Simple Approximation for Flowing Well Problem

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Abstract: The discharge and yield of a flowing well can be computed by existing complex solutions or by Duhamel's technique using kernel coefficients. The mathematical complexities of these methods can be very much simplified through numerical methods without loss of accuracy. In the present study simple equations for both the well discharge and well production functions are presented. The equation for the well discharge function has been used to find aquifer constants for known well discharge and drawdown through the error minimization method. The results are compared in order to demonstrate the relative simplicity of the proposed equations.

INTRODUCTION

Flowing wells are an uncommon manifestation of geological activities. A permeable bed, sandwiched in-between impermeable strata in a synclinal fold and exposed at the surface allowing recharge, contains water under pressure. Drilling through the upper confining bed can result in a flowing well. The water that flows from such a well is the outcome of an expansion of the water because of relief of pressure and compression of granular material of the aquifer and the included clayey beds (Jacob 1940). The drawdown at the flowing well, i.e., the difference between the water level prevailing prior to the opening of the cap of the flowing well and the level of the threshold of the flowing well is constant, whereas the well discharge varies with time. The water-yielding capacity depends on the pressure drop within the aquifer and the formation constants. Analytical treatment of the flowing well to estimate the rate of flow and change of pressure for assumed formation constants (and vice-versa) can be found in Jacob and Lohman (1952), Hantush (1959), and Glover (1978). In this technical note, the difficulties in previous solutions have been simplified by using a numerical method.

EXISTING METHODS

The rate of flow from a flowing well for constant formation parameters for a homogeneous and infinite aquifer was given by Jacob and Lohman (1952) as

$$Q = 2\pi T s_w G(\alpha) \tag{1}$$

where Q = discharge of the well; s_w = constant drawdown at the well; T = aquifer transmissivity; $\alpha = Tt/Sr_w^2$; S = storage coefficient; r_w = radius of the well; t = time measured from the instant of cap opening of the well; and $G(\alpha)$ = well discharge function. Jacob and Lohman (1952) gave the following equation for $G(\alpha)$:

$$G(\alpha) = \frac{4\alpha}{\pi} \int_0^\infty x e^{-\alpha x^2} \left\{ \frac{\pi}{2} + \tan^{-1} \left[\frac{Y_0(x)}{J_0(x)} \right] \right\} dx \tag{2}$$

where J_0 and Y_0 are Bessel's functions of 0th order and first kind and second kind, respectively. The solution, thus, is in the form of an improper integral involving Bessel's functions.

The yield of a flowing well within a given period of time can be obtained by integrating (1) with respect to time between t_1 and t_2 as

$$V = 2\pi T s_w \int_{t_1}^{t_2} G(\alpha)\, dt = 2\pi S s_w r_w^2 [H(\alpha_2) - H(\alpha_1)] \tag{3}$$

where V = total production volume of the well in time interval $t_2 - t_1$; and $H(\alpha)$ = well production function given by

$$H(\alpha) = \int_0^\alpha G(\alpha)\, d\alpha \tag{4}$$

The solution for $H(\alpha)$ based on Jacob and Lohman (1952) equation is not available.

Glover (1978) gave alternate solutions for $G(\alpha)$ and $H(\alpha)$ as a sum of infinite series involving Bessel's functions and their zeros. In these solutions the aquifer was treated as an infinite aquifer until the disturbance produced by the flow from the well reaches the outer boundary of the aquifer. Thus, in a finite circular aquifer, for a flowing well located at the center of the aquifer, the solution of Glover (1978) is valid until the moving radius of influence, R, is less than the radius of the aquifer boundary. From the series solutions given by Glover (1978), it is possible to compute $G(\alpha)$ and $H(\alpha)$ with a limited number of terms for the known radius of the flowing well and valid assumed value of R.

The flowing well problem can also be solved by Duhamel's principle using discrete kernel coefficients derived from the Theis well function. Although this technique is simpler than analytical solutions of Jacob and Lohman (1952) and Glover (1978), it involves exponential integrals in generating discrete kernel coefficients and recursive expressions for $G(\alpha)$ and $H(\alpha)$.

