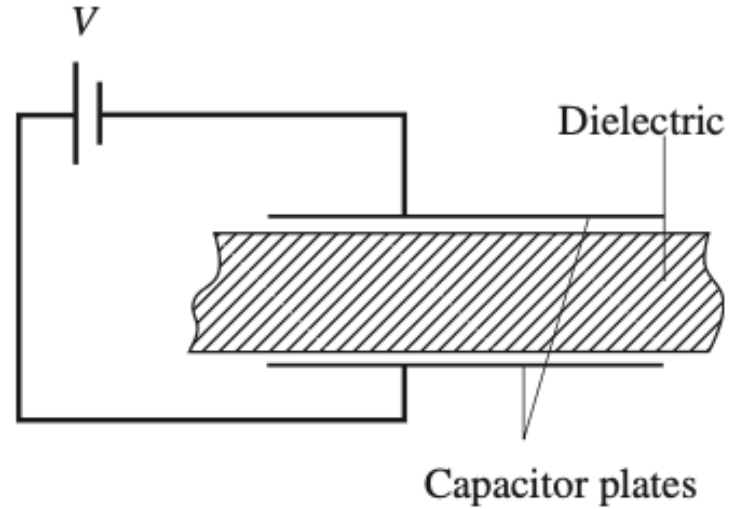


Dielectrics and Insulators

What happens when a static potential is applied across a material and no noticeable current flows?

Capacitance of a dielectric

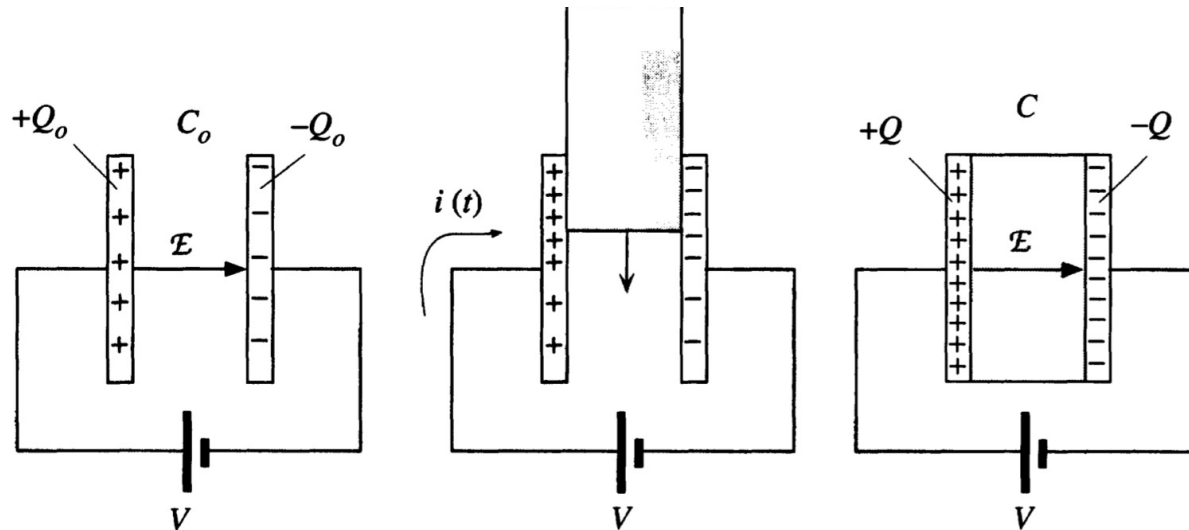


When a potential difference V is applied across an insulator of length ' l '

An electric field $E = \frac{V}{l}$ is set up. But no current flows!

This leads to a charge density on either of the terminals!

The charge density depends on the insulator (dielectric).



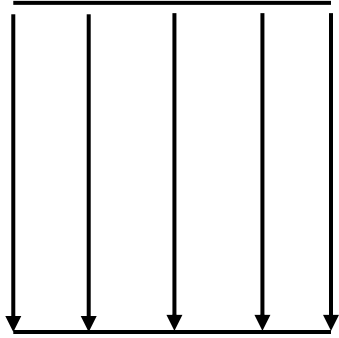
If Q_0 is the charge density when dielectric is air.

For any other dielectric, we define a constant

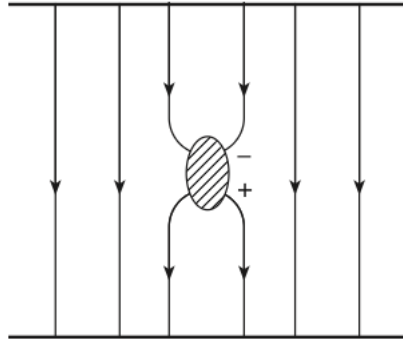
$$\epsilon_r = \frac{Q}{Q_0}$$

Called relative permittivity or dielectric constant

Polarization



Electric field lines in the absence of dielectric



A polarizable medium is one where a dipole moment is non-zero

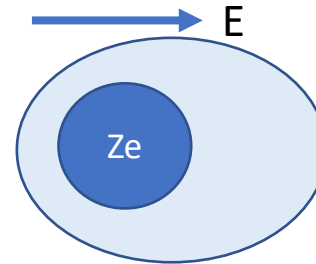
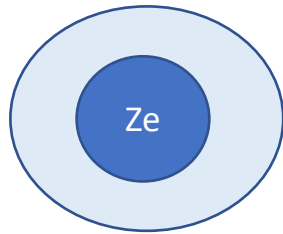
Dipole moment is

$$\mu = q * d$$

If there are N dipole moments, then we have

$$\text{Polarization } P = N\mu = Nqd$$

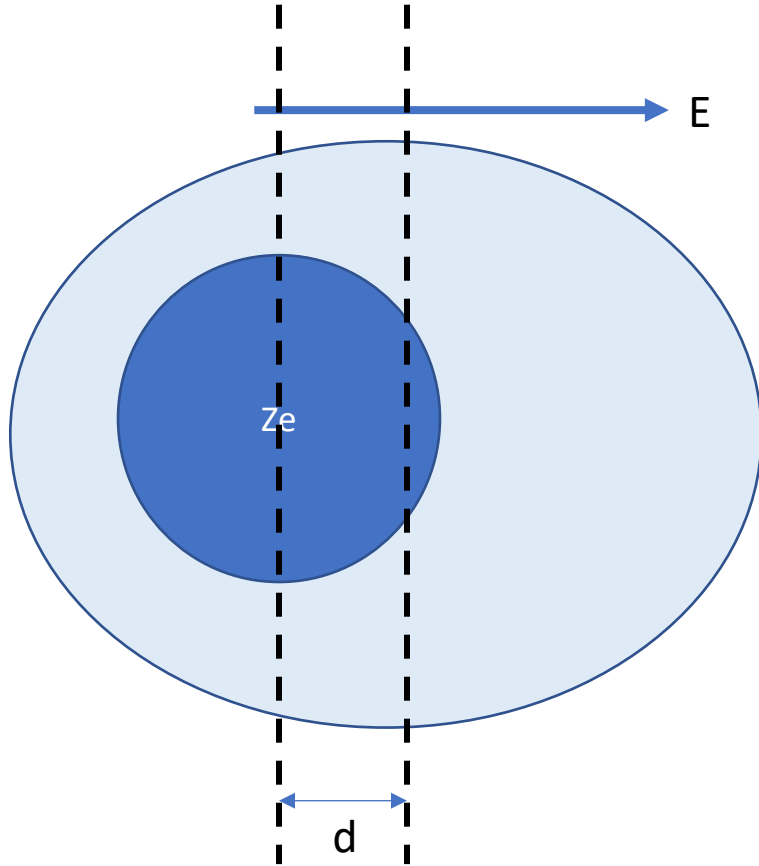
When a polarization is induced due to an applied electric field (E)



If the electron cloud is displaced by ' d '

$$\text{Force on the electrons } F = ZeE$$

$$\text{Elastic restoring force in the atom } F = \beta d$$



For a steady state electric field E ,

$$ZeE = \beta d$$

Thus, the dipole moment in each atom is $\mu = Ze * d = \frac{(Ze)^2 E}{\beta}$

If there are N such dipole moments in the field region, a polarization

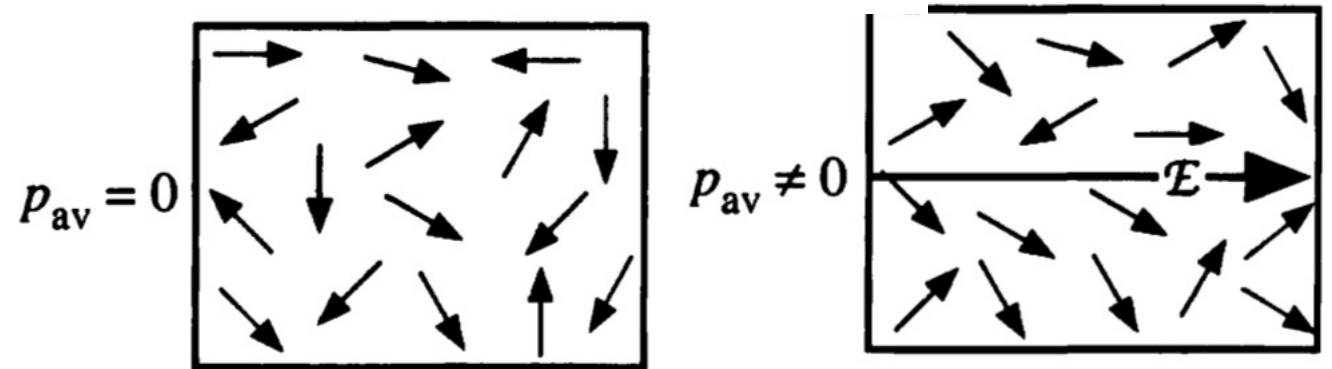
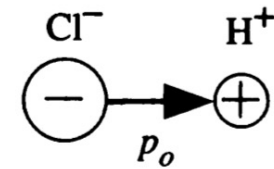
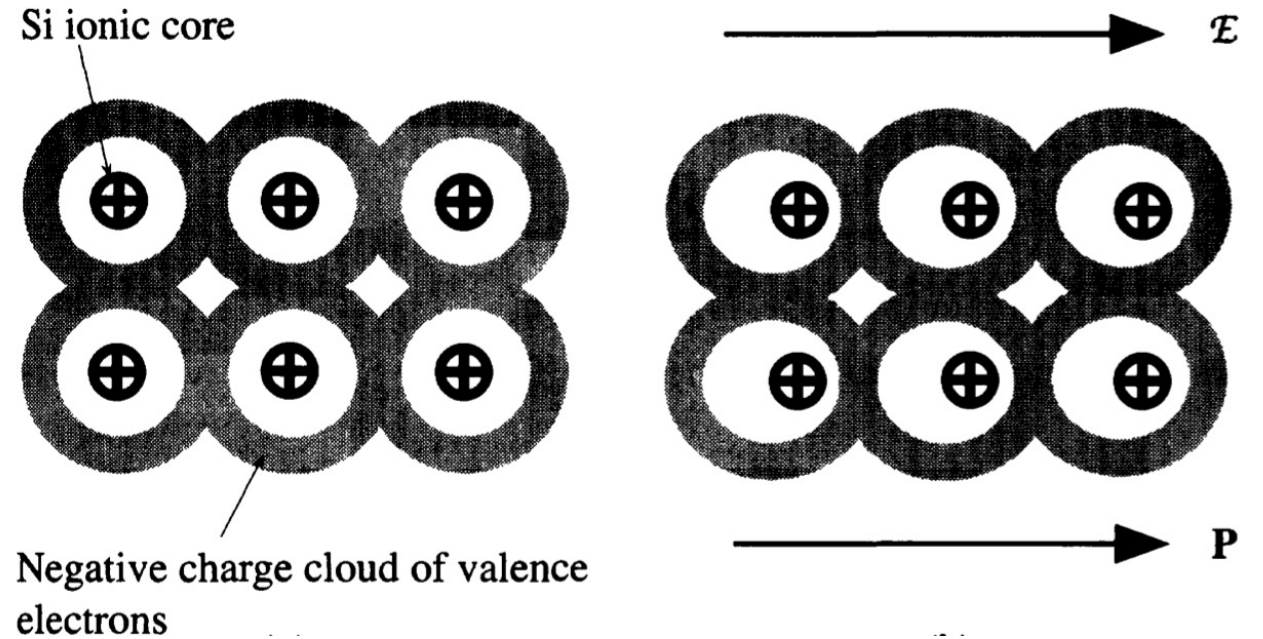
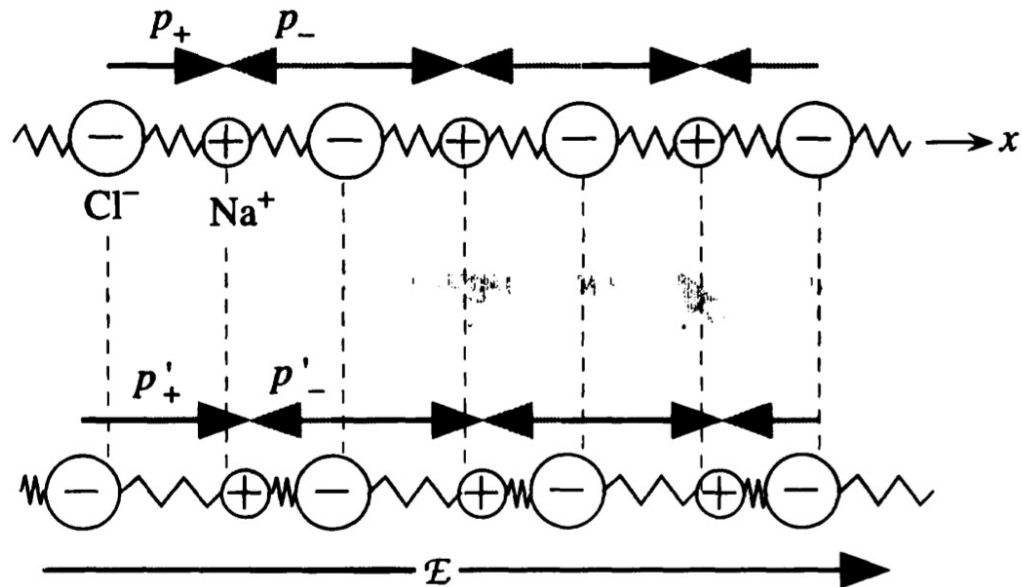
$$P = N \frac{(Ze)^2}{\beta} E$$

We write this as $P = \alpha E$ and α is called as polarizability

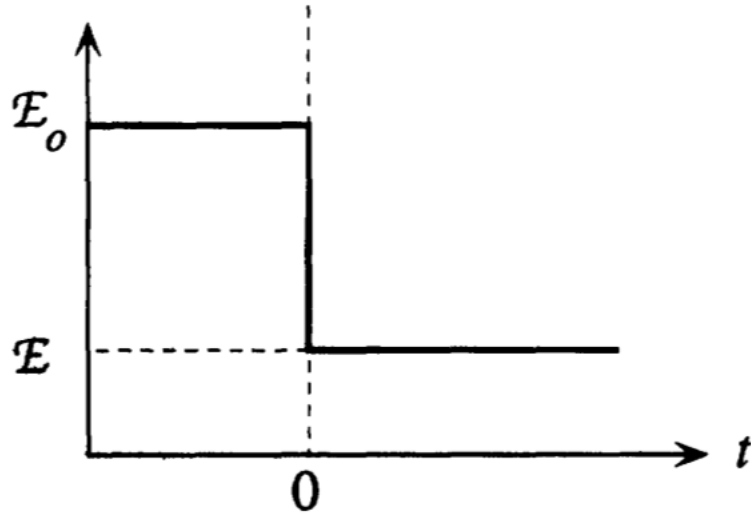
$\beta = \frac{(Ze)^2}{4\pi\epsilon r^3}$ where r is the atomic radius. This is obtained by constructing a Gaussian surface of radius 'd' surrounding the charge density center

Types of polarization

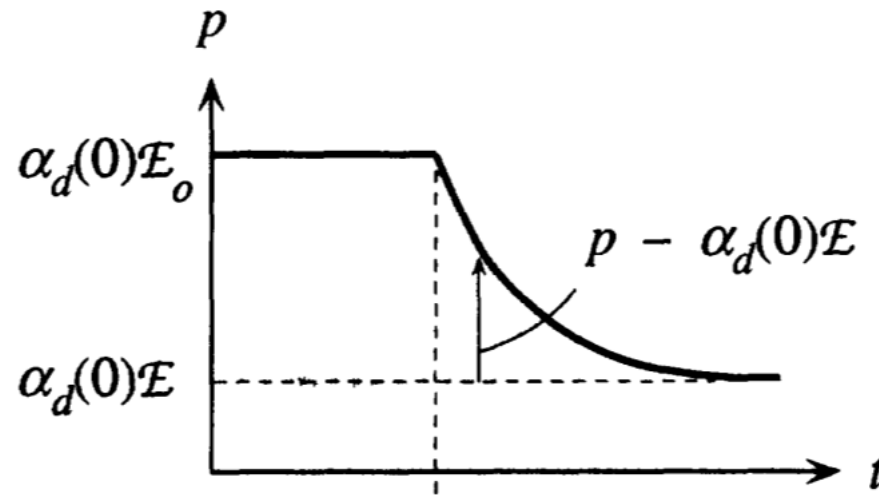
- Electronic
- Molecular
- Orientational



What happens when Electric Field Oscillates ?



