

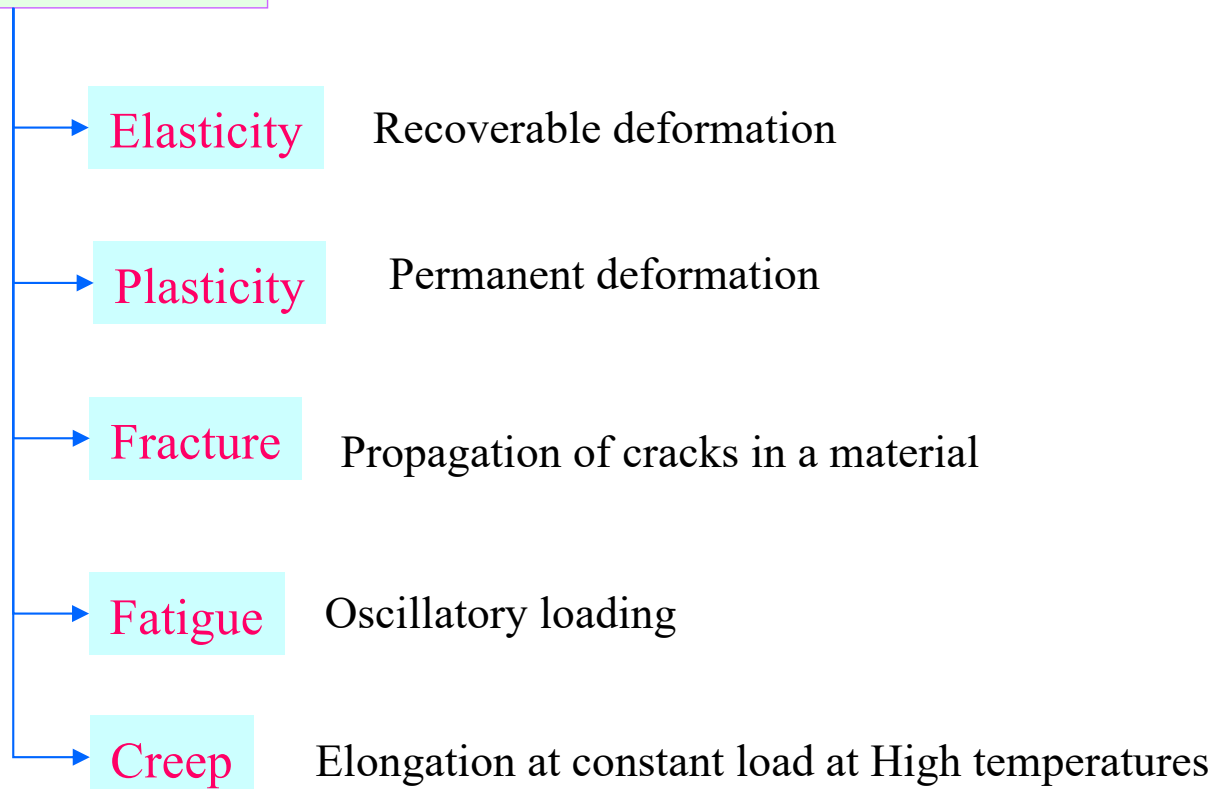
ELASTICITY

- Elasticity
- Plasticity
- Viscoelasticity

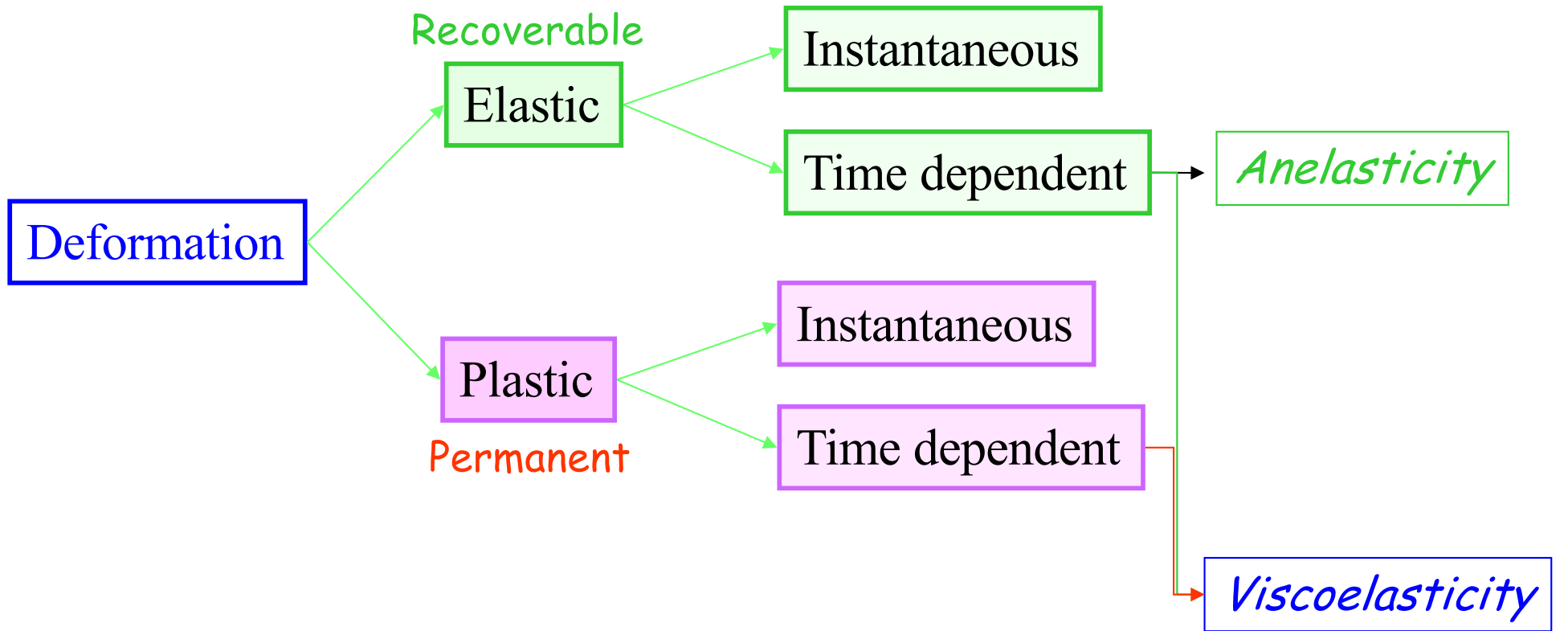
What kind of mechanical behaviour phenomena does one have to understand?

- ❑ Phenomenologically mechanical behaviour can be understood as in the flow diagram below.
- ❑ Multiple mechanisms may be associated with these phenomena (e.g. creep can occur by diffusion, grain boundary sliding etc.).
- ❑ These phenomena may lead to the failure of a material.

Mechanical Behaviour

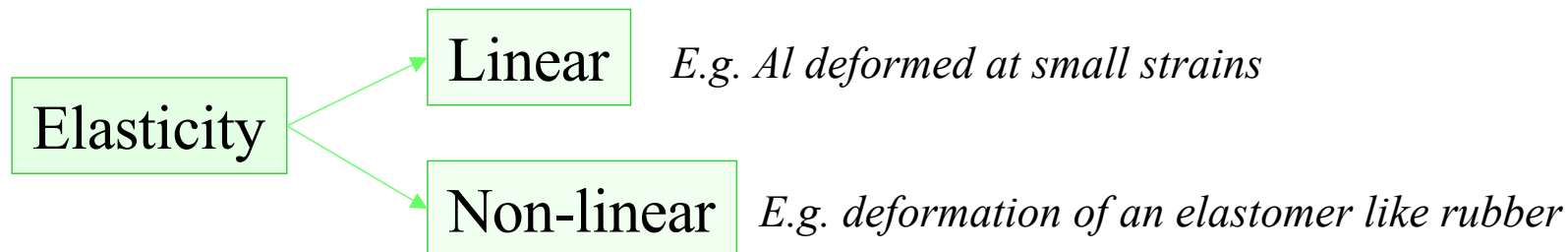


Note: above is a 'broad' classification for 'convenience'. E.g. Creep is also leads to plastic deformation!

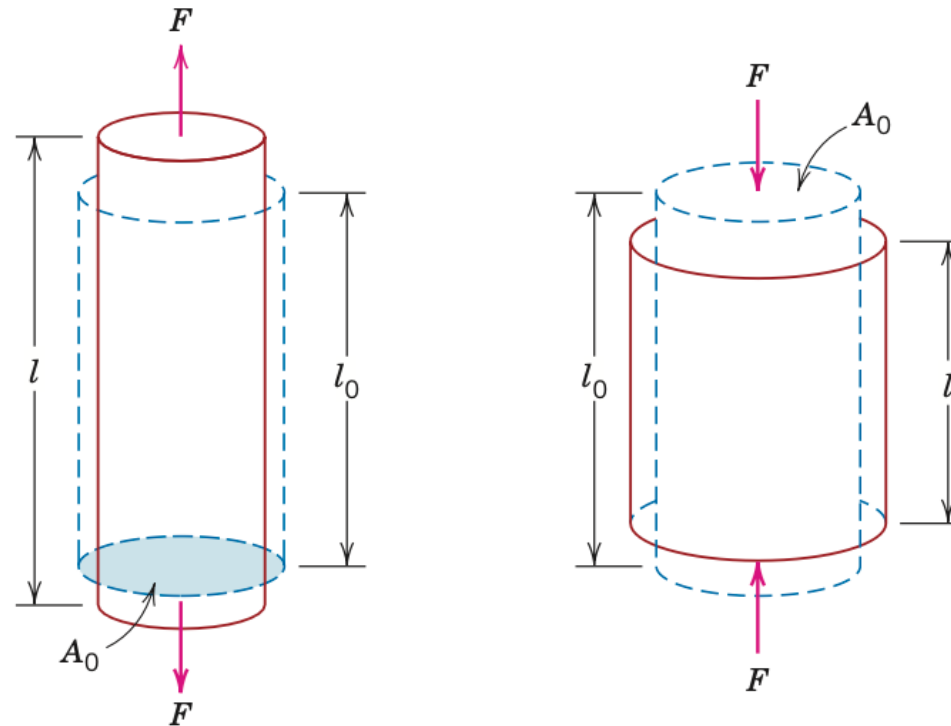


Elasticity

- ❑ Elastic deformation is reversible deformation- i.e. when load/forces/constraints are released the body returns to its original configuration (shape and size).
- ❑ Elastic deformation can be caused by tension/compression or shear forces.
- ❑ Usually in metals and ceramics elastic deformation is seen at low strains (less than $\sim 10^{-3}$).
- ❑ The elastic behaviour of metals and ceramics is usually linear.



