



भारतीय प्रौद्योगिकी संस्थान दिल्ली INDIAN INSTITUTE OF TECHNOLOGY DELHI

सभी पृष्ठों पर लिखें। Write on all pages.

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अनुवर्ती पुस्तिका संख्या
CONTINUATION BOOK No.

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Electrons as particles / Classical electrons:

Suppose we have a collection of electrons:
that hold together a solid (which is a collection of atoms).

When a potential V is applied between two ends of a conductor of length L ,

$$E = V/L \text{ is set up!}$$

Field is the force acting on an electron.

$\Rightarrow$ an electron is accelerated!

$$a = \frac{eE}{m}$$

$$\therefore \text{Current} = I = N_e \cdot e \cdot V_D$$

$\downarrow$ $\rightarrow$ drift velocity
 No. of electrons

if in the absence of field, net $V_D = 0$;

$$\text{after field } V_D(t) = a \cdot t.$$

which implies $I \rightarrow$ is a function of time!

let us write a governing diff. equation.

2

$$\frac{dV_d}{dt} = a - \frac{V_d(t)}{\tau}$$

Because, there are collisions;

let us take $\langle \tau \rangle \rightarrow$ mean time to collision.

$\hookrightarrow$ relaxation time /
collision time /
mean free time.

after $\langle \tau \rangle$, the velocity obtained by electrons is

$$a \langle \tau \rangle$$

$\downarrow$ (M)

$$\therefore \text{The drift velocity} = \frac{e \mathcal{E}}{m} \langle \tau \rangle$$

current

$$I =$$

$$N e \frac{e^2 \tau}{m} \mathcal{E}$$

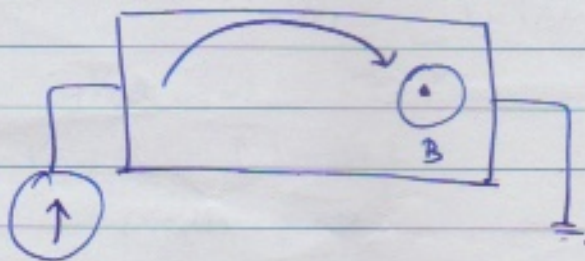
conductivity

$$\sigma = (N e e)$$

$$\left(\frac{e \tau}{m} \right)$$

mobility!

Hall Coefficient:



electrons face Lorentz force.

$$\vec{F} = -e \vec{E} - e (\vec{v} \times \vec{B})$$

$$\frac{dV_x}{dt} = -\frac{e E_x}{m} - \frac{e B v_y}{m} - \frac{v_x}{\tau}$$

$$\frac{dV_y}{dt} = -\frac{e E_y}{m} + \frac{e B v_x}{m} - \frac{V_y}{\tau}$$

@ steady state

$$E_y = B v_x ; \quad E_y = \frac{B}{N e e} \cdot (N e e \cdot v_x)$$

$$= R_H \cdot J \cdot B$$

$$\downarrow$$

$$\left(\frac{1}{N e e} \right) \rightarrow \text{Hall coefficient!}$$

Measured Hall coefficients.

	n_0/n
Li	0.8
Na	1.2
Au	1.5
Be	-0.2
Mg	-0.4
Al	-0.3

$$\frac{dV_d}{dt} = a - \frac{V_d}{\tau}$$

If the field is a time varying, but sinusoidal field,
 $E(t) = E(\omega) e^{-i\omega t}$

$$V_d(t) = V_d(\omega) e^{-i\omega t}$$

$$-i\omega \cdot V_d(t) = -\frac{V_d(t)}{\tau} - \frac{e}{m} E(t)$$

$$E(t) = \left(\frac{1}{\tau} - i\omega \right) \cdot V_d(t)$$

$$V_d(t) = \frac{-e}{m} \cdot \left(\frac{1}{\frac{1}{\tau} - i\omega} \right) \cdot E(t)$$

$$(Ne) \cdot e \cdot V_d(t) = \frac{Ne \cdot e^2}{m} \cdot \left(\frac{1}{\frac{1}{\tau} - i\omega} \right) \cdot E(t)$$

$$J(t) = \sigma(\omega) \cdot E(t)$$

$$\sigma(\omega) = \frac{Ne \cdot e^2}{m} \cdot \left[\frac{\tau}{1 - i\omega\tau} \right]$$

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}$$



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from Maxwell's

$$\nabla \cdot E = 0; \quad \nabla \cdot H = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad \nabla \times B = \mu_0 j + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t} (\nabla \times B)$$

$$= \cancel{\nabla \times} \frac{\partial}{\partial t} E - \mu_0 \epsilon_0 \frac{\partial}{\partial t} \nabla \times E$$

$$-\nabla^2 E = +\frac{\partial}{\partial t} (\mu_0 j + \mu_0 \epsilon_0 \frac{\partial E}{\partial t})$$

$$-\nabla^2 E = -\left[\mu_0 \sigma(\omega) \frac{\partial E}{\partial t} + \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \right]$$

$$-\nabla^2 E = -(i\omega \mu_0 \sigma + \omega^2 \mu_0 \epsilon_0) \cdot E$$

$$= \frac{\omega^2}{\mu_0 \epsilon_0} \left[1 + \frac{i\sigma}{\omega \epsilon_0} \right]$$

$$\nabla^2 E = \left(\frac{\omega}{c} \right)^2 \left[1 + \frac{i\sigma}{\omega \epsilon_0} \right] \cdot E$$

