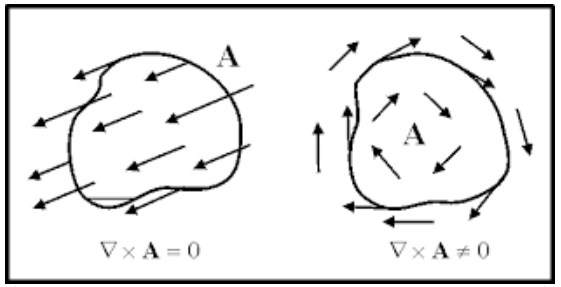

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Curl of Electric Field

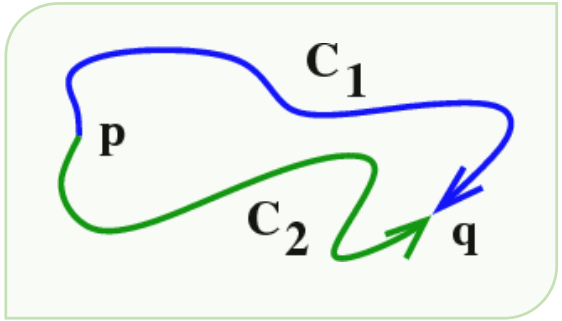
"The electric field lines do not form closed curves. The field diverges or converges from/to a charge body"

$$\nabla \times \vec{E} = 0$$

- E is a very special kind of vector.
- The three components of E are not really independent.

Using Stokes' theorem

$$\oint \vec{E} \cdot \overrightarrow{dl} = 0$$



Line integral of electric field does not depend on the paths

Curl of a Vector

"measure of how much a vector curls around a point"

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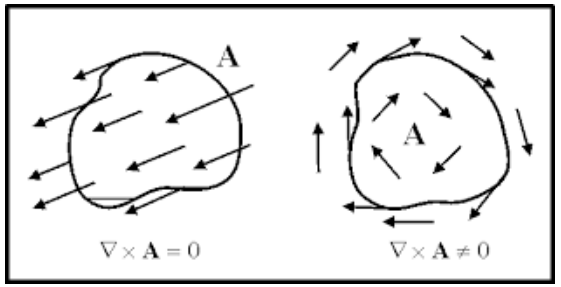

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Let us dig into the past…

Electric potential

"Reducing problem of calculating a three component vector to a scalar"

$$\nabla \times \vec{E} = 0 \quad \rightarrow \quad \vec{E} = -\nabla V$$

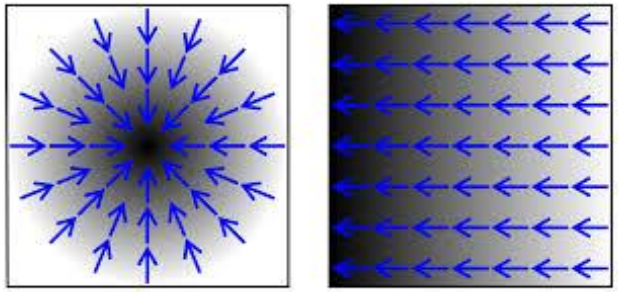
- Electric potential V is a scalar
- Can be calculated by just adding the contributions from individual charges
- All points at a distance *r* from a charge configuration have the same potential

Gradient of a Vector

"measure of the slope of the tangent of graph of function"

$$\nabla T = \left[\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] T$$

$$= \left[\hat{\imath}\frac{\partial T}{\partial x} + \hat{\jmath}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}\right]$$



Gradient of a function

Theorem

Curl-less fields can be expressed as the gradient of scalars.

"curl of a gradient is always zero"

Let us dig into the past…

Electric potential

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The electric potential at a distance *r* from a charge configuration

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

(For a point charge)

$$V(r) = \frac{1}{4\pi\epsilon_0}\int \frac{1}{r}\,dq$$

(For a continuous charge distribution)

From the top equation we can write

$$V(r) = -\int_O^r \vec{E}\cdot\overrightarrow{dl}$$

(O is a reference point, chosen in such a way that electric potential at O is zero)

Gradient of a Vector

"measure of the slope of the tangent of graph of function"

$$\nabla T = \left[\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] T$$

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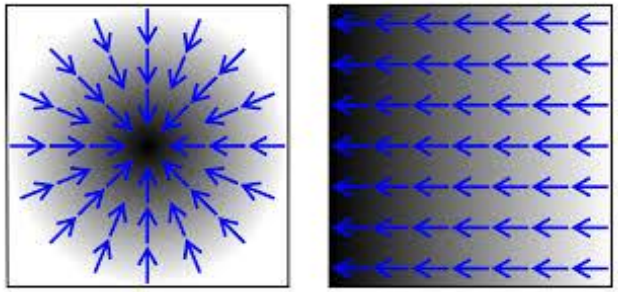