Materials in Tension



For a known elongation $l - l_0$, it is said that you need $2 \cdot F$, for an area that is $2A_0$

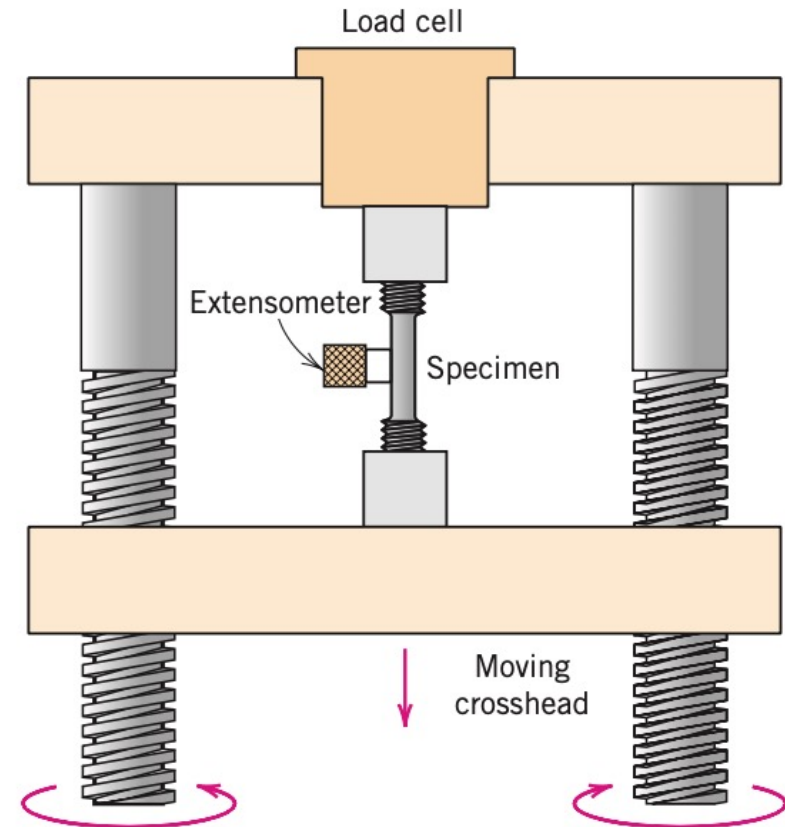
Therefore, we define $\sigma = \frac{F}{A}$ **Engineering Stress**

We also define

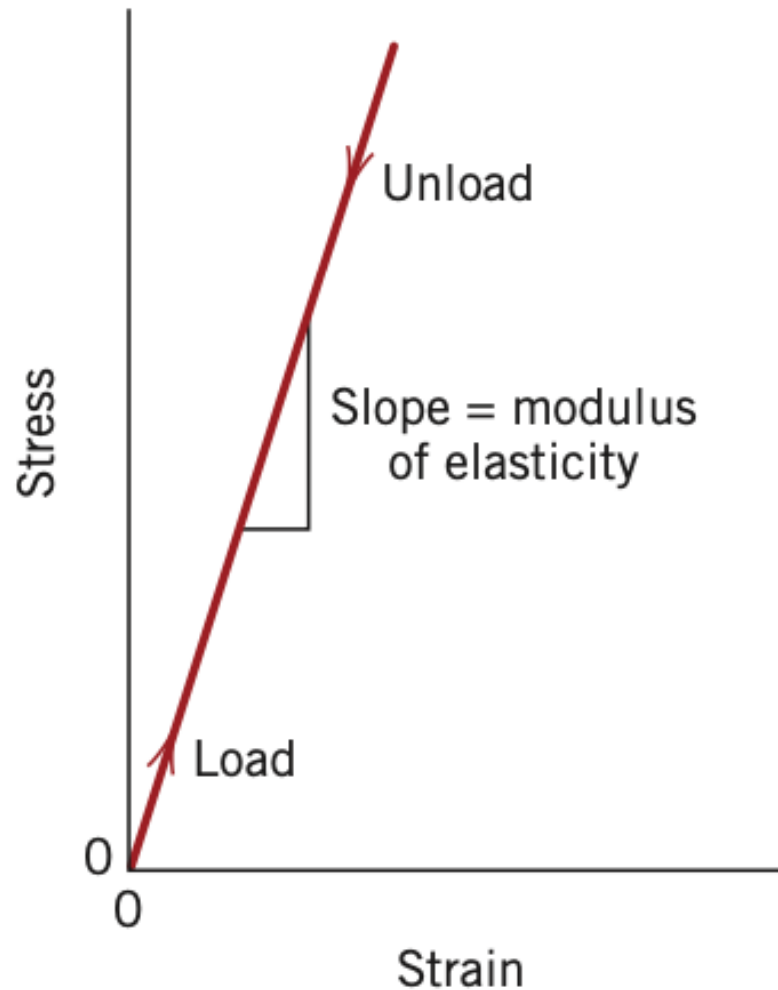
Engineering Strain

$$\epsilon = \frac{l - l_0}{l_0}$$

Dimension-less quantity



Hooke's Law



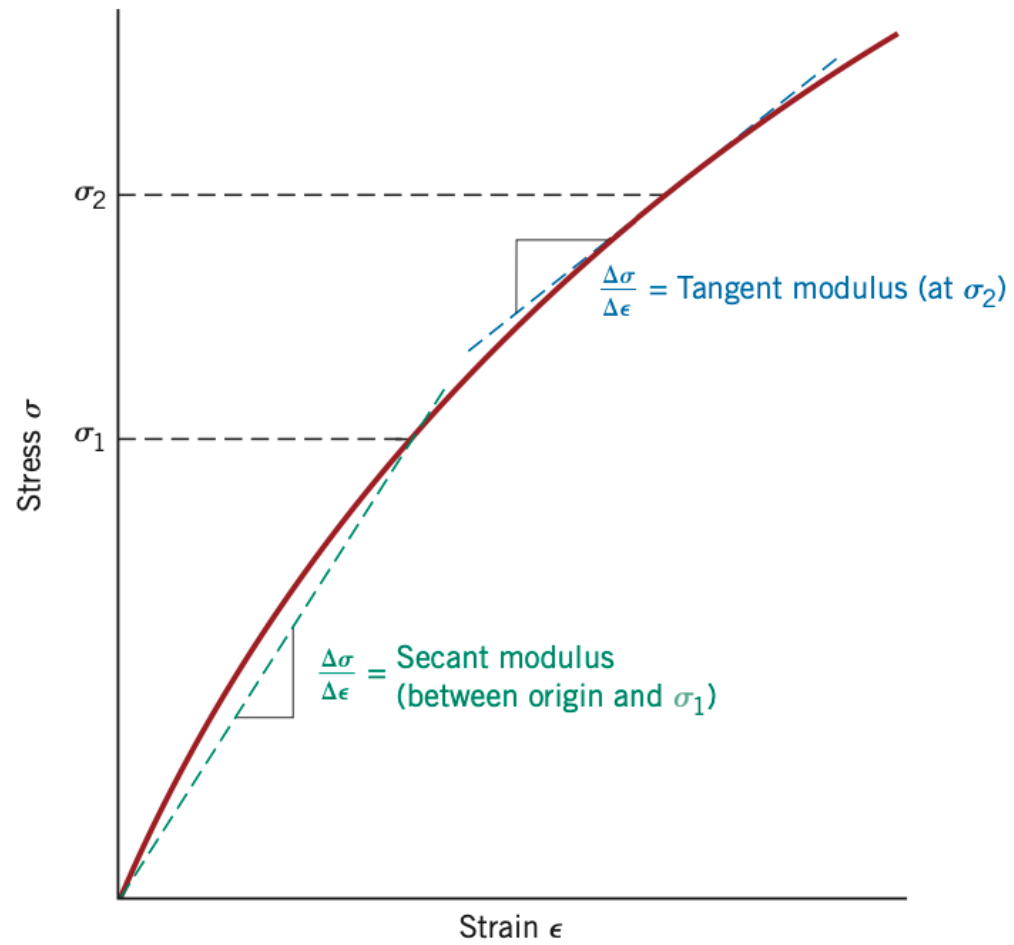
Slope of the stress-strain curve when linear is the modulus of Elasticity

Greater the modulus
-Stiffer the material

What will you do when its not/never linear ?

However, the behavior is repeatable! - elastic

Secant/ tangent methods



Atomic model for elasticity

- Let us consider the stretching of bonds (leading to elastic deformation).
- Atoms in a solid feel an **attractive force** at larger atomic separations and feel a **repulsive force** (when electron clouds ‘overlap too much’) at shorter separations. (At very large separations there is no force felt).
- The energy and the force (which is a gradient of the energy field) display functional behaviour as below.