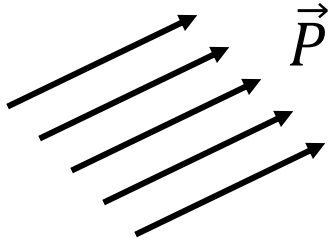
Imagine electric field changing from $E_0 \rightarrow E$



Polarization cannot change immediately due to

1. The re-orientation of dipoles require thermal assistance
2. General thermal energy leads to random disorientation of the dipoles.

The time taken to reorient the dipoles (relax dipoles) is called as relaxation time τ



At time $t=0$, if P is the existing polarization. If a changing electric field $E(\omega)$ is applied

$$\frac{dP}{dt} = -\frac{P - \alpha E(\omega)}{\tau}$$

Comparing this with the Drude's conductivity calculations done previously,
We obtain the polarizability

$$\alpha(\omega) = \frac{\alpha(0)}{1 + j\omega\tau}$$

Remembering the definitions $P = \alpha E$ and $P = \chi\epsilon_0 E$

$$\epsilon_r = 1 + \chi$$

The dielectric constant ϵ_r turns out to be complex under changing electric fields!

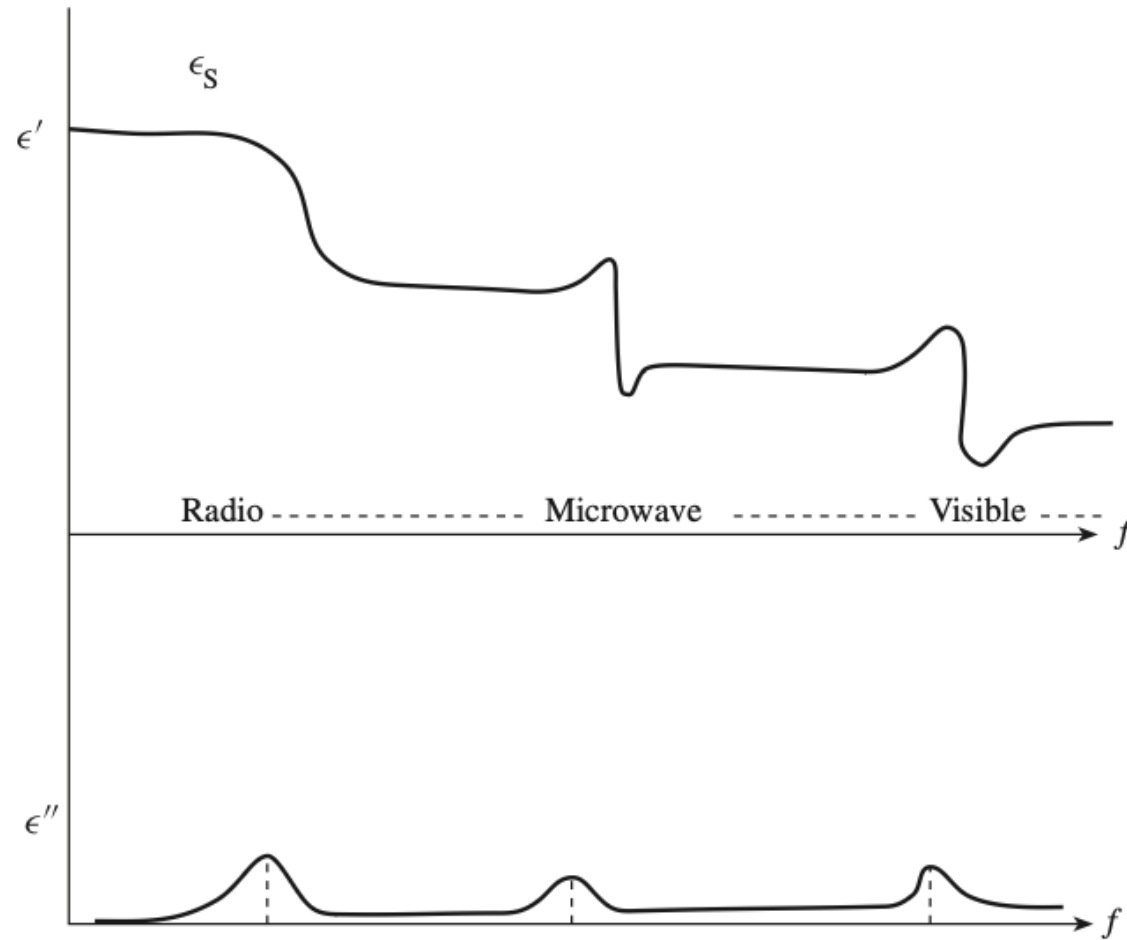
$$\epsilon_r = \epsilon'_r - j \epsilon''_r$$

Complex Dielectric Constant

$$\epsilon_r = \epsilon_r' - j \epsilon_r''$$

- At DC and low frequencies, complex dielectric constant equals relative permittivity ϵ_r
- The imaginary part ϵ_r'' represents energy lost in the medium due to P re-orientation.
- Dielectric resonance is when $\omega = \frac{1}{\tau}$
 - The rate of orientation of the dipoles (ω) matches with the rate of re-orientation ($1/\tau$)
 - There is maximal energy transfer to heat! Maximal loss!
- Loss-tangent is defined as $\tan \delta = \frac{\epsilon_r''}{\epsilon_r'}$

Typical variation of complex dielectric constant



Dielectric and Refractive indices

Maxwell's equation

$$\nabla \times H = \frac{\epsilon}{c} \frac{\partial E}{\partial t} + \frac{4\pi\sigma}{c} E$$

Vector identity

$$\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E$$

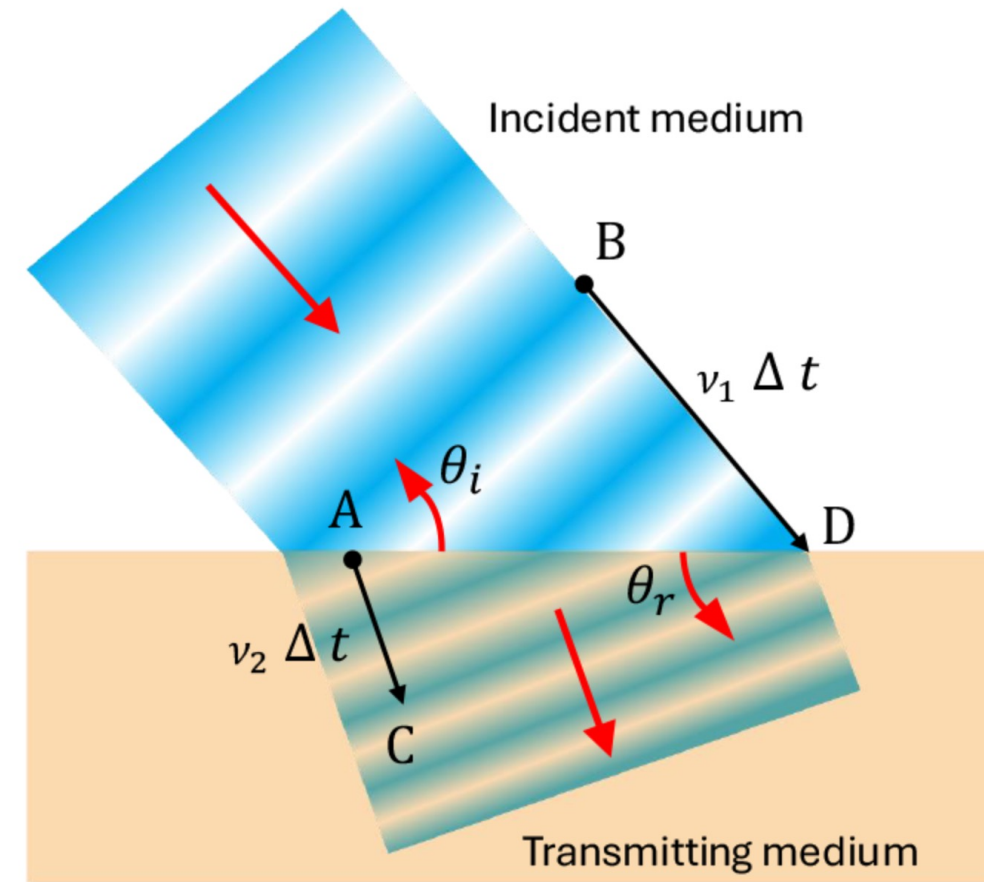
In free space without any conductivity or enclosed charges

Wave equation of light

$$\nabla^2 E = \frac{\epsilon\mu}{c^2} \frac{\partial^2 E}{\partial t^2}$$

Solution for which in 3D space is $E(r, t) = E_0 \exp i(q \cdot r - \omega t)$

When travelling in a new medium, $E'(r, t) = E_0 \exp i(q' \cdot r - \omega t)$



Solution for which in 3D space is $E(r, t) = E_0 \exp i(q \cdot r - \omega t)$

When travelling in a new medium, $E'(r, t) = E_0 \exp i(q' \cdot r - \omega t)$

$$q' = q_0(n + i\kappa)$$

Where, complex refractive index $\tilde{n} = n + i\kappa$

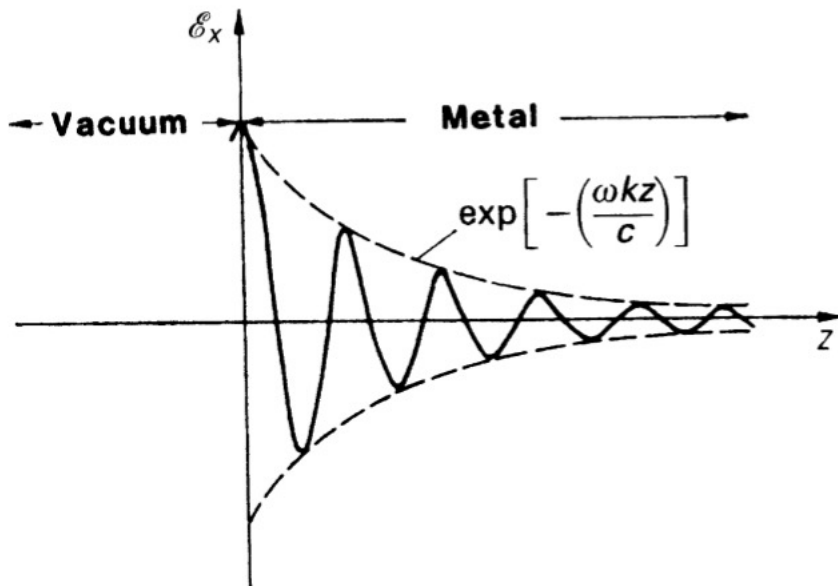
Substituting the above into the field equation:

$$E'(r, t) = E_0 e^{-\kappa q_0 \cdot r} e^{(n q_0 \cdot r - \omega t)}$$

Damped wave in the medium

Oscillating wave

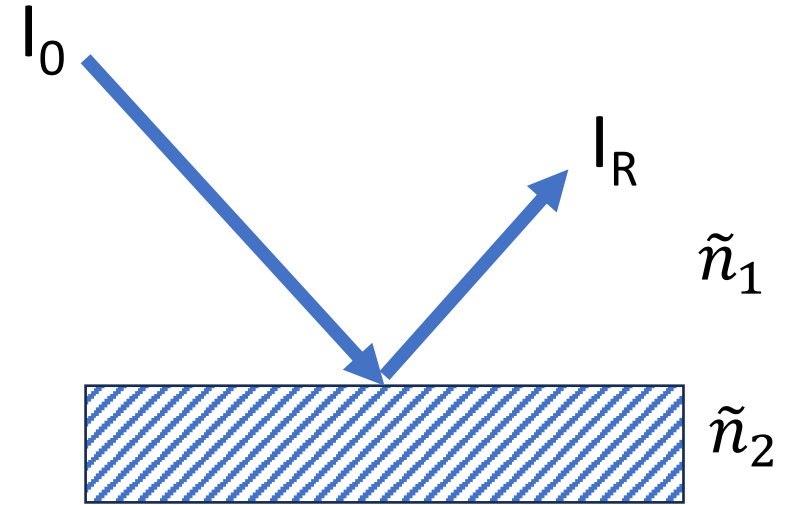
Wavelength of the oscillating wave $\lambda = \frac{\lambda_0}{n}$



Reflection

Intensity of reflected light

$$I_R = I_0 \cdot R \quad \text{Where, } R - \text{reflectivity}$$



The reflectivity according to Fresnel equations

$$R = \left| \frac{\tilde{n}_1 \cos \theta_i - \tilde{n}_2 \cos \theta_t}{\tilde{n}_1 \cos \theta_i + \tilde{n}_2 \cos \theta_t} \right|^2$$

Under the specific case of normal incidence from air

$$R = \left| \frac{\tilde{n} - 1}{\tilde{n} + 1} \right|^2$$

Absorption in the medium

The E-field EM radiation decays exponentially

$$E'(r, t) = E_o e^{-\kappa q_0 \cdot r} e^{(nq_0 \cdot r - \omega t)}$$

That means, the intensity of the EM radiation

$$I = E'^2 = I_0 \exp -2\kappa (q_0 \cdot r)$$

Characteristic penetration depth W

Distance when

$$\frac{I}{I_0} = e^{-1}$$

$$W = \frac{1}{2\kappa q_0} = \frac{\lambda_0}{4\pi\kappa}$$

Dielectrics and refractive indices

If the light with the following expression is travelling in a medium

$$E'(r, t) = E_0 \exp i(q' \cdot r - \omega t)$$

Substitute it to the Maxwell's equation for a material with some conductivity σ

$$\nabla^2 E = \frac{\epsilon \mu}{c^2} \frac{\partial^2 E}{\partial t^2} + \frac{\sigma}{\epsilon_0} \frac{\partial E}{\partial t}$$

yields

$$\tilde{n}^2 = \epsilon$$

$$\tilde{n}^2 = n^2 - \kappa^2 - i2n\kappa = \epsilon_1 - i\epsilon_2$$

Comparing the real and imaginary parts

$$\epsilon_1 = n^2 - k^2$$

$$\epsilon_2 = 2n\kappa = \frac{\sigma}{\epsilon_0\omega}$$

Equivalently

$$n^2 = \frac{1}{2} \left(\sqrt{\epsilon_1^2 + \epsilon_2^2} + \epsilon_1 \right)$$

$$\kappa^2 = \frac{1}{2} \left(\sqrt{\epsilon_1^2 + \epsilon_2^2} - \epsilon_1 \right)$$

In a metal at very small frequencies ω is small, conductivity is also high

$$\epsilon_2 = \frac{\sigma}{\epsilon_0\omega} \cong 10^5 \gg \epsilon_1$$

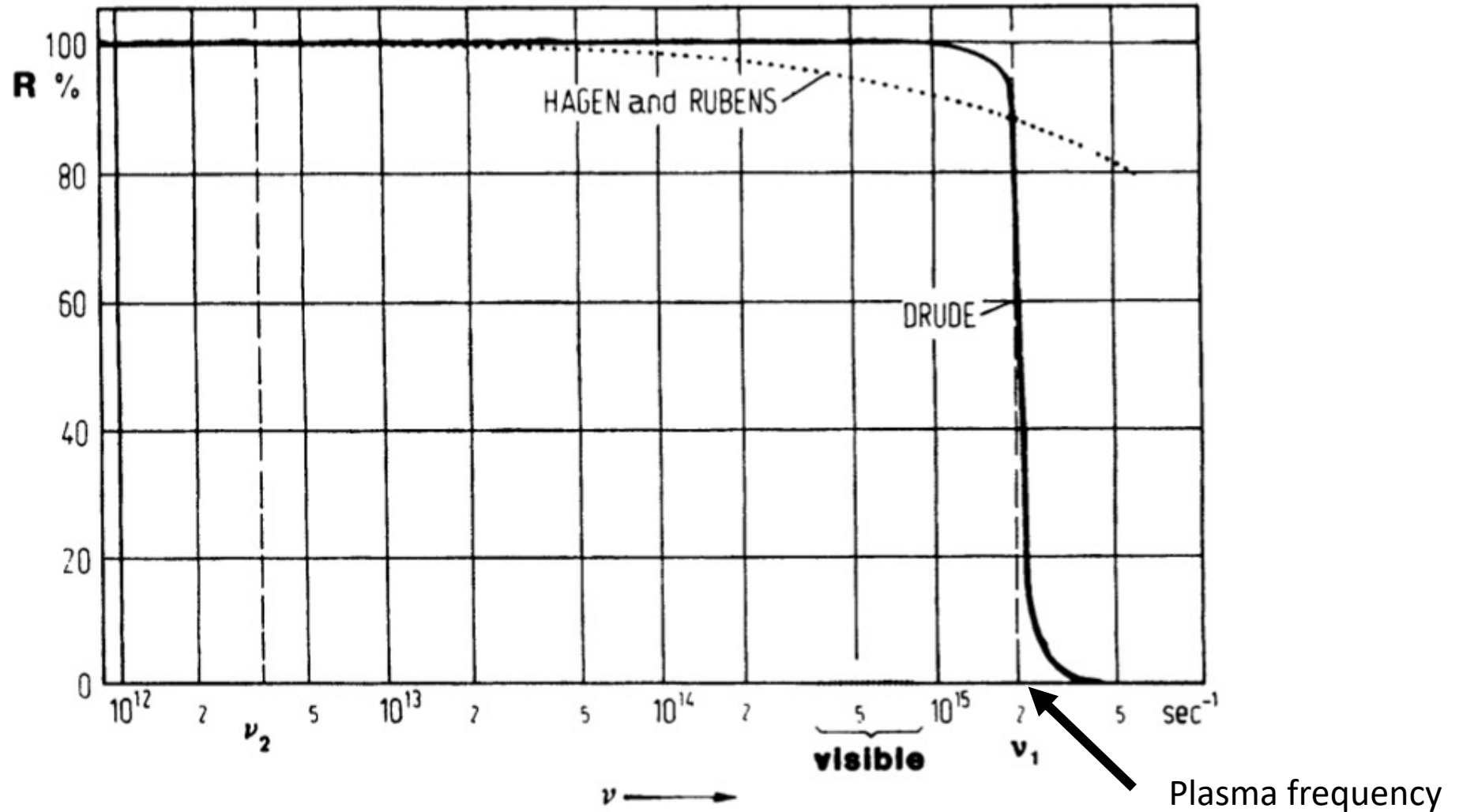
This leaves $n^2 \cong \kappa^2$

Hagen-Ruben's relation

$$R = \left| \frac{\tilde{n} - 1}{\tilde{n} + 1} \right|^2 = \frac{(n^2 + 2*n + 1 + \kappa^2 - 4*n)}{n^2 + 2n + 1 + \kappa^2} = 1 - \frac{4n}{2n^2 + 2n + 1}$$

