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Minimum Cost Design of Lined Canal Sections

PRABHATA K. SWAMEE[1], GOVINDA C. MISHRA[2] and
BHAGU R. CHAHAR[1]
[1] *Department of Civil Engineering, University of Roorkee, Roorkee 247 667, India*
[2] *Nat. Inst. of Hydrology, University of Roorkee, Roorkee 247 667, India,*
e-mail: civil@rurkiu.ernet.in

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Abstract. Though the minimum area section is generally adopted for lined canals, it is not the best section as it does not involve lining cost, and the cost of earthwork which varies with the excavation depth. On account of complexities of analysis, the minimum cost design of lined canal sections has not been attempted as yet. In this investigation, explicit equations and section shape coefficients for the design variables of minimum cost lined canal sections for triangular, rectangular, trapezoidal, and circular shapes have been obtained by applying the nonlinear optimization technique. Application of the proposed design equations along with the tabulated section shape coefficients results directly in the optimal dimensions of a lined canal without going through the conventional trial and error method of canal design. The optimal cost equation along with the corresponding section shape coefficients is useful during the planning of a canal project.

Key words: canal, canal design, earthwork, hydraulic structures, lining, optimal section, uniform flow.

Notation

The following symbols are used in this paper:

A	flow area of canal [m^2];
a	flow area up to elevation η [m^2];
b	bed width of canal [m];
C	cost per unit length of canal [\$ m^{-1}];
C_e	earthwork cost per unit length of canal [\$ m^{-1}];
c_e	unit cost of earthwork at ground level [\$ m^{-3}];
C_L	lining cost per unit length of canal [\$ m^{-1}];
c_L	unit cost of lining [\$ m^{-2}];
c_r	additional cost of excavation per unit depth [\$ m^{-4}];
D	diameter of canal [m];
F	cost of canal [\$];
g	gravitational acceleration [m s^{-2}];
k_{fs}	section shape coefficients corresponding to subscripts f and s;
L	length scale [m];

m	side slope of canal;
P	flow perimeter of canal [m];
p	penalty parameter;
Q	discharge [$\mathrm{m}^3\ \mathrm{s}^{-1}$];
R	hydraulic radius [m];
S_0	bed slope of canal;
V	average velocity [$\mathrm{m\ s}^{-1}$];
V_L	limiting velocity [$\mathrm{m\ s}^{-1}$];
y_n	normal depth of flow in canal [m];
$	dollar [monetary unit];
ε	average roughness height of canal lining [m];
η	vertical coordinate;
λ	length scale [m];
ν	kinematic viscosity [$\mathrm{m}^2\ \mathrm{s}^{-1}$];
ϕ	equality constraint;
ψ	augmented function.

Subscript

c	cost;
b	bed width;
D	diameter;
e	earthwork;
L	lining;
m	side slope;
o	depth independent earthwork case;
r	depth dependent earthwork case;
y	normal depth;
$*$	nondimensional.

Superscript

$*$	optimal.

1. Introduction

Lining of a canal is essential for efficient use of land and water resources. Control of seepage saves water for further extension of the irrigation network as well as reduces the water logging in the adjoining areas. The smooth surface of lining reduces the friction slope, which enables the canal to be laid on a flatter bed slope. This increases the command area of the canal. On the other hand, as the lining permits higher average velocities, the canal can be laid on steeper slopes to save the cost of earthwork in formation. As the lining provides a rigid boundary, it ensures protection against bed and bank erosion. When the canal is constructed in an area containing black cotton soil, the canal lining becomes essential for stability of the canal banks. Thus, the maintenance cost gets reduced.

Canals conveying water in arid and semi arid regions need to be lined prior to conveying water in them. This is because, for a newly completed irrigation project, seepage from the canal is maximum as the ground water level is likely to be at a larger depth below the canal bed. Also in arid and semi arid regions the ground water is likely to be brackish and the seepage water which joins the ground water may not be withdrawn by pumping as the pumped water is unlikely to satisfy the irrigation water standards. Seepage control for existing canals with conventional lining is found to be prohibitive because of material costs and restriction on closure of the canal. If lining is envisaged in the planning stage, a smaller cross section could be adopted and lining can be justified from an economic point of view.