Energy

$$U = -\frac{A}{r^n} + \frac{B}{r^m}$$

Attractive

Repulsive

$A, B, m, n \rightarrow$ constants
 $m > n$

$$F = -\frac{dU}{dr}$$

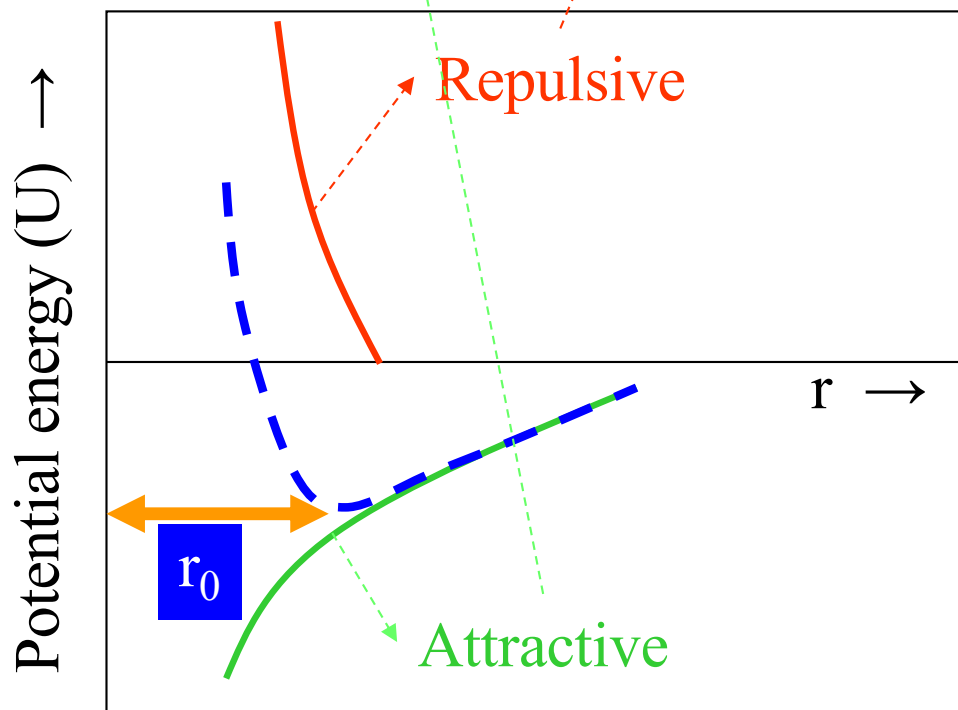
Force

$$F = -\frac{nA}{r^{n+1}} + \frac{mB}{r^{m+1}}$$

$$F = -\frac{A'}{r^p} + \frac{B'}{r^q}$$

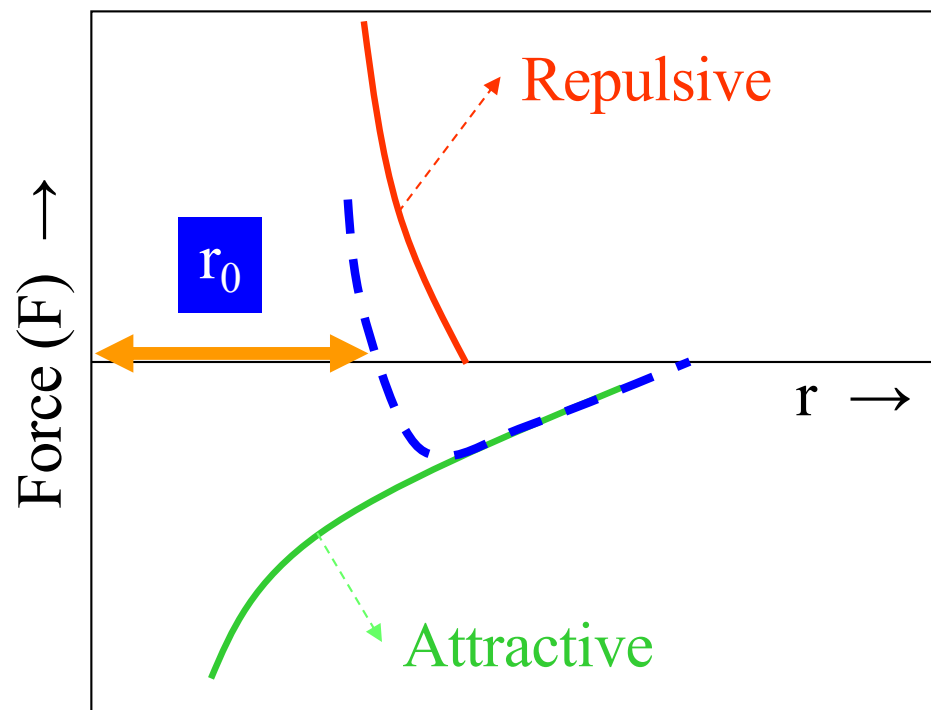
The plots of these functions is shown in the next slide

$$U = -\frac{A}{r^n} + \frac{B}{r^m}$$

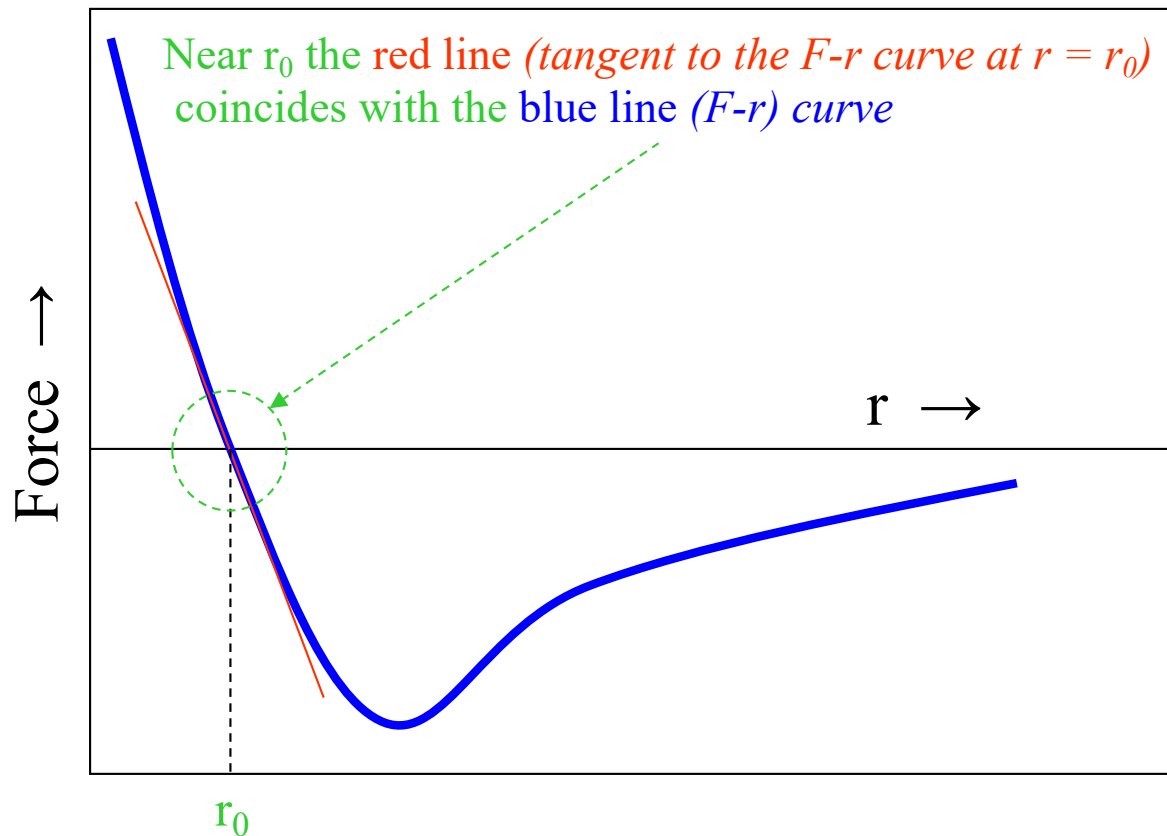


r_0 → Equilibrium separation

$$F = -\frac{A'}{r^p} + \frac{B'}{r^q}$$



□ Elastic modulus is the slope of the Force-interatomic spacing curve (F-r curve), at the



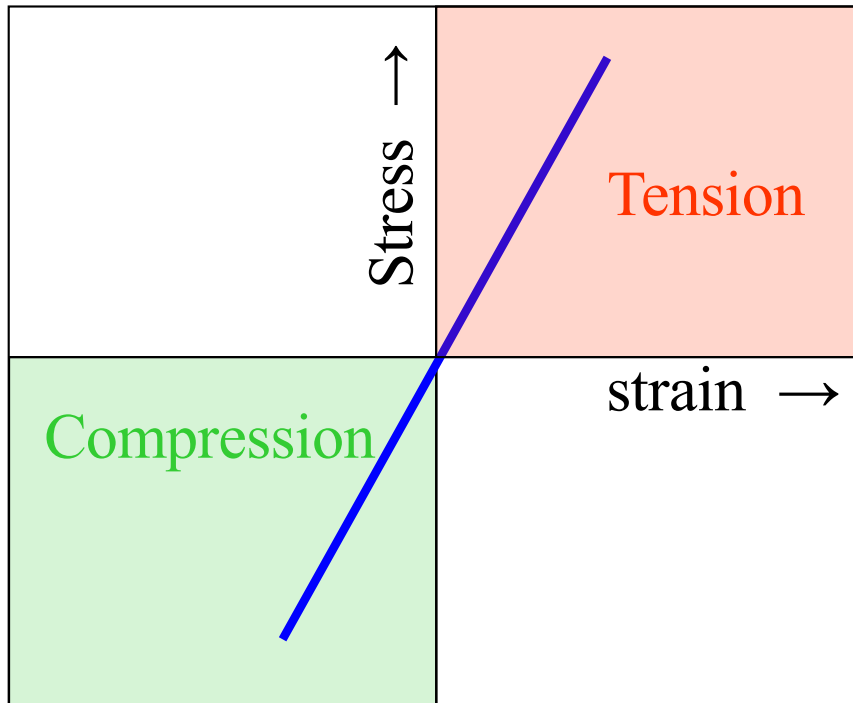
For displacements around $r_0 \rightarrow$ Force-displacement curve is approximately linear
 \rightarrow THE LINEAR ELASTIC REGION

