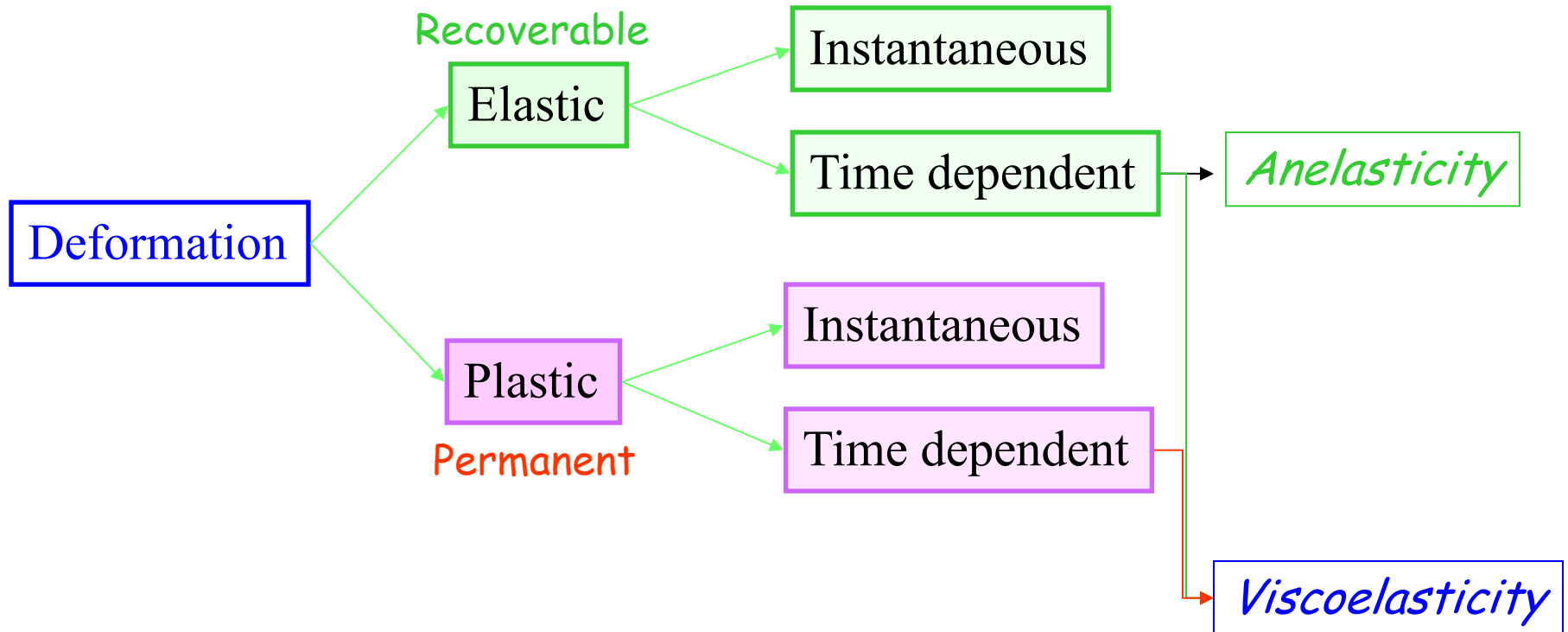




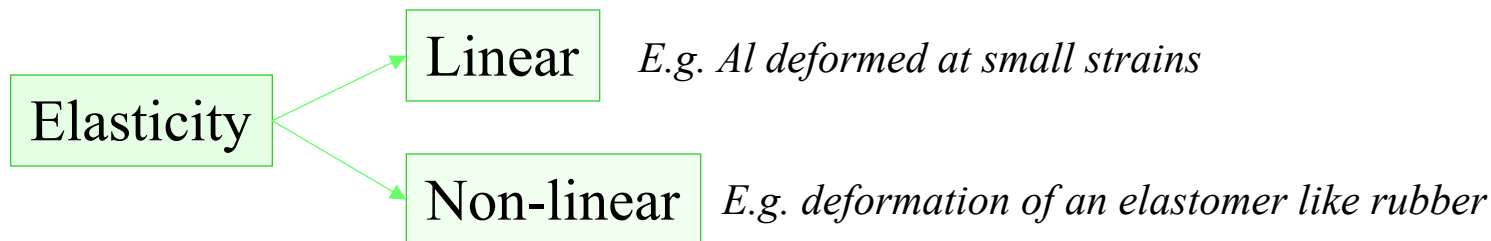
What it takes to build reliable structures ?

An understanding of mechanical properties of materials

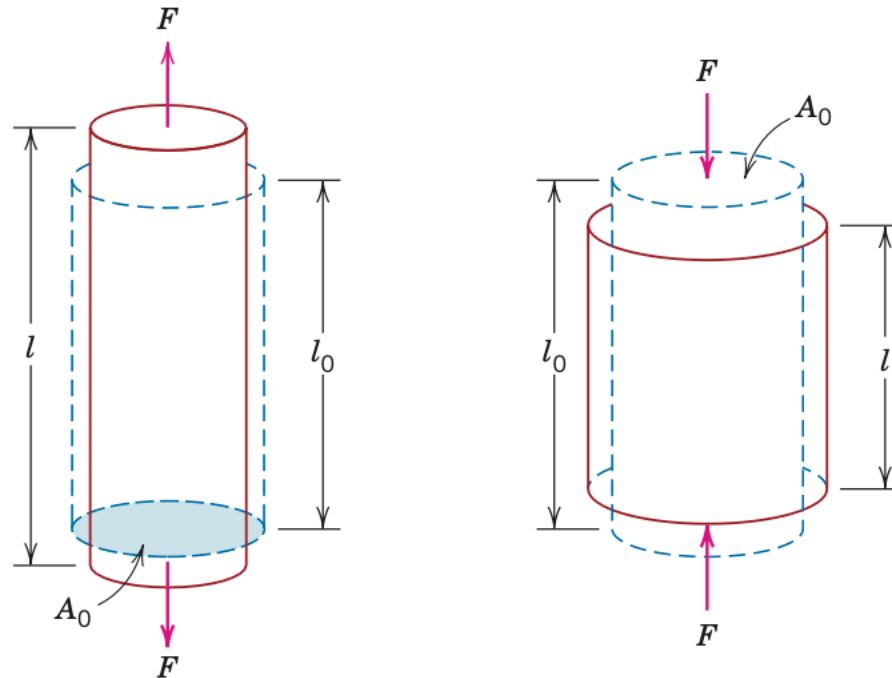


Elasticity

- ❑ Elastic deformation is reversible deformation- i.e. when load/forces/constraints are released the body returns to its original configuration (shape and size).
- ❑ Elastic deformation can be caused by tension/compression or shear forces.
- ❑ Usually in metals and ceramics elastic deformation is seen at low strains (less than $\sim 10^{-3}$).
- ❑ The elastic behaviour of metals and ceramics is usually linear.