Finally we come to tabular solutions. The improper integral involving Bessel's functions in (2) cannot be evaluated by any of the ordinary means of integration. Using numerical methods, Jacob and Lohman (1952) tabulated $G(\alpha)$ for α ranging from 10^{-4} to 10^{12}. Glover (1978) also tabulated $G(\alpha)$ and $H(\alpha)/4\alpha$ for α ranging from 2.25 to 2.5×10^7, and R/r_w ranging from 25 to 25,600. The interpolated values of $G(\alpha)$ from these tables differed slightly.

SIMPLIFIED EXPRESSIONS

The methods given by Jacob and Lohman (1952) and Glover (1978) have been simplified, using numerical methods to avoid mathematical complexities of the solutions or inconvenience in interpolations of the tabulated values. The tabulated values of $G(\alpha)$ as given by Jacob and Lohman (1952) were fitted to the following equation:

$$G(\alpha) = \frac{1}{\sqrt{\pi\alpha}} + 2\{\ln[(1 + 4e^{-\gamma}\alpha)(1 + 30\alpha^{-0.45})]\}^{-1} \tag{5}$$

where γ = Euler's constant = 0.577216. The maximum error involved in the equation is 2.32% at $\alpha = 0.03$ with mean absolute error about 0.66%, as shown in Fig. 1.

The following equation, for the well production function

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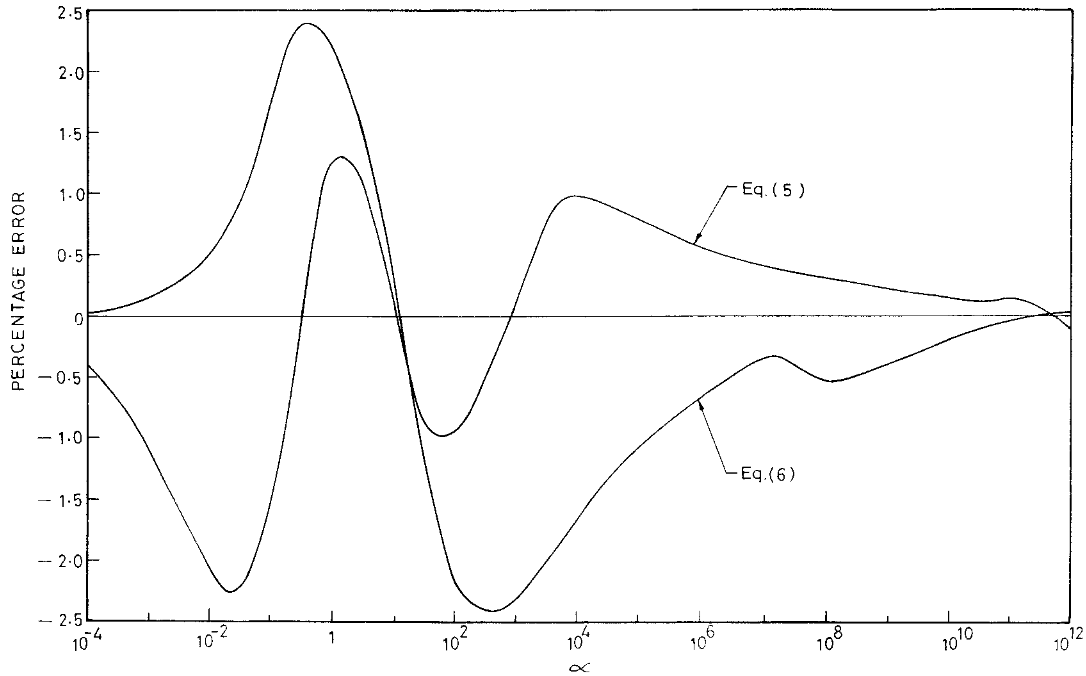