Young's modulus (Y / E)**

- Young's modulus is :: to the -ve slope of the F-r curve at $r = r_0$

$$Y \propto -\frac{dF}{dr}$$

$$Y \propto -\frac{dF}{dr} \square \frac{d^2U}{dr^2}$$



Bonding and Elastic modulus

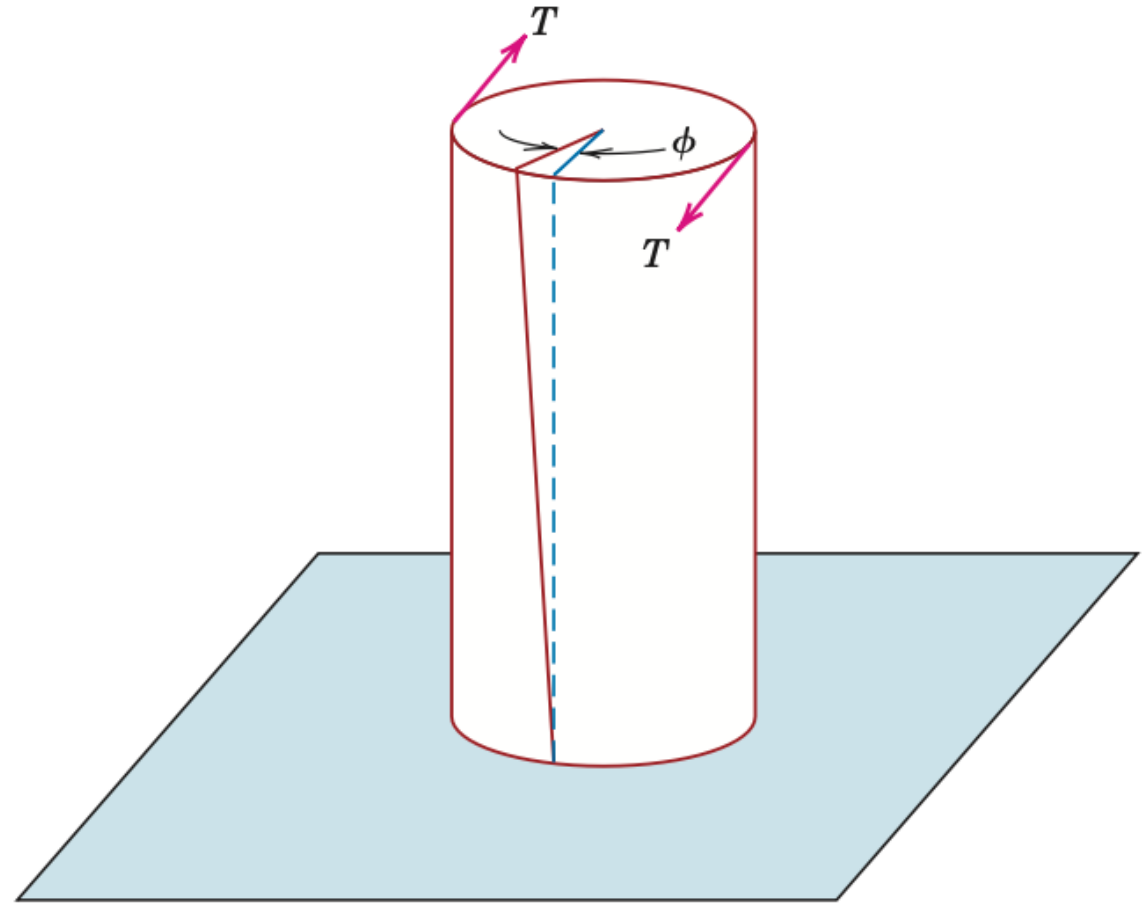
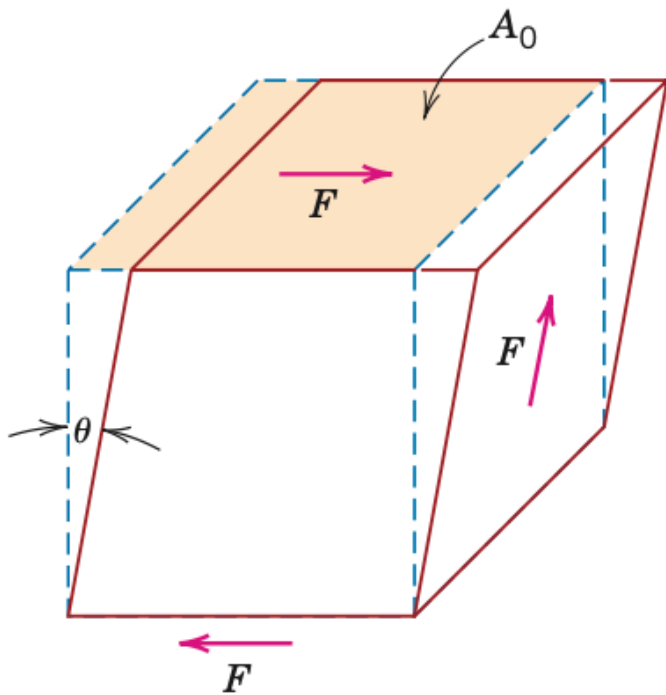
- ❑ Materials with strong bonds have a deep potential energy well with a high curvature \Rightarrow high elastic modulus
- ❑ Along the period of a periodic table the covalent character of the bond and its strength increase \Rightarrow systematic increase in elastic modulus
- ❑ Down a period the covalent character of the bonding $\downarrow \Rightarrow \downarrow$ in Y
- ❑ On **heating** the elastic modulus **decrease**: $0\text{ K} \rightarrow M.P.$, 10-20% \downarrow in modulus

Along the period \rightarrow	Li	Be	B	C _{diamond}	C _{graphite}
Atomic number (Z)	3	4	5	6	6
Young's Modulus (GN / m ²)	11.5	289	440	1140	8

Down the row \rightarrow	C _{diamond}	Si	Ge	Sn	Pb
Atomic number (Z)	6	14	32	50	82
Young's Modulus (GN / m ²)	1140	103	99	52	16

Shear stresses:

Torsional stresses are a variation of the shear



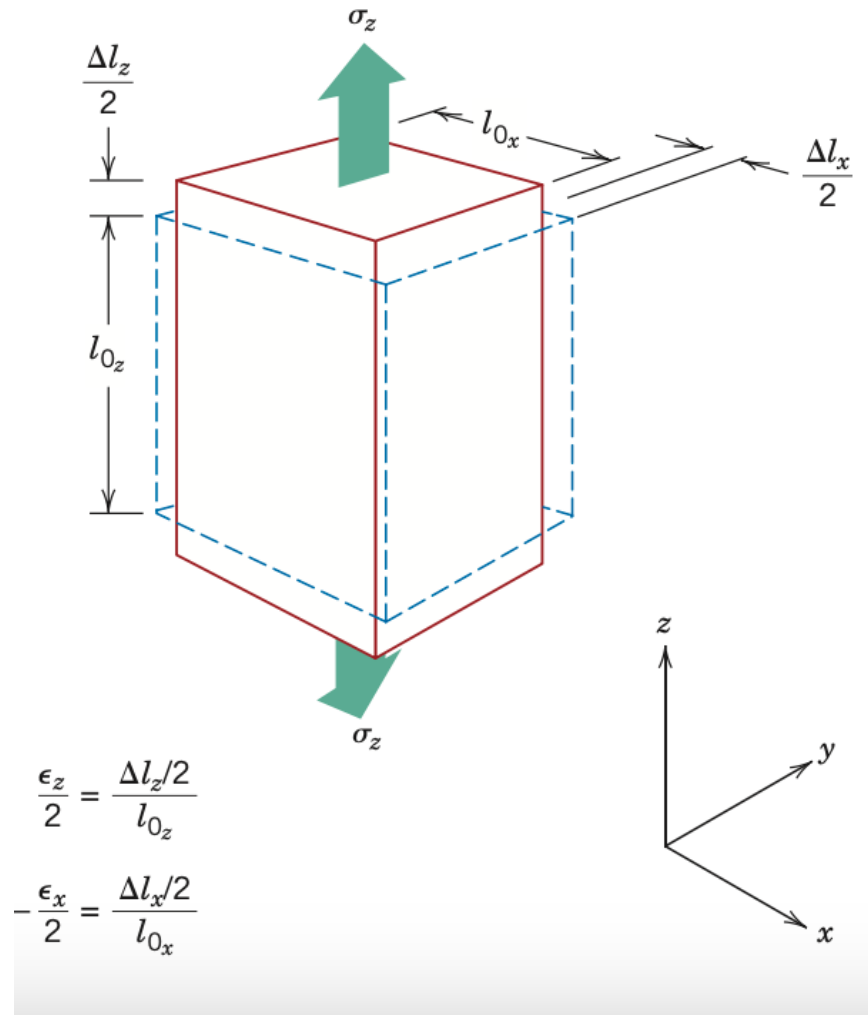
Shear stress when the force is tangential to the face