Several types of materials are used for canal lining (Sharma and Chawla, 1975). The choice of material mainly depends on the degree of water tightness required. Though less watertight, soil-cement lining and boulder lining are preferred on account of their low initial cost. Another low cost lining is composed of polyethylene plastic or alkathene sheets spread over the boundary surface with adequate earth cover. This type of lining is used for stable channels. Brick lining and brunt clay tile lining are the popular linings as they provide reasonable water tightness along with strength. For improving these qualities, double layers of brick or the brunt clay tiles are also provided. For canals carrying large discharges, *in situ* concrete lining or concrete tile linings are used. Low-density polyethylene (LDPE) or high-density polyethylene (HDPE) films of thickness 100 μm, 200 μm or more are sandwiched between two layers of canal linings to act as a second line of defense.

Lined canals are designed for uniform flow formula considering hydraulic efficiency, practicability, and economy (Streeter, 1945). Chow (1973) and French (1994) have listed various properties of the most hydraulically efficient sections. Swamee and Bhatia (1972) expressed all the channel dimensions in term of a length scale comprising independent design variables and developed curves for the optimal design of trapezoidal, rounded bottom and rounded corner sections. Guo and Hughes (1984) found that a channel narrower than the hydraulically best section results in minimum excavation when free board is taken into consideration. Monadjemi (1994) and Swamee (1995b) have done a comprehensive investigation for the optimal dimensions for various canal shapes. Canal cost and practicability requirements are combined into objective function and constraints in obtaining the optimal channel section by several investigators (Trout, 1982; Flynn and Marimno, 1987; Loganathan, 1991; Imam *et al*., 1991; Froehlich, 1994). Manning's equation as a uniform flow equation was used in the optimal channel design by the above investigators. A more general resistance equation based on roughness height was used in the optimal design of irrigation canals by Swamee (1995a). Thus the review of literature reveals that though considerable work has been reported on the design of minimum area cross section, practically no work has been done on the minimum cost lined canal sections. Minimum cost design of lined canals involves minimization of the sum of depth-dependent excavation cost and cost of lining subject to uniform flow condition in the canal, which results in nonlinear objective function and nonlinear equality constraint making the problem hard to solve analytically. In this paper, generalized empirical equations for design of minimum cost lined sections are obtained for triangular, rectangular, trapezoidal, and circular canals.

2. Resistance Equation

Uniform open channel flow is governed by the resistance equation. The most commonly used resistance formula is Manning's equation (Chow, 1973) which is applicable for rough turbulent flow, and in a limited bandwidth of relative roughness (Christensen, 1984). Relaxing these restrictions, Swamee (1994) gave the following resistance equation

$$V = -2.457\sqrt{gRS_0}\,\ln\left(\frac{\varepsilon}{12R} + \frac{0.221\nu}{R\sqrt{gRS_0}}\right), \tag{1}$$

where V = average flow velocity (m s^{-1}); g = gravitational acceleration (m s^{-2}); R = hydraulic radius (m) defined as the ratio of the flow area A (m^2) to the flow perimeter P (m); ε = average roughness height of the canal lining (m); and ν = kinematic viscosity of water (m^2 s^{-1}). Similar to the case of resistance equation for pipe flow, Equation (1) involves physically conceivable parameters ε and ν. The discharge Q (m^3 s^{-1}) was given by the continuity equation

$$Q = AV. \tag{2}$$

Combining Equations (1) and (2) the discharge was given by

$$Q = -2.457A\sqrt{gRS_0}\,\ln\left(\frac{\varepsilon}{12R} + \frac{0.221\nu}{R\sqrt{gRS_0}}\right). \tag{3}$$

3. Cost Structure

The objective function consists of the cost of the canal's unit length. This includes the costs of lining and of the earthwork. Considering the unit cost of lining (cost per unit surface area covered) to be independent of the depth of placement, the cost of lining C_L (monetary unit per unit length, e.g., \$ m^{-1}) was expressed as

$$C_L = c_L P. \tag{4}$$

where c_L = unit cost of lining (monetary unit per unit area of lining, e.g., \$ m^{-2}).