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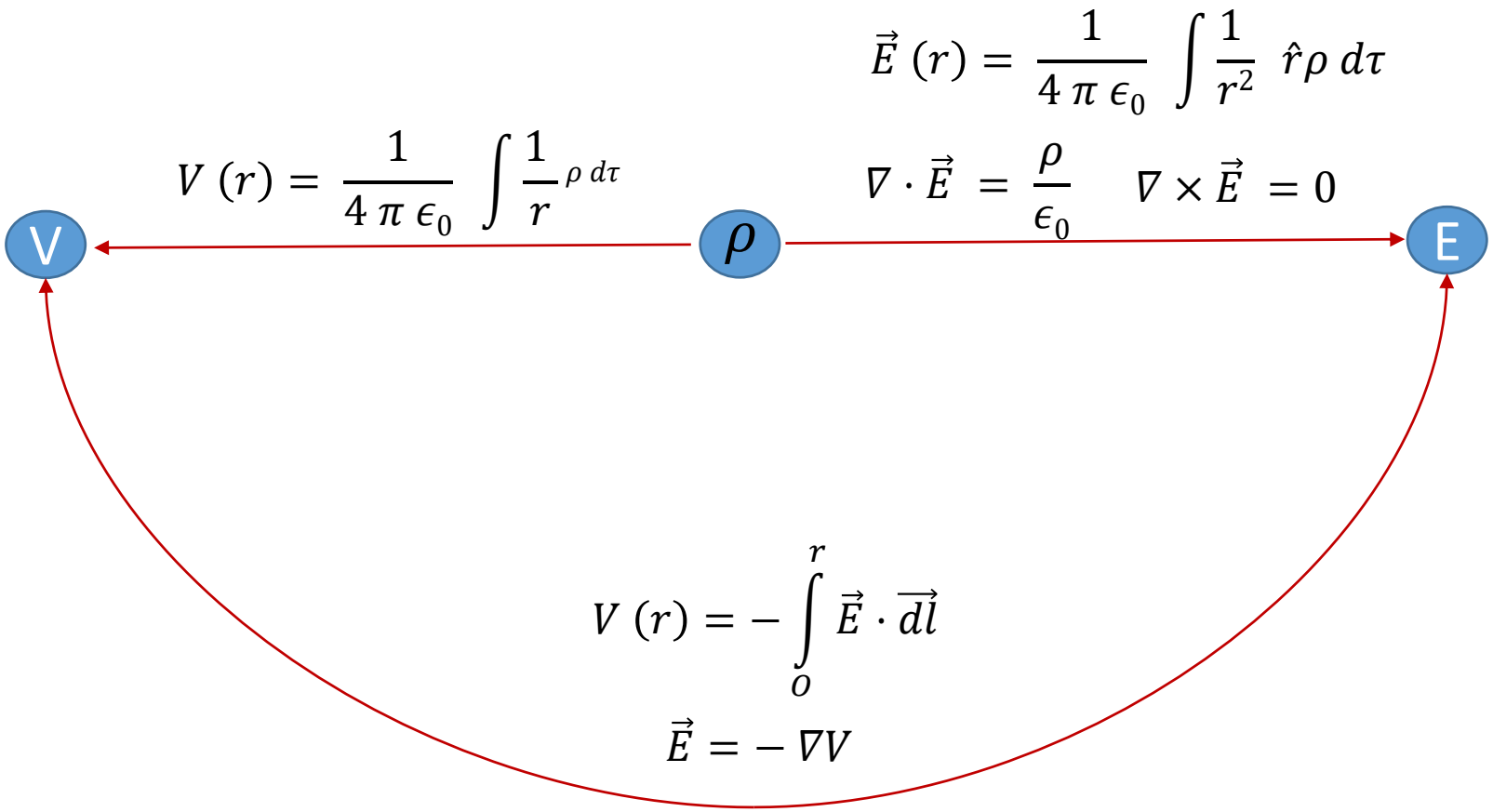
Summary of Electrostatics

"charges are stationary"

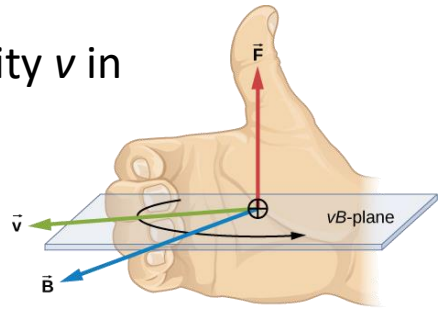


Magnetic Force

"charges are moving"

The magnetic force on a charge *q* moving with velocity *v* in a magnetic field *B* is

$$\vec{F}_{mag} = q(\vec{v} \times \vec{B})$$



- No force if no magnetic field
- Force maximum when charge moves perpendicular to *B*
- Direction of force is perpendicular to both *v* and *B*
- Also depends on the polarity of charge

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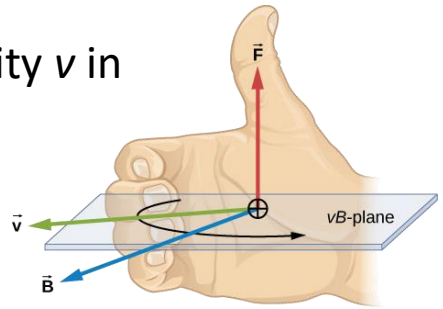

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Supposing a charge *q* moves a distance *dl* then the work done by the magnetic field

$$dW = \vec{F}_{mag} \cdot \overrightarrow{dl} = q(\vec{v} \times \vec{B}) \cdot \vec{v}\, dt = 0$$

Comparing with Electric Force:

- Electric force is always in the direction of field, magnetic force is always perpendicular to the magnetic field
- Magnetic force acts when charges are in motion
- Electric force expends energy in displacing a charge particle, magnetic force does no work
- Magnetic field can only change the direction of the particle NOT the speed.