Materials in Tension



For a known elongation $l - l_0$, it is said that you need $2 \cdot F$, for an area that is $2A_0$

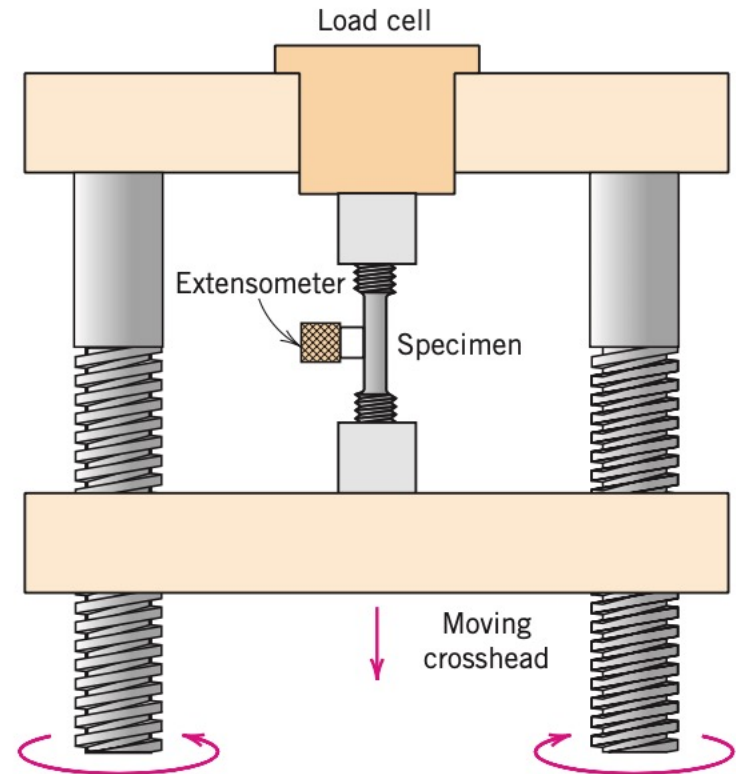
Therefore, we define $\sigma = \frac{F}{A}$ **Engineering Stress**

We also define

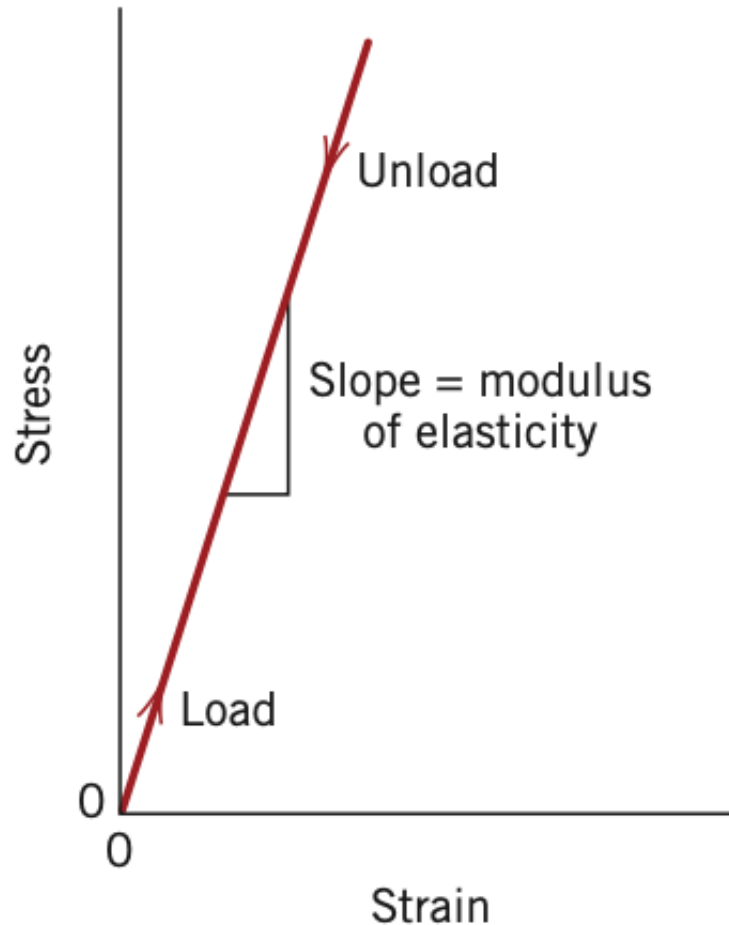
Engineering Strain

$$\epsilon = \frac{l - l_0}{l_0}$$

Dimension-less quantity



Hooke's Law



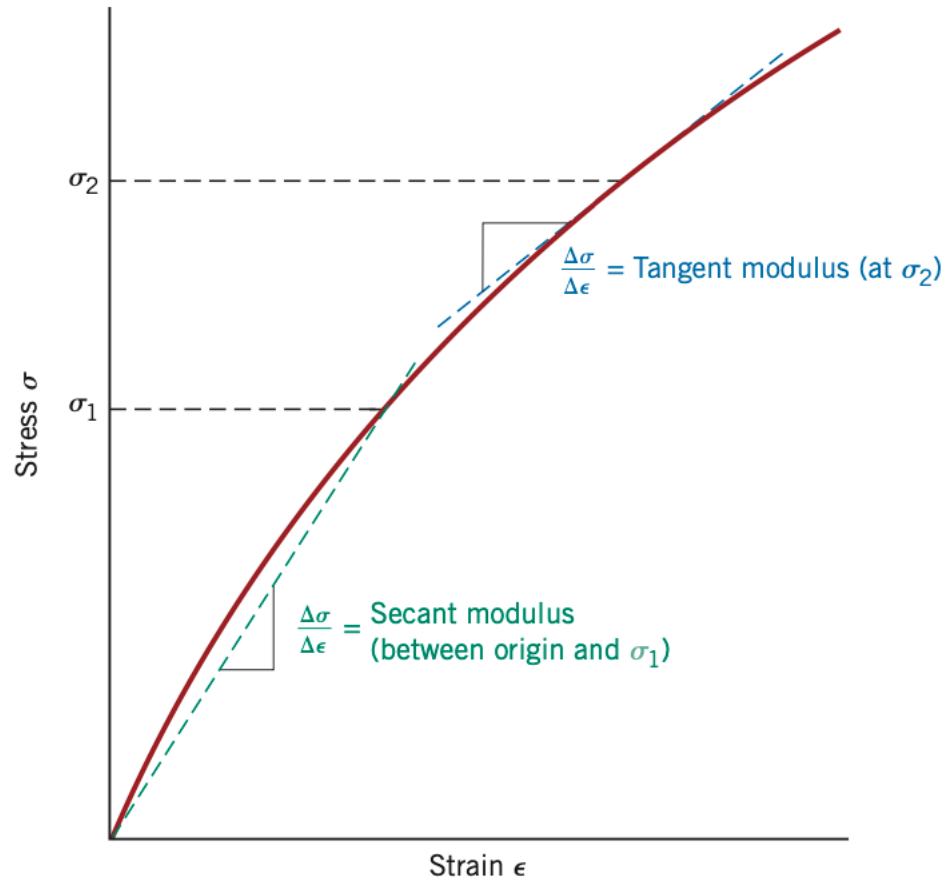
Slope of the stress-strain curve when linear is the modulus of Elasticity

Greater the modulus
-Stiffer the material

What will you do when its not/never linear ?