$$\text{Shear Stress } \sigma = \frac{F}{A_0}$$

$$\text{Shear Strain } \gamma = \tan \theta$$

$$\text{Shear modulus } G = \sigma / \gamma$$

Poisson's ratio



If the material is isotropic

And stress uniaxial σ_z

The orthogonal strains

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z}$$

Theoretical Poisson's ratio = 0.25
for isotropic materials

maximum value for no net-volume change is 0.5

However, typical value lies between 0.25 – 0.35

Other elastic moduli

$$\square \sigma = E \cdot \varepsilon$$

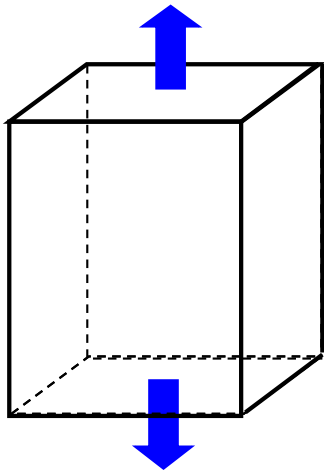
$E \rightarrow$ Young's modulus

$$\square \tau = G \cdot \gamma$$

$G \rightarrow$ Shear modulus

$$\square \sigma_{\text{hydrodynamical}} = K \cdot \text{volumetric strain}$$

$K \rightarrow$ Bulk modulus

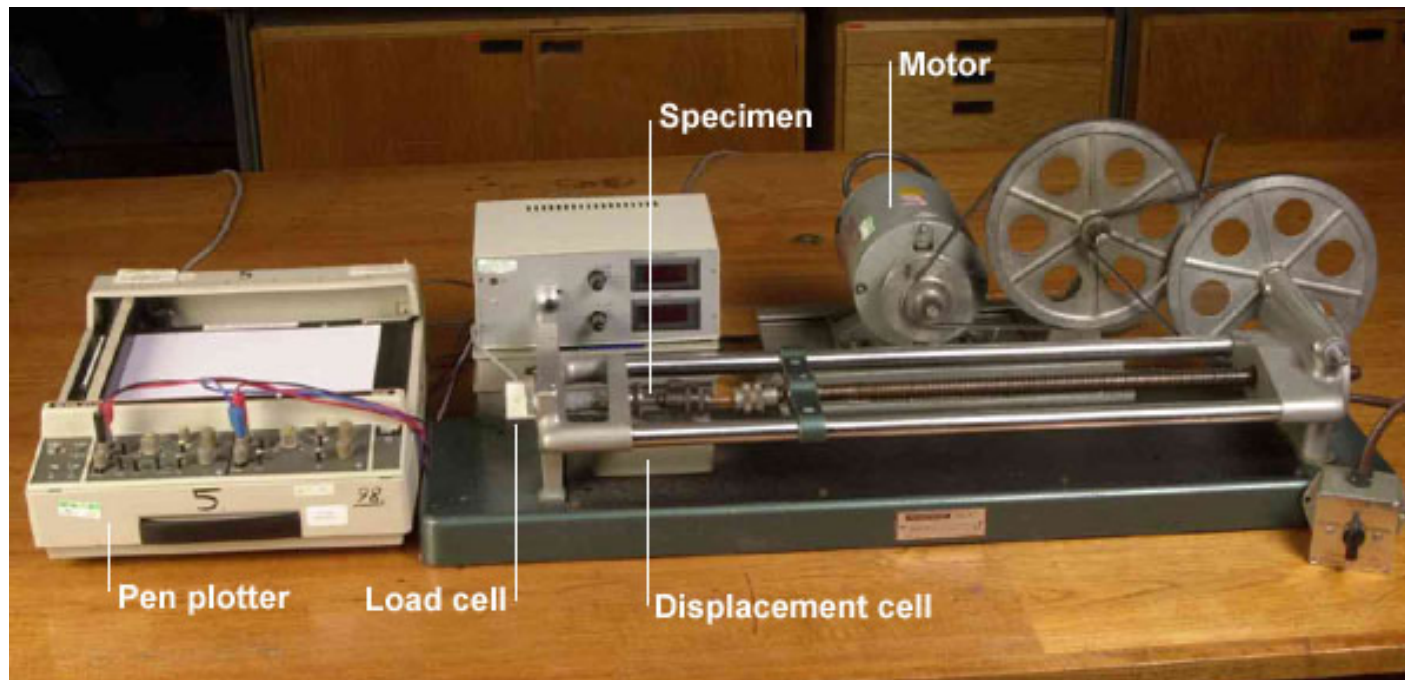
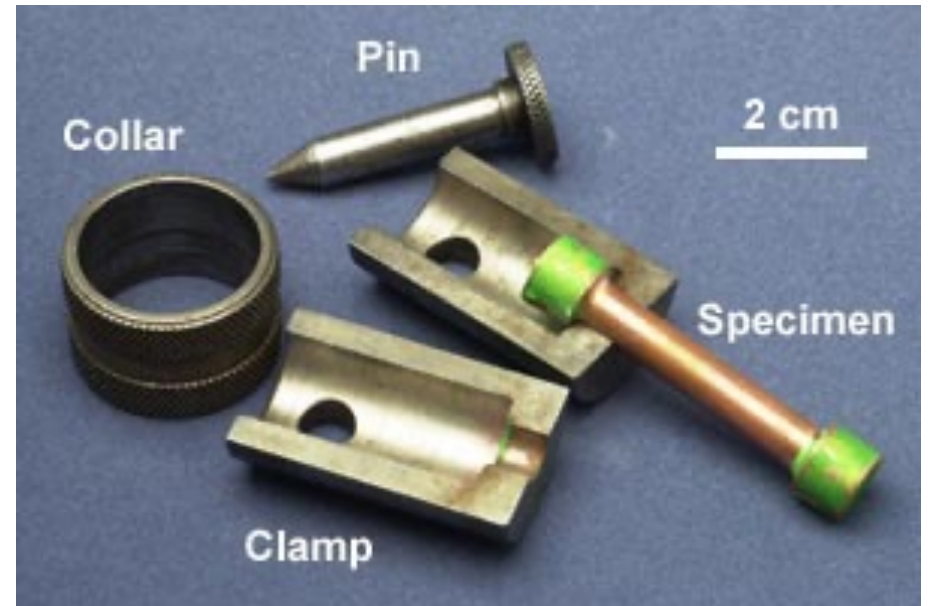
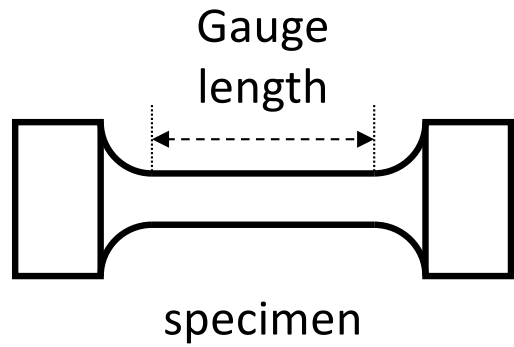


$$\nu \square - \frac{\varepsilon_t}{\varepsilon_l}$$

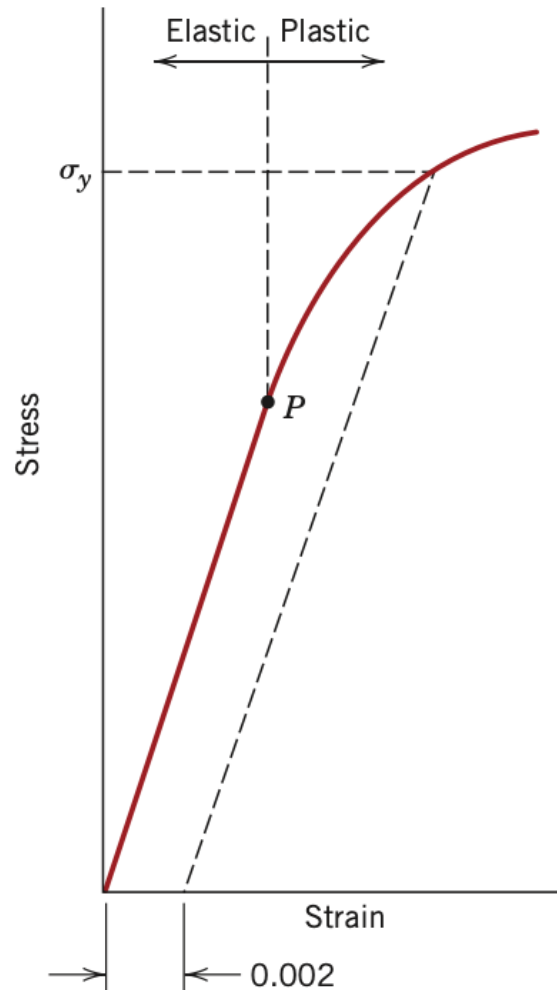
$$G \square \frac{E}{2(1 + \nu)}$$

$$K \square \frac{E}{3(1 - 2\nu)}$$

Tensile Testing



Tensile Stress-Strain Curves



Below point P
Linear region – Obey Hooke's law
Elastic in the linear region.