Let us dig into the past…

Magnetic Force

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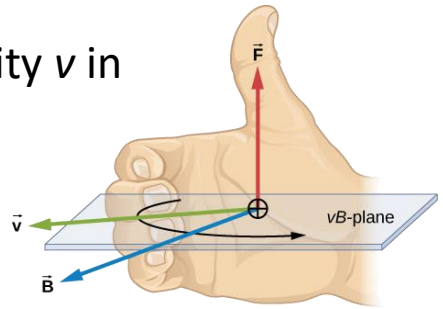

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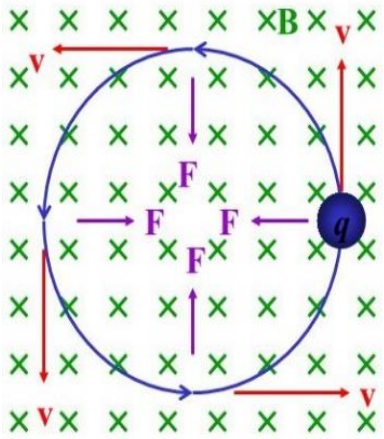
Cyclotron Motion

"experiment to determine the momenta of elementary particles"



- Motion is uniform circular motion
- Centripetal force formula
- Centripetal force is equal to magnetic force

$$F = \frac{mv^2}{R} = qvB$$

- Let's find how long it takes to go around:

$$T = \frac{2\pi R}{v} \qquad T = \frac{2\pi m}{qB} \qquad \omega = \frac{2\pi}{T} \qquad \omega = \frac{qB}{m}$$

Comparing with Electric Force:

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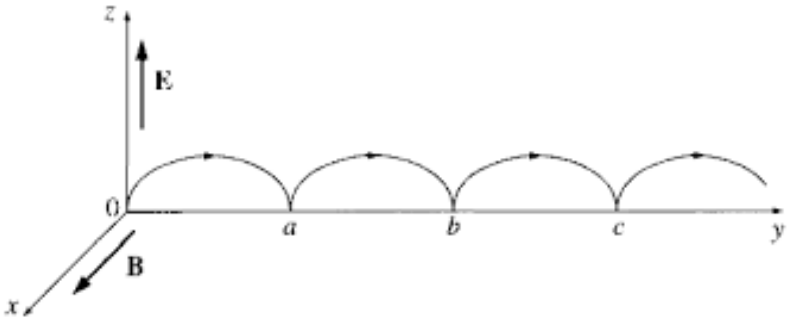
Electromagnetic Force

The magnetic force on a charge *q* moving with an instantaneous velocity *v* in external fields *B* and *E* is

$$\vec{F}_{mag} = q\,\vec{E} + q(\vec{v} \times \vec{B})$$ (Lorentz Force)



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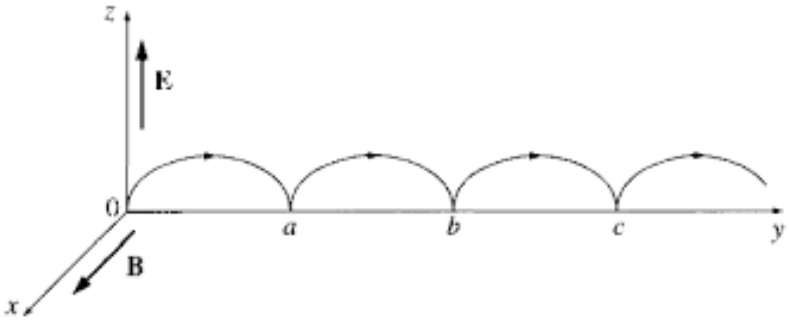
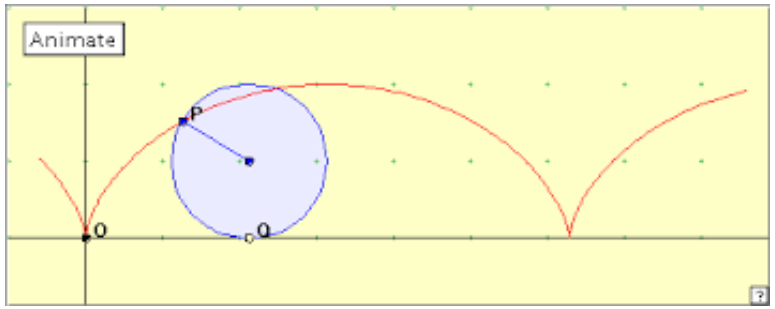


