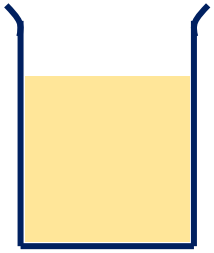
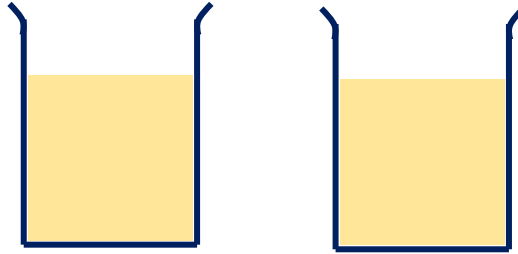


System at Equilibrium

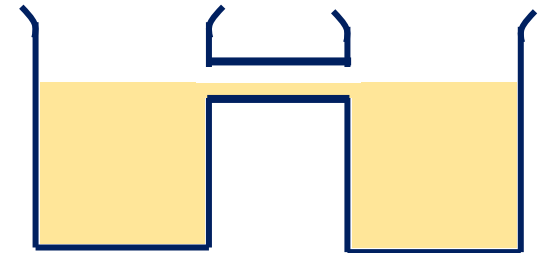
- Whats difference between equilibrium and steady state



Liquid at equilibrium in a container

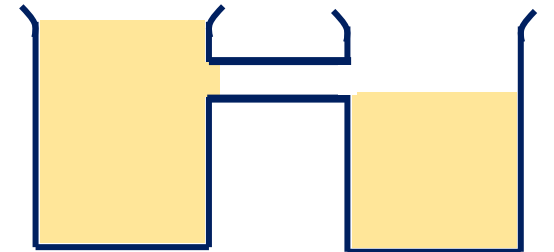


Two independent jars at equilibrium



Two connected jars at equilibrium

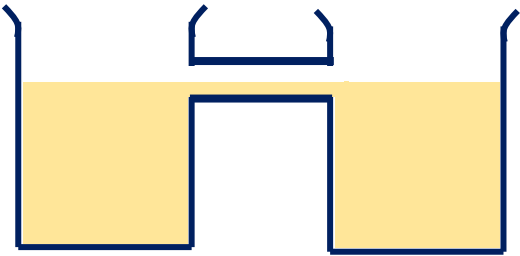
At equilibrium, the water level does not change.
There is no net motion of water.



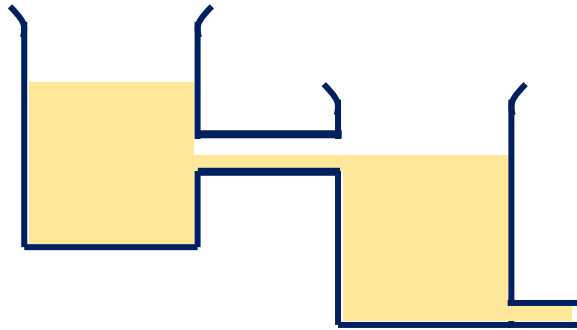
Two connected jars not at equilibrium

Systems at steady state

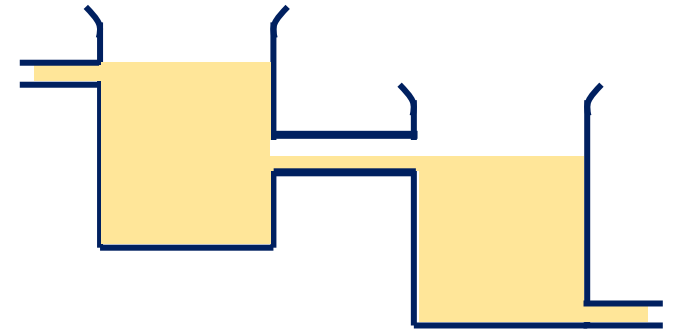
- Whats difference between equilibrium and steady state



Two connected jars at equilibrium



Two connected jars not at steady state

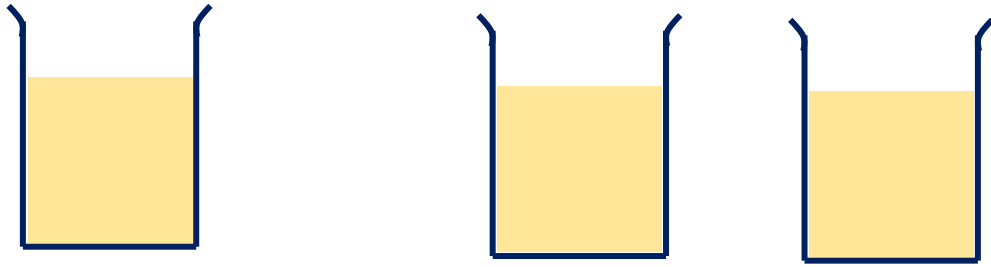


Two connected jars at steady state

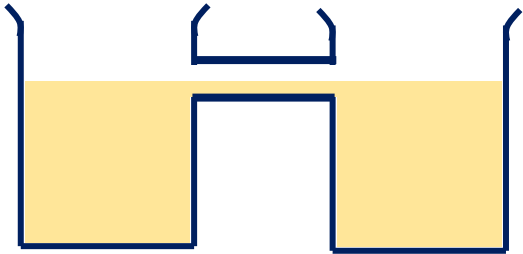
At steady state, the water level does not change.

There is constant net motion of water.

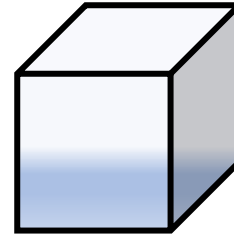
Equivalence with electron systems



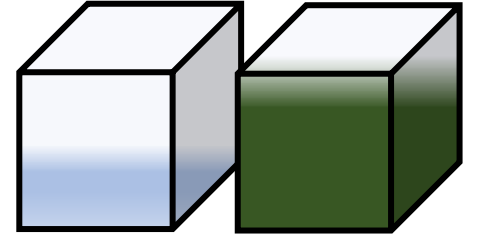
Water at equilibrium
in a jar



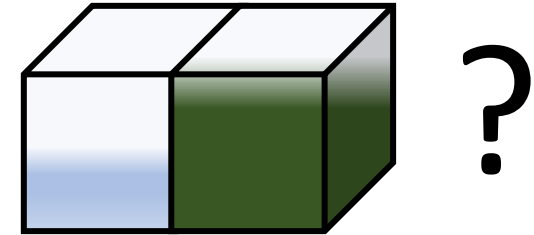
Two connected jars at equilibrium
The water level is constant!
Why ? – Total potential energy minima in gravitational force field



Electrons at equilibrium
in a crystal



Electrons at equilibrium
in two disconnected crystals

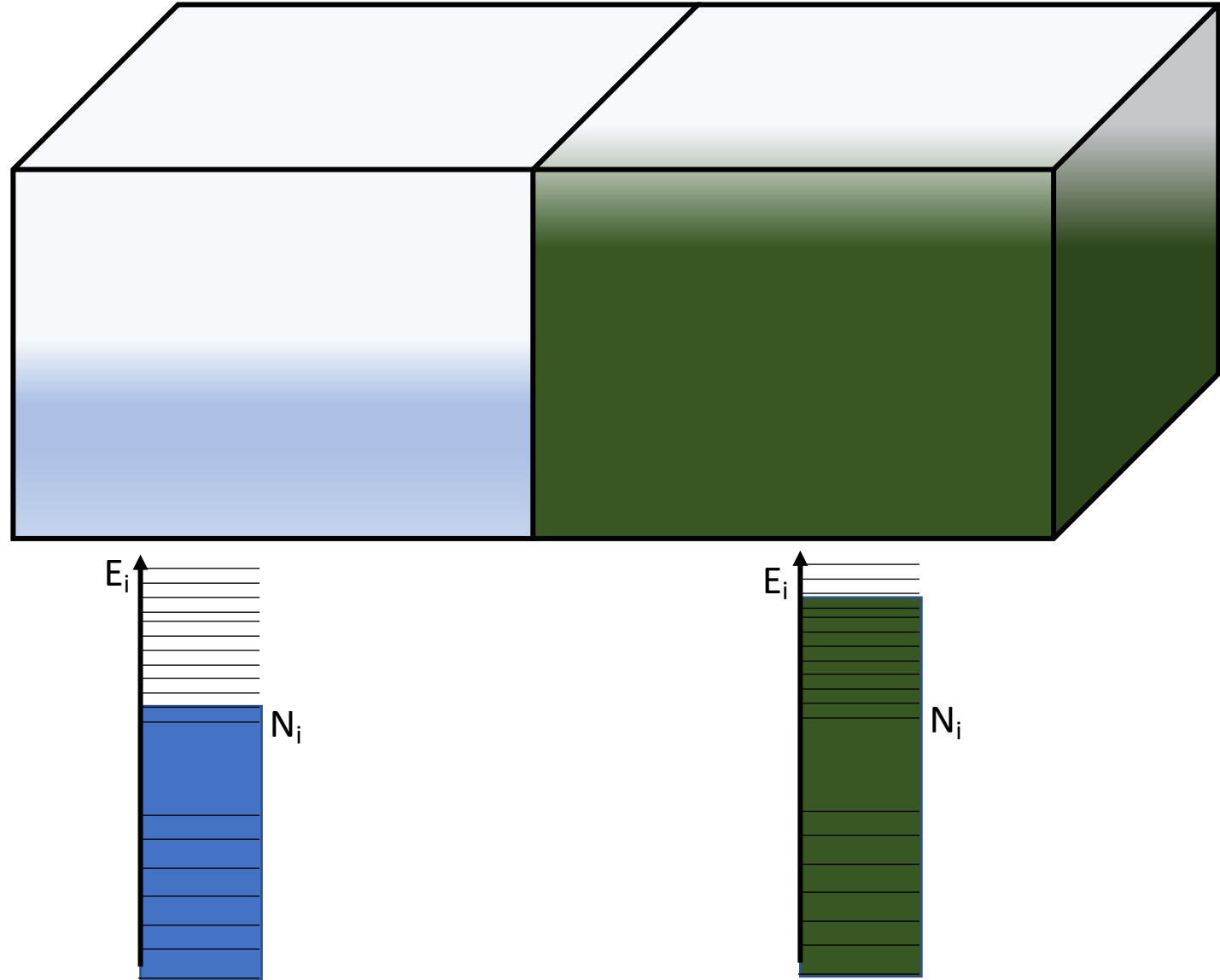


What should be constant when two materials
with different Electronic density meet ?