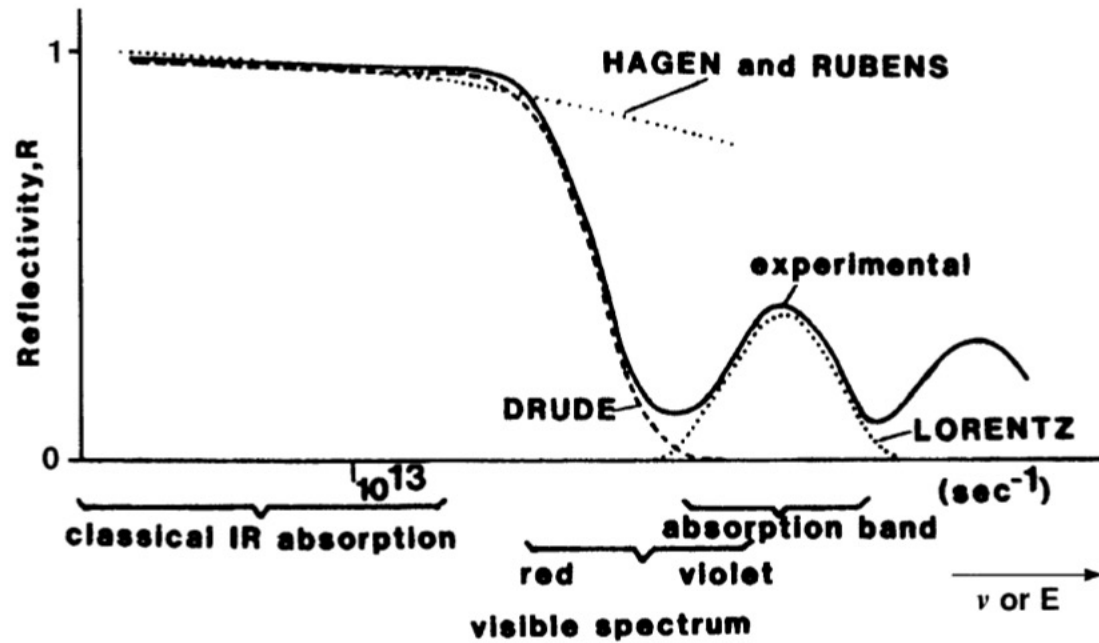
$$R = 1 - \frac{2}{n}$$

Experimental Reflectivity measurement in metals

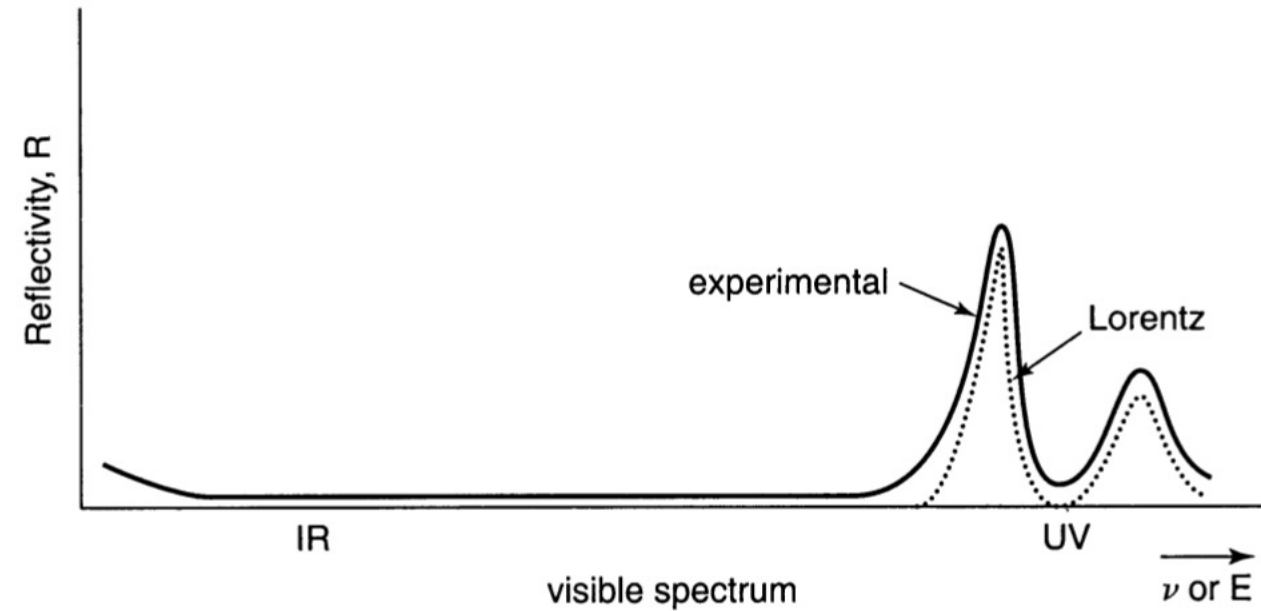


Lorentz Oscillators

Metals

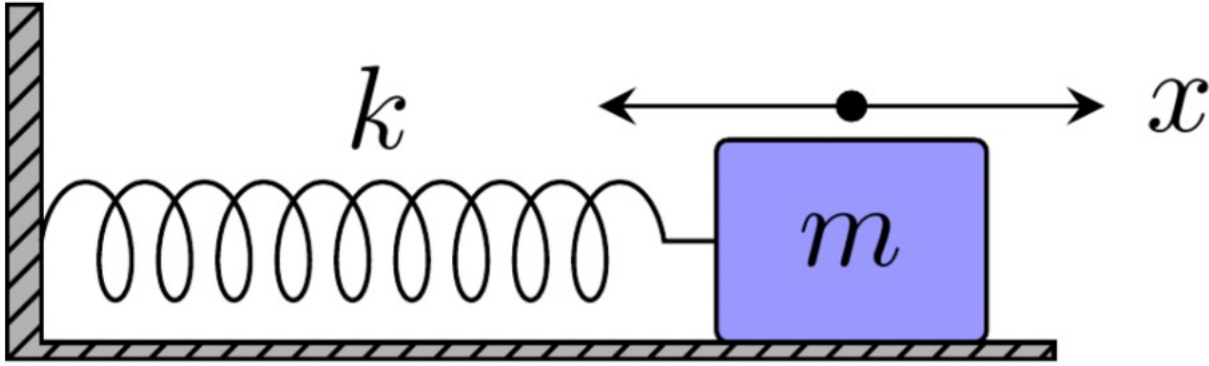


Dielectrics



Important difference in reflectivity of metals and dielectrics

Imagine a simple spring mass system

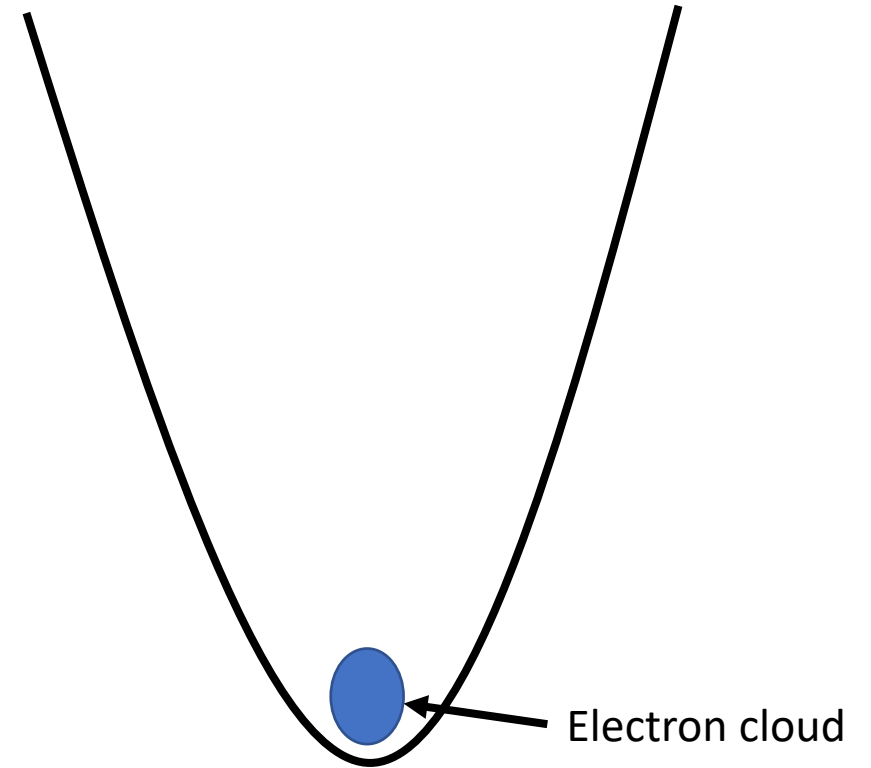


If the driving force is $F_d = E_0 \cos \omega t$

A restoring spring force $F_s = -kx$

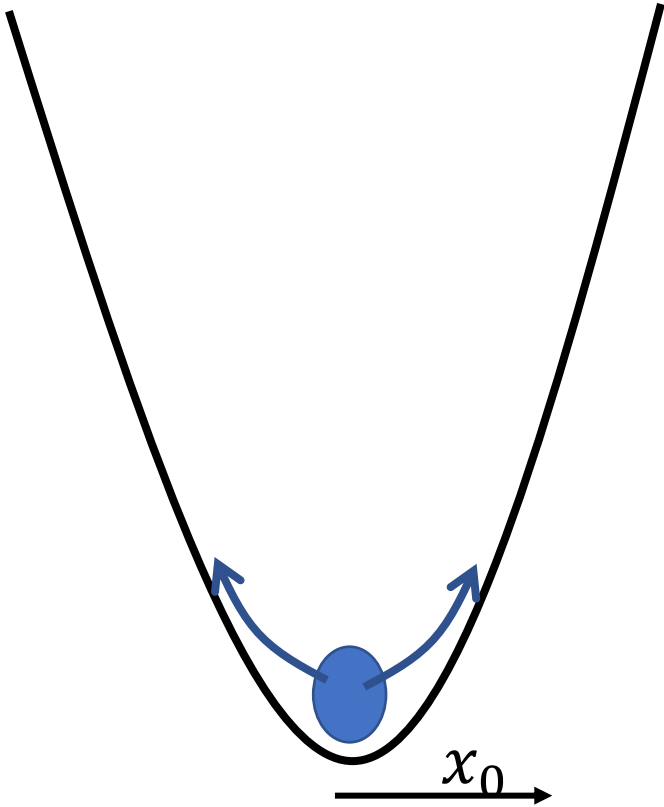
A damping force $F_d = \gamma m \frac{dx}{dt}$

$$F = m \frac{d^2 x}{dt^2}$$



The solution is oscillatory. The electron cloud moves

$$x = x_0 e^{-i\omega t}$$



The solution is oscillatory. The electron cloud moves

$$x = x_0 e^{-i\omega t}$$

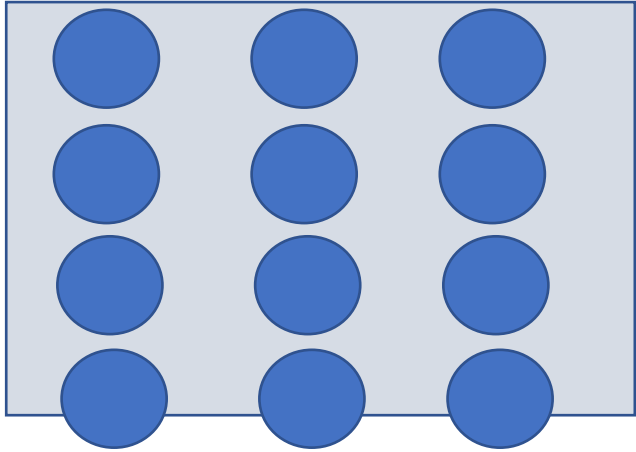
The maximum excursion of the electron cloud is obtained after substitution into the force equation

$$m\ddot{x} = qE_0 \cos \omega t - kx - \gamma m\dot{x}$$

Amplitude of oscillation is obtained by substitution

$$x_0 = \frac{qE_0}{m} \frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

$$\text{polarization } P = Nx_0q = \epsilon_0\chi E$$

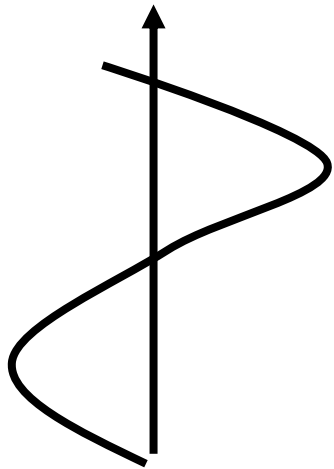
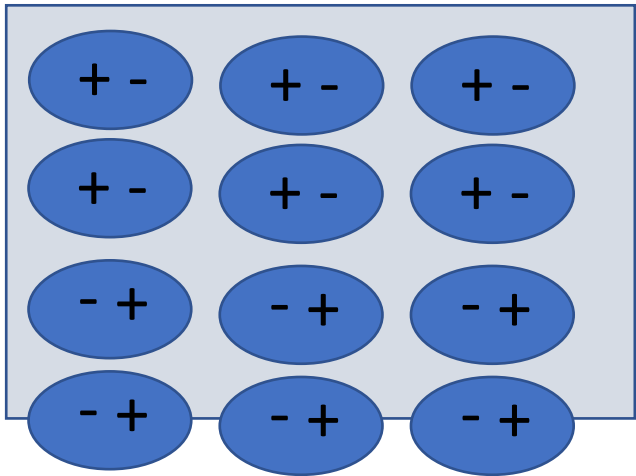


polarization $P = Nx_0q = \epsilon_0\chi E$

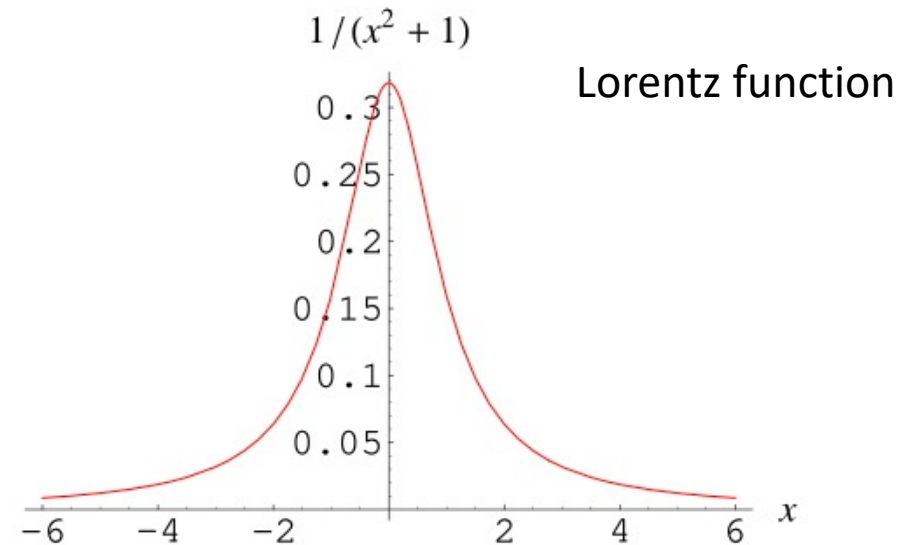
$$\chi = \frac{Nq^2}{m\epsilon_0} \frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