The equation of motion can be derived as

$$(y - R\omega t)^2 + (z - R)^2 = R^2$$

Where $v = R\omega = \dfrac{\vec{E}}{\vec{B}}$

Equation of a circle with radius R, whose center (0, Rwt, R) travels in the y-direction

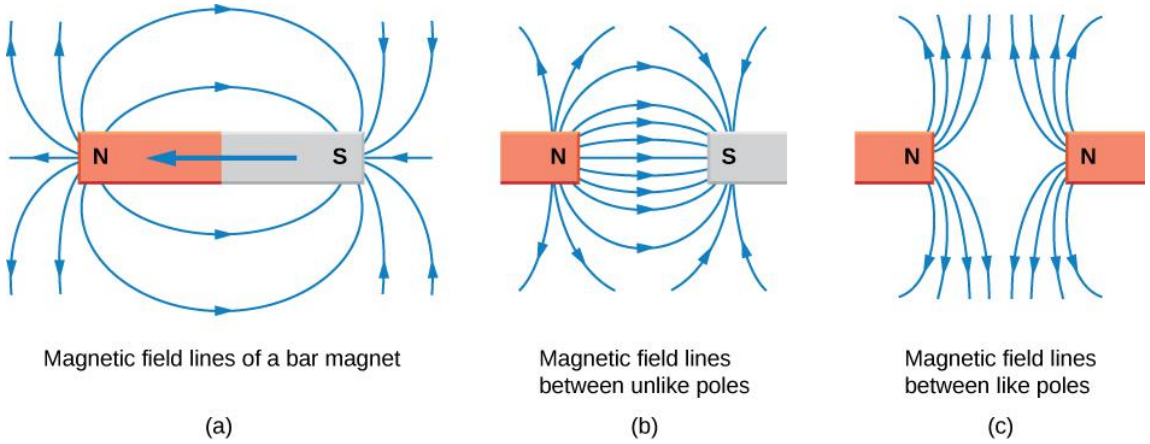


A point on the rim of a circular ring rolling down along y-axis

Magnetic Field Lines

"magnetic field lines are used to represent the strength and direction of magnetic field at a point is space"

- The direction of magnetic field at a point is a tangent to the field lines at that point.
- Strength of magnetic field is proportional to the density of the field lines.
- Magnetic field lines can never cross, meaning that the field is unique at any point in space.
- Magnetic field lines are continuous, forming closed loops without a beginning or end. They are directed from the north pole to the south pole.
- No such thing as a magnetic monopole

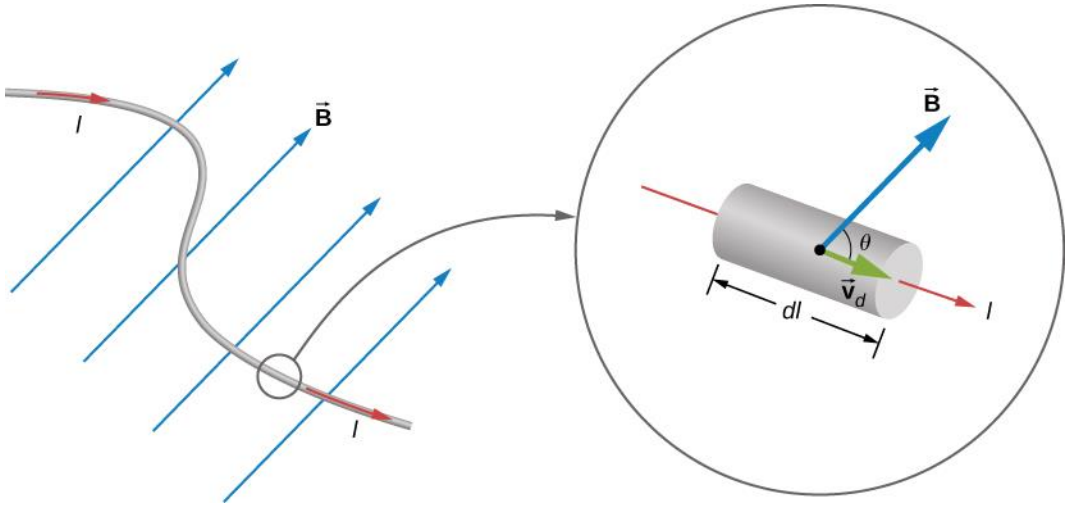


Bar magnet

No magnetic monopole

Magnetic Force Due to Current

"charges are moving in wire"



Current carrying wire

- If n is the charge per unit volume in the wire, the total number of charge contained in the small segment = $nA.\,dl$

- Then the magnetic force in the segment is

$$\overrightarrow{dF}_{mag} = \left(\mathrm{n}\,\overrightarrow{A}.\,\overrightarrow{dl}\right) \times q\left(\vec{v}_d \times \vec{B}\right)$$
$$= I\,(\overrightarrow{dl} \times \vec{B})$$

Where $I = nqvdA$ is the current in the wire,

dl

and I both points in the same direction.