Equilibrium in solid state

- For systems to be in equilibrium,
 - No net motion of electrons from one system to other!
- If at each energy E , number of electrons moving from material 1 to material 2 at same energy = number of electrons moving from material 2 to material 1

Then there won't be any net electron motion



Number of electrons moving from M1 -> M2 per unit time

$$R_{\rightarrow} \propto F_1(E)N_1(E) [1 - F_2(E)]N_2(E)$$

Number of electrons moving from M2 -> M1 per unit time

$$R_{\leftarrow} \propto F_2(E)N_2(E) [1 - F_1(E)]N_1(E)$$

At equilibrium, they need to match

$$R_{\rightarrow} = R_{\leftarrow}$$

$$F_1(E)N_1(E) [1 - F_2(E)]N_2(E) = F_2(E)N_2(E) [1 - F_1(E)]N_1(E)$$

$$F_1N_1N_2 - F_1F_2N_2N_1 = F_2N_2N_1 - F_1F_2N_2N_1$$

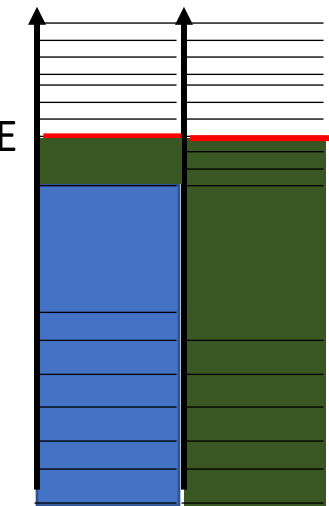
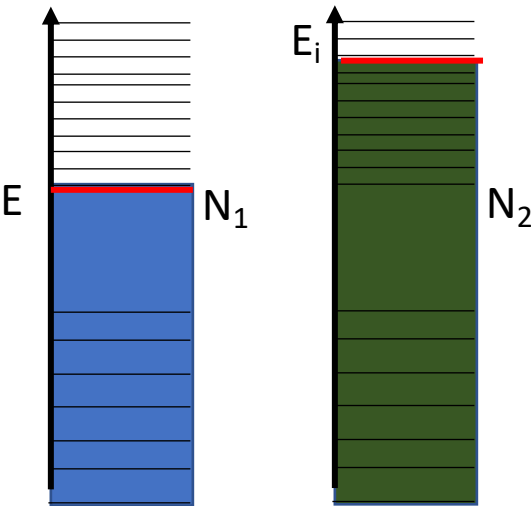
This can be true for any N1 and N2 iff $F_1 = F_2$

$$\left[1 + e^{\frac{E-E_{F1}}{KT}}\right] = \left[1 + e^{\frac{E-E_{F2}}{KT}}\right]$$

$$E_{F1} = E_{F2}$$

At equilibrium, the Fermi levels must match!

At thermal equilibrium, when there is no net motion of electrons, $\frac{dE_F}{dr} = 0$



Processes at equilibrium

- Generation and Recombination

At equilibrium, $\frac{dn(t)}{dt} = \frac{dp(t)}{dt} = 0$

We showed previously, recombination rate $R = \alpha n_i^2$

So, at thermal equilibrium, generation rate should also be $R = G = \alpha n_i^2$

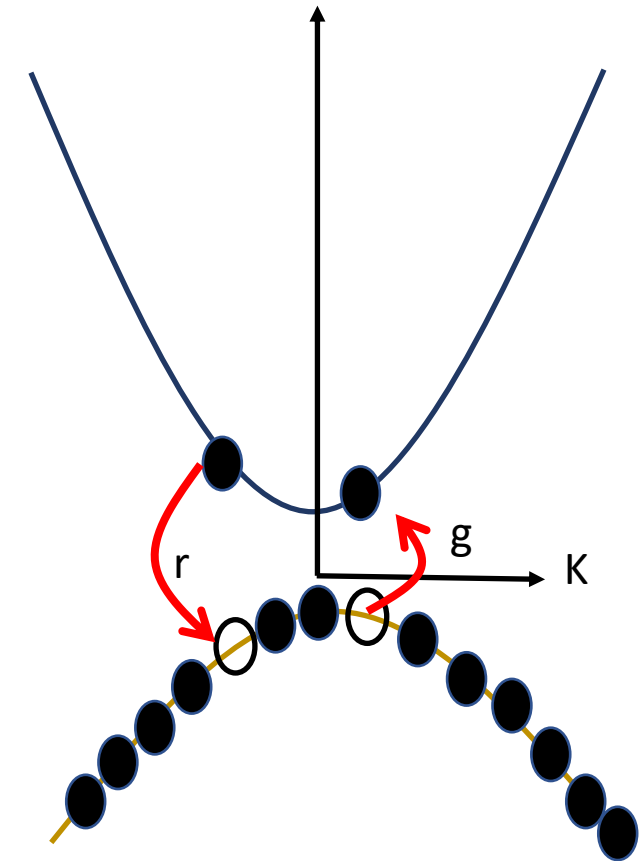
If, things are not at equilibrium,

$$\frac{dn(t)}{dt} = \alpha_g n_i^2 - \alpha_r n(t)p(t)$$

The change in the carrier densities can be taken to be small:

$$n(t) = n_0 + \delta n(t) \text{ and } p(t) = p_0 + \delta p(t)$$

$\delta n(t)$ and $\delta p(t)$ are called as excess carrier densities



Generation is assumed only by thermal means

$$\frac{d\delta n(t)}{dt} = \alpha_g n_i^2 - \alpha_r [n_0 + \delta n(t)][p_0 + \delta p(t)]$$

$$\frac{d\delta n(t)}{dt} = \alpha_g n_i^2 - \alpha_r [n_0 + \delta n(t)][p_0 + \delta p(t)]$$

$$\frac{d\delta n(t)}{dt} = -\alpha_r [p_0 \delta n(t) + n_0 \delta p(t) + \delta n(t) \delta p(t)]$$

Assume some disturbance in a material at equilibrium

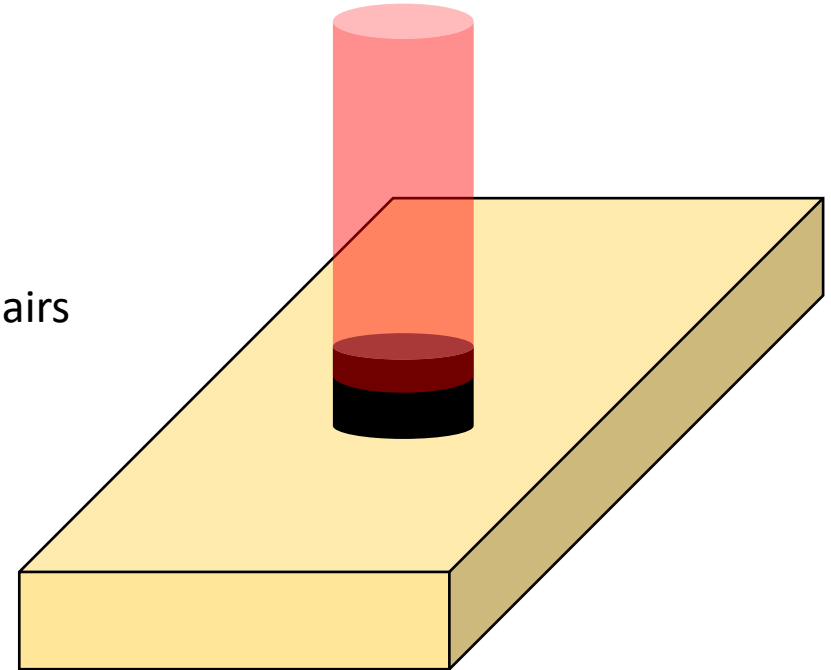
Like shining light, excess carriers , electrons and holes are created in pairs

$$\delta n(t = 0) = \delta p(t = 0)$$

If we assume, the change in density is small, and the material is of say p-type

$$\delta n(t) \delta p(t) \rightarrow 0$$

In a p-type material, $p_0 \gg n_i$
 Law of mass action $n_0 p_0 = n_i^2$
 When, $n_0 = n_i^2 / p_0$, $n_0 \ll p_0$



The excess carriers in a p-type semiconductor

$$\frac{d\delta n(t)}{dt} = -\alpha_r p_0 \delta n(t)$$

The solution to this equation is an exponentially decaying function

The excess carriers will die exponentially if the light (perturbation) is switched off.

$$\delta n(t) = Ae^{-\alpha_r p_0 t} = Ae^{-t/\tau}$$

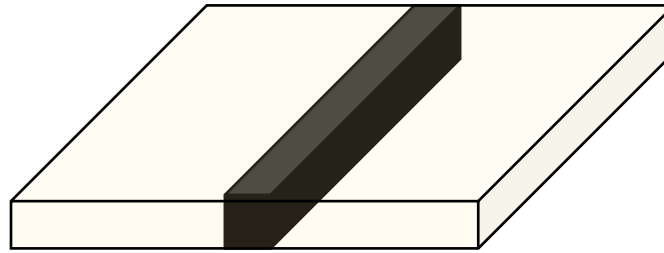
Where $\tau = \frac{1}{\alpha_r p_0}$ is called as the relaxation time constant.