$$\chi = \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

$$\omega_p^2 = \frac{Nq^2}{m\epsilon_0}$$

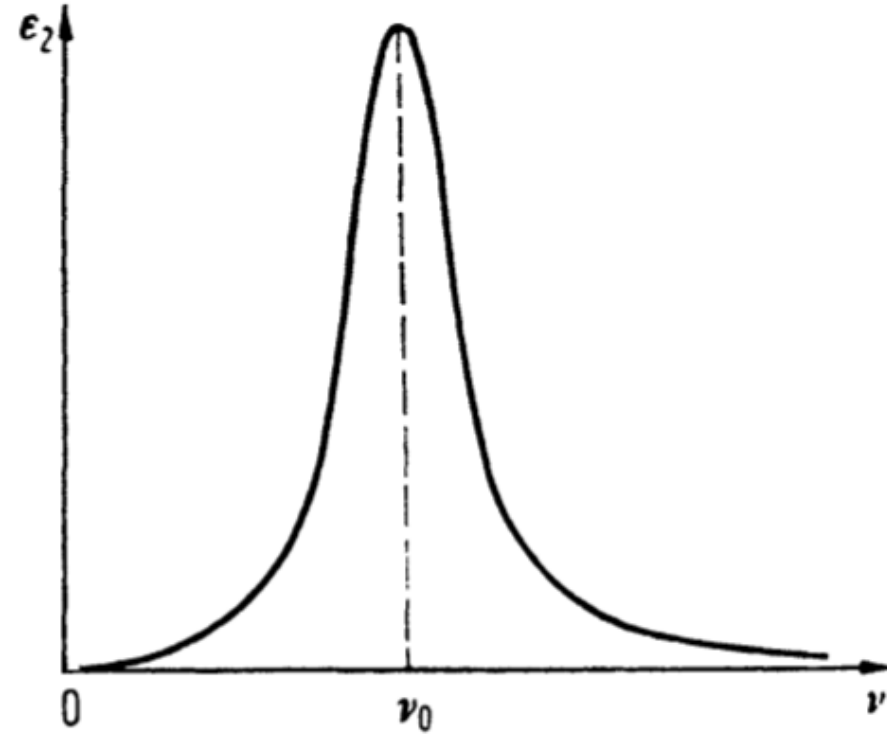
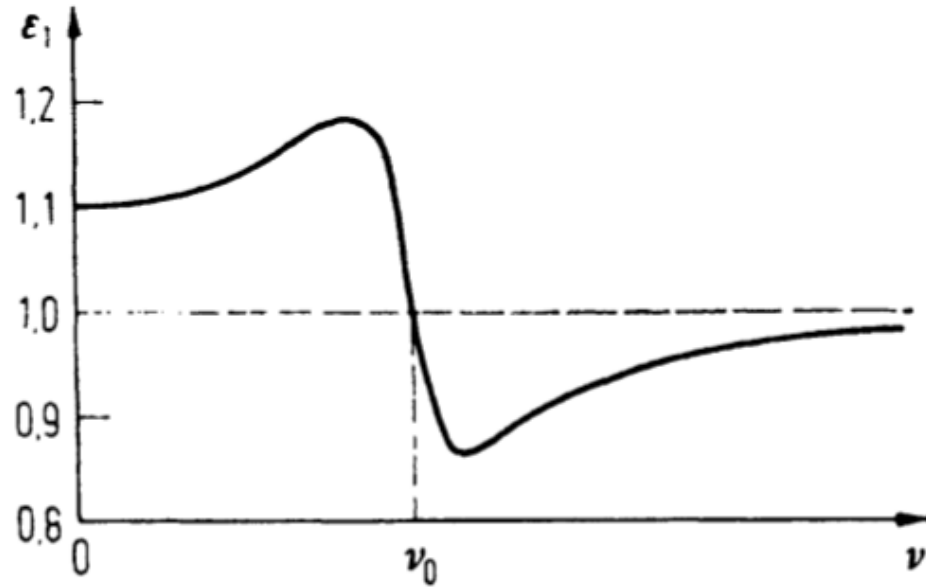


Complex susceptibility



Complex Dielectric Constants

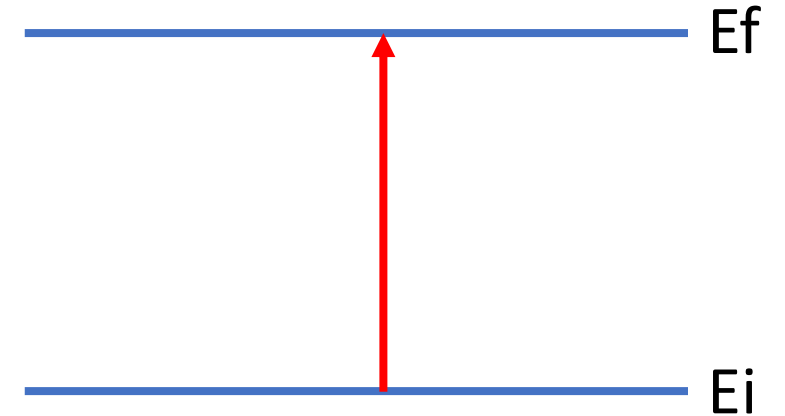
ϵ



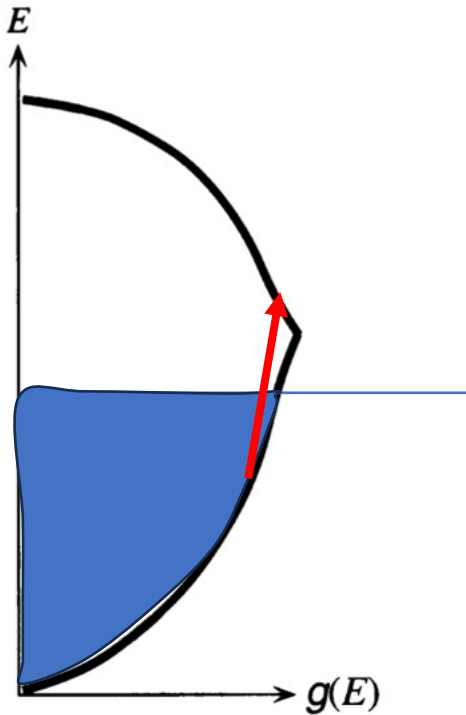
ω

Quantum mechanical Treatment

- Electronic Transitions
 - Inter-band
 - Intra-band transitions
- Necessary condition
 - Filled initial state of the electron
 - Existing vacant final state of the electron
 - Difference in energy level matches photon energy



Intra-band transitions: Example in a metal



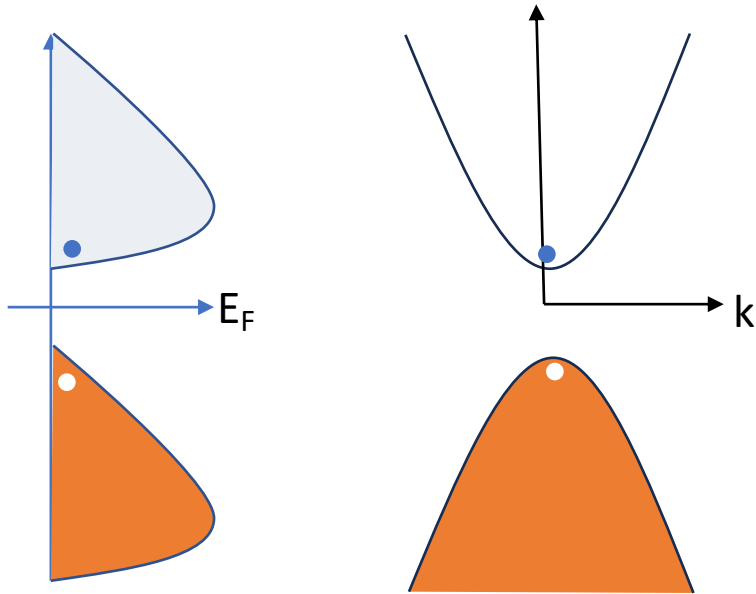
In a metal, there are a plenty of vacant states near by filled states

A photon of even small energies can be absorbed.

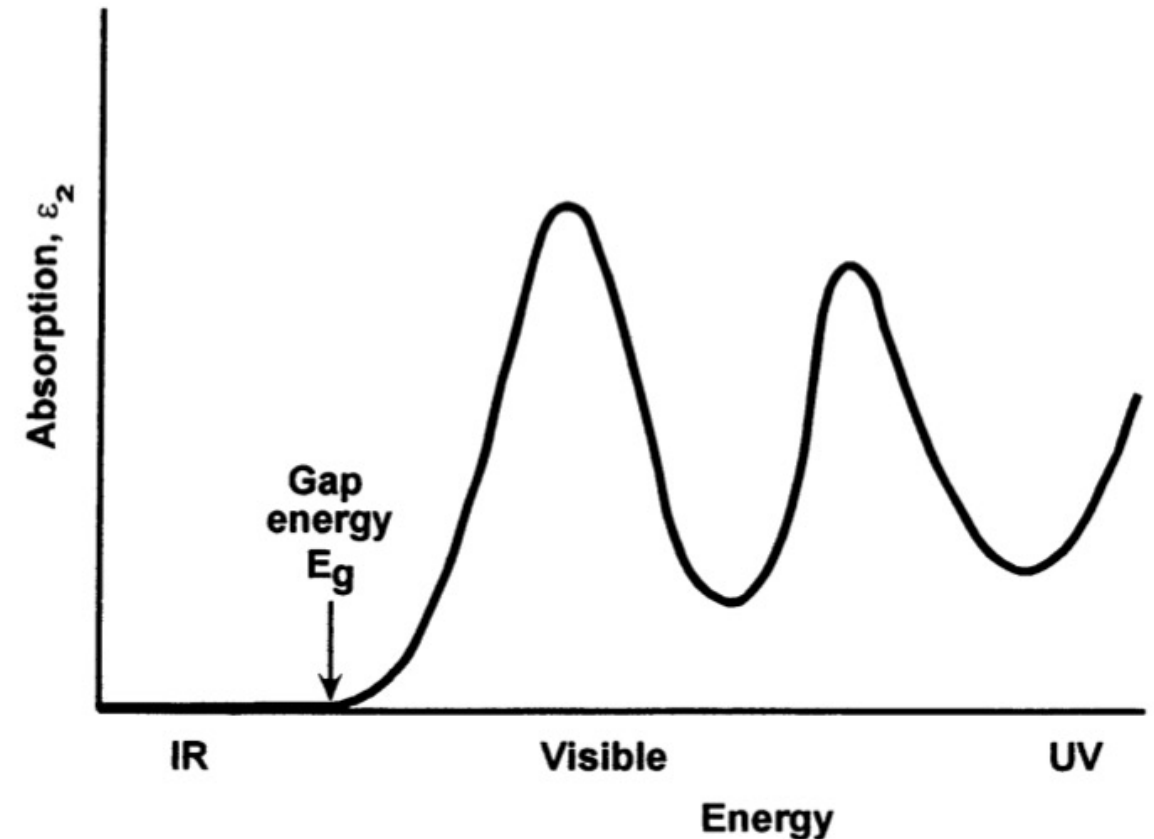
Continuous absorption with no specific threshold.

Inter-band transitions

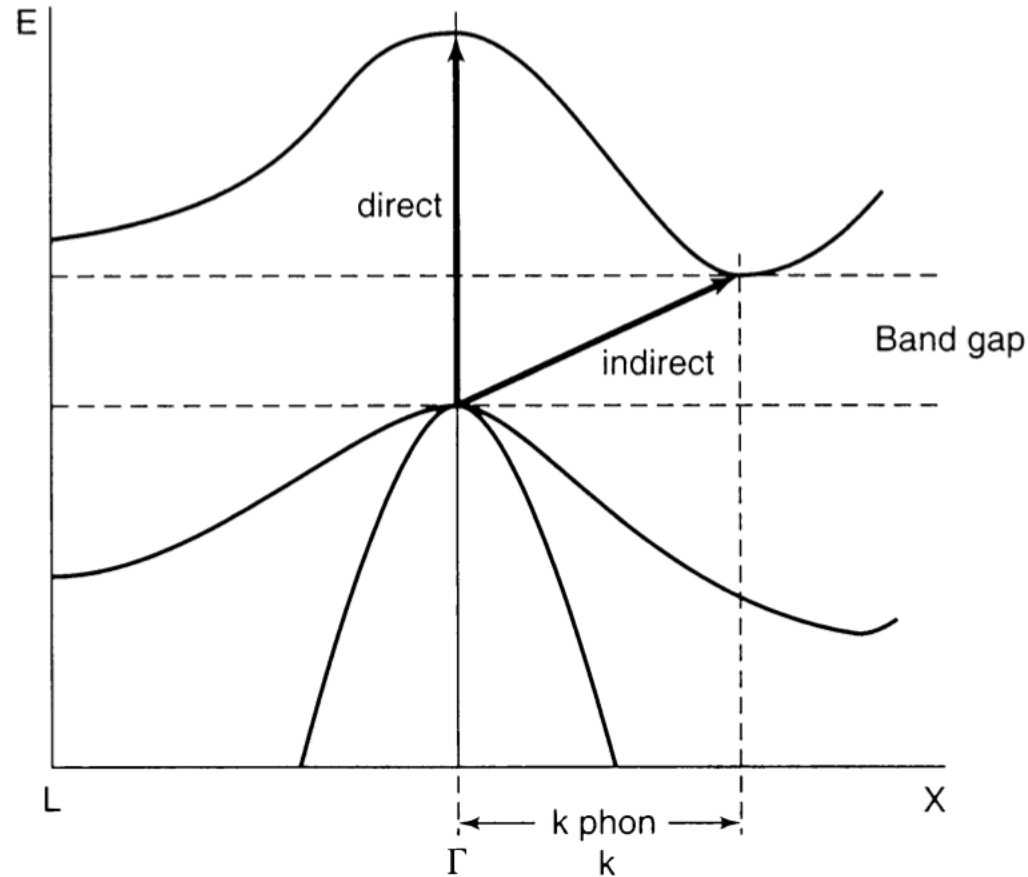
- An electron is excited from the valence band to conduction band



Absorption happens only at specific energy levels



Direct and In-direct Semiconductors

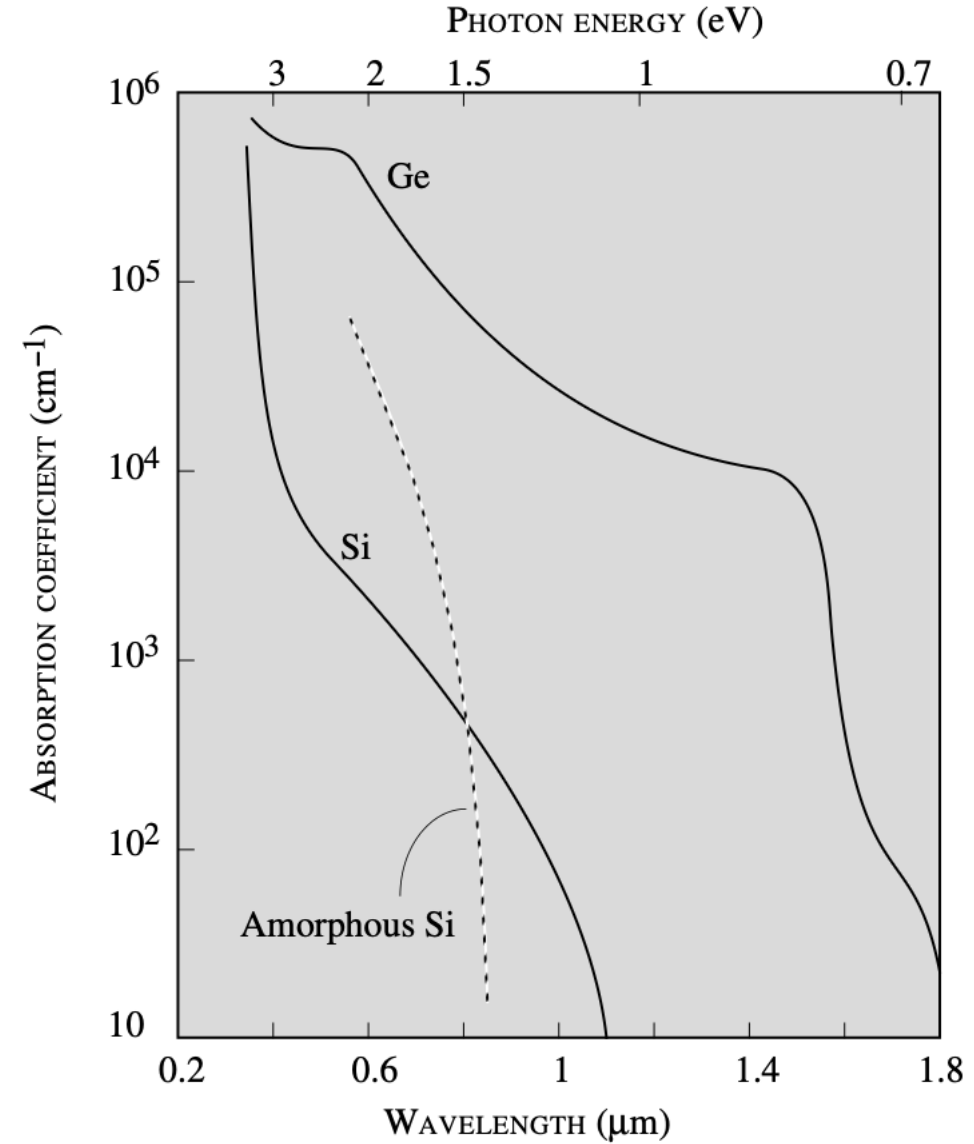
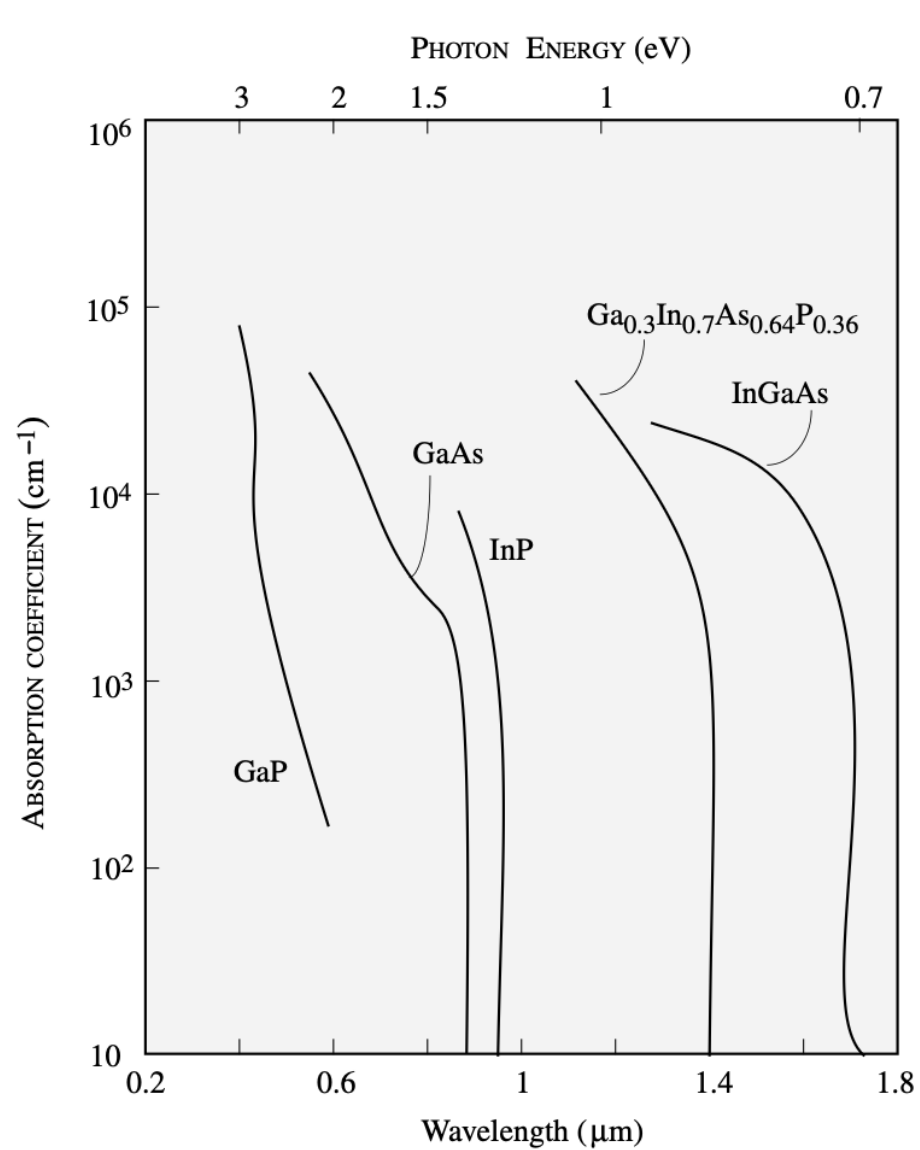