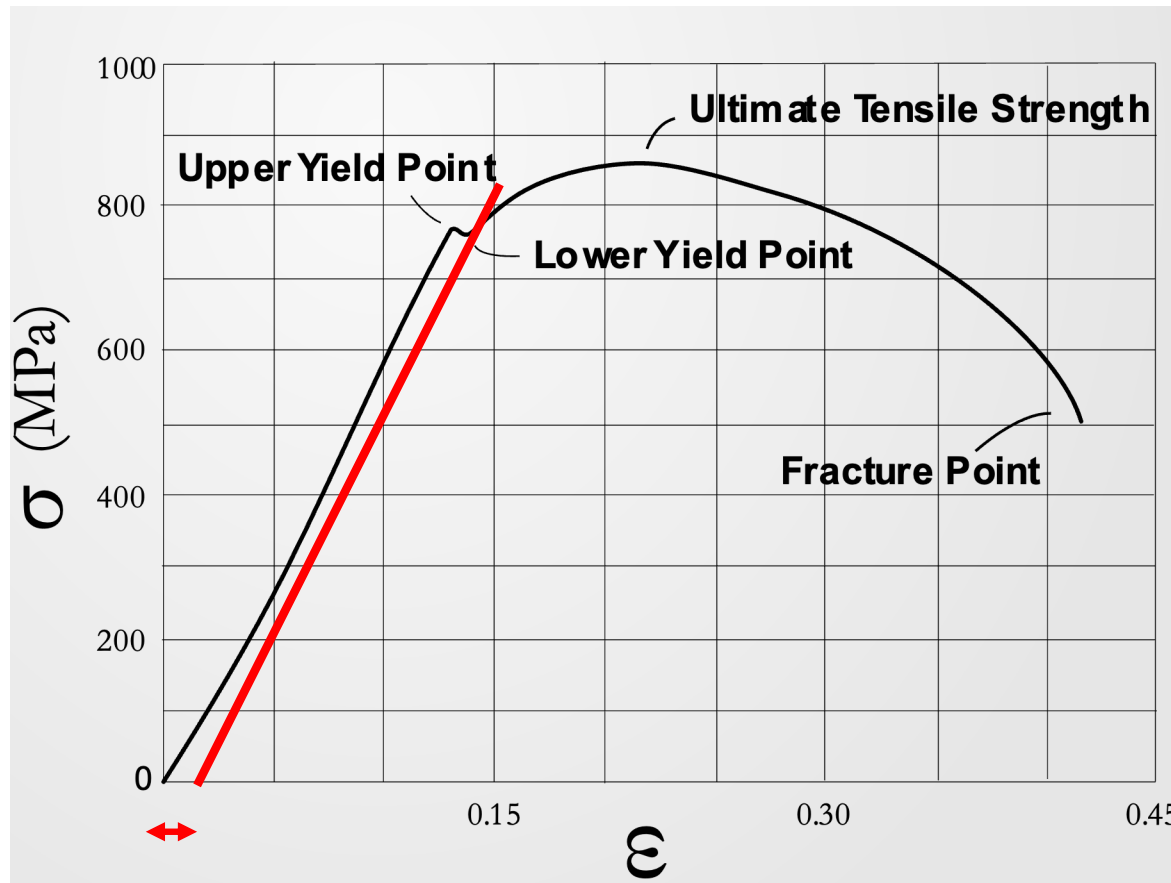
Beyond P
Deviation from linearity
Permanent – non recoverable damage
Atoms finding new equilibrium

In crystalline solids deformation is via plane slips
In non-crystalline solids – viscous flow

Note the strain ranges!

Yielding

Yield stress – The stress point at which the deviation from linearity happens

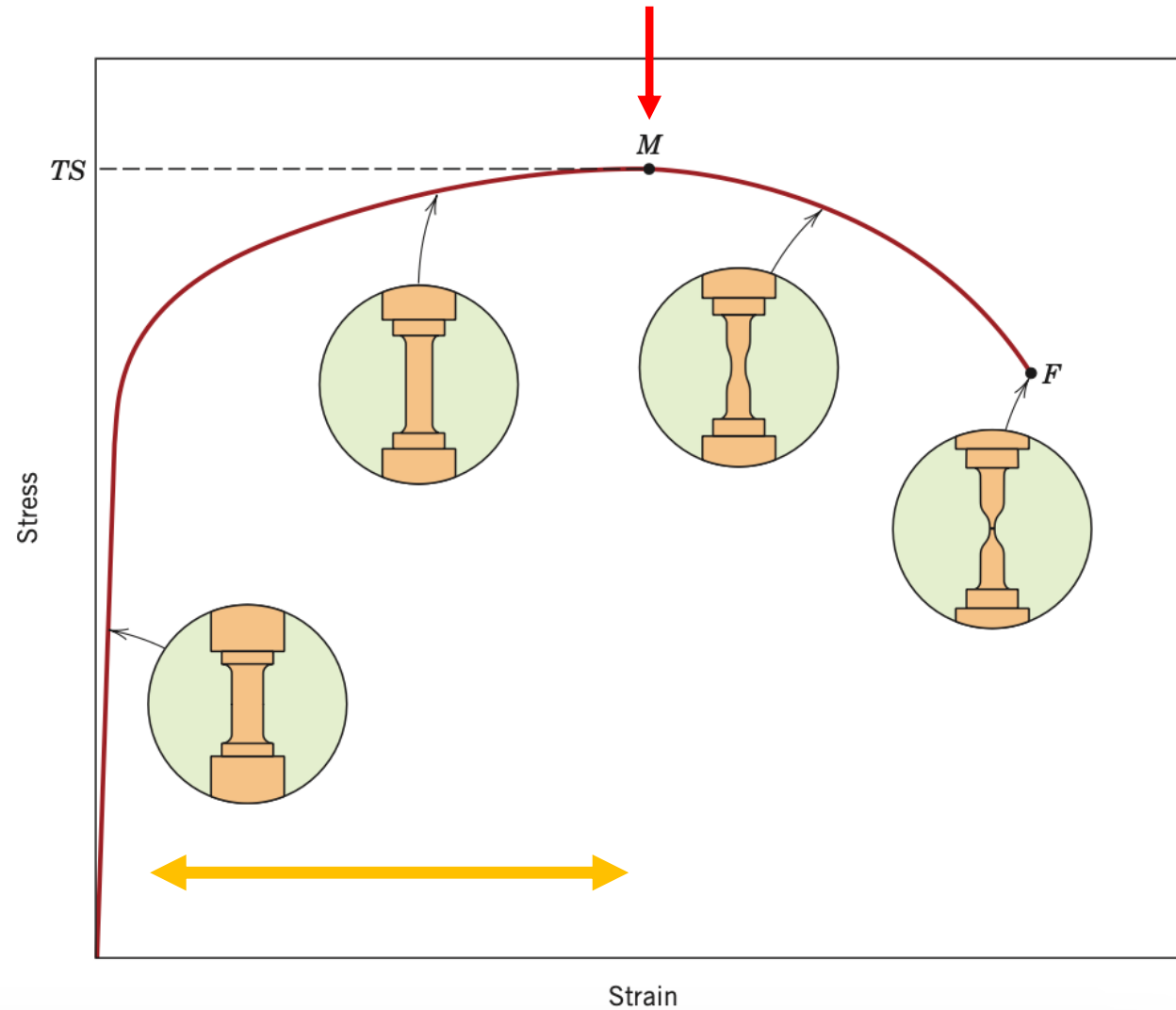


0.2%

In places where the yielding point is not clear, a convention (0.2%) is employed.

The limits

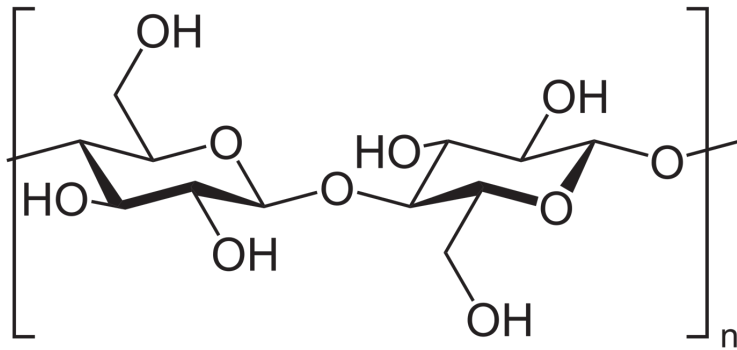
Maximum strength!
The strength then decays to eventual fracture



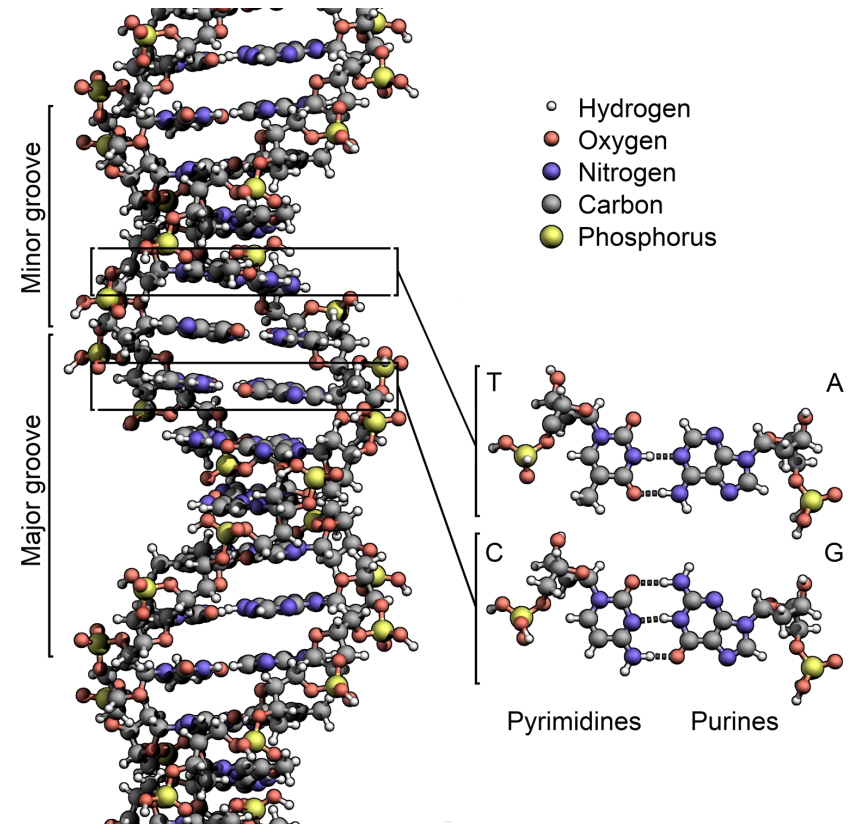
Strain hardening can also be observed. However, a maximal stress point is achieved

What happens in polymers?

Polymers are long chain molecules of repeating units



Cellulose –
Present in cotton, wood, cell walls ...



DNA, RNA, ...

Elastomers

Long chain molecules when bent tend to cross-link.