Table I. Lining and earthwork cost coefficients

Types of strata	c_L/c_e (m)									c_e/c_r (m)
	Type of lining									
	Concrete tile			Brick tile			Brunt clay tile			
	With LDPE film		Without film	With LDPE film		Without film	With LDPE film		Without film	
	100 μ	200 μ		100 μ	200 μ		100 μ	200 μ		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Ordinary soil	12.75	13.02	12.24	6.39	6.67	5.88	6.08	6.35	5.57	6.96
Hard soil	10.00	10.22	9.60	5.01	5.23	4.62	4.77	4.99	3.37	8.86
Impure lime nodules	8.90	9.10	8.55	4.47	4.66	4.11	4.25	4.44	3.89	9.96
Dry shoal with shingle	6.56	6.71	6.30	3.29	3.43	3.03	3.13	3.27	2.86	13.50
Slush and lahel	6.40	6.54	6.14	3.21	3.35	2.95	3.05	3.19	2.79	13.86

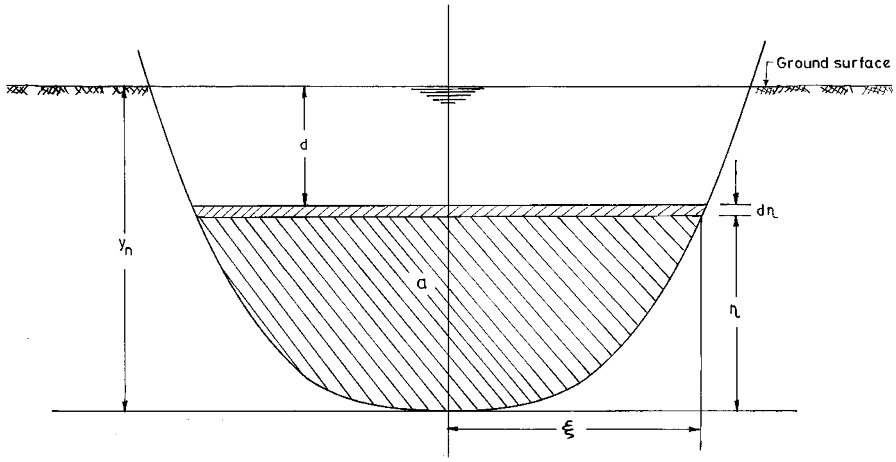

Figure 1. Definition sketch.

For a canal section with the normal water surface at the (average) ground level, as shown in Figure 1, the earthwork cost C_e (monetary unit per unit length, e.g., \$ m^{-1}) was written as

$$C_e = c_e A + c_r \int_0^A (y_n - \eta)\mathrm{d}a, \tag{5}$$

where c_e = unit cost of earthwork at ground level (monetary unit per unit volume of earthwork, e.g., \$ m^{-3}); c_r = the additional cost per unit excavation per unit depth (monetary unit per unit volume of excavation per unit depth, e.g. \$ m^{-4}); η = vertical coordinate; y_n = normal depth (m); a = flow area (m^2) at height η. As c_L/c_e and c_e/c_r have length dimension, they remain unaffected by the monetary unit chosen. Using 'Schedule' (1997) and 'U.P.' (1992) the c_L/c_e and c_e/c_r ratios were obtained for various types of linings and the soil strata. These ratios are listed in Table I. Integrating by parts, Equation (5) was reduced to

$$C_e = c_e A + c_r \int_0^{y_n} a\mathrm{d}\eta. \tag{6}$$

Adding Equations (4) and (6), the cost function C (monetary unit per unit length, e.g., \$ m^{-1}) was obtained as

$$C = c_e A + c_L P + c_r \int_0^{y_n} a\mathrm{d}\eta. \tag{7}$$

4. Optimization Algorithm

The problem of determination of optimal canal section shape was reduced to

$$\text{minimize} \quad C = c_e A + c_L P + c_r \int_0^{y_n} a \mathrm{d}\eta, \tag{8}$$

$$\text{subject to} \quad \phi = Q + 2.457 A \sqrt{g R S_0} \ln \left(\frac{\varepsilon}{12R} + \frac{0.221\nu}{R\sqrt{g R S_0}} \right) = 0, \tag{9}$$

where ϕ = equality constraint function. The constrained optimization problem (8)–(9) was solved by minimizing the augmented function ψ given by

$$\psi = C + p\phi^2, \tag{10}$$

where p = a penalty parameter. Adopting small p, Equation (10) was minimized using grid search algorithm. Increasing p five-fold, the minimization was carried through various cycles till the optimum stabilized.