If the top of valence band and bottom of the conduction band are at the same 'k' points, then we have a direct bandgap semiconductor.

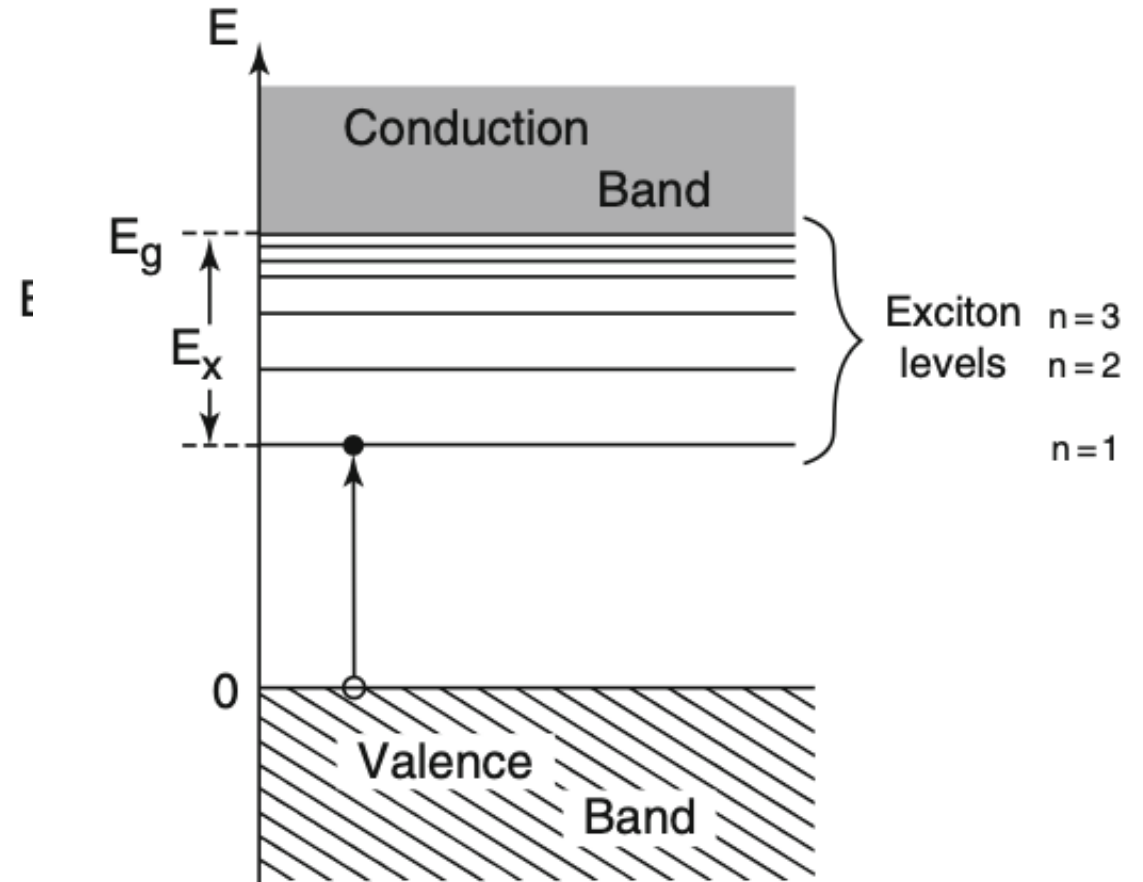
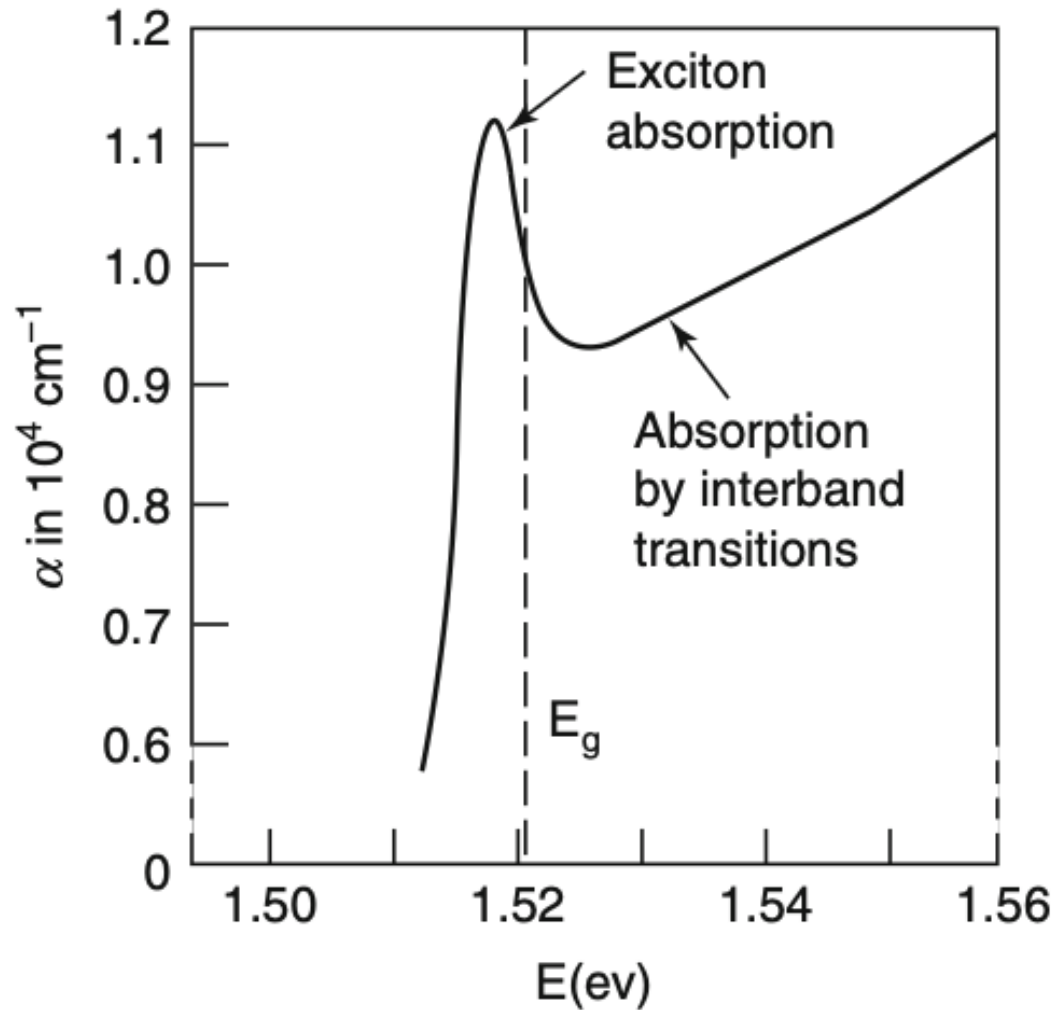
A significant difference in the K-values of the two points leads to an indirect semiconductor.

There is a need for phonon assistance in an indirect semiconductor.

Comparison of Direct and In-Direct



Absorption from Exciton



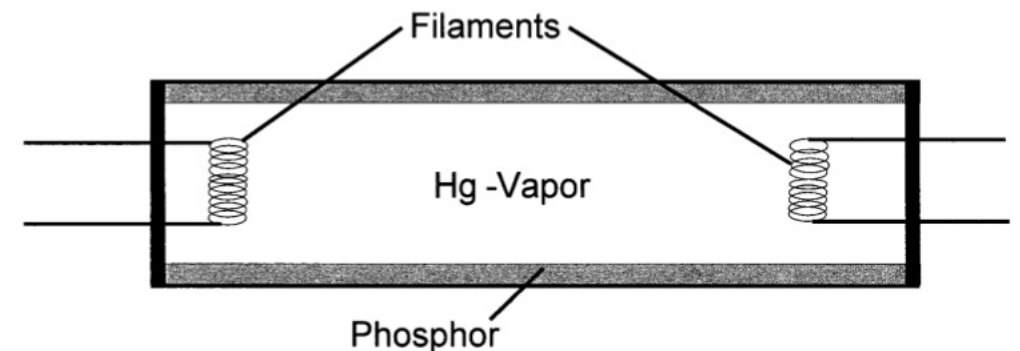
Applications of absorption measurements

- Absorption measurements is very commonly used in materials research for the following applications
 - Determination of bandgap of semiconductors
 - Identification of localized states, defect induced states and its occupation
 - Direct/In-direct nature of semiconductor
 - Estimation of reflectivity and electrical conductivity

Photon Emission

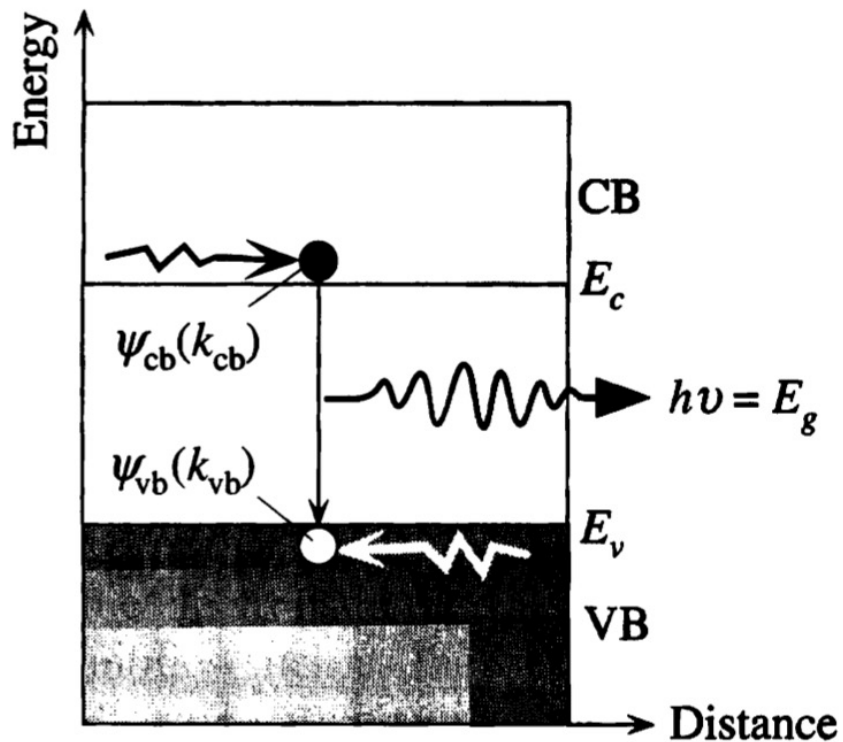
- Light emission is also a very popular characteristic of material
- An electron in a material that underwent excitation recovers back
- A recovery using photons is called as luminescence!
- Photoluminescence
 - Emission of light from a material which was excited by another light
 - Electroluminescence is the emission of light from material excited by electric current

Example is fluorescence lamps



Recombination by light emission

An excited electron in a semiconductor returns back to the valence band by a process called as recombination.



The criteria for recombination is the same as that of absorption.

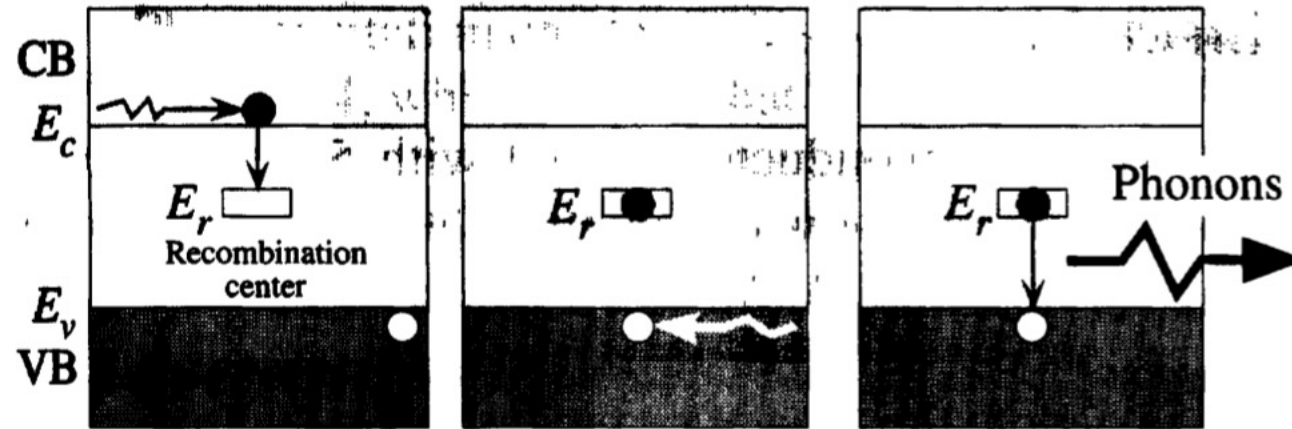
Conservation of linear momentum leads to