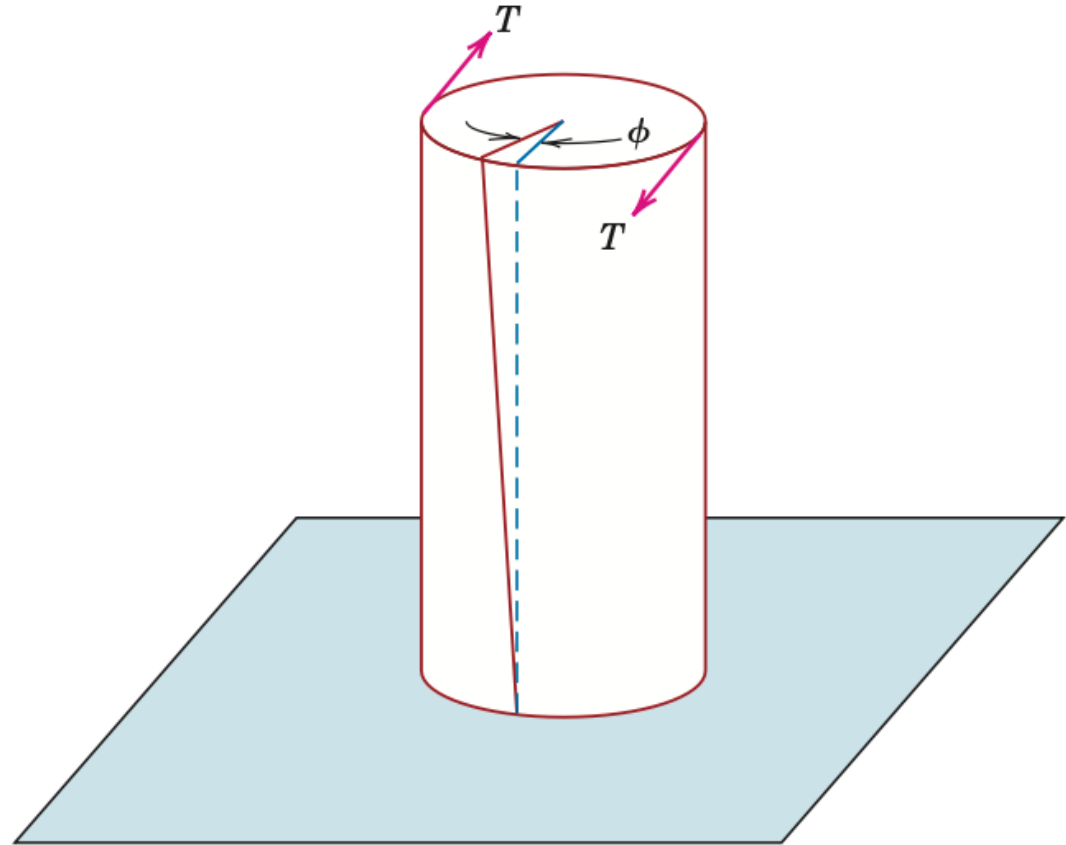
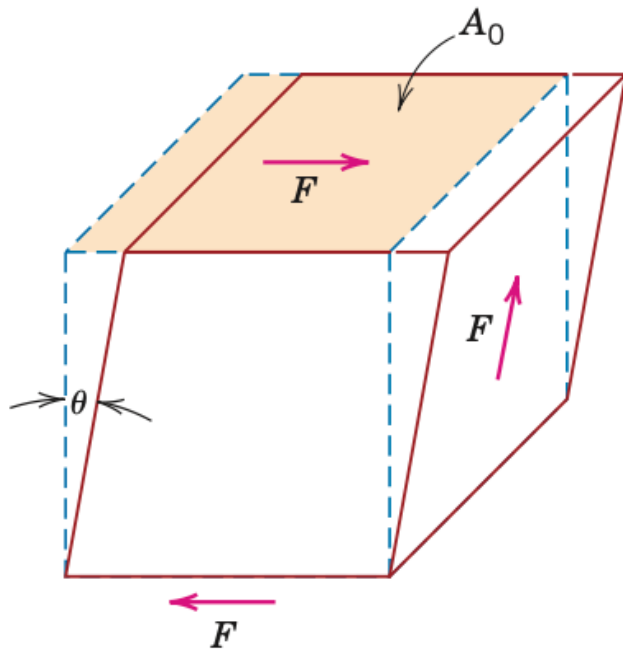
However, the behavior is repeatable! - elastic

Secant/ tangent methods



Shear stresses:

Torsional stresses are a variation of the shear



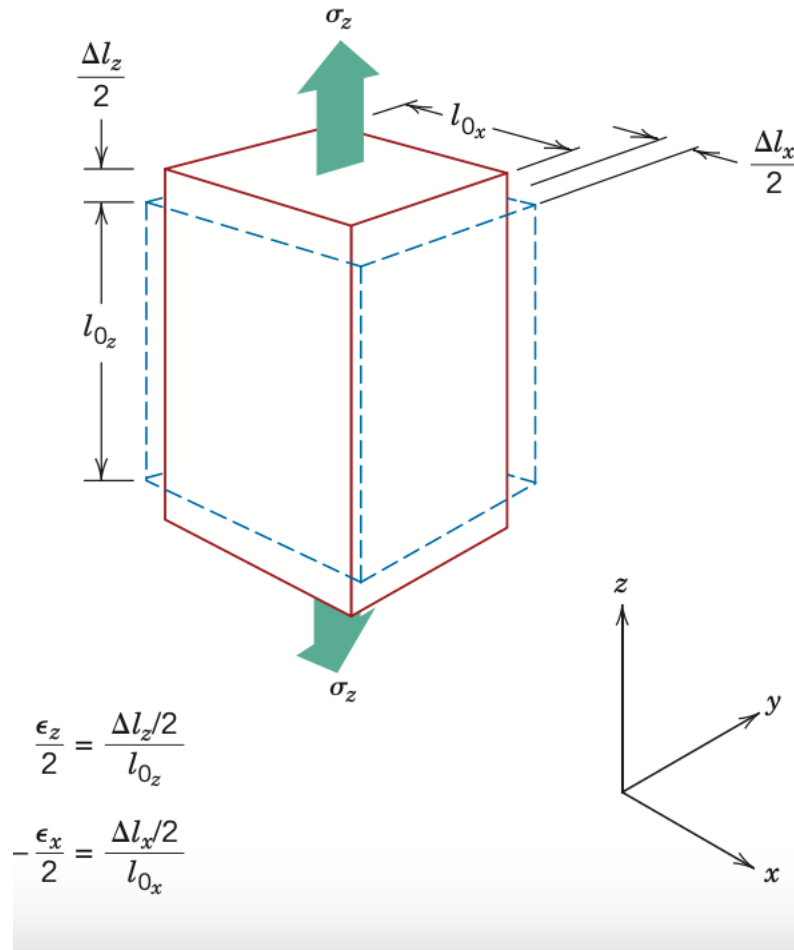
Shear stress when the force is tangential to the face