That means, in a heavily doped semiconductor, the excess carriers dies much faster than a lightly doped semiconductor.

Diffusion in semiconductors

What happens, if we disturb the equilibrium in a material, introduce inhomogeneity

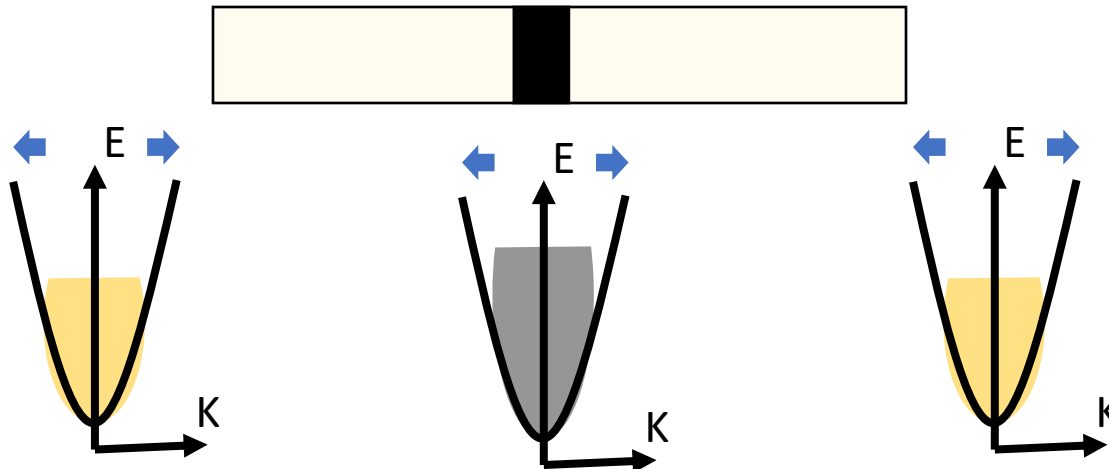
Introduce excess carriers in the Middle of a semiconductor.

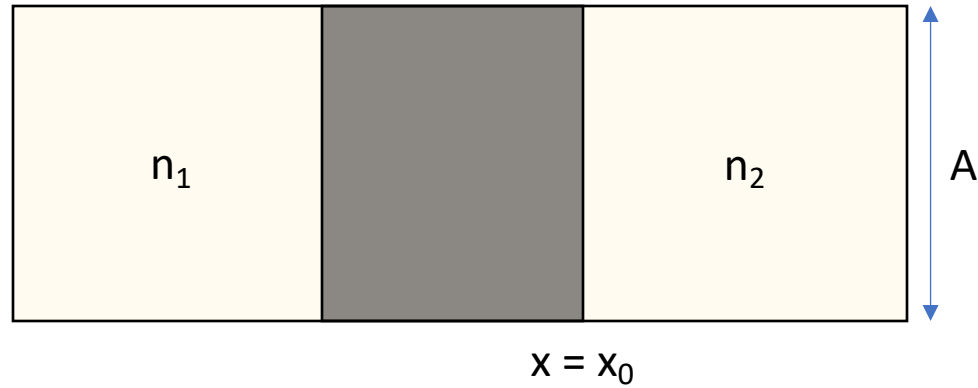


What happens to the carriers ?

Why do they behave in the way you expect ?

Assume we are at $T = 0\text{K}$, all states below Fermi level is filled as per the definition.
We have three regions





If $\bar{t}$ is the mean scattering time of electrons, and $\bar{l}$ is the scattering length

$$\text{Number of electrons passing } x_0 \text{ from left} = \frac{1}{2} n_1 \bar{l} A$$

$$\text{Number of electrons passing } x_0 \text{ from right} = \frac{1}{2} n_2 \bar{l} A$$

$$\text{Rate at which electrons go from black to white is } \phi_n(x_0) = \frac{\bar{l}}{2\bar{t}} (n_1 - n_2)$$

If the mean scattering length is very small $\bar{l} = \delta x$

$$\left. \frac{dn}{dx} \right|_{x=x_0} = (n_2 - n_1) / \delta x$$

$$\phi_n(x_0) = - \frac{\bar{l}^2}{2\bar{t}} \frac{dn}{dx}$$

The electron flux

$$\phi_n(x_0) = -\frac{\bar{l}^2}{2\bar{t}} \frac{dn}{dx}$$

The hole flux

$$\phi_p(x_0) = -\frac{\bar{l}^2}{2\bar{t}} \frac{dp}{dx}$$

The term $D_n = \frac{\bar{l}^2}{2\bar{t}}$ is called as diffusion constant of electrons

The negative sign means the charge flux is opposite to the gradient.
Or the charge flux is in direction of reducing charge concentration.
The flux direction is same irrespective of electrons/holes.

$$\text{Current } J_n = -q\phi_n$$

$$J_n = qD_n \frac{dn}{dx}$$

$$\text{Current } J_p = q\phi_p$$

$$J_p = -qD_p \frac{dp}{dx}$$

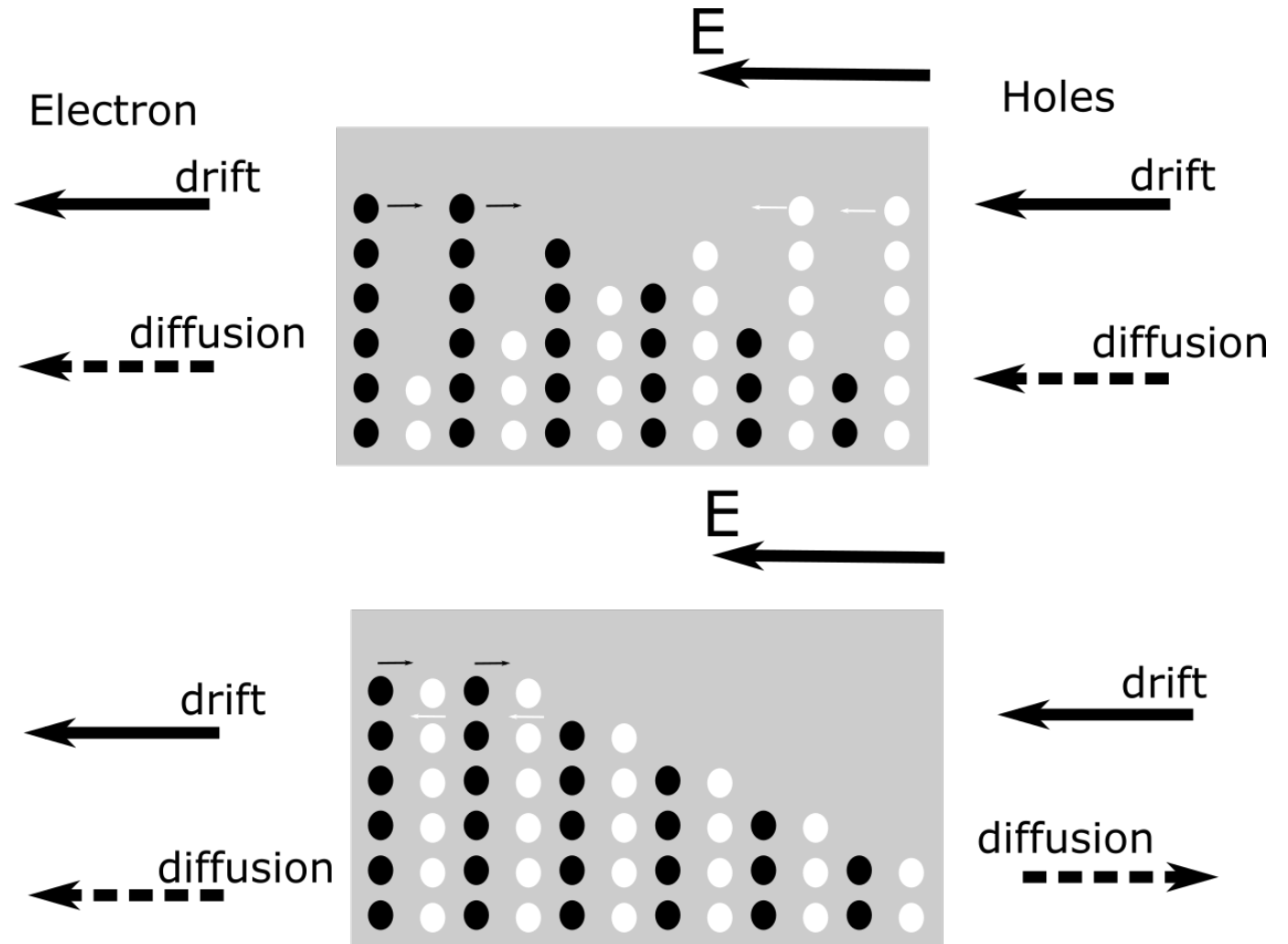
Total current in a material:

If there exists electric field E and a carrier gradients $\frac{dn}{dx} \neq 0$ and $\frac{dp}{dx} \neq 0$

The total current in the material

$$J_n(x) = qn(x)\mu_n E(x) + qD_n \frac{dn(x)}{dx}$$

$$J_p(x) = qp(x)\mu_p E(x) - qD_p \frac{dp(x)}{dx}$$



Minority carriers contribute strongly to current through diffusion

Effect of electric field

Can be studied in three situations: 1. thermal equilibrium, 2. steady state and 3. transient

Here let us study the first two: Field in a material at thermal equilibrium.

Assume a constant field across a material which does not have any loops

No current in the material at thermal equilibrium.