Thermodynamically favored to cross-link and stay bent

Higher configurational entropy \rightarrow lower ΔG

So, when stretched, work is done on a polymer

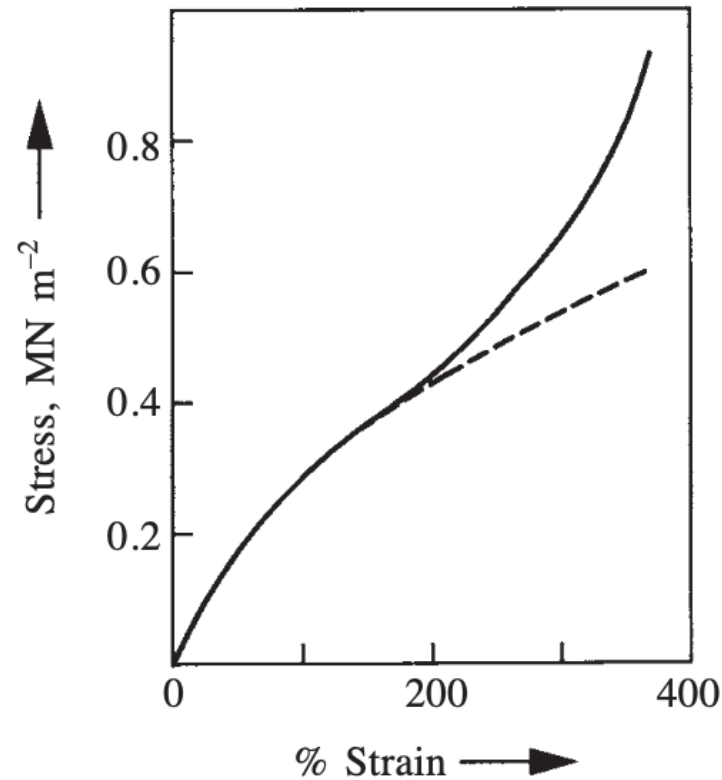
- No bonds are stretched
- The chain is straightened (reduced entropy) at T

Calculations yield:

$$F = -T \frac{\partial S}{\partial L}$$

$$F = \frac{N_0 k T}{L_0} \left[\left(\frac{L}{L_0} \right) - \left(\frac{L_0}{L} \right)^2 \right]$$

Stress-strain curve for an elastomer



Check the strain values

The dotted line is from the equation $F = \frac{N_0 kT}{L_0} \left[\left(\frac{L}{L_0} \right) - \left(\frac{L_0}{L} \right)^2 \right]$

However, after a large strain, there is deviation – largely due to bond stretching.!

Anisotropy in the Elastic modulus

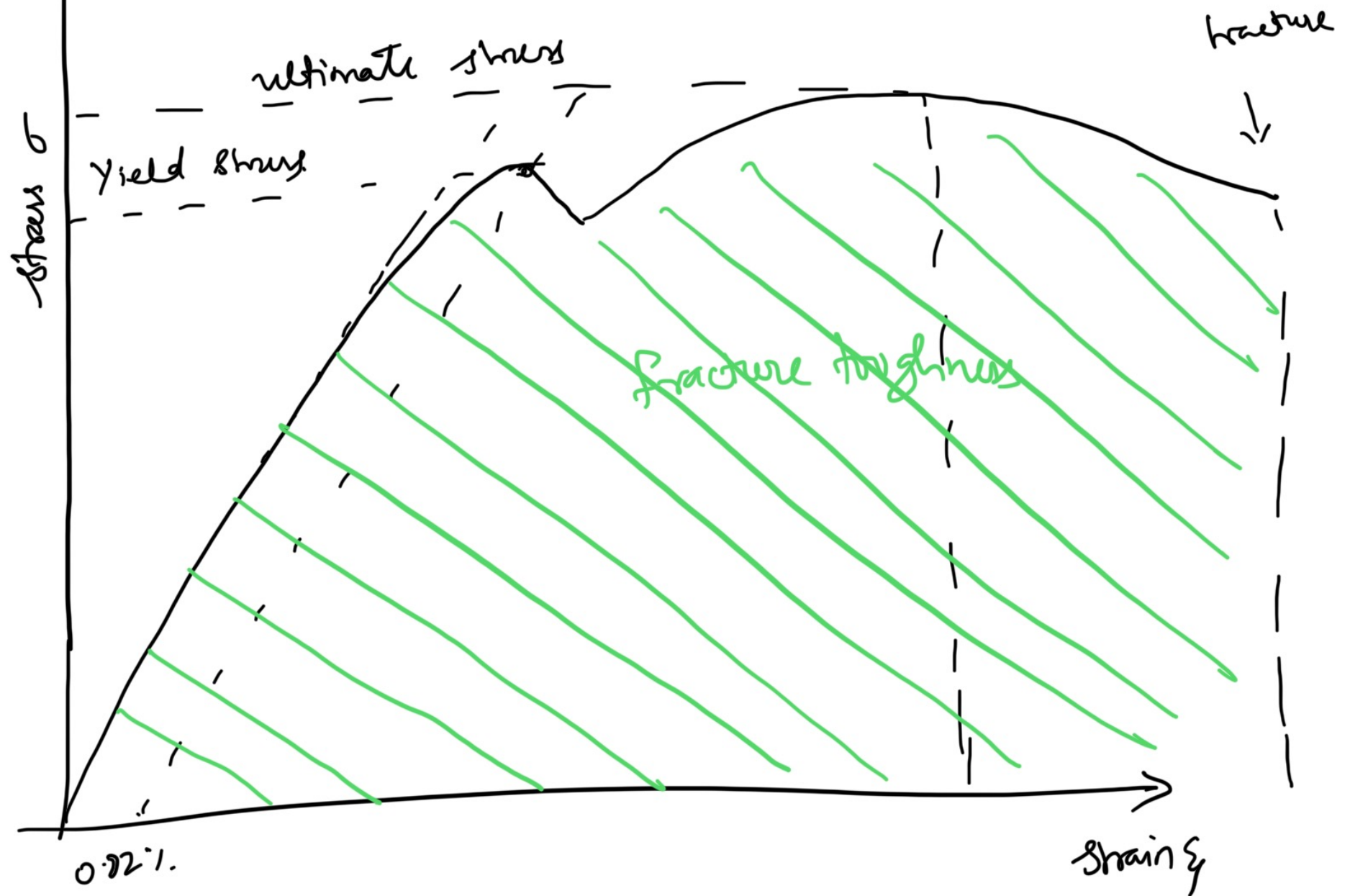
- In a crystal the interatomic distance varies with direction
→ elastic anisotropy

<i>Metal</i>	<i>Modulus of Elasticity (GPa)</i>		
	<i>[100]</i>	<i>[110]</i>	<i>[111]</i>
Aluminum	63.7	72.6	76.1
Copper	66.7	130.3	191.1
Iron	125.0	210.5	272.7
Tungsten	384.6	384.6	384.6

- Elastic anisotropy is especially pronounced in materials with
 - ▶ two kinds of bonds

E.g. in graphite $E [10 \bar{1}0] = 950 \text{ GPa}$, $E [0001] = 8 \text{ GPa}$

Estimation of mechanical properties



Elastic modulus – Slope of the stress-strain curve

Yield Strength – The stress point of deviation from linearity

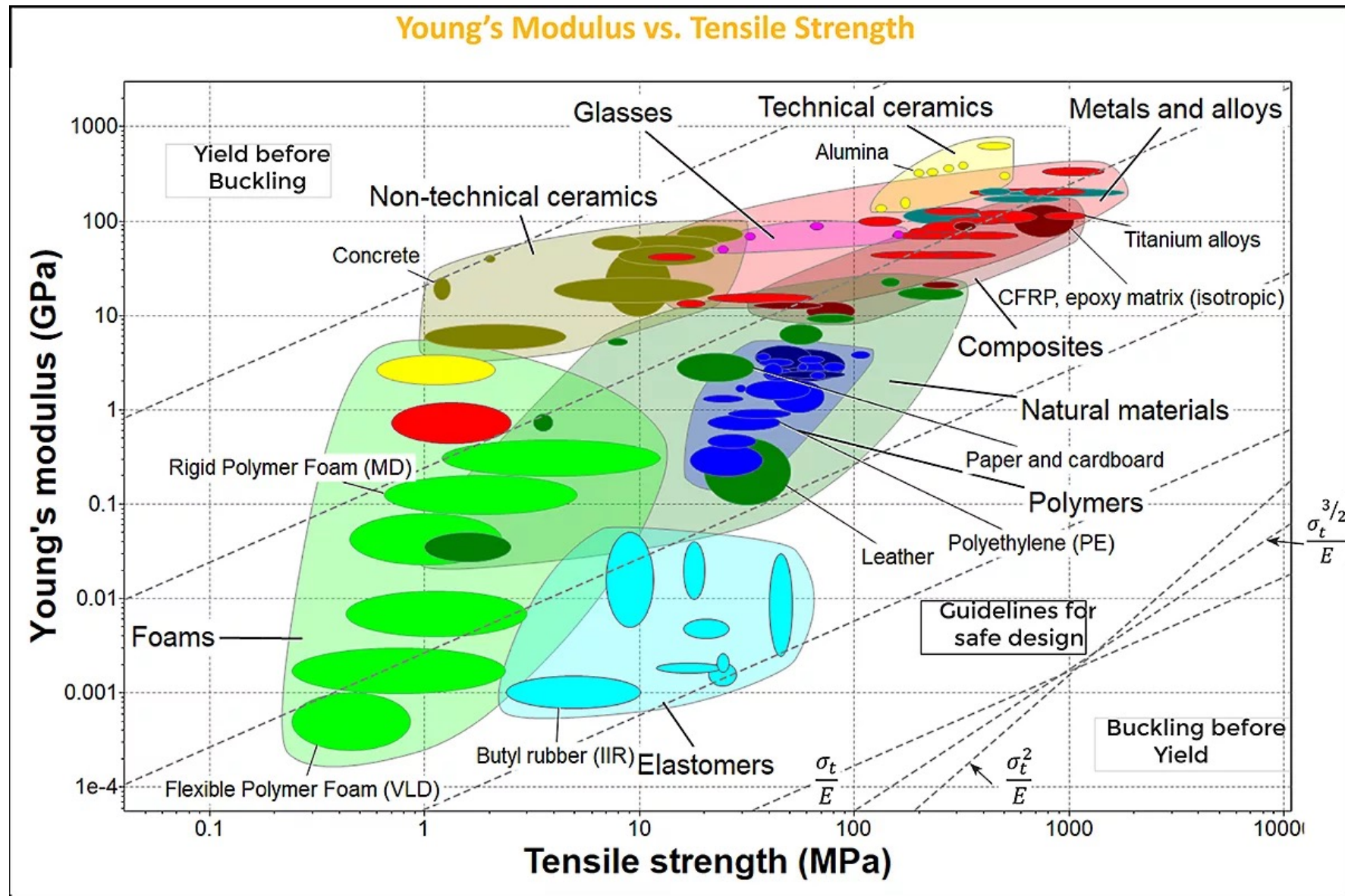
Ultimate Tensile Strength – Stress at the maximum of engineering stress-Strain curve