$$\text{Shear Stress } \sigma = \frac{F}{A_0}$$

$$\text{Shear Strain } \gamma = \tan \theta$$

$$\text{Shear modulus } G = \sigma / \gamma$$

Poisson's ratio



If the material is isotropic

And stress uniaxial σ_z

The orthogonal strains

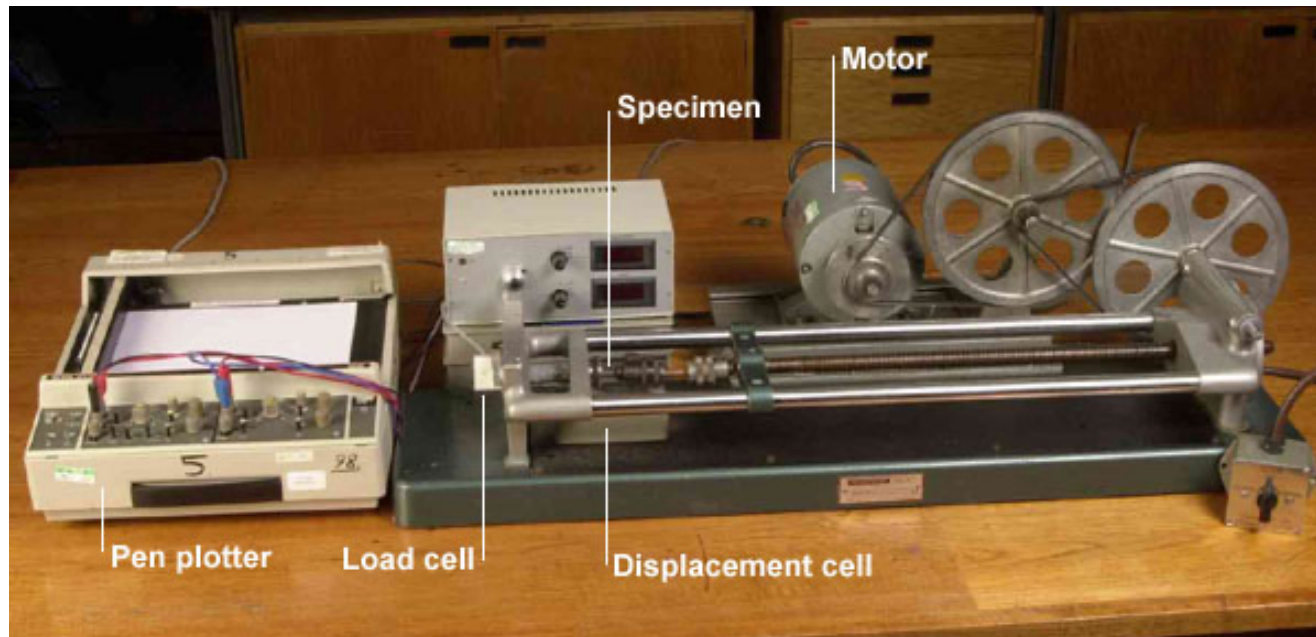
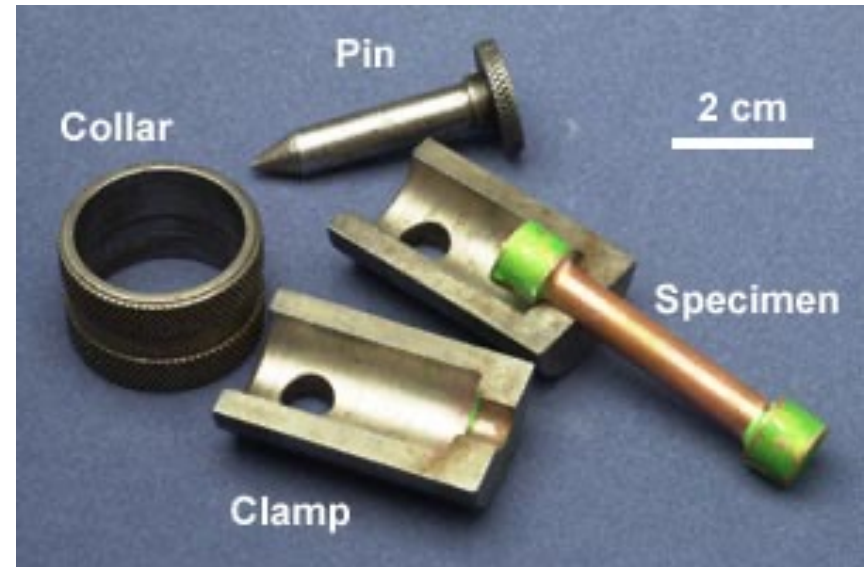
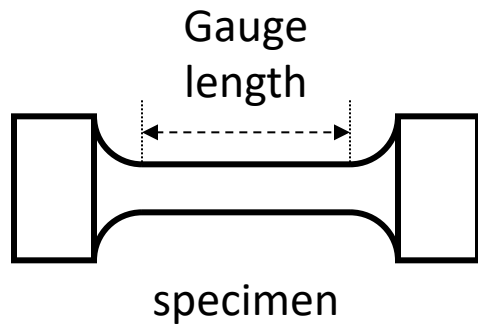
$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z}$$

Theoretical Poisson's ratio = 0.25
for isotropic materials

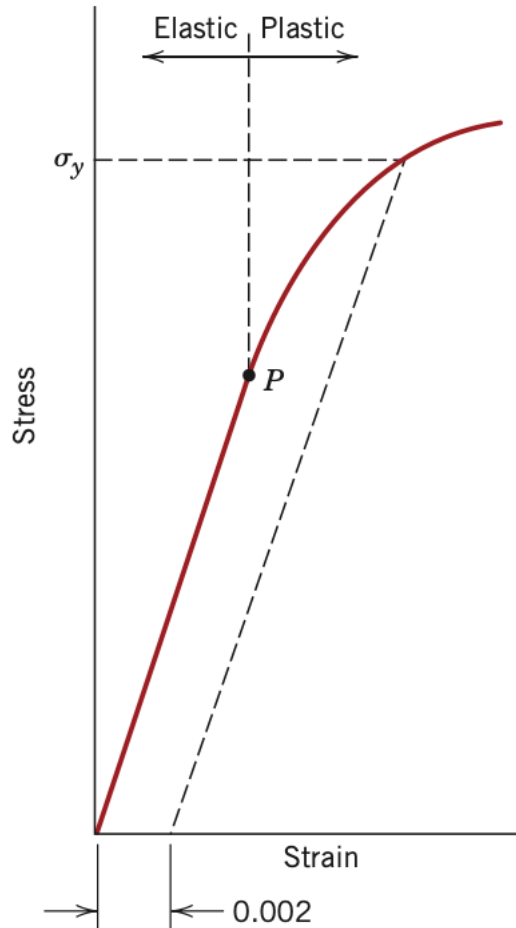
maximum value for no net-volume change is 0.5

However, typical value lies between 0.25 – 0.35

Tensile Testing



Tensile Stress-Strain Curves



Below point P
Linear region – Obey Hooke's law
Elastic in the linear region.

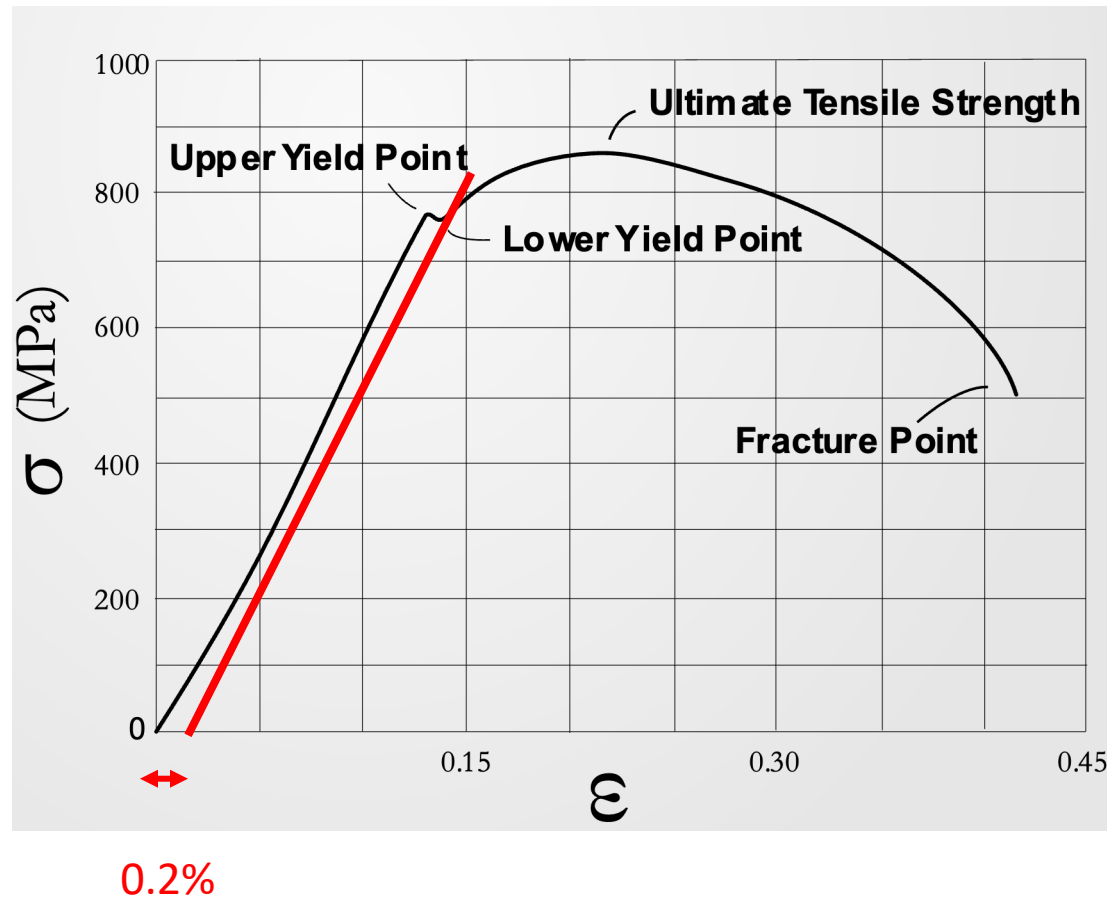
Beyond P
Deviation from linearity
Permanent – non recoverable damage
Atoms finding new equilibrium

In crystalline solids deformation is via plane slips
In non-crystalline solids – viscous flow

Note the strain ranges!