5. Optimal Section Shapes

Defining the length scale λ as

$$\lambda = \left(\frac{Q^2}{g S_0} \right)^{0.2}, \tag{11}$$

the following nondimensional variables were obtained:

$$C_* = \frac{C}{c_e \lambda^2}, \tag{12a}$$

$$c_{L^*} = \frac{c_L}{c_e \lambda}, \tag{12b}$$

$$c_{r^*} = \frac{c_r \lambda}{c_e}, \tag{12c}$$

$$y_{n^*} = \frac{y_n}{\lambda}, \tag{12d}$$

$$\varepsilon_* = \frac{\varepsilon}{\lambda}, \tag{12e}$$

$$\nu_* = \frac{\upsilon \lambda}{Q}. \tag{12f}$$

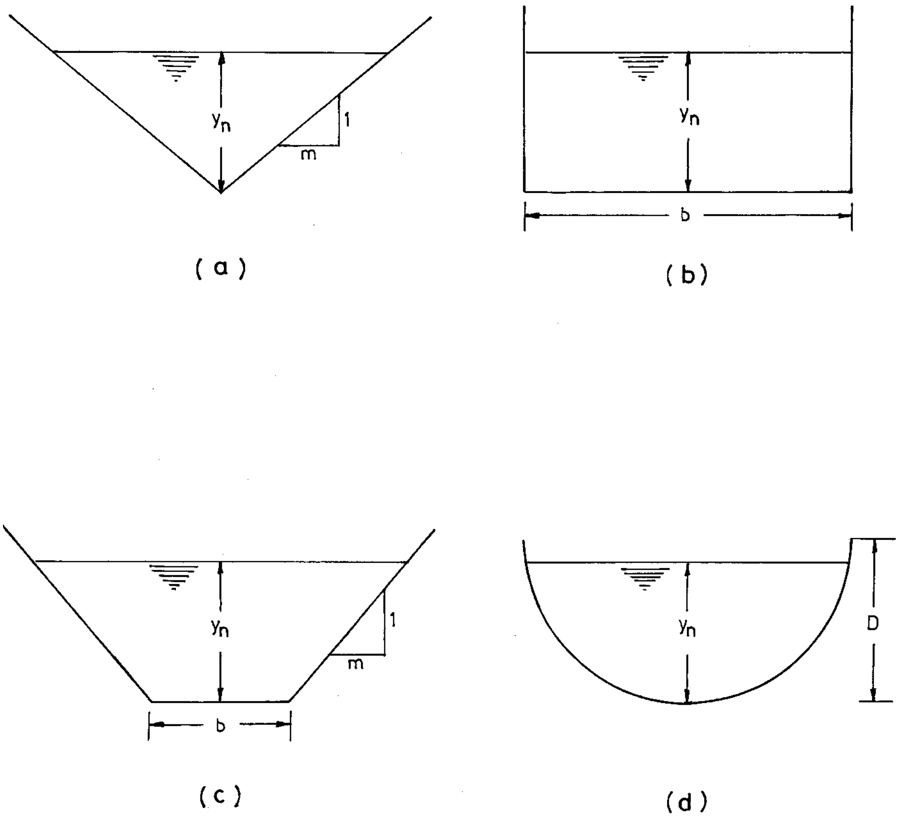

Figure 2. Canal sections: (a) triangular section, (b) rectangular section, (c) trapezoidal section, (d) circular section.