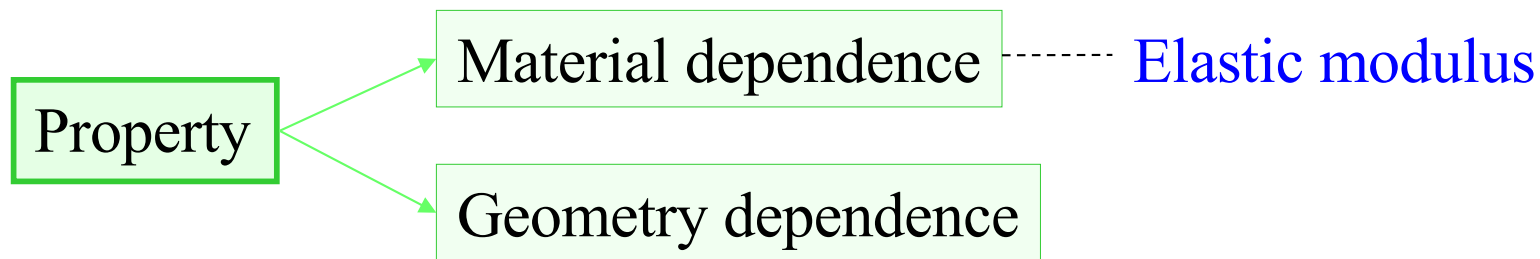
Fracture Strength – Point of strain at which fracture is witnessed in the engineering stress-Strain curve

Fracture Toughness – Total area under the engineering stress-strain curve – energy/volume required for fracture

Ductility – Percentage elongation at/near fracture $D\% = \frac{l_f - l_0}{l_0} * 100$

Parameters for design:





Elastic modulus in design

- ❑ **Stiffness** of a material is its ability to resist elastic deformation of deflection on loading → depends on the geometry of the component.
- ❑ High modulus in conjunction with good ductility should be chosen (*good ductility avoids catastrophic failure in case of accidental overloading*)
- ❑ **Covalently bonded materials**- e.g. diamond have high E (1140 GPa)
BUT brittle
- ❑ **Ionic solids** are also very brittle

Ionic solids →	NaCl	MgO	Al ₂ O ₃	TiC	Silica glass
Young's Modulus (GN / m ²)	37	310	402	308	70

❑ METALS

- ▶ First transition series → good combination of ductility & modulus (200 GPa)
- ▶ Second & third transition series → even higher modulus, but higher density (cost)

❑ POLYMERS

- ▶ Polymers can have good plasticity → but low modulus dependent on
 - the nature of secondary bonds- Van der Waals / hydrogen
 - presence of bulky side groups
 - branching in the chains
 - *Unbranched polyethylene $E = 0.2$ GPa,*
 - *Polystyrene with large phenyl side group $E = 3$ GPa,*
 - *3D network polymer phenol formaldehyde $E = 3-5$ GPa*
 - cross-linking

Increasing the modulus of a material

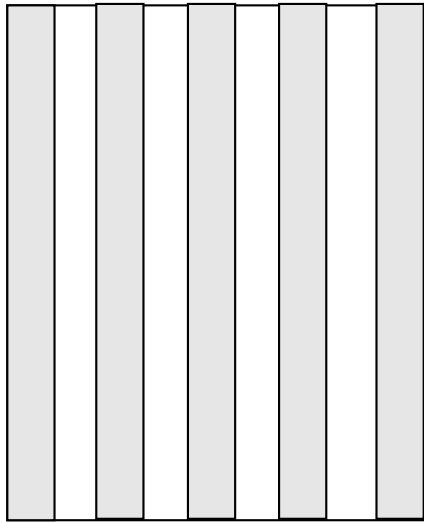
☐ METALS

- ▶ By suitably alloying the Young's modulus can be increased
- ▶ But E is a **structure** (*microstructure*) insensitive property
⇒ the increase is \propto fraction added
- ▶ TiB_2 (\sim spherical, in equilibrium with matrix) added to Fe to increase E

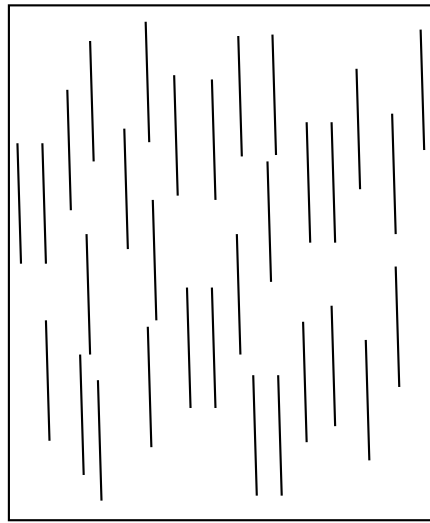
☐ COMPOSITES

- ▶ A second phase (*reinforcement*) can be added to a low E material to \uparrow E
(*particles, fibres, laminates*)
- ▶ The second phase can be brittle and the ductility is provided by the matrix \rightarrow if reinforcement fractures the crack is stopped by the matrix (Examples are Boron and Al composites)

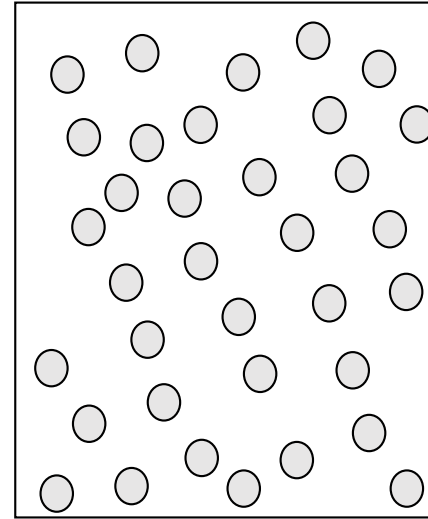
COMPOSITES



Laminate composite



Aligned fiber composite



Particulate composite

Modulus parallel to the direction of the fibers

$$E_c \square E_f V_f \square E_m V_m$$

Volume fractions

- Under iso-strain conditions
- I.e. *parallel* configuration
- m-matrix, f-fibre, c-composite

Composite modulus in isostress and isostrain conditions

$$E_c \square E_f V_f \square E_m V_m$$

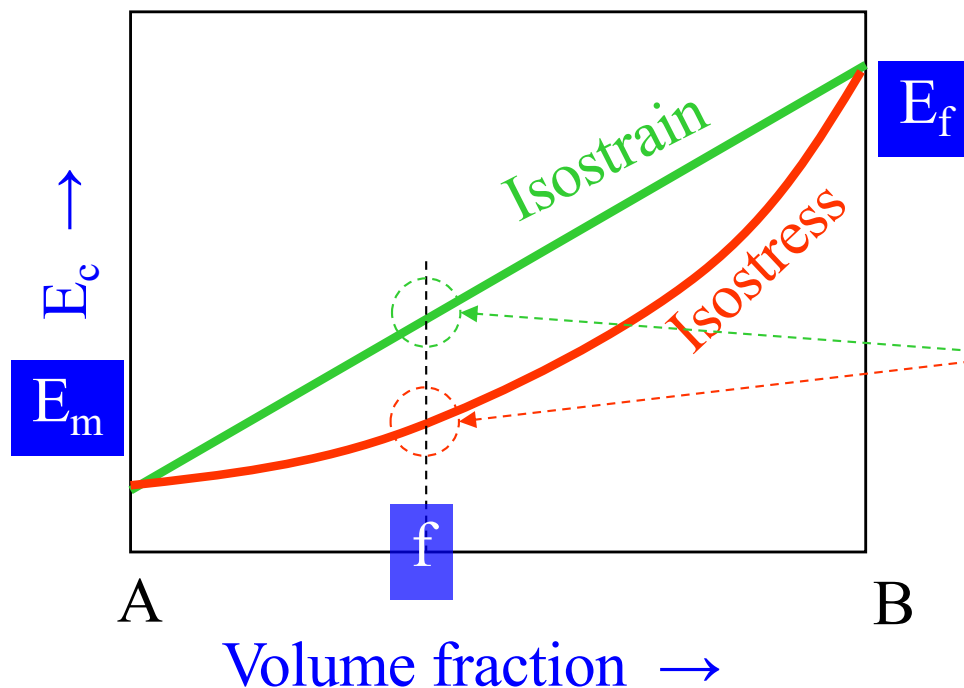
Voigt averaging

- Under iso-strain conditions [$\epsilon_m = \epsilon_f = \epsilon_c$]
- I.e. \sim resistances in series configuration

$$\frac{1}{E_c} \square \frac{V_f}{E_f} \square \frac{V_m}{E_m}$$

- Under iso-stress conditions [$\sigma_m = \sigma_f = \sigma_c$]
- I.e. \sim resistances in parallel configuration
- Usually not found in practice

Reuss averaging



By rule of mixtures

For a given fiber fraction f , the moduli of various conceivable composites lie between an upper bound given by **isostrain** condition and a lower bound given by **isostress** condition