Yielding

Yield stress – The stress point at which the deviation from linearity happens

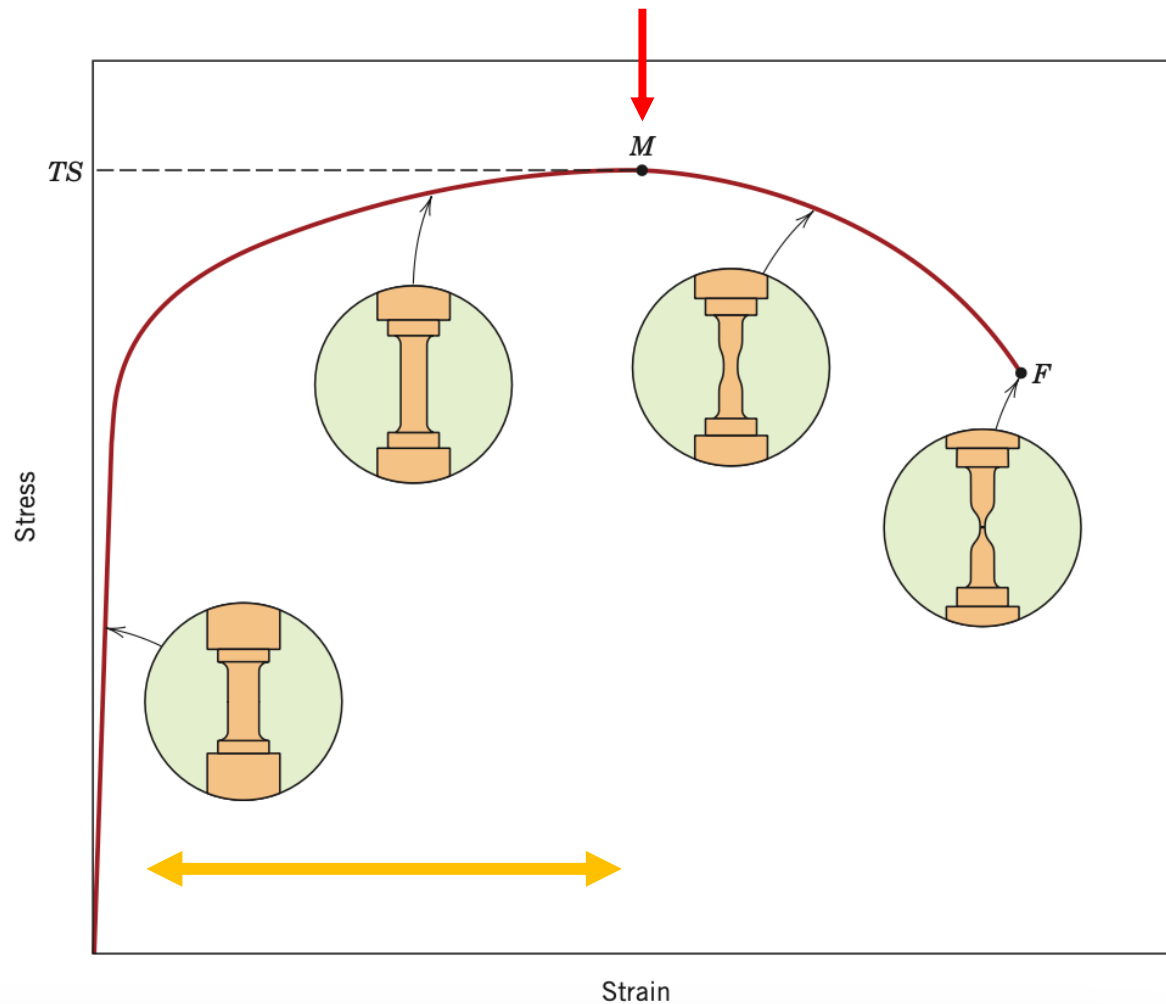


In places where the yielding point is not clear, a convention (0.2%) is employed.

The limits

Maximum strength!

The strength then decays to eventual fracture



Strain hardening can also be observed. However, a maximal stress point is achieved

Elastic modulus – Slope of the stress-strain curve

Yield Strength – The stress point of deviation from linearity

Ultimate Tensile Strength – Stress at the maximum of engineering stress-Strain curve

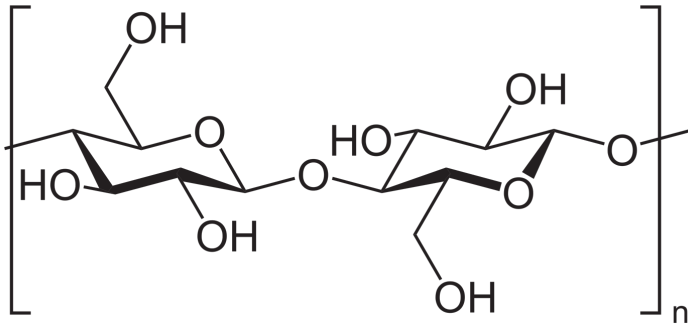
Fracture Strength – Point of stress at which fracture is witnessed in the engineering stress-Strain curve

Fracture Toughness – Total area under the engineering stress-strain curve – energy/volume required for fracture

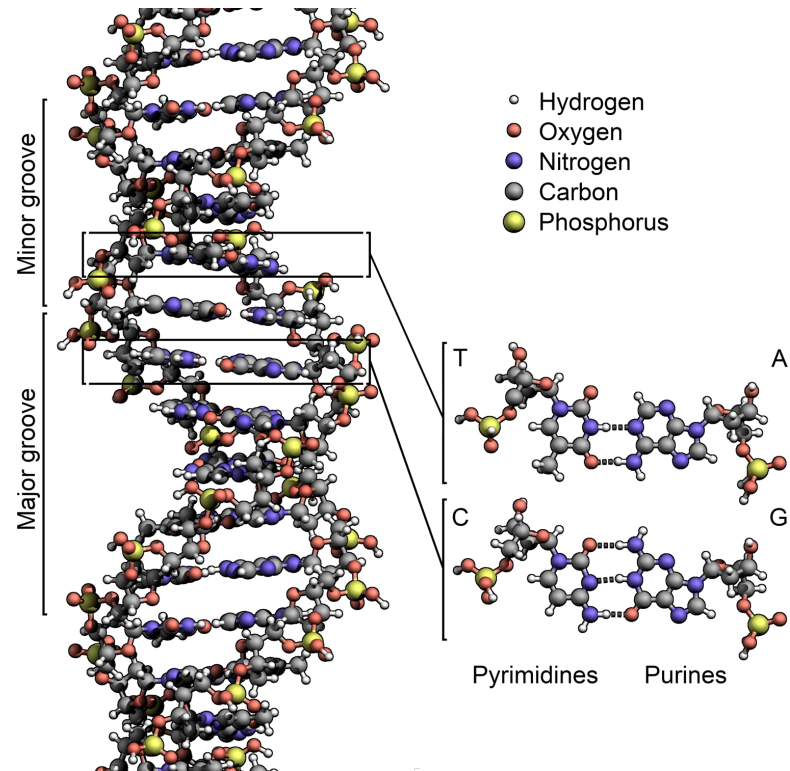
Ductility – Percentage elongation at/near fracture $D\% = \frac{l_f - l_0}{l_0} * 100$

What happens in polymers?