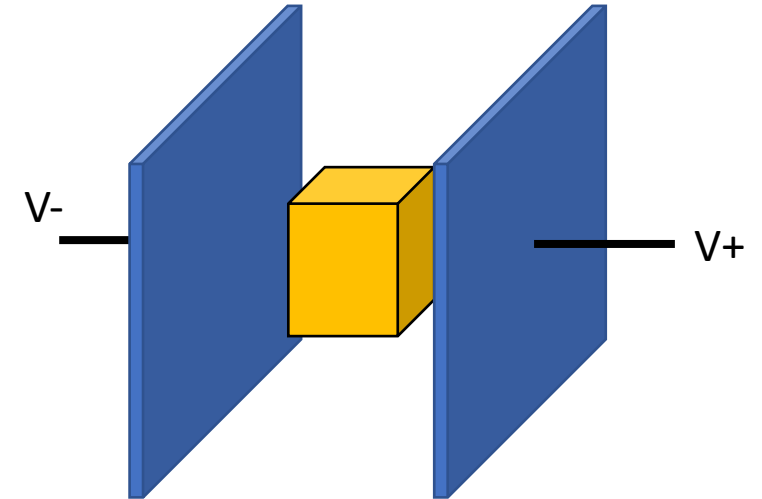
What happens to the electric field ?

$$\text{Electric field } \mathcal{E} = -\frac{dV(x)}{dx}$$

In the bands, the electron energy E is related to the potential (V) by $V = \frac{E}{-q}$

$$\text{Therefore the field at any point } x \text{ is given by } \mathcal{E} = -\frac{d}{dx} \left[\frac{E(x)}{-q} \right] = \frac{1}{q} \frac{dE}{dx}$$

Therefore, when there is an external applied electric field, the electron energies in the band has a nonzero spatial gradient



At thermal equilibrium $J = 0$

$$J_n = J_p = 0$$

For holes

$$J_n = qn(x)\mu_n\mathcal{E} + qD_n\frac{dn(x)}{dx} = 0$$

$$J_p = qp(x)\mu_p\mathcal{E} - qD_p\frac{dp(x)}{dx} = 0$$

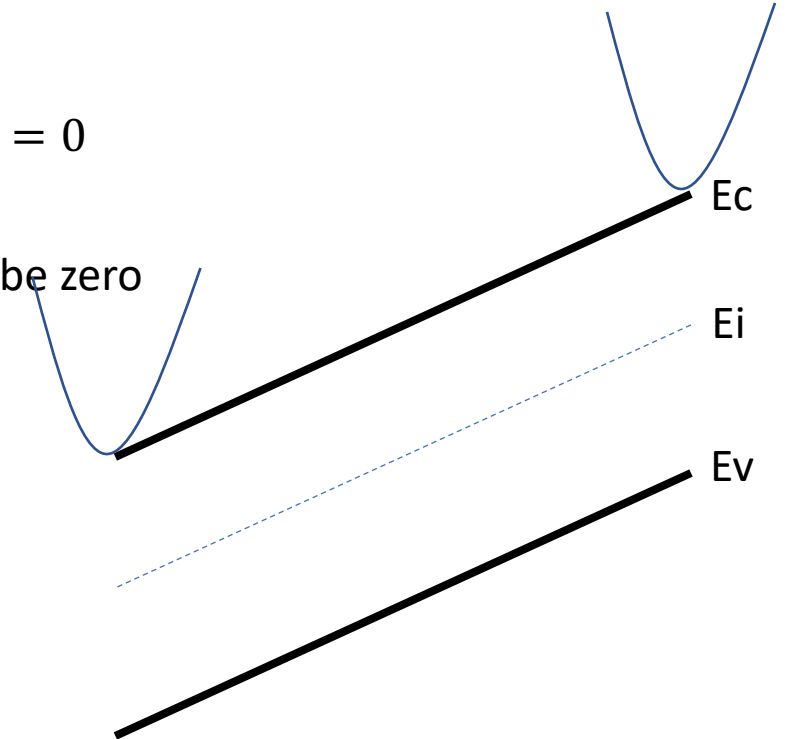
Both the electron and hole current at equilibrium needs to be zero

$$\mathcal{E} = -\frac{D_n}{\mu_n} \frac{1}{n(x)} \frac{dn(x)}{dx}$$

In any material, we know that $n = n_i e^{(E_f - E_i)/KT}$

$$\frac{dn(x)}{dx} = n_i e^{(E_F - E_i)/KT} \left[\frac{1}{KT} \left(\frac{dE_F}{dx} - \frac{dE_i}{dx} \right) \right]$$

$$\frac{1}{n(x)} \frac{dn(x)}{dx} = \left[\frac{1}{KT} \left(\frac{dE_F}{dx} - \frac{dE_i}{dx} \right) \right]$$



At thermal equilibrium, the Fermi level lies flat: $\frac{dE_F}{dx} = 0$

$$\text{Thus, } \frac{1}{n(x)} \frac{dn(x)}{dx} = \left[\frac{1}{KT} \left(-\frac{dE_i}{dx} \right) \right]$$

We saw that Electric field induces gradient in the band at equilibrium

$$\mathcal{E} = -\frac{d}{dx} \left[\frac{E(x)}{-q} \right] = \frac{1}{q} \frac{dE}{dx}$$

$$\frac{1}{n(x)} \frac{dn(x)}{dx} = \left[\frac{1}{KT} - q\mathcal{E} \right]$$

We started with

$$\mathcal{E} = -\frac{D_n}{\mu_n} \frac{1}{n(x)} \frac{dn(x)}{dx} \qquad \mathcal{E} = -\frac{D_n - q\mathcal{E}}{\mu_n KT}$$

Einstein's Diffusion equation

$$\frac{D_n}{\mu_n} = \frac{KT}{q}$$

D_n is the ease of diffusion

μ_n is the ease of drift

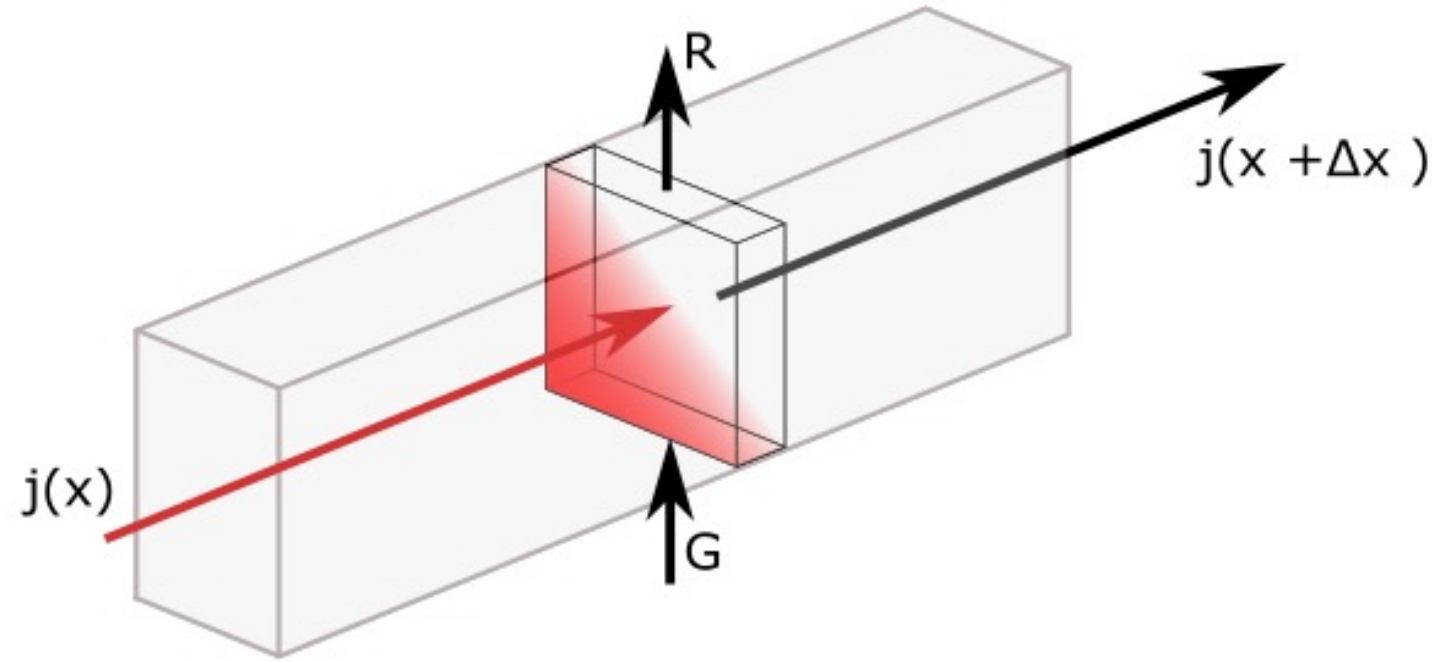
They are related by simple thermodynamic constant

Away from equilibrium

Consider a current J across a cross section A

A small section Δx taken in the proximity of x

P is the density of holes per unit volume, and the change in the density of holes per unit volume around the point x is given by



$$\frac{dp}{dt} = G - R + \frac{[j(x) - j(x + \Delta x)]}{q\Delta x}$$

If Δx is infinitesimally small, $\frac{dJ}{dx} = \frac{j(x+\Delta x) - j(x)}{\Delta x} \big|_{\Delta x \rightarrow 0}$

$$\frac{dp}{dt} = G - R + \frac{1}{q} \frac{dJ}{dx}$$

$$\frac{dp}{dt} = G - R + \frac{1}{q} \Delta \cdot J$$

Generalized continuity equation

Generation in semiconductor $G = \alpha_R n_i^2$

Recombination in semiconductor $R = \frac{\delta p}{\tau_p}$

Divergence of current $\Delta J = \frac{d}{dx} \left[qp(x)\mu_p E - qD_p \frac{dp}{dx} \right]$

For a holes

$$\frac{dp}{dt} = G - \frac{\delta p}{\tau_p} - \mu_p \frac{d(pE)}{dx} + D_p \frac{d^2 p}{dx^2}$$

For a electrons

$$\frac{dn}{dt} = G - \frac{\delta n}{\tau_n} + \mu_n \frac{d(nE)}{dx} + D_n \frac{d^2 n}{dx^2}$$