FIG. 1. Error Diagram

$H(\alpha)$, was obtained by fitting the tabulated values of Glover (1978) given for the range $2.25 \leq \alpha \leq 2.5 \times 10^7$ and the computed values obtained by numerical integration of (5) for the ranges $10^{-4} \leq \alpha < 2.25$ and $2.5 \times 10^7 < \alpha \leq 10^{12}$:

$$H(\alpha) = \sqrt{\frac{4\alpha}{\pi}} + \frac{2\alpha(1 + 0.088\alpha^{-0.03})}{\ln[(1 + 4e^{-\gamma}\alpha)(1 + 30\alpha^{-0.45})]} \tag{6}$$

The maximum error in the use of (6) is 2.45% at $\alpha = 0.5$ and 400 with average absolute error = 0.77% (Fig. 1).

Glover (1978) adopted a different dimensionless parameter α_G and a well production function $H_G(\alpha_G)$, which are related to α and $H(\alpha)$ as

$$\alpha_G = \sqrt{4Tt/Sr_w^2} = 2\sqrt{\alpha} \tag{7}$$

$$H_G(\alpha_G) = \frac{H(\alpha)}{4\alpha} \tag{8}$$

Using (5)–(8), the following expressions for the Glover's well discharge function $G_G(\alpha_G)$ and Glover's well production function were obtained:

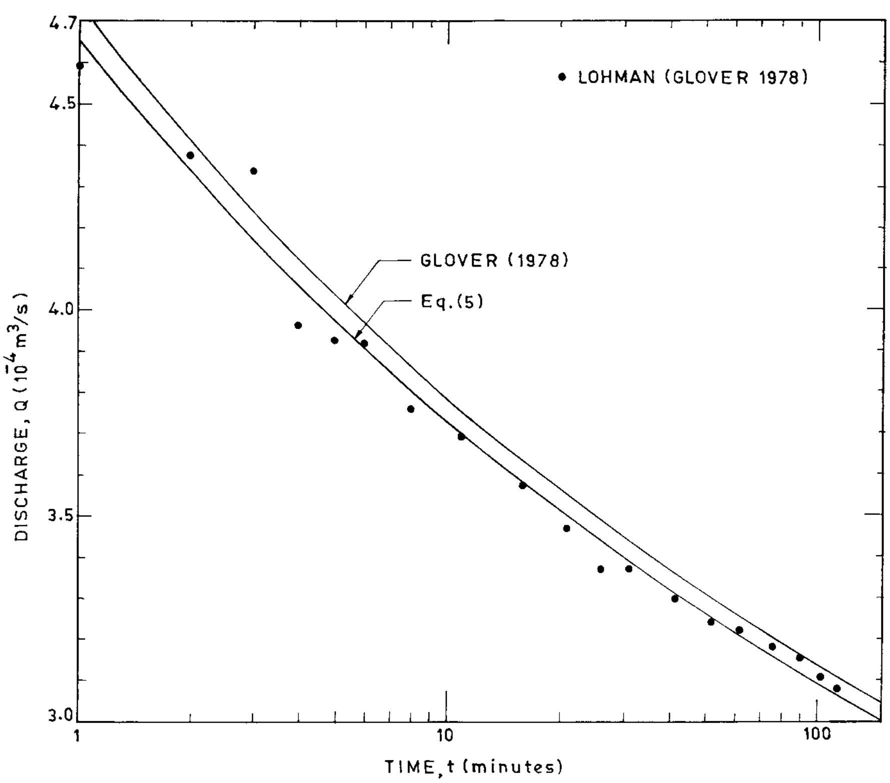

FIG. 2. Discharge Comparison

$$G_G(\alpha_G) = \frac{2}{\alpha_G\sqrt{\pi}} + 2\{\ln[(1 + e^{-\gamma}\alpha_G^2)(1 + 56\alpha_G^{-0.9})]\}^{-1} \quad (9)$$

$$H_G(\alpha_G) = \frac{1}{\alpha_G\sqrt{\pi}} + \frac{1 + 0.092\alpha_G^{-0.06}}{2\ln[(1 + e^{-\gamma}\alpha_G^2)(1 + 56\alpha_G^{-0.9})]} \quad (10)$$

$G(\alpha)$ and $G_G(\alpha_G)$ are the same for any particular t. Eqs. (9) and (10) approximately replace the tables of $G_G(\alpha_G)$ and $H_G(\alpha_G)$ given by Glover (1978).