Polymers are long chain molecules of repeating units



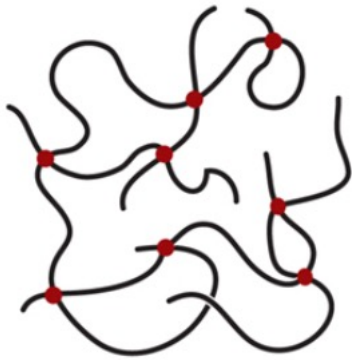
Cellulose –
Present in cotton, wood, cell walls ...



DNA, RNA, ...

Elastomers

Long chain molecules when bent tend to cross-link.



Thermodynamically favored to cross-link and stay bent

Higher configurational entropy $\rightarrow$ lower ΔG

So, when stretched, work is done on a polymer

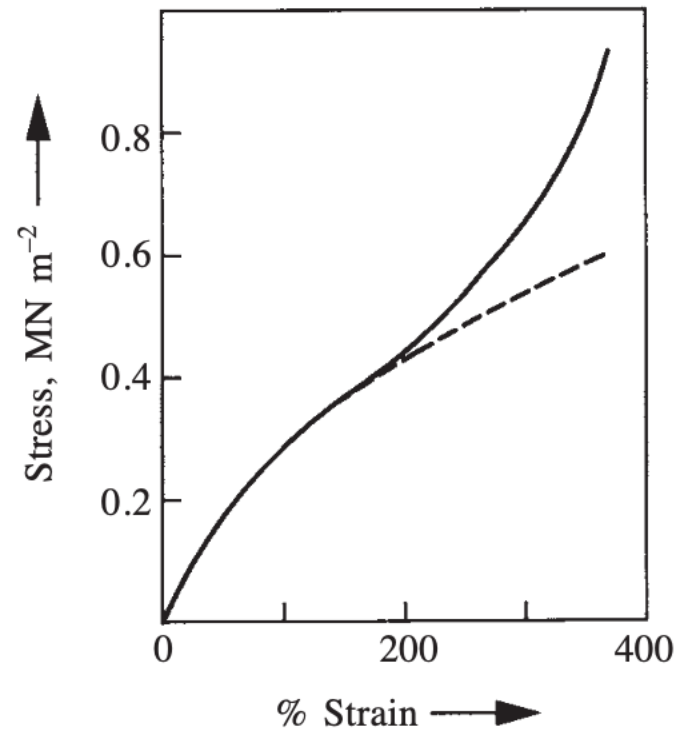
- No bonds are stretched
- The chain is straightened (reduced entropy) at T

$$F = -T \frac{\partial S}{\partial L}$$

Calculations yield:

$$F = \frac{N_0 k T}{L_0} \left[\left(\frac{L}{L_0} \right) - \left(\frac{L_0}{L} \right)^2 \right]$$

Stress-strain curve for an elastomer

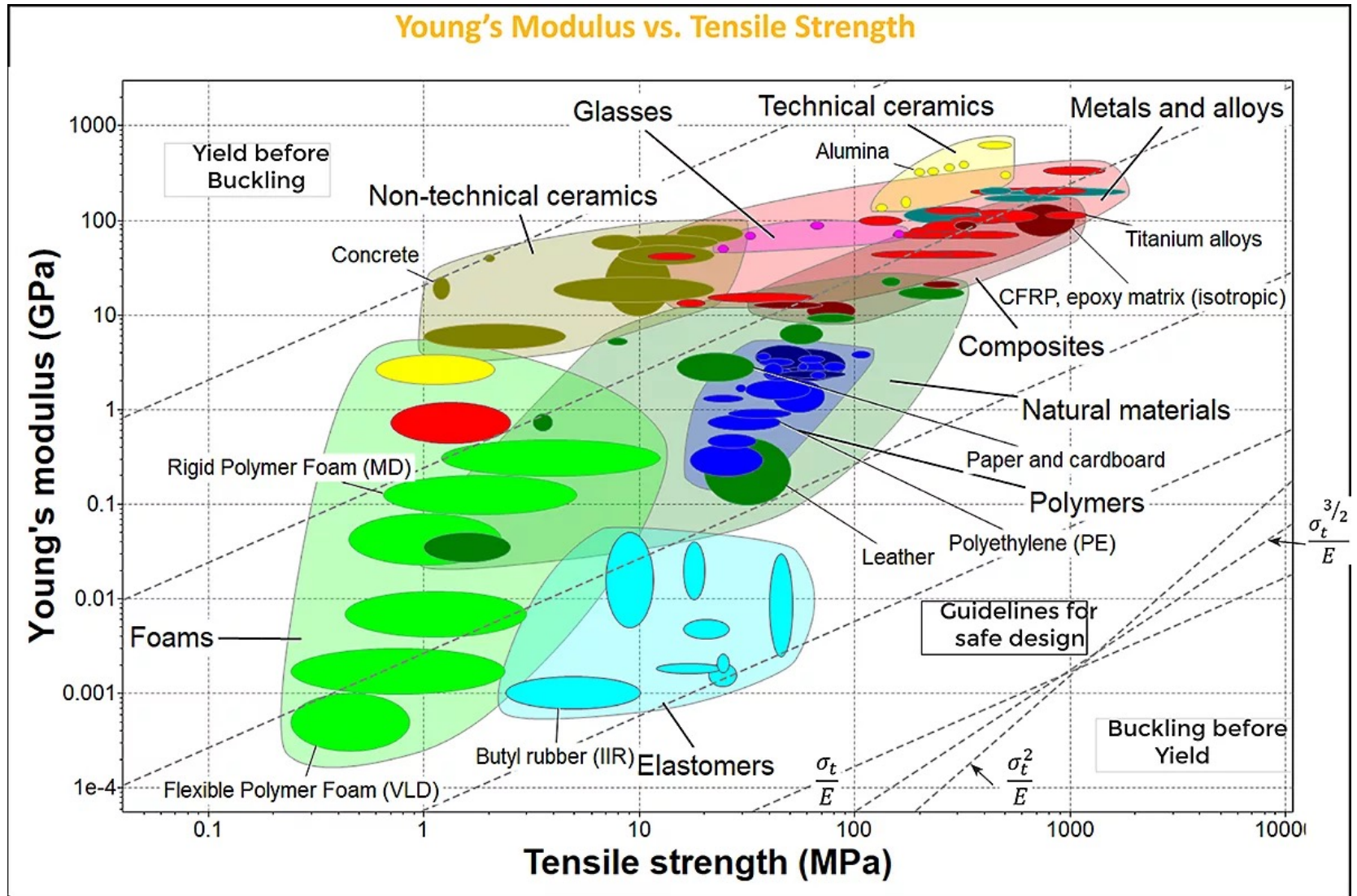


Check the strain values

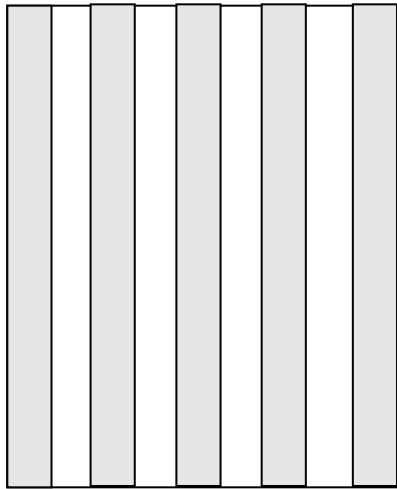
The dotted line is from the equation $F = \frac{N_0 k T}{L_0} \left[\left(\frac{L}{L_0} \right) - \left(\frac{L_0}{L} \right)^2 \right]$

However, after a large strain, there is deviation – largely due to bond stretching.!