- Then the total force due to current in entire wire is

$$\vec{F}_{mag} = I \int (\overrightarrow{dl} \times \vec{B})$$

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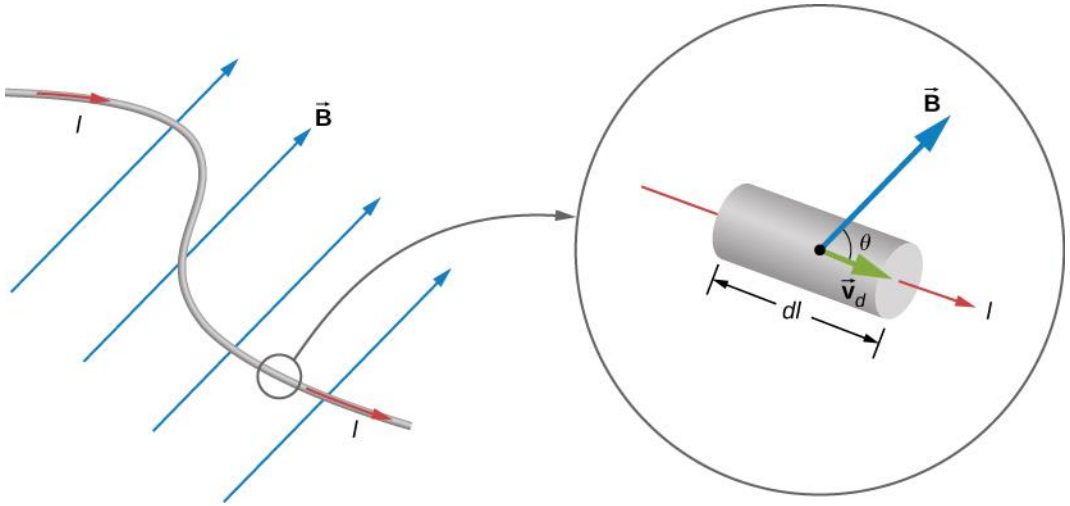
$$\vec{F}_{mag} = I \int (\overrightarrow{dl} \times \vec{B})$$

- The total magnetic force on any closed current loop in an uniform magnetic field is $\vec{F}_{mag} = I \oint \left(\overrightarrow{dl}\right) \times \vec{B}\big) = 0$



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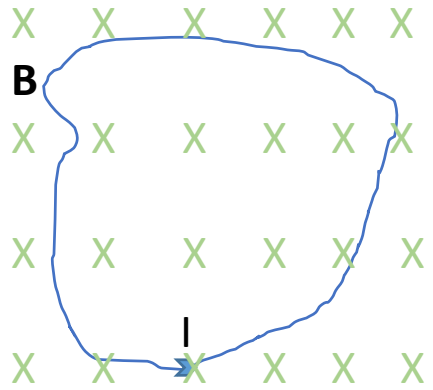


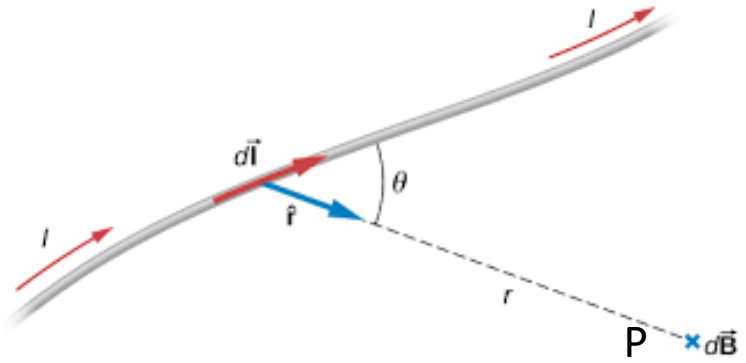
Total Force is Zero

Magnetic Field Due to Currents (The Biot-Savart Law)

"steady currents"

The magnetic at a distance *r* from a current carrying element *dl* is

$$\overrightarrow{dB}(r) = \frac{\mu_0}{4\pi} \frac{I\,\overrightarrow{dl}\times\hat{r}}{r^2}$$



- Magnetic field points perpendicular to the plane containing $\overrightarrow{dl} \times \hat{r}$.
- So, $\overrightarrow{dB}$ in this case points out of the screen/page.
- For given *r*, $\overrightarrow{dB}$ will be maximum at points in the plane perpendicular to $\overrightarrow{dl}$.
- Fields due to collection of source currents is obtained by superposition principle.