$$\hbar k_e - \hbar k_p \cong 0$$

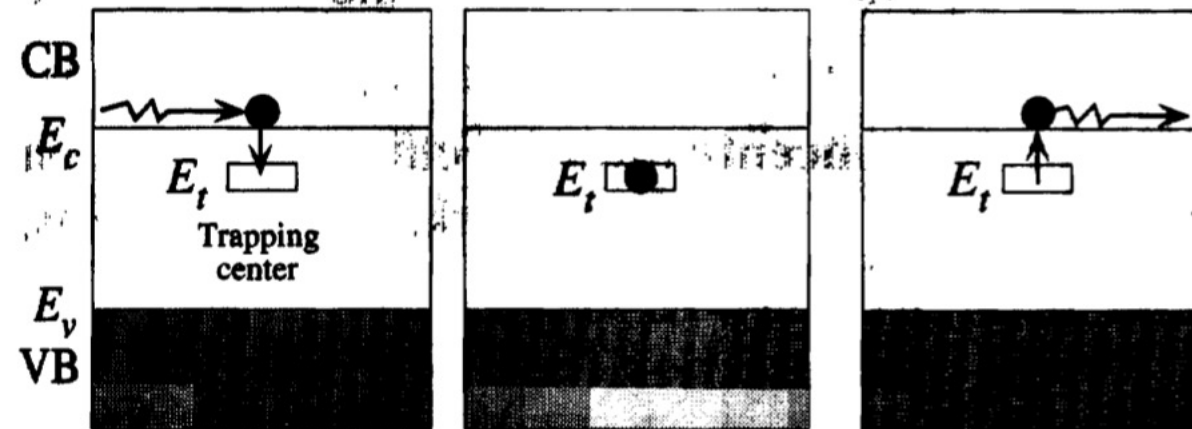
Conservation of energy

$$E_c - E_v = h\nu$$

Recombination by Trapping



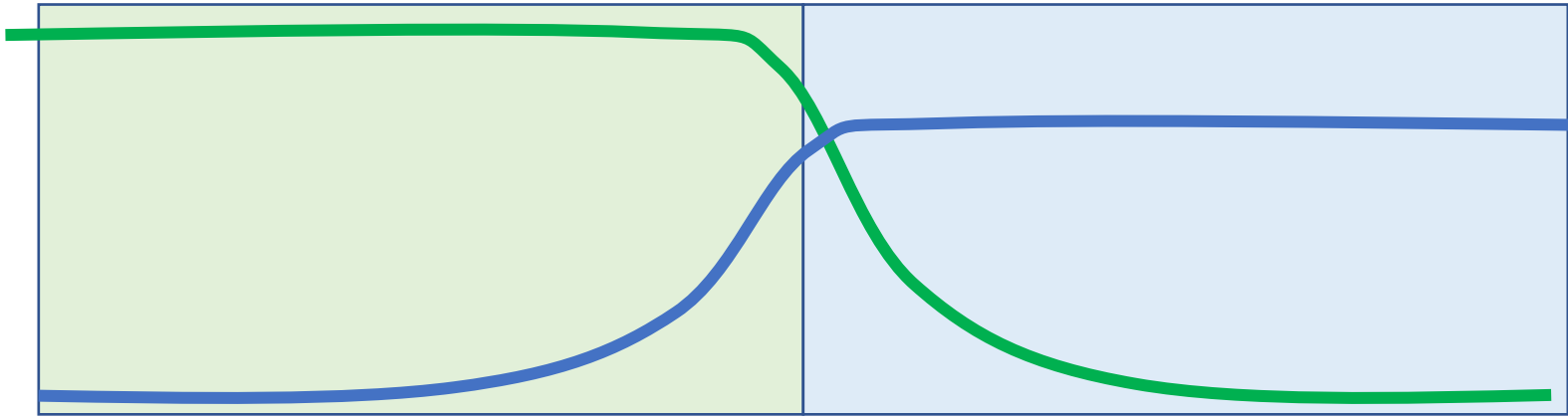
(a) Recombination



(b) Trapping

- Certain defects enable efficient recombination by trapping the electron/hole.
- This will permit the other charge carrier to arrive at the electron/hole for recombinations

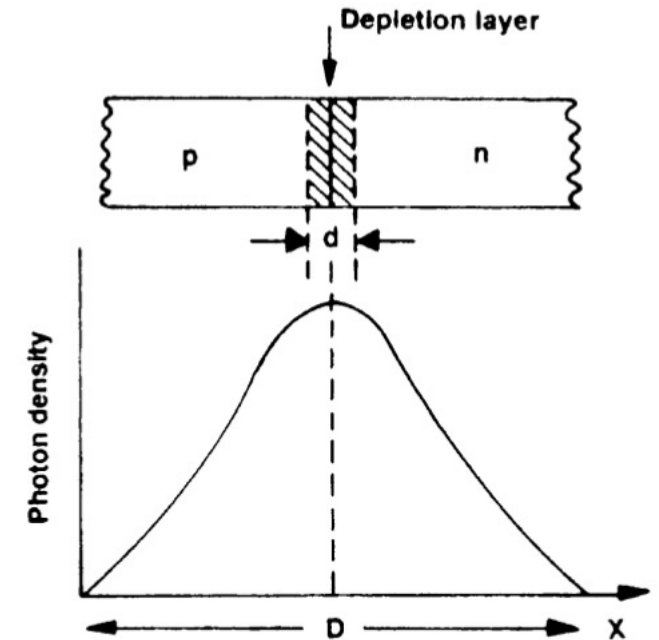
Light Emitting Diode



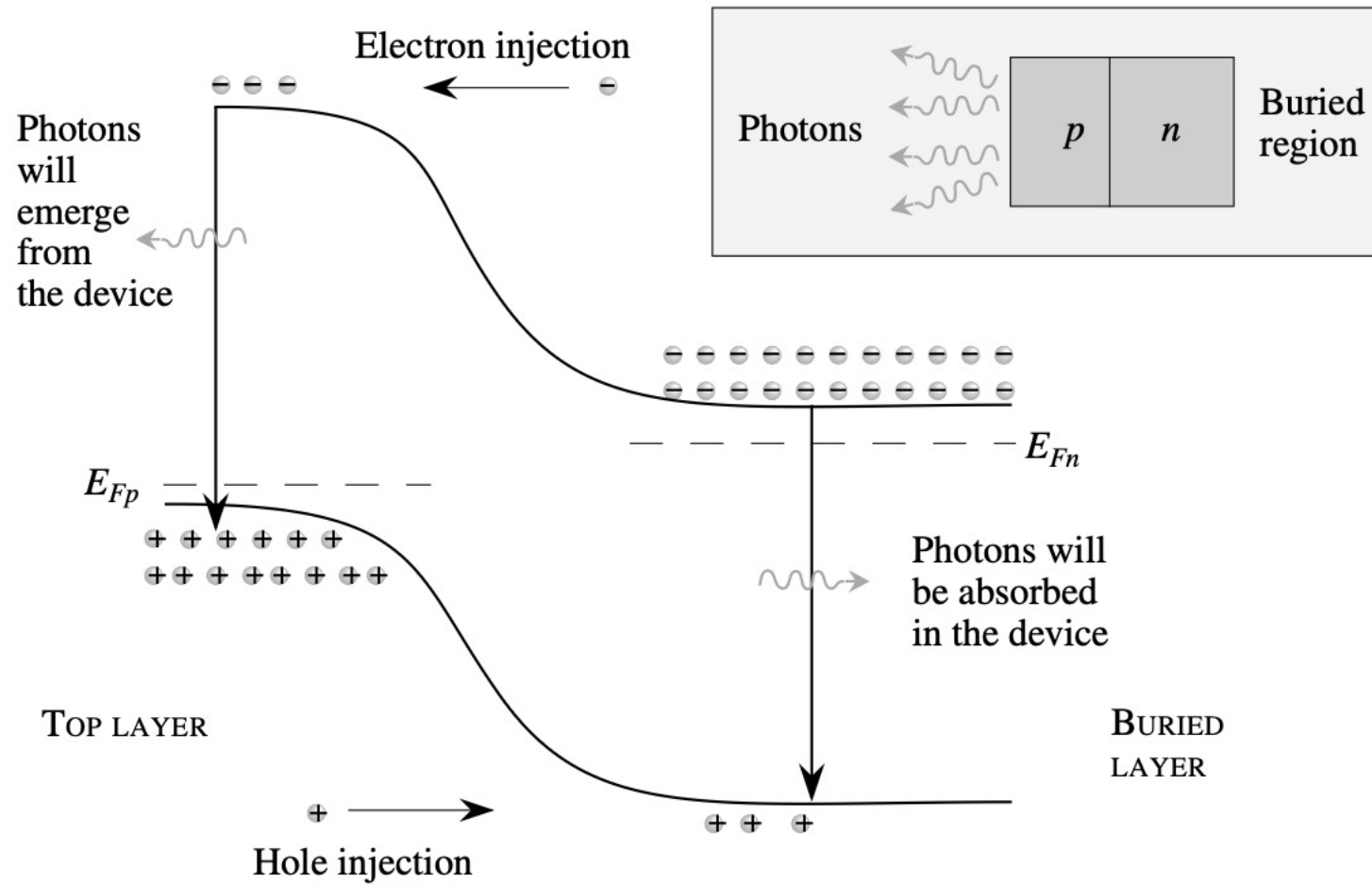
Largest carrier density change in the PN junction diode happens at the depletion!

The carrier density change can only be done by recombination. And in a semiconductor, it will lead to light emission

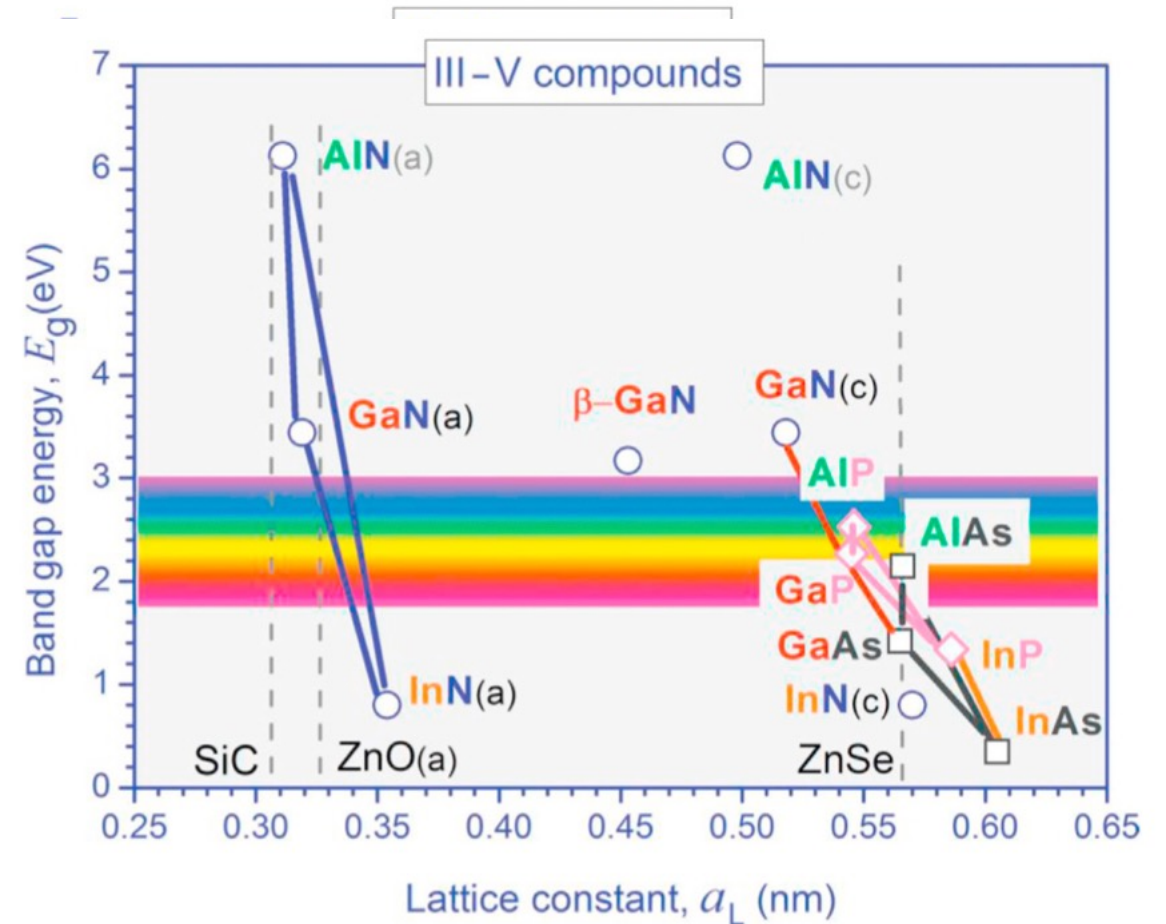
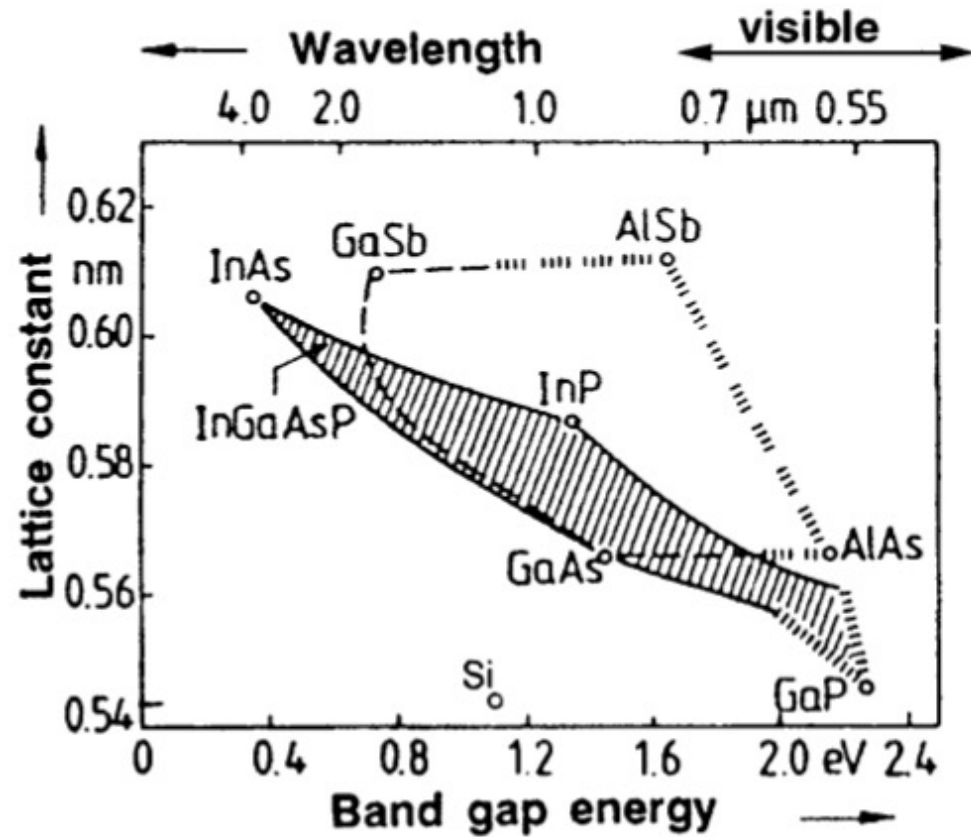
Therefore, every PN junction is LED and every LED is a type of PN junction!



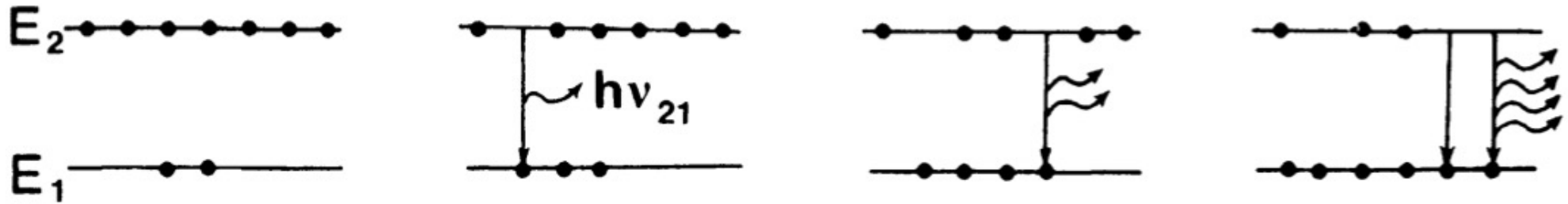
An LED under forward bias



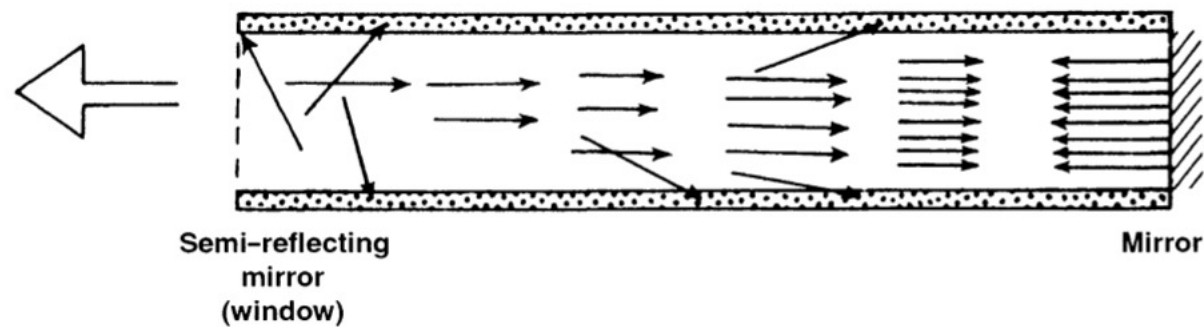
Various Light emitting diodes



Stimulated Emission



General Schematics of a laser

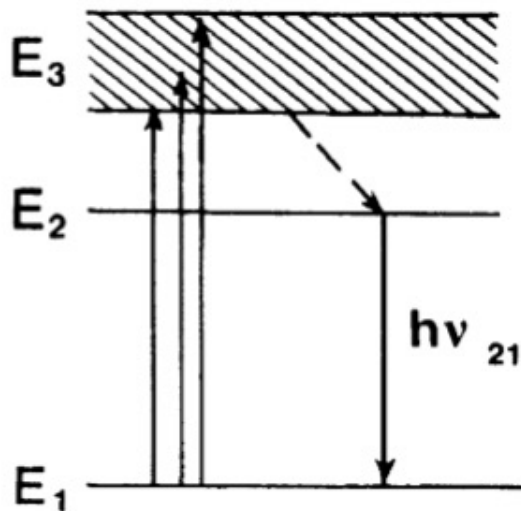
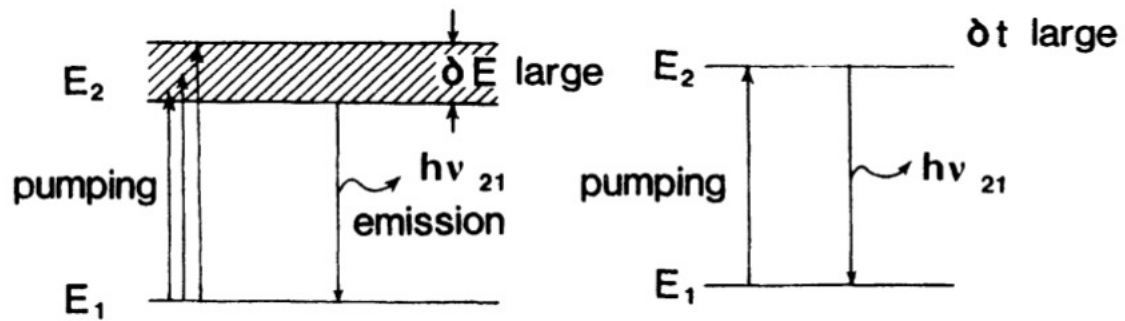


Requirements:

1. Population inversion
2. Light amplification and gain

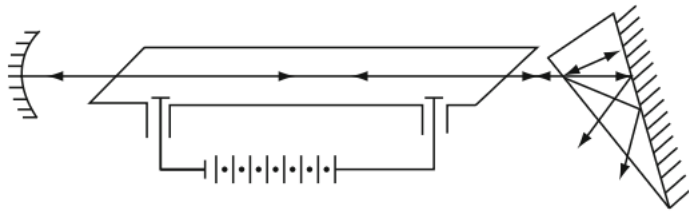
An emitted photon forces other excited electrons to oscillate at the same frequency and phase as itself. Forcing emission of a photon with same energy and phase. Leading to amplification.

Practical laser systems

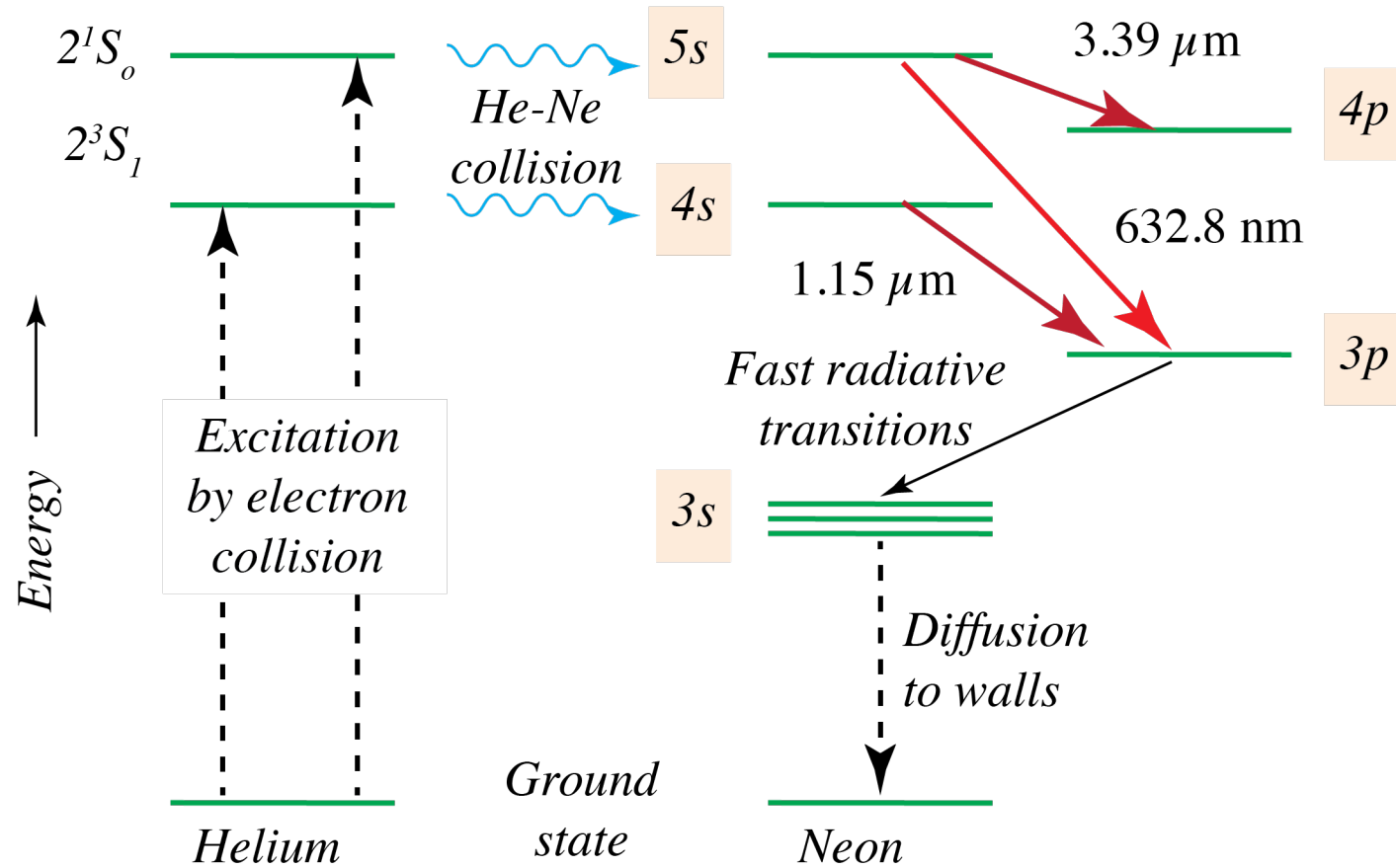


- A three level system
- Choose a level E_3 whose band is large!
 - Much easier to achieve inversion!
- E_3 electrons relax to E_2
- E_2 gets inverted. Starts to lase from E_2 .
- Photon of energy E_2 cannot excite to E_3 .

An example – He-Ne laser

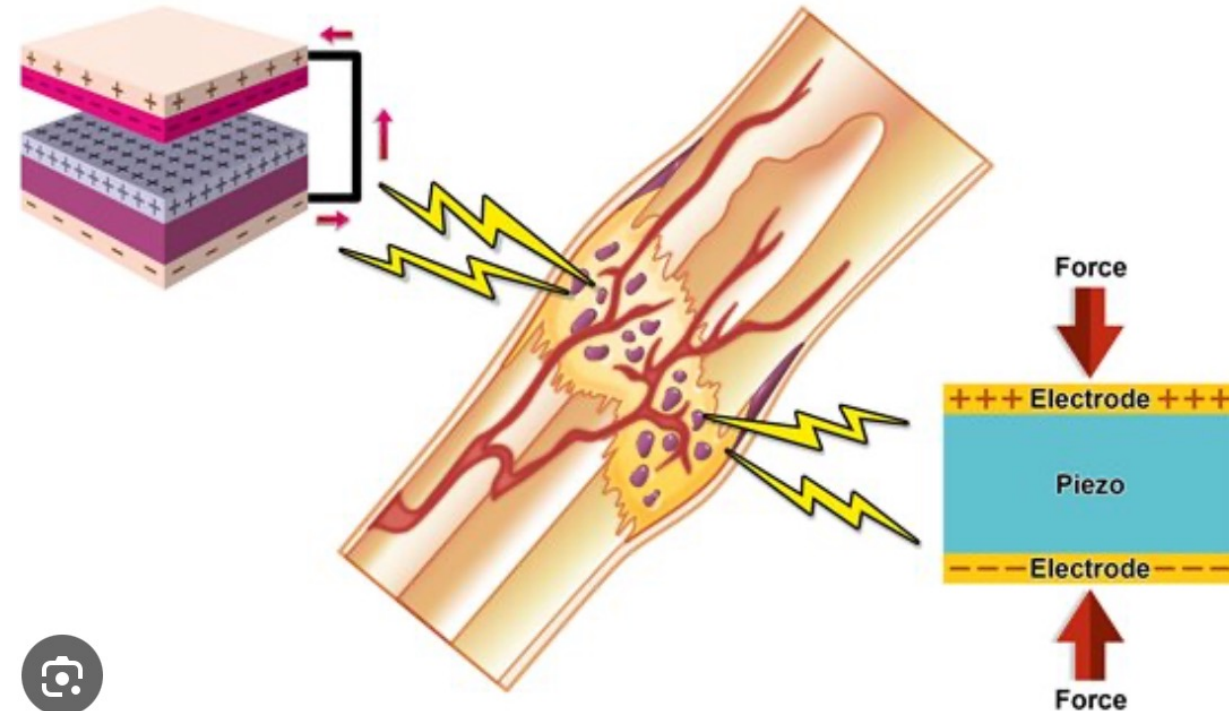
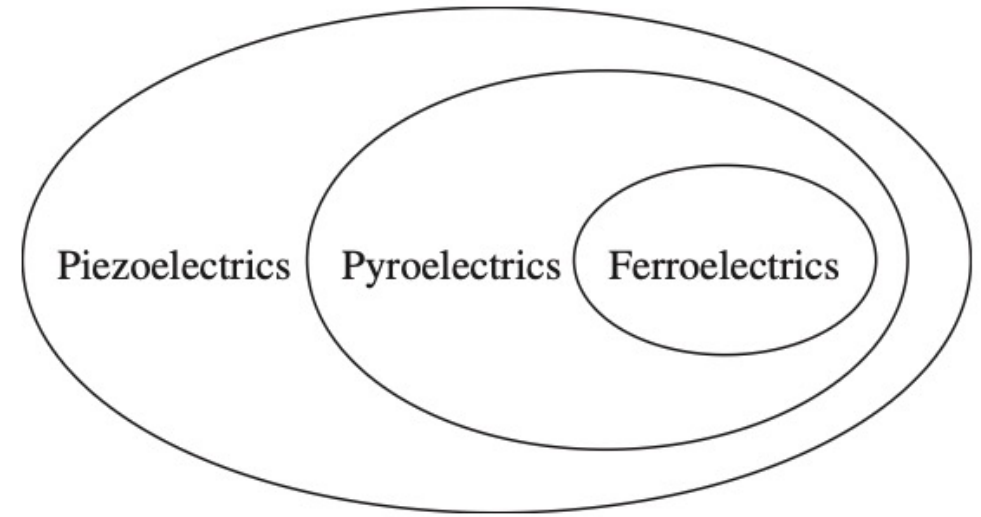


- He gas is ionized.

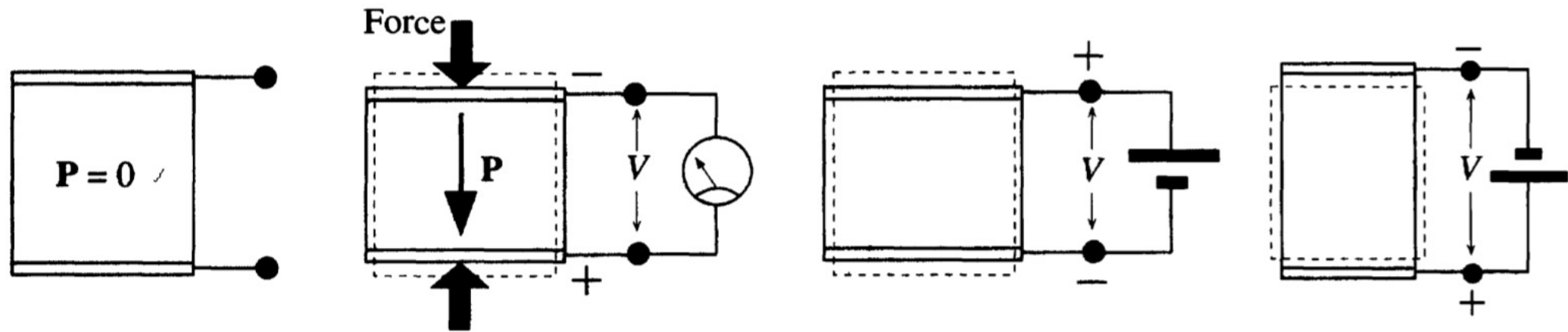


Other Dielectrics

- Piezo-Electric
- Ferro-Electric
- Pyro- Electric



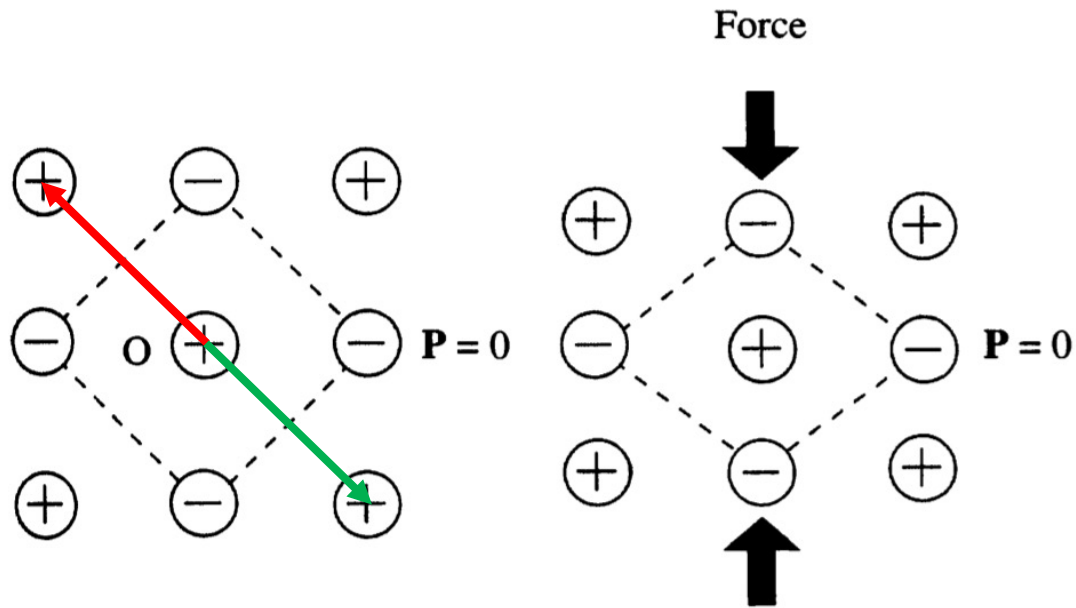
Piezo electricity



- Piezo – Greek for squeeze or press

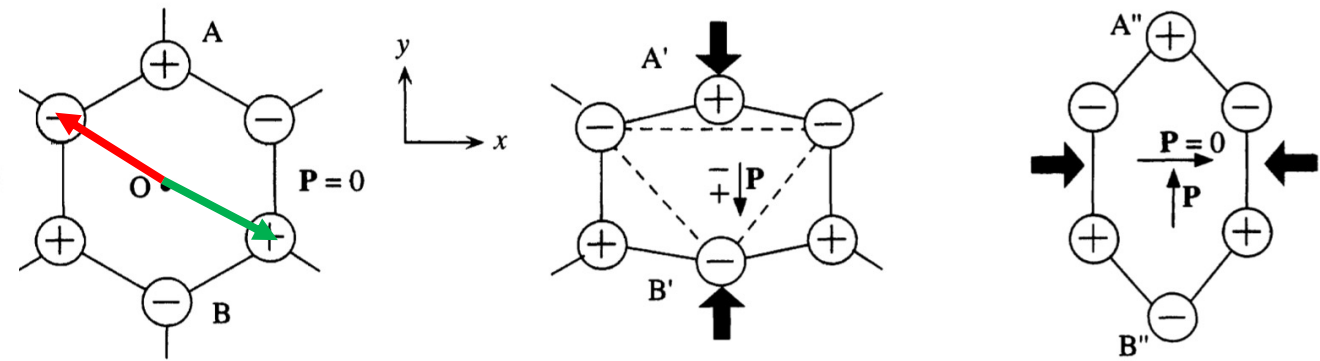
Origin of polarization

NaCl



System with center of symmetry

A hexagonal ionic lattice



System without a center of symmetry

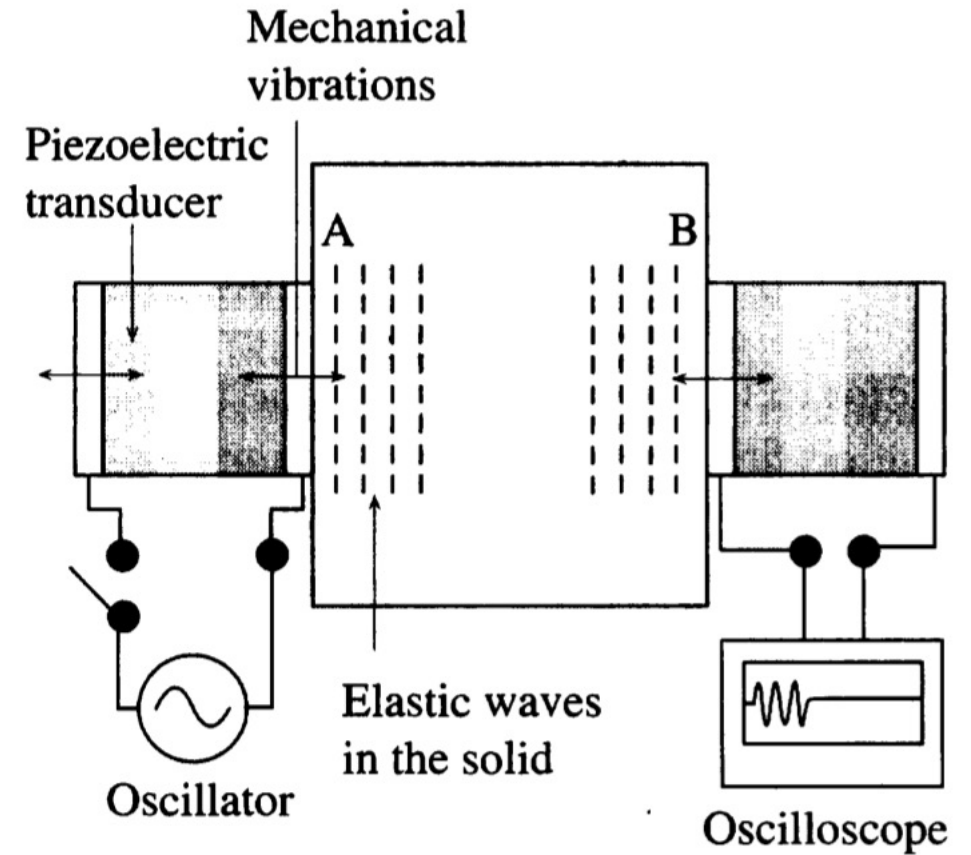
Constitutive relations

- If T_i is the applied pressure along direction i
- A polarization P_j is obtained along j -direction.
- A piezoelectric coefficient $P_j = d_{ji}T_i$

$$K^2 = \frac{\text{Mechanical Energy obtained}}{\text{Electrical energy spent}}$$

Exact inverse is also true.

Application of voltage leads to piezo-induced strain. (Eg) ultrasonic transducers



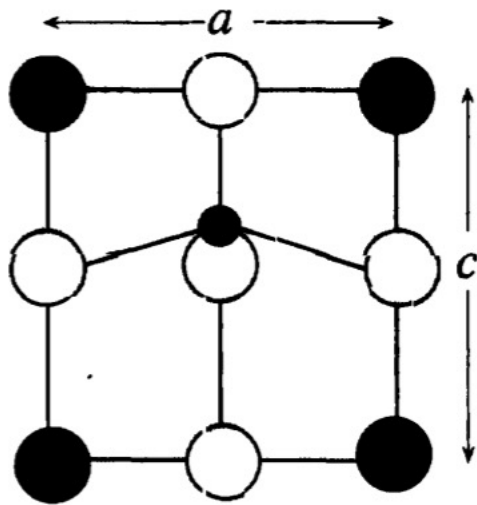
Commonly used piezo-electric materials

Crystal	d (m V ⁻¹)	k	Comment
Quartz (crystal SiO ₂)	2.3×10^{-12}	0.1	Crystal oscillators, ultrasonic transducers, delay lines, filters
Rochelle salt (NaKC ₄ H ₄ O ₆ · 4H ₂ O)	350×10^{-12}	0.78	Accelerometers
Barium titanate (BaTiO ₃)	190×10^{-12}	0.49	
PZT, lead zirconate titanate (PbTi _{1-x} Zr _x O ₃)	480×10^{-12}	0.72	
Polyvinylidene fluoride (PVDF)	18×10^{-12}	—	Wide range of applications including earphones, microphones, spark generators (gas lighters, car ignition), displacement transducers, accelerometers Must be poled; heated, put in an electric field and then cooled. Large area and inexpensive

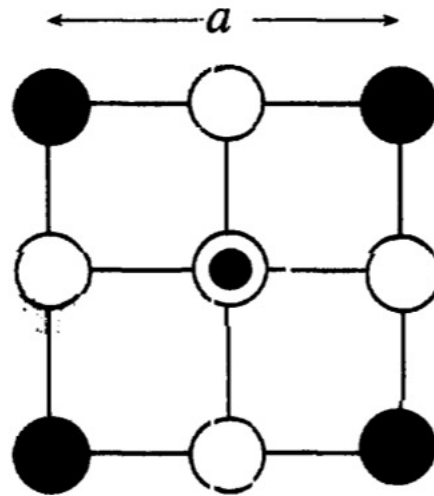
Read about electrostriction – close relative to piezoelectricity

Ferro-Electric Materials

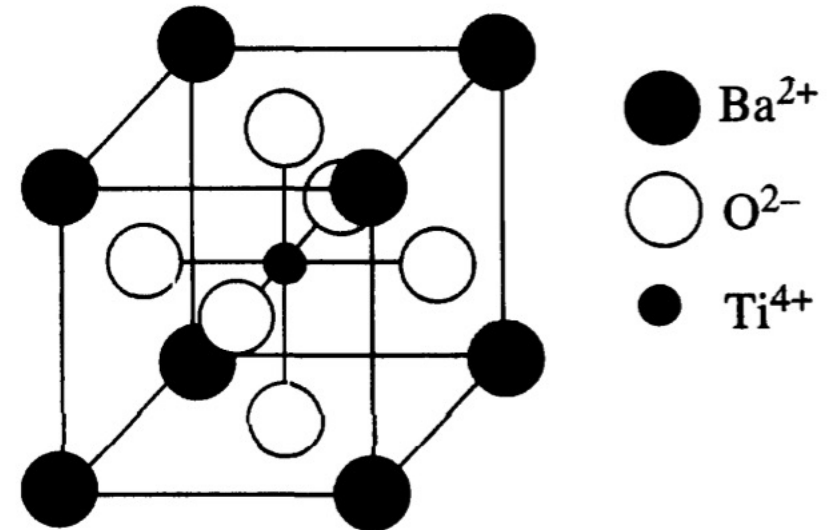
- Materials with permanent polarization



Tetragonal lattice at room conditions



Cubic at higher temperatures



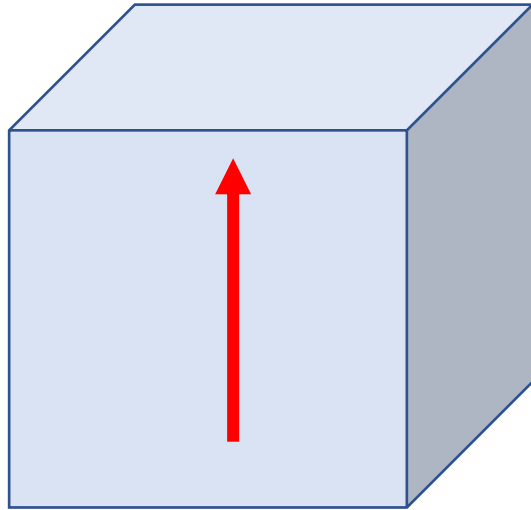
Properties of Ferro-Electric Materials

1. Characterized by permanent dipole moments – polarization
2. Polarization even in the absence of electric fields
3. Originate due to minor deviations in crystal symmetry or crystal asymmetry
4. They are electrical analogues of ferro-magnetic materials
5. There exists a temperature above which any remnant polarization is lost

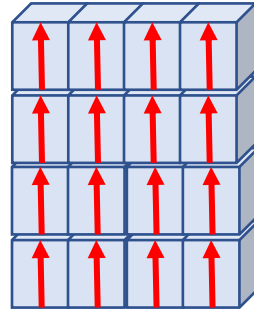
Ferro-electric becomes para-electric

The curie point/temperature is defined as the temperature of this transition

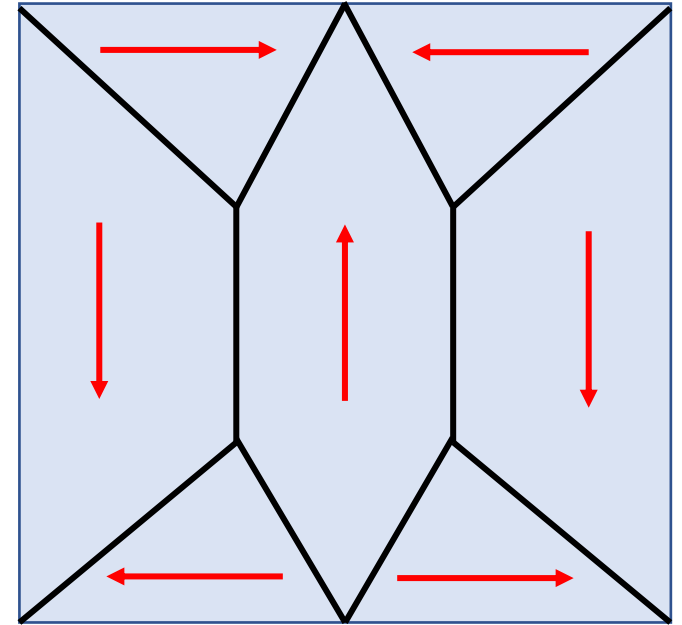
Clustering and Domains



Ferro-electric unit cell

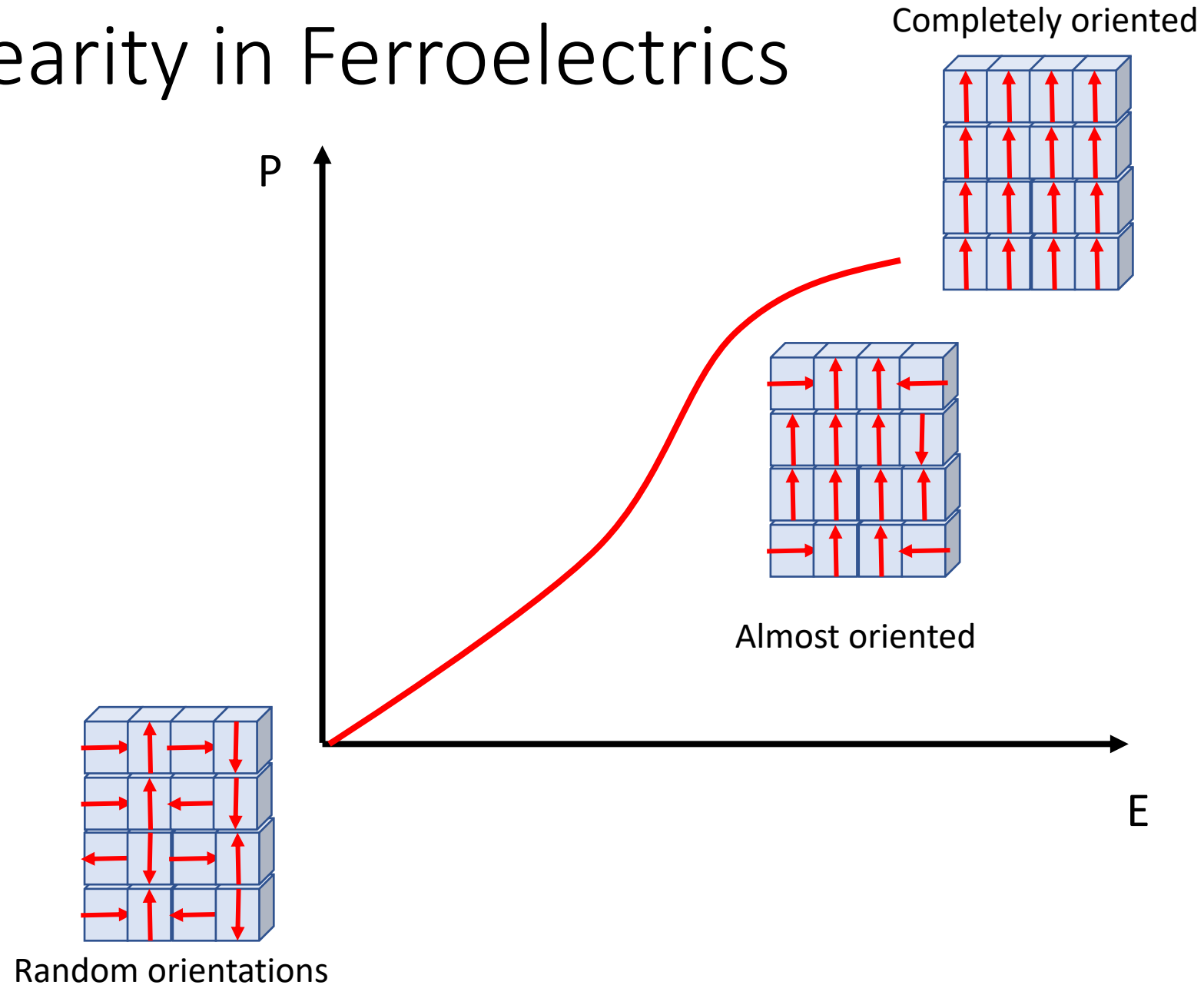


Clustering of similarly oriented dipoles leads to a domain having a net polarization

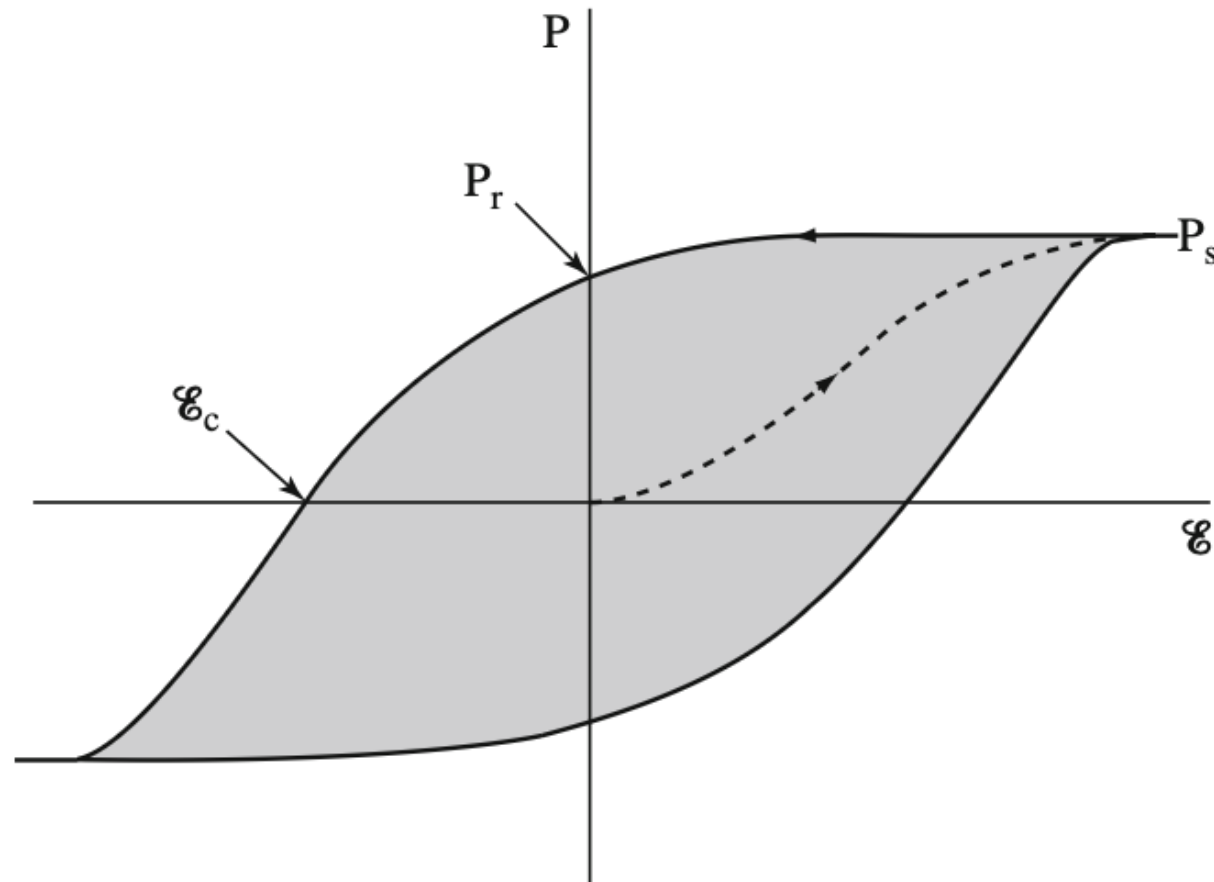


Domain alignment leading to net zero polarization in the absence of applied field

Non-linearity in Ferroelectrics



Hysteresis in Ferroelectrics



P_r – remnant polarization
 E_c – Coercive field

Curie-Weiss Law

We previously saw

$$P = N\alpha E = \epsilon(\chi - 1)E$$

If a material has remnant polarization, the local field is different than applied field

$$E' = E + P/3\epsilon_0$$

Substituting in the previous equation

$$P = \epsilon(\chi - 1)E'$$

$$P = \frac{N\alpha E}{1 - \frac{N\alpha}{3\epsilon_0}}$$

Therefore, when $N\alpha = 3\epsilon_0$, Polarization goes to infinity!

Since, the polarizability (α) is known to be proportional to temperature

$$\alpha = \frac{C}{kT} \qquad N\alpha = 3\epsilon_0$$

Therefore at a temperature $T = T_C = \frac{NC}{3k\epsilon_0}$, The polarization diverges to infinity

T_C is called as the Curie Temperature

$$\chi(T) = \epsilon_r - 1(T) = \frac{P}{\epsilon_0 E} = \frac{3T_C}{T - T_C}$$

Curie-Weiss Law