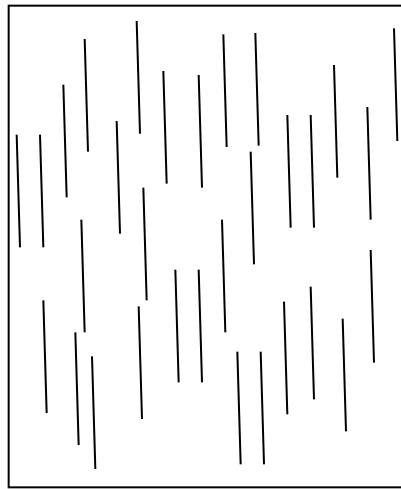
Parameters for design:



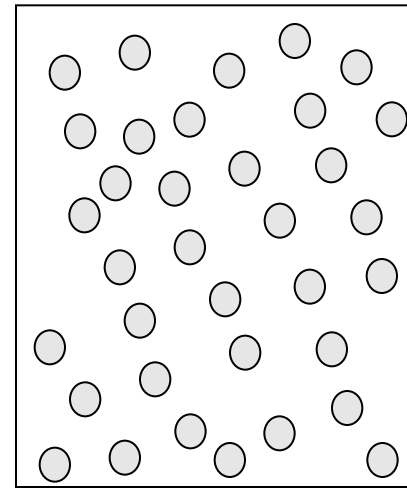
COMPOSITES



Laminate
composite



Aligned
fiber
composite



Particulate
composite

Modulus parallel to the direction of the fibers

$$E_c = E_f V_f + E_m V_m$$

Volume fractions

- Under iso-strain conditions
- I.e. *parallel* configuration
- m-matrix, f-fibre, c-composite

Composite modulus in isostress and isostrain conditions

Voigt averaging

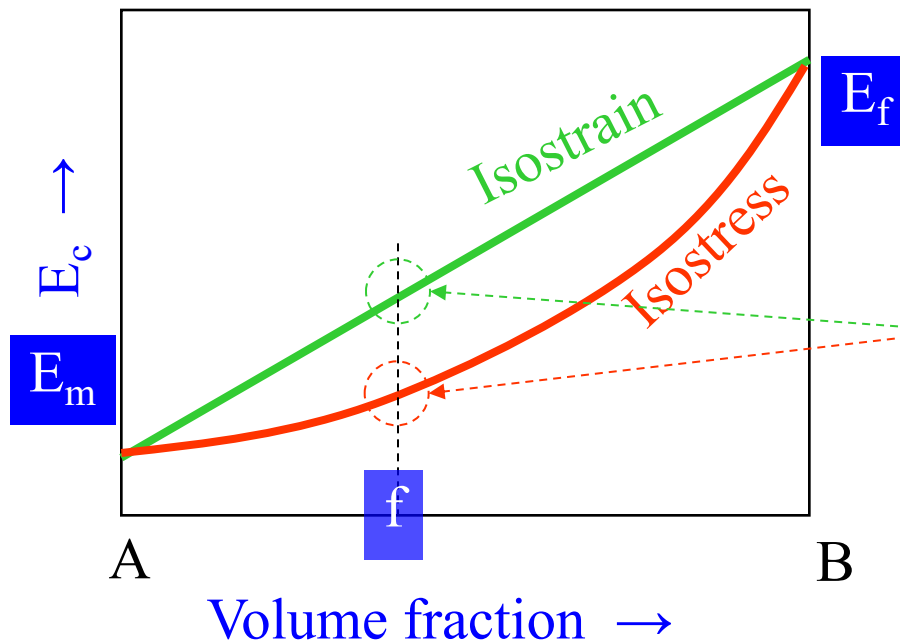
$$E_c = E_f V_f + E_m V_m$$

- Under iso-strain conditions [$\epsilon_m = \epsilon_f = \epsilon_c$]
- I.e. $\sim$ resistances in series configuration

$$\frac{1}{E_c} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

- Under iso-stress conditions [$\sigma_m = \sigma_f = \sigma_c$]
- I.e. $\sim$ resistances in parallel configuration
- Usually not found in practice

Reuss averaging



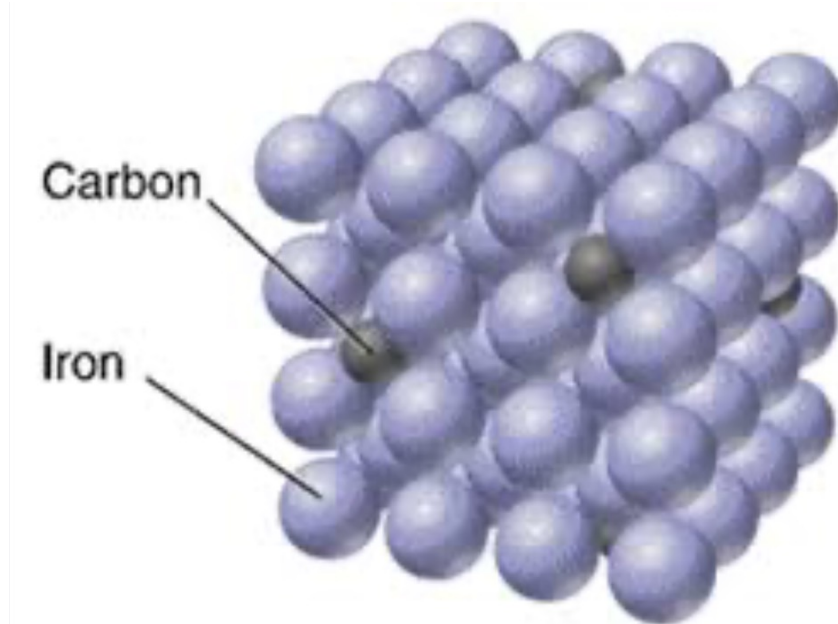
By rule of mixtures

For a given fiber fraction f , the moduli of various conceivable composites lie between an upper bound given by **isostrain** condition and a lower bound given by **isostress** condition

Anelastic Behavior

If time scale for relaxation of stress is comparable to time of stress application

The stress-strain relation is time dependent.



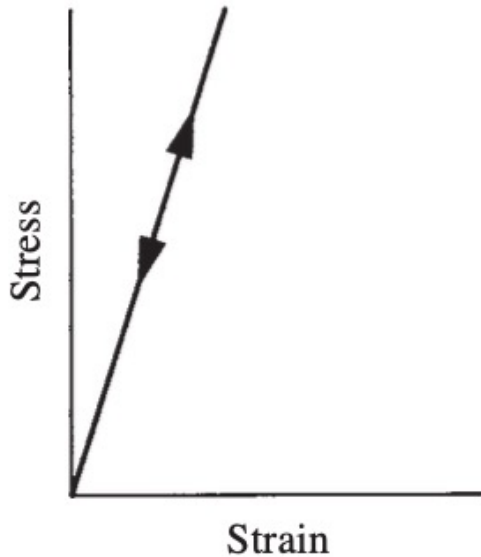
At room temperature, carbon size is much larger than interstitial space

This leads to huge lattice stress.