For a triangular section of side slope m horizontal to 1 vertical (see Figure 2(a)), Equations (8) and (9) were reduced to

$$\text{minimize} \quad C_* = 2c_{L^*} y_{n^*} \sqrt{1+m^2} + m y_{n^*}^2 + \frac{c_{r^*} m y_{n^*}^3}{3} \tag{13}$$

$$\text{subject to} \quad \phi = 1.737 \frac{m^{1.5} y_{n^*}^{2.5}}{(1 + m^2)^{0.25}} \ln\left(\frac{\varepsilon_* (1+m^2)^{0.5}}{6 m y_{n^*}} + \right.$$

$$\left. + 0.625 \nu_* \frac{(1+m^2)^{0.75}}{(m y_{n^*})^{1.5}} \right) + 1 = 0. \tag{14}$$

Using the optimization algorithm for a number of values of ε_*, ν_*, c_{L^*}, and c_{r^*} varying in the ranges

$$10^{-6} \leqslant \varepsilon_* \leqslant 10^{-3}, \tag{15a}$$

$$10^{-7} \leqslant \nu_* \leqslant 10^{-5}, \tag{15b}$$

$$0 \leqslant c_{L*} < \infty, \tag{15c}$$

$$0 \leqslant c_{r*} \leqslant 1.0 \quad \text{or} \quad 0 \leqslant c_{r*} \leqslant 50 c_{L*}, \tag{15d}$$

a large number of optimal sections were obtained. Analysis of these sections yielded the following empirical equations:

$$C^* = 1.42486 c_L L + 0.25302 c_e L^2 + 0.03965 c_r L^3, \tag{16a}$$

$$m^* = 1 + \frac{0.30389 c_r L^2}{c_e L + 15.0491 c_L}, \tag{16b}$$

$$y_n^* = 0.50301 L \left(1 + \frac{0.13973 c_r L^2}{c_e L + 15.03886 c_L}\right)^{-1}, \tag{16c}$$

where * indicated optimality; and L = length scale (Swamee, 1995a) given by

$$L = \lambda(\varepsilon_* + 8\nu_*)^{0.04}. \tag{17}$$

Following the above procedure for rectangular, trapezoidal, and circular canal sections as depicted in Figures 2(b)–(d), generalized optimal equations for all the four canal shapes were expressed as

$$C^* = k_{cL} c_L L + k_{ce} c_e L^2 + k_{cr} c_r L^3, \tag{18a}$$

$$m^* = k_{m0} + \frac{k_{mr} c_r L^2}{c_e L + k_{mL} c_L}, \tag{18b}$$

$$b^* = k_{b0} L + \frac{k_{br} c_r L^3}{c_e L + k_{bL} c_L}, \tag{18c}$$

$$D^* = k_{D0} L + \frac{k_{Dr} c_r L^3}{c_e L + k_{DL} c_L}, \tag{18d}$$

$$y_n^* = k_{y0} L \left(1 + \frac{k_{yr} c_r L^2}{c_e L + k_{yL} c_L}\right)^{-1}, \tag{18e}$$

where b = bed width (m); D = diameter (m); k = section shape coefficients in which the first subscripts c, m, b, D and y denote cost function, side slope, bed width, diameter and normal depth respectively; and the second subscript L, e, r, and o denote lining, earthwork, and earthwork increasing rate, and the case $c_r = 0$, respectively. Table II lists the section shape coefficients.

For a given set of data, the use of Equations (17) and (18), along with Table II results in the optimal canal section. For this section the average flow velocity V can be obtained by Equation (2). This velocity should be greater than the nonsilting velocity but less than the limiting velocity V_L. The limiting velocity depends on the lining material as given in Table III (Sharma and Chawla, 1975). If V is greater than V_L, a superior lining material having larger V_L should be selected.

Table II. Properties of optimal canal sections

Entity	Section shape coefficients	Section shape			
		Triangular	Rectangular	Trapezoidal	Circular
(1)	(2)	(3)	(4)	(5)	(6)
Side slope	k_{m0}	1.00000		0.57735	
	k_{mL}	15.0491		14.2772	
	k_{mr}	0.30389		0.12485	
Bed width or diameter	k_{b0} or k_{D0}		0.71136	0.43407	0.78065
	k_{bL} or k_{DL}		15.0284	14.2425	13.6232
	k_{br} or k_{Dr}		0.22772	0.15121	0.19375
Normal depth	k_{y0}	0.50301	0.35568	0.37592	0.39032
	k_{yL}	15.0389	15.0234	14.2274	12.9379
	k_{yr}	0.13973	0.30657	0.22332	0.12631
Cost	k_{ce}	0.25302	0.25302	0.24476	0.23932
	k_{cL}	1.42486	1.42396	1.30367	1.22652
	k_{cr}	0.03965	0.03961	0.03723	0.03712