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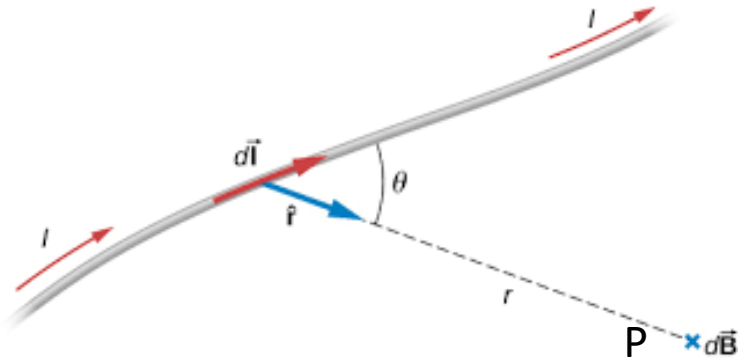

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The total magnetic at a distance *r* due to entire wire is

$$\vec{B}(r) = \frac{\mu_0 \mathrm{I}}{4\pi}\int\frac{\overrightarrow{dl}\times\hat{r}}{r^2}$$ (Integration is over the length of the wire)

Similarly for surface and volume currents

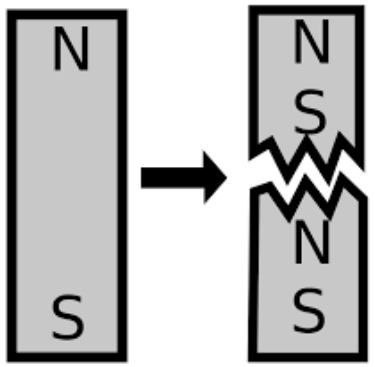
$$\overrightarrow{dB}(r) = \frac{\mu_0 \mathrm{I}}{4\pi}\int\frac{\vec{K}(r\prime)\times\hat{r}}{r^2} \quad \text{and} \quad \overrightarrow{dB}(r) = \frac{\mu_0 \mathrm{I}}{4\pi}\int\frac{\vec{J}(r\prime)\times\hat{r}}{r^2}$$

Divergence of $\vec{B}$ (Gauss's Law for Magnetism)

$\nabla \cdot \vec{B} = 0$ (Differential form)

$\oint \vec{B} \cdot \overrightarrow{da} = 0$ (Integral form)

Magnetic monopoles do not exist



Let us dig into the past…

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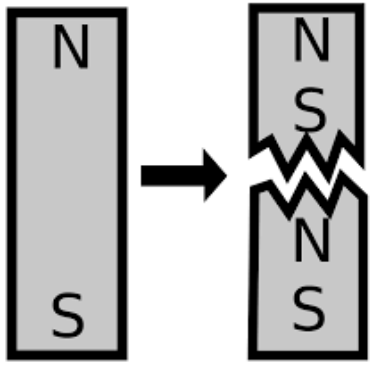
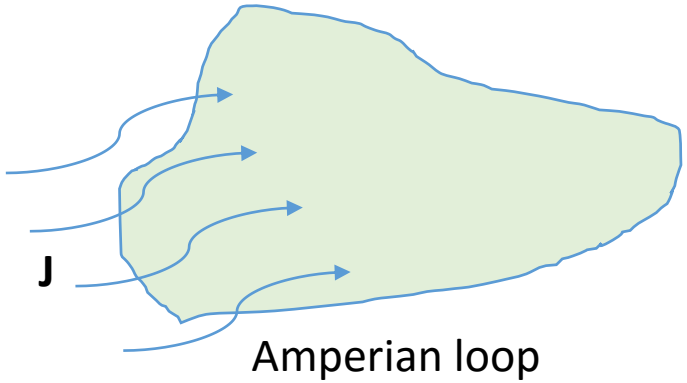


Curl of $\vec{B}$ (Ampere's Law)

$\nabla \times \vec{B} = \mu_0 \vec{J}$ (Differential form)

$\oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 \, Ien_{closed}$ (Integral form)

$\vec{J}$ is the current density and $I_{enclosed}$ is total current enclosed by the Amperian loop.



Very useful in calculating magnetic field. However, applicable to the symmetrical current configurations only.

Let us dig into the past…

Magnetic Vector Potential

In electrostatics

$$\nabla \times \vec{E} = 0 \quad \rightarrow \quad \vec{E} = -\nabla V \longrightarrow \text{a scalar potential}$$

In magnetostatics

$$\nabla \cdot \vec{B} = 0 \quad \rightarrow \quad \vec{B} = -\nabla \times \vec{A} \longrightarrow \text{a vector potential}$$

Theorem

Divergence-less fields can be expressed as the curl of vectors.

"divergence of a curl is always zero"

Magnetic Vector Potential

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In magnetostatics

$$\nabla \cdot \vec{B} = 0 \quad \rightarrow \quad \vec{B} = -\nabla \times \vec{A} \longrightarrow \text{a vector potential}$$

The magnetic vector potential due to volume current distribution $\vec{j}(r)$ at a distance *r* can be derived as

$$\vec{A}(r) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(r')}{r} \, d\tau'$$

Magnetic vector potential is not as useful in calculating $\vec{B}$ as electric potential in calculating $\vec{E}$.

Theorem

Divergence-less fields can be expressed as the curl of vectors.