Typically, majority carriers are so large in number that their time dynamics are mostly insignificant

Therefore, for minority holes in a n-type semiconductor

$$\frac{dp}{dt} = G - \frac{\delta p}{\tau_p} + D_p \frac{d^2 p}{dx^2}$$

For minority electrons in p-type semiconductor

$$\frac{dn}{dt} = G - \frac{\delta n}{\tau_n} + D_n \frac{d^2 n}{dx^2}$$

An Example:

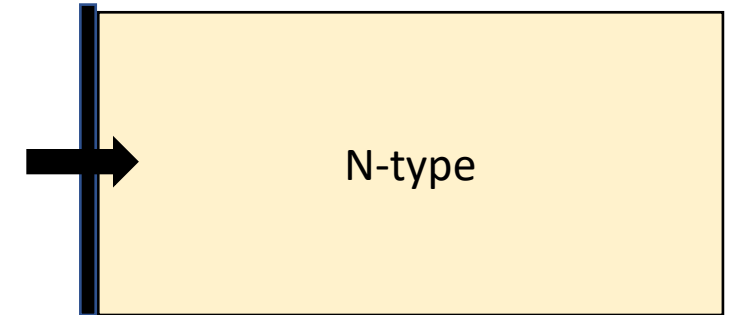
Steady state injection of minority carriers in a semiconductor

Assume a n-type semiconductor with doping density $N_D \text{ cm}^{-3}$

If we assume all the dopants are ionized, at given temperature,
The equilibrium number of electrons are given by

$$n_0 = N_D$$

The equilibrium number of holes $p_0 = \frac{n_i^2}{n_0}$



If we assume a fictional source of holes from one side, capable of injecting a constant density of holes into the n type semiconductor

At $x= 0$, saw a density Δp is injected

Given all the processes in a semiconductor, how do we expect the carriers to behave ?

We are looking at steady state solutions:

The property of steady state is no variations in time.

$$\frac{dp}{dt} = 0$$

Substituting this into the continuity equation

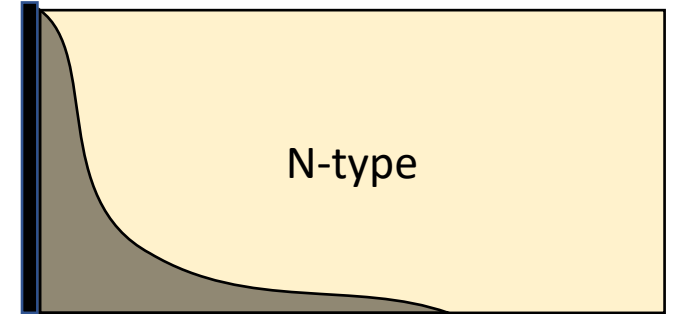
$$\frac{dp}{dt} = G - \frac{\delta p}{\tau_p} + D_p \frac{d^2 p}{dx^2} = 0$$

We again assume, the generation is not strong enough to influence the time dynamics, atleast near the source.

$$D_p \frac{d^2 p}{dx^2} = \frac{\delta p}{\tau_p}$$

The hole concentration can be written as $p = p_0 + \delta p$

$$\frac{d^2 \delta p}{dx^2} = \frac{\delta p}{D_p \tau_p}$$



Therefore,

$$\delta p = Ae^{-x/L_p} + C$$

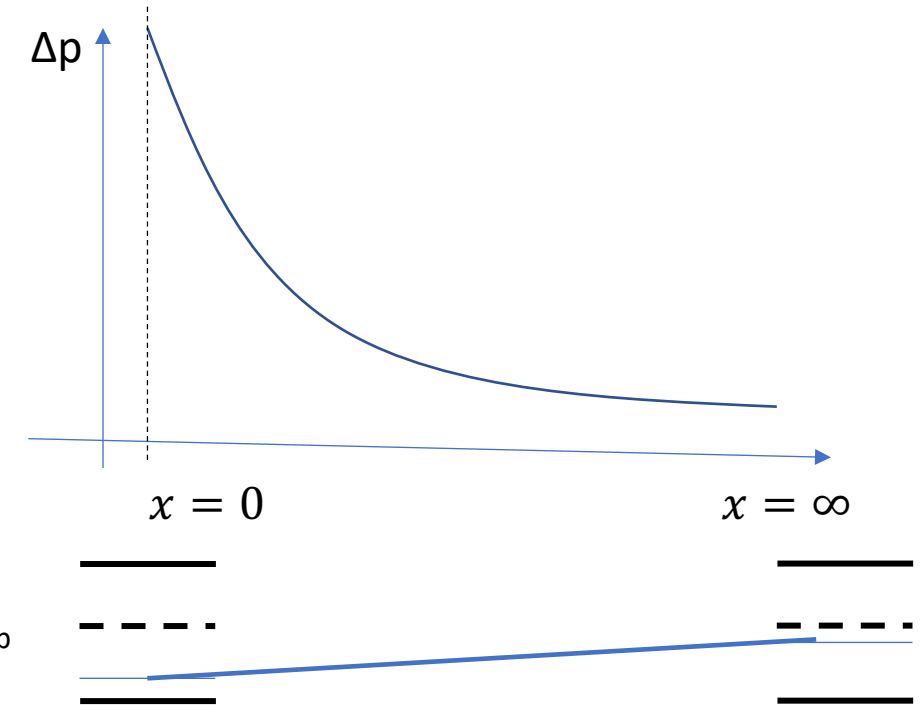
Is a solution to the problem. We use the boundary conditions

$$\delta p(x = 0) = \Delta p$$

$$\delta p(x \rightarrow \infty) = 0$$

Therefore the solution is $\delta p = \Delta p e^{-\frac{x}{L_p}}$

The full solution is $p(x) = p_0 + \Delta p e^{-\frac{x}{L_p}}$



A few conclusions from this:

1. The average length an injected hole travels before recombination: L_p
2. Diffusion current of holes $J_p = \frac{qD_n\Delta P}{L_p} e^{-\frac{x}{L_p}}$
3. $p(x) = n_i e^{\frac{E_i - E_F}{KT}}$. Since, the carrier density is not constant, the Fermi level is not constant. Hence, a quasi-Fermi level is used to describe this behavior.
4. Since, the carrier density varies exponentially with distance, density is exponential of QFL, QFL varies linearly with distance!

$$p(x) = n_i e^{\frac{E_i - F_p}{KT}}$$

Diffusion current is given by $J_p = -qD_p \frac{d}{dx} \left(n_i e^{\frac{E_i - F_p}{KT}} \right)$

In a general case: $J_p = qp(x)\mu_p \mathcal{E} - \frac{qD_p p(x)}{KT} \left[\frac{dE_i}{dx} - \frac{dF_p}{dx} \right]$

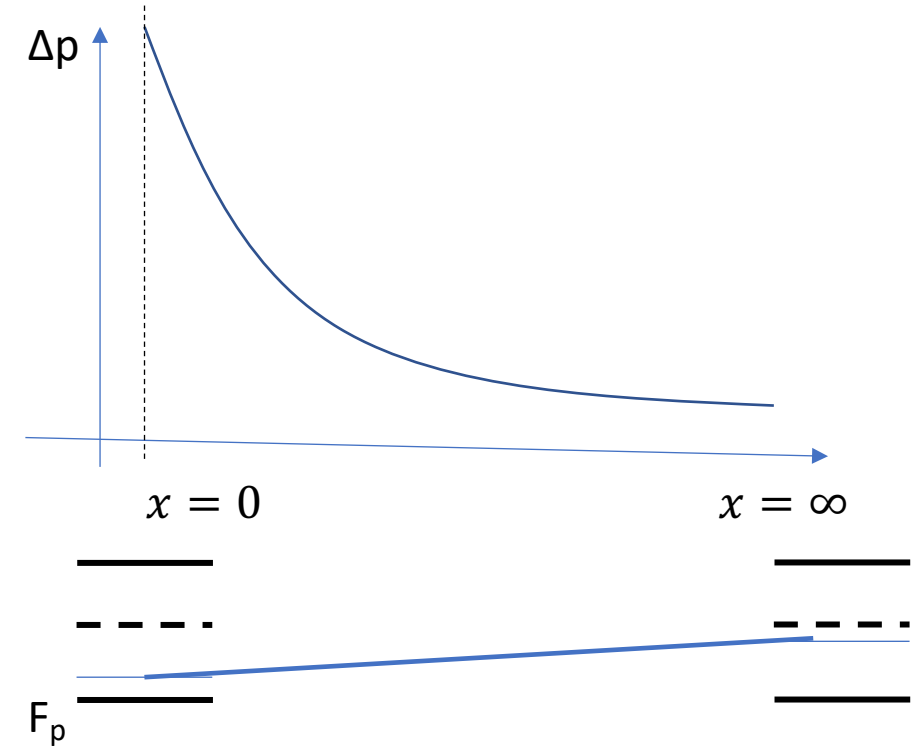
$$J_p = qp(x)\mu_p \mathcal{E} - \mu_p p(x) \left[\frac{dE_i}{dx} - \frac{dF_p}{dx} \right]$$

$$J_p = qp(x)\mu_p \mathcal{E} - \mu_p p(x) \left[q\mathcal{E} - \frac{dF_p}{dx} \right]$$