Table III. Limiting velocities

Lining material	Limiting velocity (m s^{-1})
(1)	(2)
Boulder	1.0–1.5
Brunt clay tile	1.5–2.0
Concrete tile	2.0–2.5
Concrete	2.5–3.0

6. Salient Points

Equation (18a) gives the optimal cost per unit length of the canal. Dividing a canal alignment into various reaches with constant Q, S_0, ν, ε, c_e, c_L, and c_r, the canal cost F can be worked out by using Equation (18a). Thus,

$$F = \sum_{i=1}^{n} \left(k_{cL} c_{Li} L_i + k_{ce} c_{ei} L_i^2 + k_{cr} c_{ri} L_i^3 \right), \tag{19}$$

where n = number of reaches. Considering various alignments the minimum cost alignment can be finalized. Further by changing the shape of cross-section, the coefficients k_{cL}, k_{ce}, and k_{cr}, occurring in Equation (19) can be altered. Thus, the canal shape yielding minimum F can be found out. Similarly, by trying various types of linings having different roughness, one may arrive at the appropriate lining. Thus, Equation (19) can be used at the planning stage of a water resources project.

Equations (18b)–(18e) indicate that for c_r = 0, the optimal section is the minimum area section. However, with the increase in c_r the canal section becomes wide and shallow. On the other hand, with the increase in c_e and/or c_L, the canal section approaches to the corresponding minimum area section.

7. Design Example

Design a concrete lined trapezoidal canal section for carrying a discharge of 125 m^3 s^{-1} on a longitudinal slope of 0.0002. The canal passes through a stratum of ordinary soil for which c_e/c_r = 7 m. Further, it is proposed to provide concrete lining with c_L/c_e = 12 m.

7.1. DESIGN STEPS

For the design g = 9.80 m s^{-2}; $\nu = 1 \times 10^{-6}$ m^2 s^{-1} (water at 20 °C); and ε = 1 mm are adopted.

Using Equation (11), λ = 24 m; using Equations (12b)–(12c) c_{L^*} = 0.5; and c_{r^*} = 3.429; and using Equation (17) L = 16.06 m.

For a trapezoidal section Table II gave: k_{mo} = 0.57735; k_{mL} = 14.2772; k_{mr} = 0.12485; k_{bo} = 0.43407; k_{bL} = 14.2425; k_{br} = 0.15121; k_{yo} = 0.37592; k_{yL} = 14.2274; k_{yr} = 0.22332; k_{ce} = 0.24476; k_{cL} = 1.30367; and k_{cr} = 0.03723.

With these coefficients Equations (18b), (18c) and (18e) yielded: m^* = 0.602; b^* = 7.461 m; and y_n^* = 5.783 m. Further Equation (18a) gave: lining cost per meter C_L = 251.2433c_e; and the excavation cost per meter C_e = 63.1294c_e + 22.0309c_e = 85.1603c_e. Thus, the canal cost per meter = C_e + C_L = 336.4036c_e. It can be seen that the lining shares the major portion of the total cost. These dimensions yield $A = y_n(b + my_n)$ = 63.280 m^2. Thus, V = 125/63.280 = 1.975 m s^{-1}, which is within the permissible limit (Table III).

8. Conclusions

Explicit design equations and section shape coefficients have been presented for the minimum cost design of lined canals of triangular, rectangular, trapezoidal, and circular shapes. These equations and coefficients have been obtained by applying the nonlinear optimization technique. Using the optimal design equations along with the tabulated section shape coefficients, the optimal dimensions of a canal and the corresponding cost can be obtained. The method avoids the trial and error method of canal design and overcomes the complexity of the minimum cost design of lined canals. The optimal design equations show that on account of additional cost of excavation with canal depth the optimal section is wider and shallower than the minimum area section. On the other hand, for increased lining cost the optimal canal section approaches to the minimum area section. A useful extension of the method in the planning of a canal project has also been presented.

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