PARAMETER ESTIMATION

Jacob and Lohman (1952) and Glover (1978) recommended the curve-matching method for estimation of aquifer parameters S and T. The curve-matching method is subject to errors of judgment. For removing subjectiveness in parameter estimation, $G(\alpha)$ from (5) can be used in minimizing the average absolute error E given by

$$E = \frac{1}{n}\sum_{i=1}^{n} |Q_i - 2\pi T s_w G(\alpha_i)| \quad (11)$$

where Q_i = observed discharge at time t_i; n = number of observations; and

$$\alpha_i = Tt_i/Sr_w^2 \quad (12)$$

EXAMPLE

Lohman (Glover 1978) reported discharge data, Q_i, of a flowing well of radius = 0.084 m at the threshold of which a constant drawdown s_w = 28.142 m prevailed. For the trial value of S and T in (12), α_i was obtained for various times t_i. Using α_i so obtained, $G(\alpha_i)$ was computed from (5). Then discharge data Q_i and $G(\alpha_i)$ were used to obtain E in (11). Varying S and T by the lattice search method (Burley 1974), E was minimized yielding $S = 3.88 \times 10^{-5}$ and $T = 1.16 \times 10^{-5}$ m^2/s. Using the curve-matching method Glover (1978) estimated $S = 4.14 \times 10^{-5}$ and $T = 1.18 \times 10^{-5}$ m^2/s. Fig. 2 depicts the variation of discharge with time corresponding to the estimated parameters. The figure also shows the discharges as obtained by using the parameters estimated by Glover (1978). It can be seen from Fig. 2 that the discharges computed by the present method have better agreement with the observed discharge data than do those obtained by the method of Glover (1978).

For $T = 1.16 \times 10^{-5}$ m^2/s and $S = 3.88 \times 10^{-5}$, at t_1 = 31.0 min, $\alpha_1 = 7.8576 \times 10^4$; and at t_2 = 91.0 min, $\alpha_2 = 2.3066 \times 10^5$. From (6), $H(\alpha_1) = 1.39465 \times 10^4$ and $H(\alpha_2) = 3.73984 \times 10^4$. Using (3), the total production volume V = 1.139 m^3. Using Glover's (1978) estimation of S and T, V = 1.177 m^3.

CONCLUSIONS

The solutions given by Jacob and Lohman (1952) and Glover (1978) have been simplified to compute the discharge and production volume of a flowing well. In this method, interpolations from the tables of the well discharge function and the well production function are not required. Further, it does not require computations of the improper integrals involving Bessel's function. In aquifer parameter estimation, the error minimization method—which uses the proposed equation for well discharge—is more convenient than the conventional curve-matching method.

APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

E = average absolute error;
$G(\alpha)$ = well discharge function;
$G_G(\alpha_G)$ = Glover's well discharge function;
$H(\alpha)$ = well production function;
$H_G(\alpha_G)$ = Glover's well production function;
J_0 = Bessel's function of 0th order and first kind;
n = number of observations;
Q = discharge of well at time t;
Q_i = discharge of well at time t_i;
R = radius of influence or radius of aquifer;
r_w = radius of well;
S = aquifer storage coefficient;
s_w = constant drawdown at well;
T = aquifer transmissivity;
t = time measured from cap opening of well;
t_i = time of ith observation for discharge;
V = production volume of well;
x = dummy variable;
Y_0 = Bessel's function of 0th order and second kind;
α = Tt/Sr_w^2;
α_i = Tt_i/Sr_w^2;
α_G = Glover's parameter = $\sqrt{4Tt/Sr_w^2}$; and
γ = Euler's constant = 0.577216.