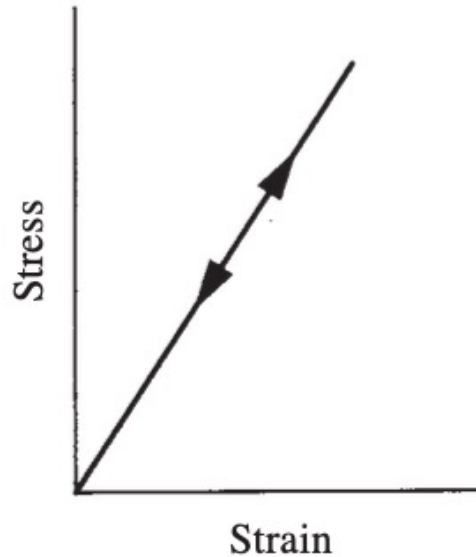
Upon tensile straining,
Carbon rearranges, slowly.

Typical time scale is ~ 100 s at RT

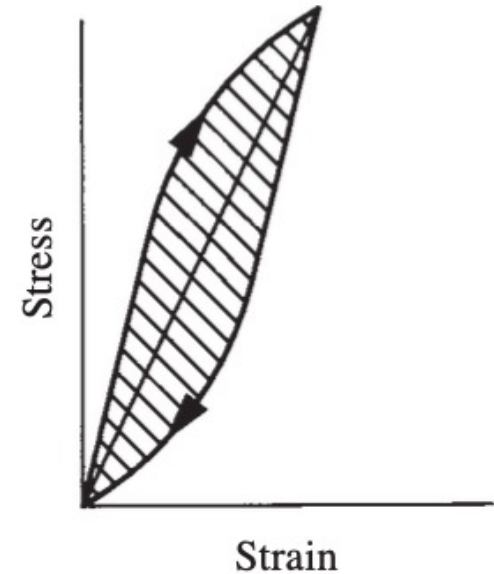
Three different cases



Purely elastic deformation



Anelastic;
But deformation is very
slow
Because of carbon
finding low energy sites,
the stress required to
deform is lower
Lesser stiffer material

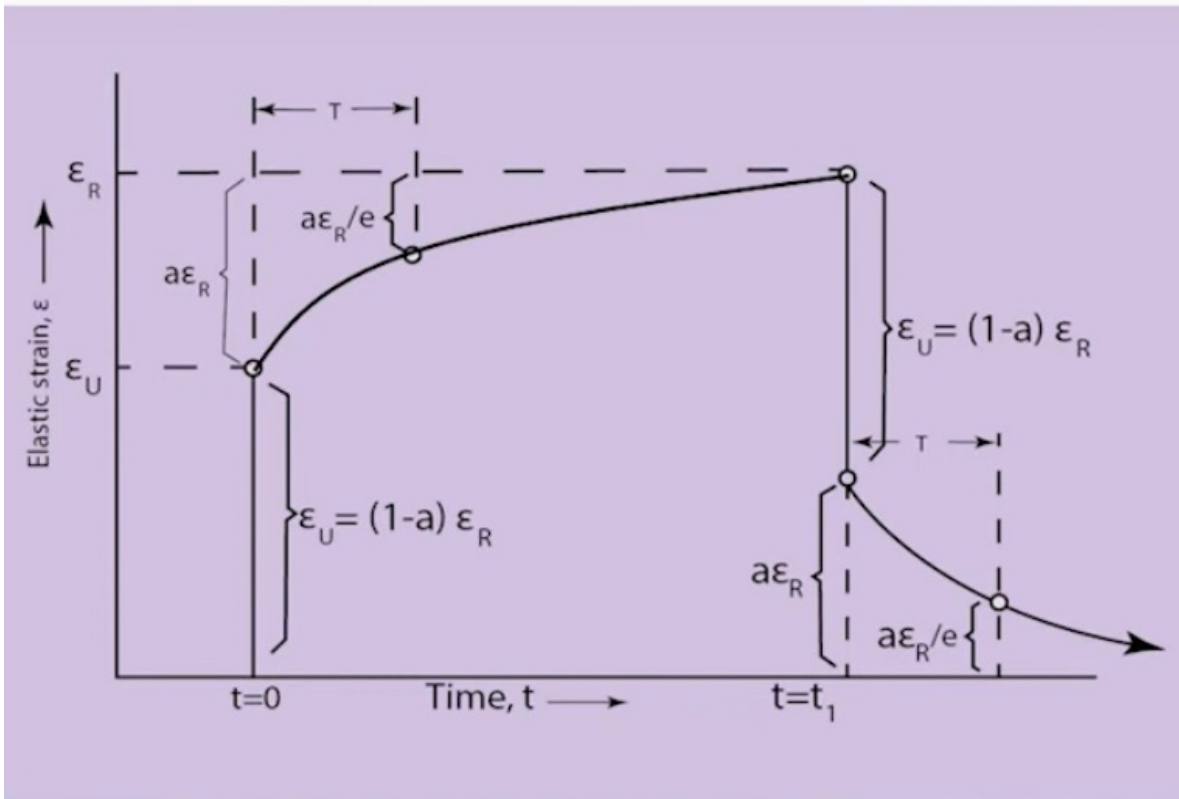


Anelastic;
But deformation is fast.
There is hysteresis.
Strain during loading is
always **lower** than that
of unloading!

We can then define relaxation time:

Time taken to recover back by atleast 63%.

If the question is why 63 ?



If instantaneous stress is applied;

A instantaneous strain ϵ_U is seen.

A gradual strain $\epsilon_A = \epsilon_R - \epsilon_U$
Is also observed.

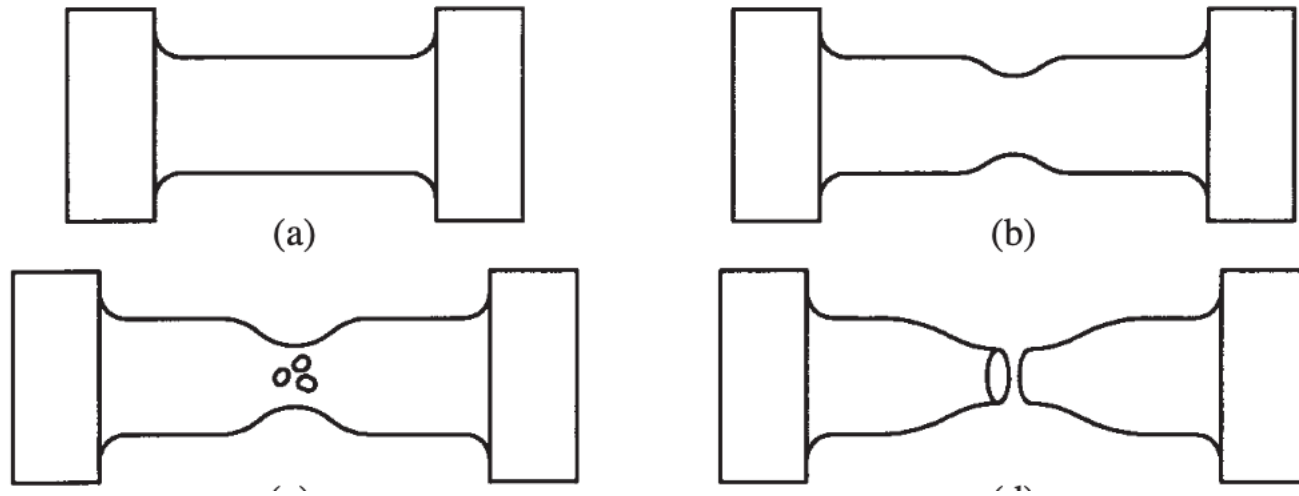
Since a exponential dependence is
seen;

Total strain at any time after
loading

$$\epsilon(t) = \epsilon_U + a\epsilon_R \left(1 - e^{-\frac{t}{T}}\right)$$

Where $a = \frac{\epsilon_A}{\epsilon_R}$

Ductile -> Brittle transformations



Even in simple ductile materials, close to necking, cracks start to appear on brittle particles.

Eg: Cementite in steel.

Therefore, it's important to know about the various phases of materials.
And also about phase transformations.