"divergence of a curl is always zero"

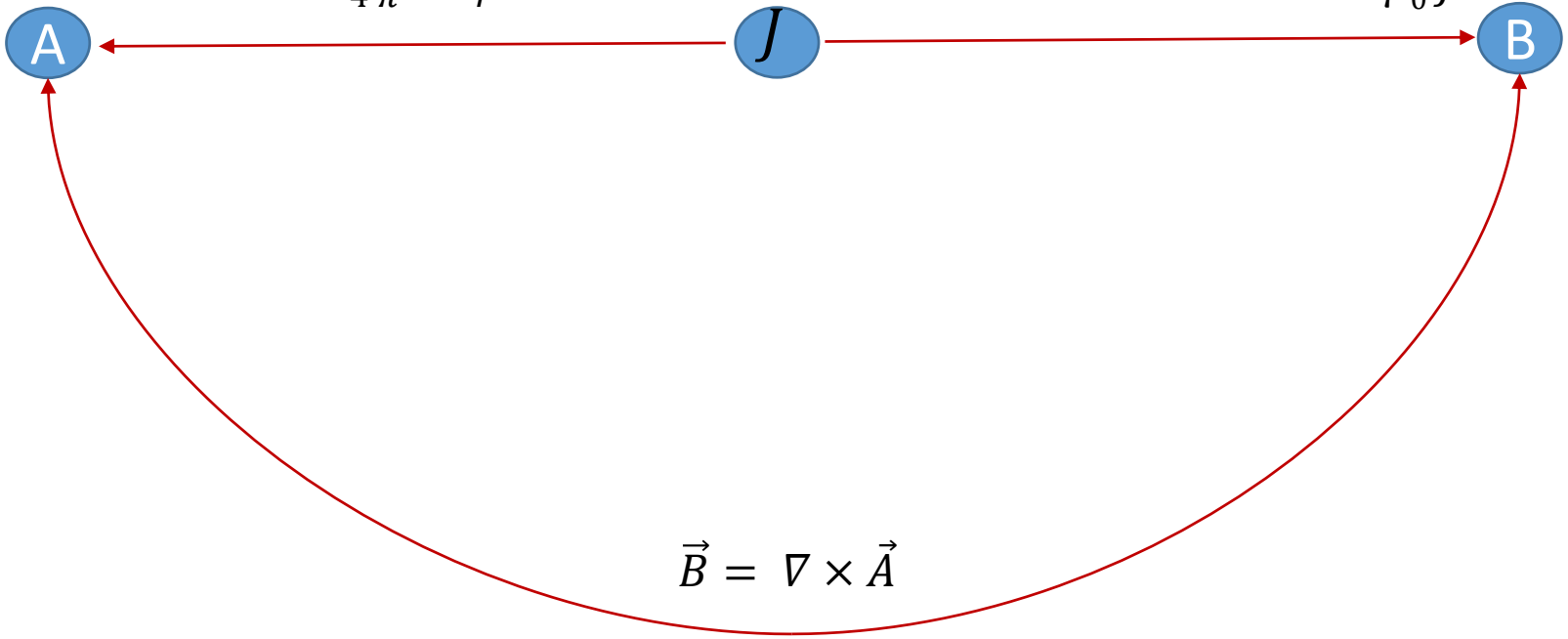
Summary of Magnetostatics

"steady currents"

$$\vec{B}(r) = \frac{\mu_0}{4\pi} \int \frac{\vec{J} \times \hat{r}}{r^2} \, d\tau$$

$$A(r) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}}{r} \, d\tau$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu_0 \vec{J}$$



Summary of Electrostatics + Magnetostatics

"stationary charges" + "steady currents"

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

Maxwell's equation for electrostatics and magentostatics

Let us dig into the past…

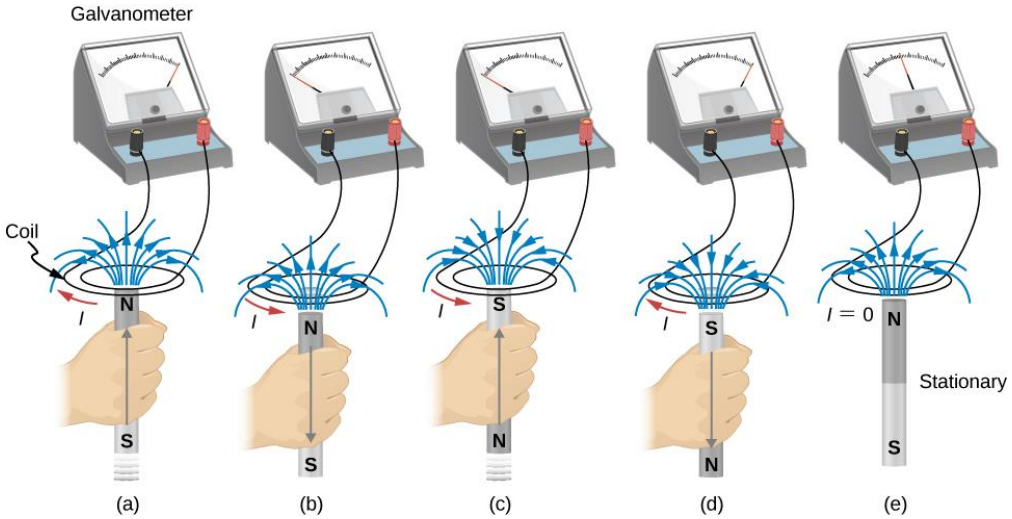
Time Varying Magnetic Fields (Faraday' Laws)