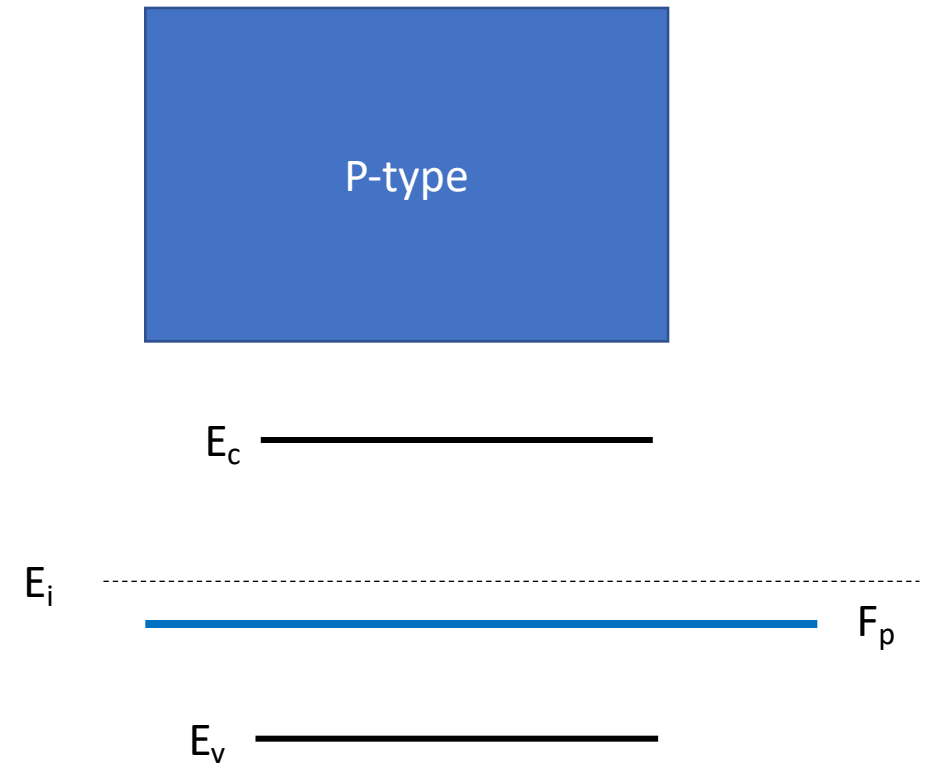
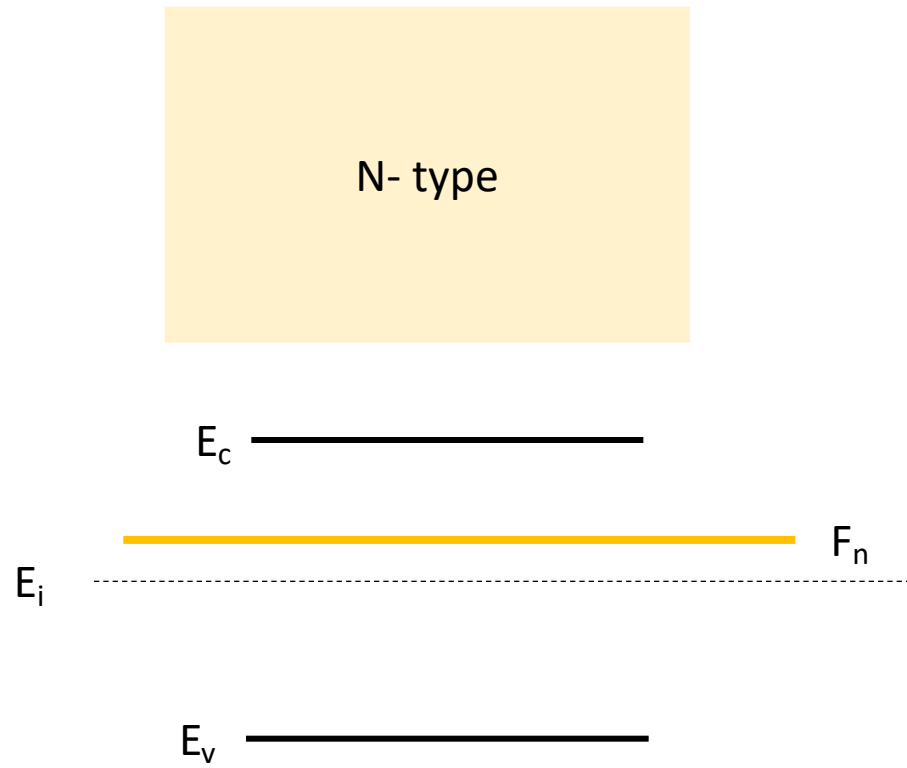
$$J_p = \mu_p p(x) \left[\frac{dF_p}{dx} \right]$$

Thus the total current is rewritten as $J = q\mu_p p(x) \frac{dF_p/q}{dx}$

This is a re-interpretation of the Ohms law $J = \sigma \frac{dF_p/q}{dx}$



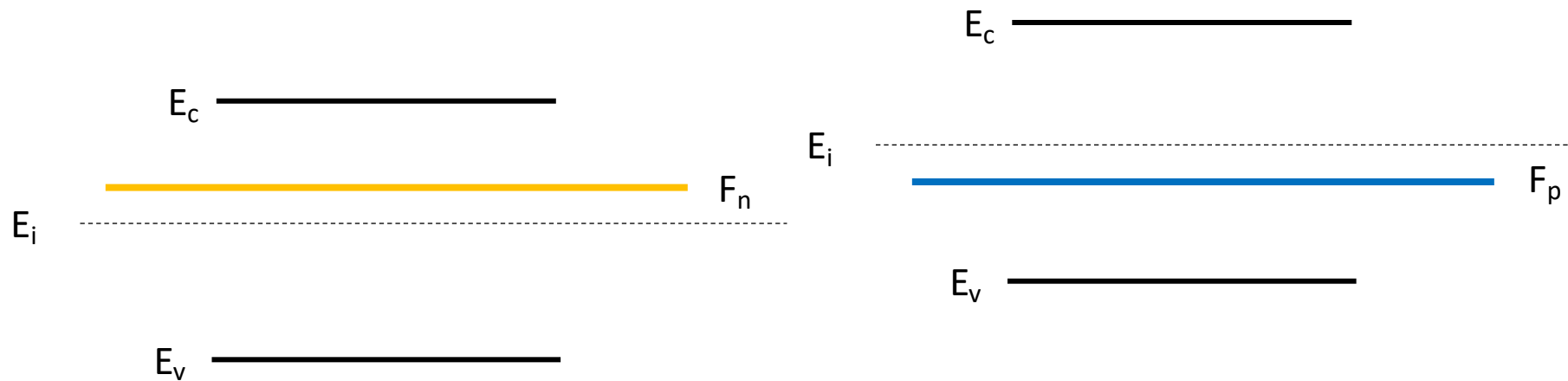
Consider a junction formation between two types of semiconductors



We look at the thermal equilibrium between the two materials:

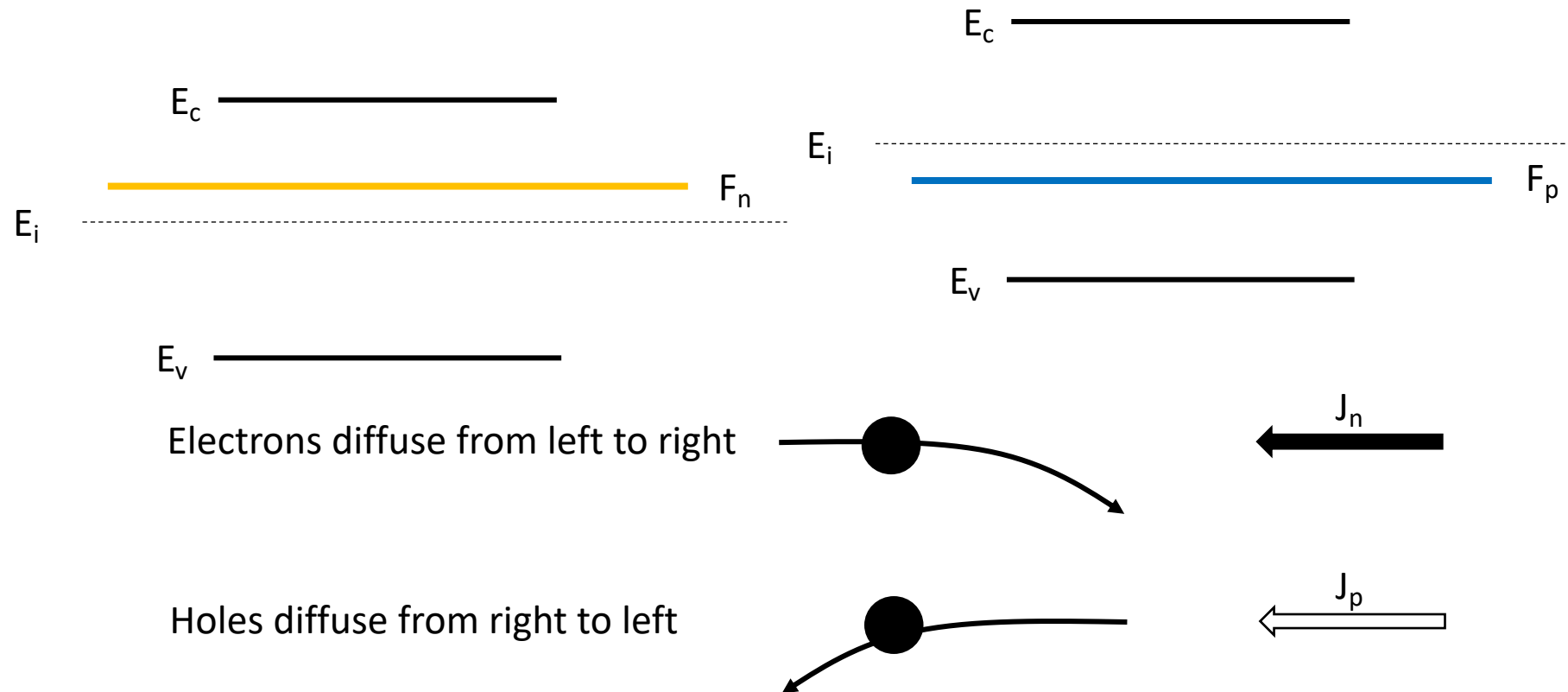


Net current should be zero at equilibrium. -> Fermi levels (quasi ones) should be flat!