Sub $\sigma = \frac{\sigma_0}{1 - i\omega \tau}$;

$$\nabla^2 E = \left(\frac{\omega}{c} \right)^2 \left[1 + \frac{i\sigma_0}{\omega \epsilon_0 (1 - i\omega \tau)} \right] E$$

if we take $\omega z \gg 1$

$$\nabla^2 E = + \frac{\omega^2}{c^2} \left[1 - \frac{\sigma_0}{\omega^2 \epsilon_0} \right] E$$

$$= + \frac{\omega^2}{c^2} \left[1 - \frac{\omega_p^2}{\omega^2} \right] E;$$

$$\boxed{\omega_p = \frac{Ne e^2}{m \cdot \epsilon_0}}$$

$$\nabla^2 E = -k^2 E$$

$$\Rightarrow k^2 = \frac{\omega^2}{c^2} \left[1 - \frac{\omega_p^2}{\omega^2} \right]$$

plasma frequency.

if $\omega < \omega_p$,

$k^2 \rightarrow -ve$; k is imaginary

$e^{-ikx} \rightarrow$ is then real

if $\omega > \omega_p$, then there is a propagation.

Heat:

specific heat capacity:

$$C_V = \frac{dE_T}{dt}$$

$$E_T = Ne \cdot E \quad C_V = Ne \cdot \frac{dE}{dt}$$

if taking kinetic energy $E = \frac{3}{2} k_B T$; $\frac{dE}{dT} = \frac{3}{2} k_B$

$$C_V = \frac{3}{2} Ne k_B$$

An example .:

say $N_e = 6 \times 10^{28} \text{ m}^{-3}$

$$f = \frac{\omega}{2\pi} = \left(\frac{N_e e^2}{m \epsilon_0} \right)^{1/2}$$

$$= \left[\frac{6 \times 10^{28} \cdot (1.6 \times 10^{-19})^2}{9.31 \times 10^{-31} \times 8.85 \times 10^{-12}} \right]^{1/2}$$

$$\approx 2.2 \times 10^{15} \text{ Hz}$$

$$\lambda = c/f = \underline{\underline{136 \text{ nm}}}$$

$\lambda < 136 \text{ nm}$; That metal is transparent!



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Schrodinger's equation.

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V \right] \psi = i\hbar \frac{\partial \psi}{\partial t}$$

Say for a time independent system.

$$\psi(r, t) = \psi(r) \cdot \omega(t)$$

substituting!

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V \psi = \frac{i\hbar}{\omega} \frac{\partial \omega}{\partial t}$$

spatially varying

time varying

They can be equal iff, they are equal to a third quantity.

$$i\hbar \frac{\partial \omega}{\partial t} = E \omega$$

$$-\frac{\hbar^2}{2m} \nabla^2 \psi(r) + V \psi(r) = E \psi(r)$$

Each of these are eigen equations.

Soln: to $\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar} \psi$ is

$$\psi = A e^{\frac{-iE}{\hbar} t}$$

if we substitute $E = \hbar \omega$,

~~then~~

$$\psi = A e^{-i\omega t}$$

for the second part, more is needed.

$$\left[-\frac{\hbar^2 \nabla^2}{2m} + V \right] \psi = E \psi$$

for simplest case of $V=0$.

$$-\frac{\hbar^2 \nabla^2 \psi}{2m} = E \psi$$

$$\psi(x) = A e^{ikx} + B e^{-ikx}$$

$$E = \frac{\hbar^2 k^2}{2m}$$

$$\int \psi^* \psi = 1 \Rightarrow$$

$$\int_{-\infty}^{\infty} A^* A e^{ikx} e^{-ikx} = 1$$

(cannot be normalized)

$$0 < x < a.$$

$$V = \begin{cases} 0 & -\frac{a}{2} < x < \frac{a}{2} \\ \infty & \text{other wise.} \end{cases}$$

$$\psi = A \sin kx + B \cos kx.$$

$$\psi(x=0) = \psi(x=a) = 0$$

$$0 = B;$$

$$A \sin ka = 0$$

$$k = \frac{n\pi}{a} \quad n = 0, 1, 2, \dots$$

$$\int_0^a A^2 \sin^2 kx \, dx = 1.$$

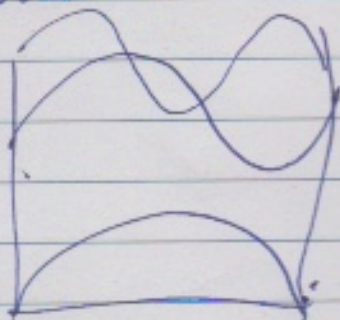
$$|A|^2 \frac{a}{2} = 1 \quad ; \quad |A|^2 = \frac{2}{a} \quad ; \quad A = \sqrt{\frac{2}{a}}$$

$$\psi(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x.$$

$$E = \frac{\hbar^2 k^2}{2m}$$

$$\therefore \text{Energies} = 0, \frac{\hbar^2 \pi^2}{2ma^2}, \frac{\hbar^2 4\pi^2}{2ma^2}, \dots$$

Solutions:



1. all solutions are orthonormal;

$$\int_0^a \frac{2}{a} \sin \frac{m\pi x}{a} \cdot \frac{2}{a} \sin \frac{n\pi x}{a} dx$$

$$\frac{2}{a} \int_0^a \left[\cos \frac{m-n}{a} \pi x - \cos \frac{m+n}{a} \pi x \right] dx$$

$$= \frac{2}{a} \left[\frac{\sin \frac{(m-n)\pi x}{a}}{\left(\frac{m-n}{a}\right)\pi} - \frac{\sin \frac{(m+n)\pi x}{a}}{\frac{(m+n)\pi}{a}} \right]$$

$$= 0$$