"fields are time dependent"

The induced electromotive force (emf) is equal to the rate of change of the magnetic flux

$$\epsilon = \oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d\Phi}{dt}$$

Φ is the magnetic flux. Negative sign indicates that induced emf opposes the change in flux (Lenz's Law)



Faraday's Experiment (1831)

Let us dig into the past…

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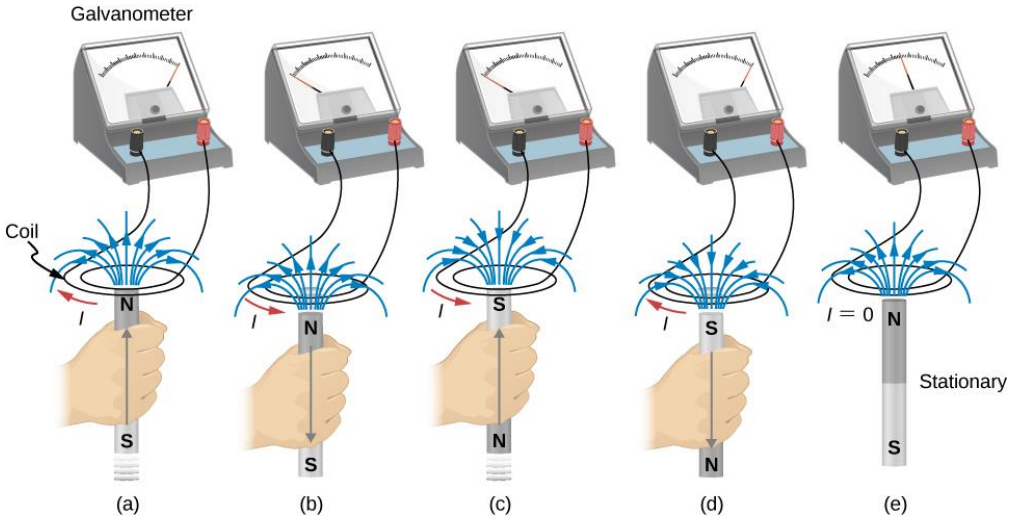


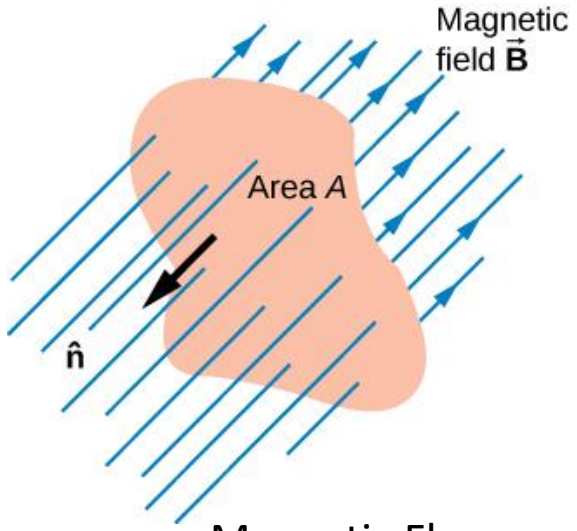
Faraday's Experiment (1831)

Using Stokes theorem we can write the above equation as

$$\int (\nabla \times \vec{E}) \cdot \overrightarrow{da} = -\int (\frac{\partial \vec{B}}{\partial t}) \cdot \overrightarrow{da}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{(Faraday's Law)}$$

Gives $\nabla \times \vec{E}$ = for static magnetic field



Magnetic Flux

Let us dig into the past…

Time Varying Electric Fields (Maxwell's Correction)

"fields are time dependent"

The Ampere's law: $\nabla \times \vec{B} = \mu_0 \vec{J}$

Taking divergence both sides

$$\nabla \cdot (\nabla \times \vec{B}) = \mu_0 \ \nabla \cdot \vec{J}$$

Always zero

Zero only for the "steady currents"

Continuity Equation

"total charge is conserved"

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Let us dig into the past…

Time Varying Electric Fields (Maxwell's Correction)

"fields are time dependent"

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Taking divergence both sides

$$\nabla \cdot (\nabla \times \vec{B}) = \mu_0 \nabla \cdot \vec{J}$$

Always zero

Zero only for the "steady currents"

From continuity equation we can write using Gauss's law

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} = -\nabla \cdot (\epsilon_0 \frac{\partial \vec{E}}{\partial t})$$

Displacement current

The Ampere's law after Maxwell's correction: $\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

Continuity Equation

"total charge is conserved"

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Maxwell's Equations in Free Space

"time varying electric and magnetic fields"

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Maxwell's equation in Electrodynamics in free space