Charge inhomogeneity should lead to diffusion in the carriers from high concentration to lower concentration



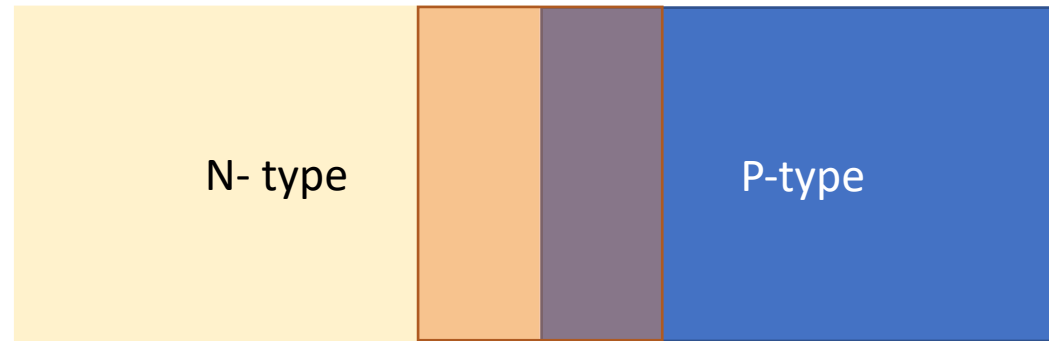
Still at equilibrium, so the total current should be zero.

Since, both the electron and hole current are in the same direction

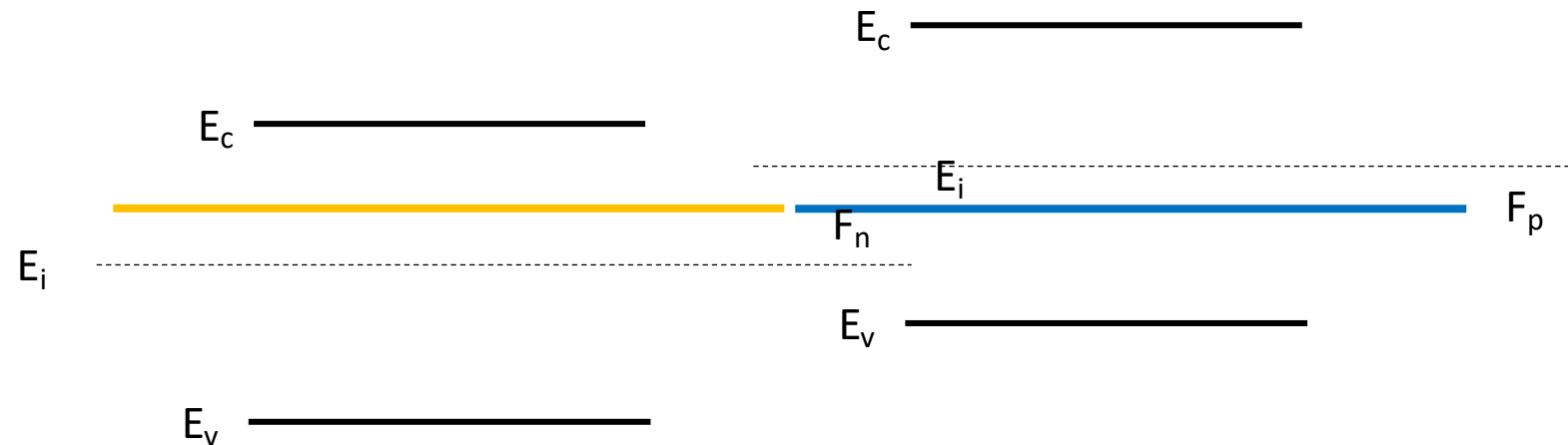
$$J_n = J_p = 0$$

$$J_n = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx} = 0$$

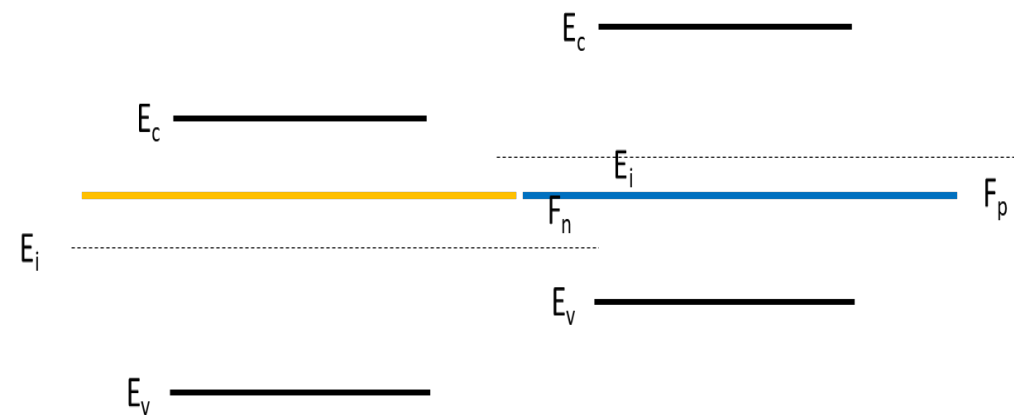
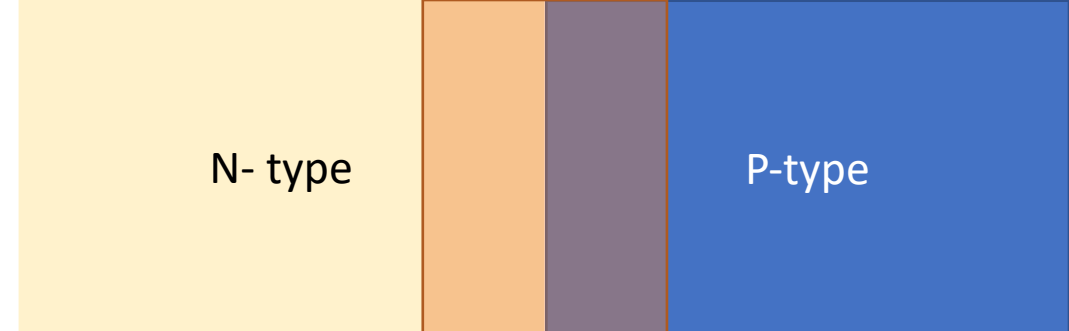
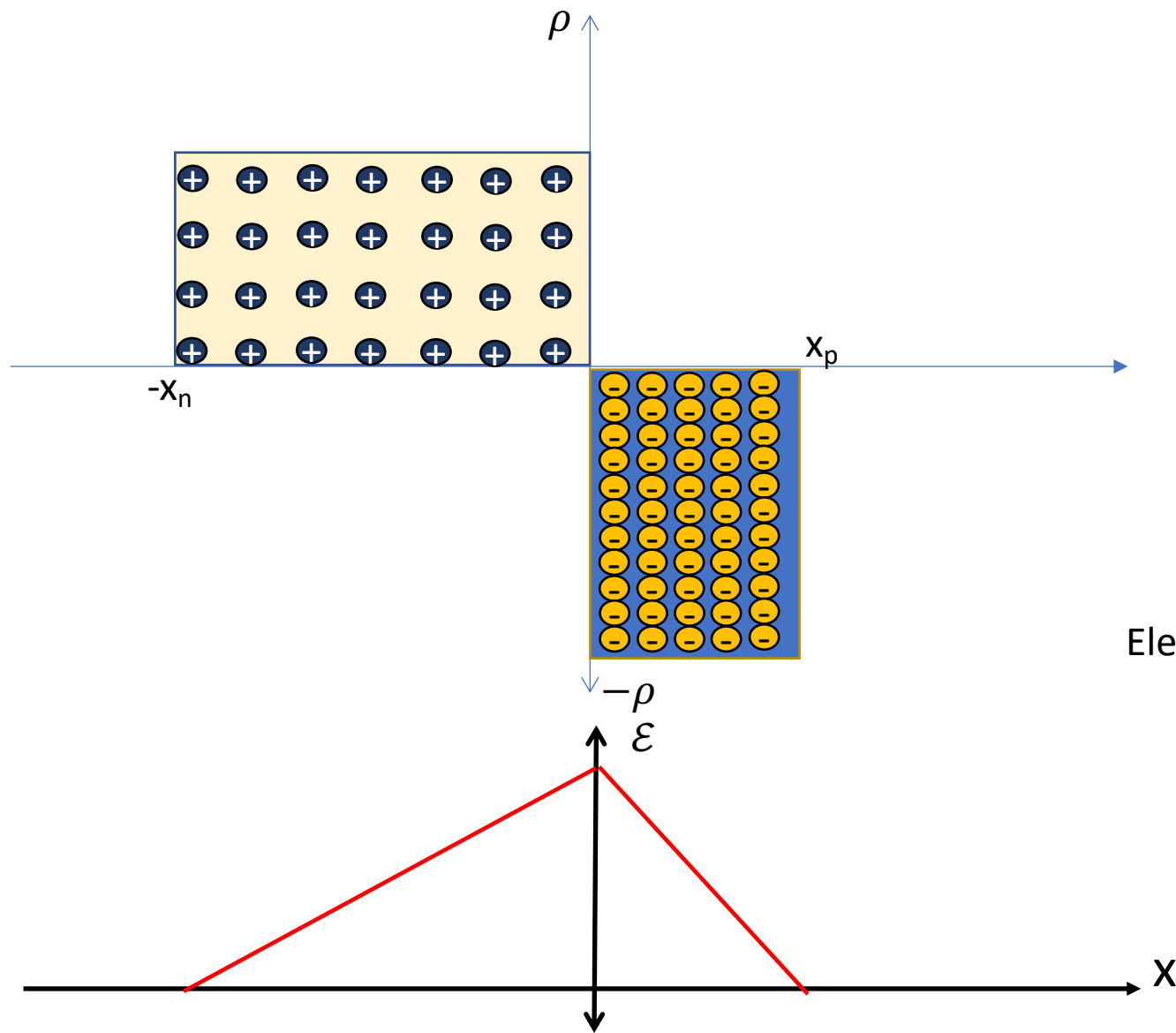
Once equilibrium is reached: an electric field $\mathcal{E}$ is developed at the interface such that the drift current is compensated



So what happens at the interface ?



The diffused charges leave behind charged impurities

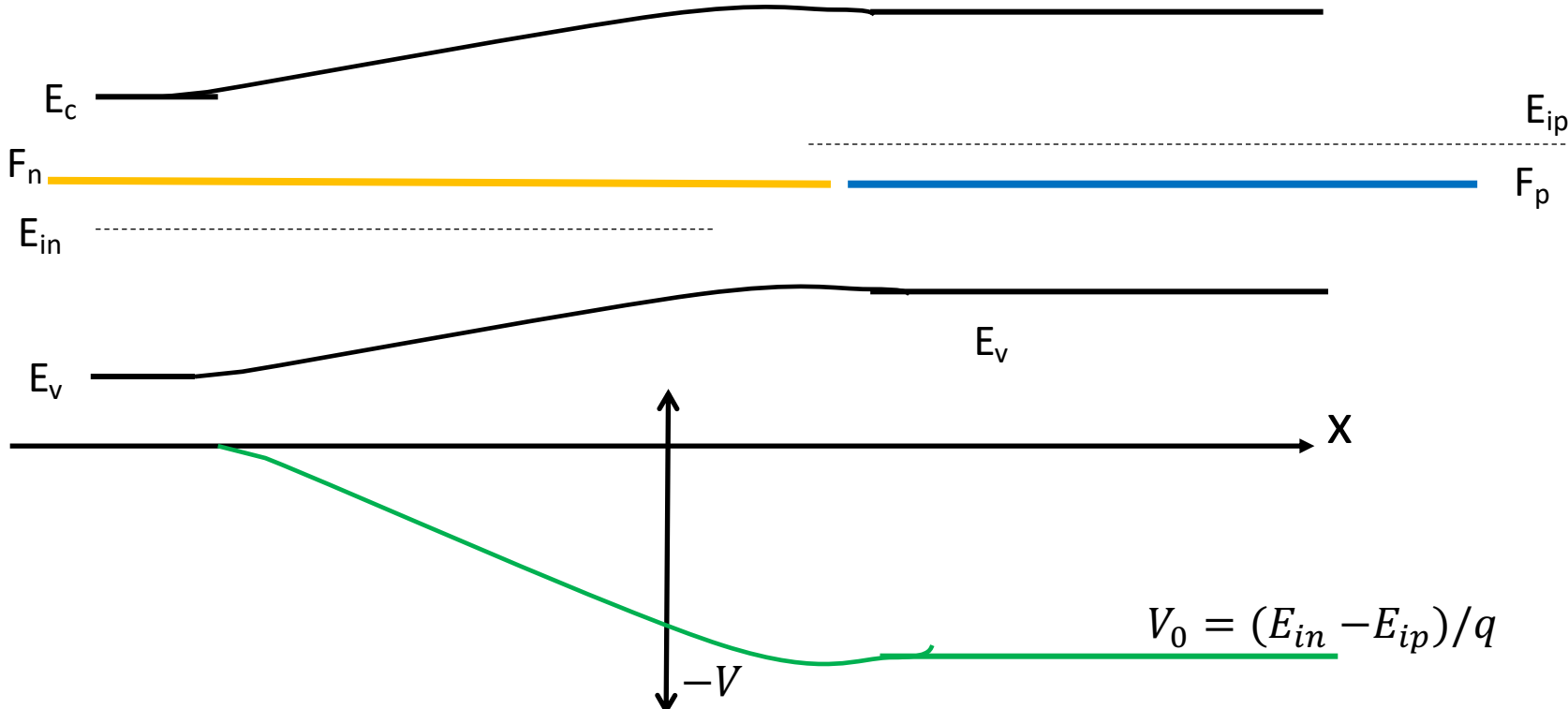
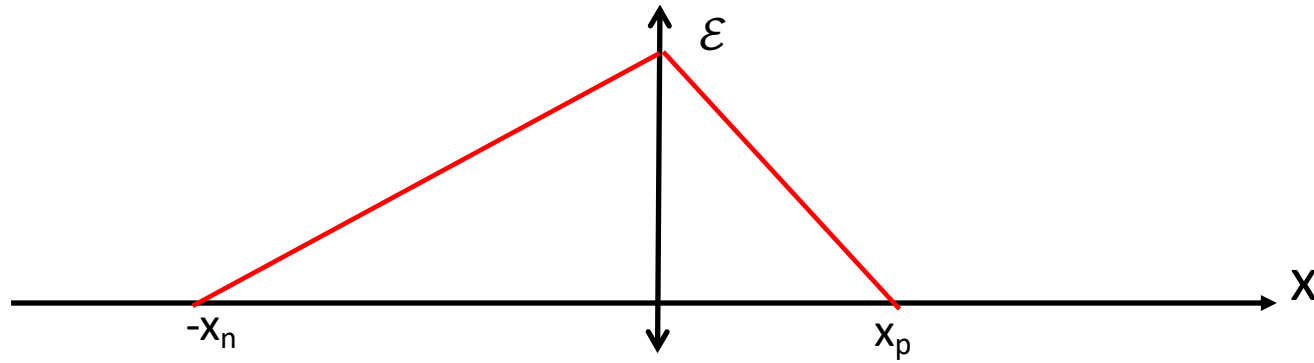


Electric field gradient is proportional to charge density

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon}$$

$$\rho = q[p - n + N_D^+ + N_A^-]$$

$$\text{For } -x_n < x < 0 \quad \mathcal{E} = qN_D(x + x_n)$$



$$\text{For } -x_n < x < 0 \quad \mathcal{E} = qN_D(x + x_n)$$

$$\mathcal{E}_m = qN_D x_n$$

$$\text{For } 0 < x < x_p \quad \mathcal{E} = -qN_A(x - x_p)$$

$$\mathcal{E}_m = qN_A x_p$$

$$-\frac{dV_0}{dx} = \mathcal{E}$$

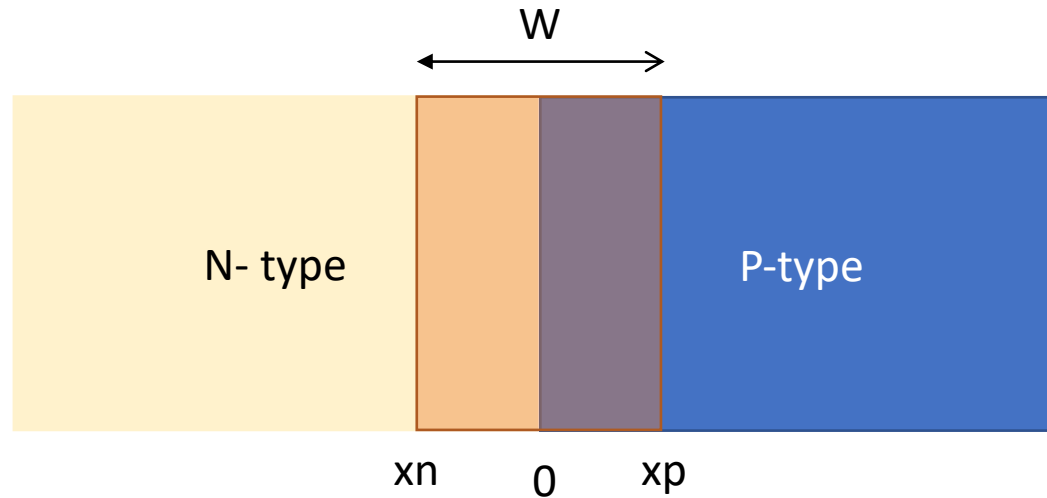
$$V_0(x) = \int -E \, dx$$

$$V_0(x) = \mathcal{E}_m \left(x - \frac{x^2}{2W} \right)$$

$$\text{Where } W = x_n + x_p$$

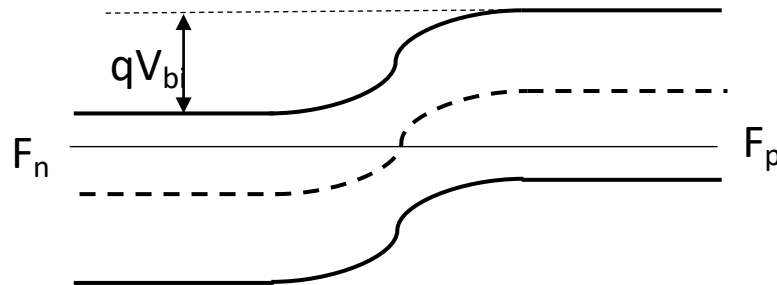
$$\text{At } x = x_p \quad V_0(x) = V_{bi} = -\frac{1}{2}\mathcal{E}_m W$$

At equilibrium, the charges are depleted near the junction forming depletion width and built-in potential



$$V_{bi} = \frac{1}{2} \epsilon_m W = \frac{1}{2} q N_D x_n W$$

$$V_{bi} = \frac{1}{2} \frac{q}{\epsilon} \frac{(N_A N_D)}{N_A + N_D} W^2$$



The region sandwiched between the two materials is depleted of carriers as the intrinsic level comes close to the quasi-Fermi levels indicating the reduction in the carrier density

Assumption 1: There is no potential drop in the neutral regions, contacts

Common question: do you have such built-in potential in all interfaces ? If so, how do we even measure using a multimeter?
You always have a built-in potential affecting your measurement?

Assumption 1: There is no potential drop in the neutral regions, contacts

What happens in the presence of bias:

