DESIGN OF MINIMUM SEEPAGE LOSS CANAL SECTIONS

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ABSTRACT: The minimum area section is a thoroughly investigated problem in the hydraulics literature. However, because of the complexities of the analysis, the design of a minimum seepage loss section has not been attempted as yet. In this investigation, using previously derived results, simplified algebraic equations for computation of seepage loss from triangular, rectangular, and trapezoidal canals have been presented, which replace accurately the cumbersome evaluation of complex integrals. Using these seepage loss equations and the general uniform flow equation, explicit equations for the design variables of minimum seepage loss canal sections have been obtained for each of the three canal shapes by applying nonlinear optimization technique. The optimal trapezoidal section has the least seepage loss and cross-sectional area among the three optimal sections. A step-by-step design procedure for rectangular and trapezoidal canal sections has been presented. The analysis also includes the sensitivity of the seepage loss to design variables around the optimum value.

INTRODUCTION

Canals continue to be major conveyance systems for delivering water for irrigation in the alluvial plains of India. But the seepage loss from irrigation canals constitutes a substantial percentage of the usable water. By the time the water reaches the field, it has been estimated that the seepage losses are of the order of 45% of the water supplied at the head of the canal (Sharma and Chawla 1975). According to the Indian Standard (''Measurement'' 1980), the loss of water by seepage from unlined canals in India generally varies from 0.3 to 7.0 m^3/s per 10^6 m^2 of wetted surface. The transit losses are more accentuated in alluvial canals. It has been estimated (Sharma and Chawla 1975) that if the seepage loss is prevented, about 6,000,000 ha of additional area could be irrigated. The seepage loss results not only in depleted freshwater resources but also causes water logging, salinization, and ground-water contamination. Canals in alluvium are lined in general and reduce the seepage in particular. Seepage from a lined canal occurs at a reduced rate. The perfect lining would prevent all the seepage loss, but a canal lining deteriorates with time. An examination of canals by Wachyan and Rushton (1987) indicated that even with the greatest care the lining does not remain perfect. A well-maintained canal with a 99% perfect lining reduces seepage about 30–40% (Wachyan and Rushton 1987); seepage from a canal cannot be controlled completely. Significant seepage losses do occur from a canal even if it is lined. Therefore, a canal cross section should be designed in such a shape and with dimensions that minimize the seepage loss. This paper addresses the design of a minimum seepage section.

The seepage loss from canals is governed by hydraulic conductivity of the subsoils, canal geometry, hydraulic gradient between the canal and the aquifer underneath, and initial and boundary conditions. The seepage loss from a canal in an unconfined flow condition is finite and maximum when the water table lies at a very large depth. Canal seepage has been estimated for different sets of specific conditions (Harr 1962; Polubarinova-Kochina 1962; Morel-Seytoux 1964; Garg and Chawla 1970; Subramanya et al. 1973; Sharma and Chawla 1979; Wolde-Kirkos and Chawla 1994). However, the methods adopted by various investigators are applicable to known canal dimensions. An exact mathematical solution to unconfined steady-state seepage from a trapezoidal canal in a homogeneous isotropic porous medium of large depth has been given by Vedernikov (Harr 1962). The solution has been obtained using inversion of the hodograph and conformal mapping technique. The triangular canal is a particular case of the trapezoidal canal. A family of curves for flat canal banks has been presented. However, seepage from a rectangular canal cannot be computed from the analytical solution given for a trapezoidal canal. The case of a rectangular canal has been dealt with by Morel-Seytoux (1964), and the solution has been obtained by conformal mapping and the use of Green functions. The analytical form of these solutions, which contain improper integrals and unknown implicit state variables, is not convenient in estimating seepage from the existing canals and in designing canals. These methods have been simplified by numerical methods for easy computation of seepage in this study.

Though considerable work has been reported on the design of minimum area cross section, practically no work has been done on the minimum seepage loss canal sections. Swamee (1995) reviewed the existing literature on minimum area canal sections.

Presented herein are the three explicit equations for the seepage loss from triangular, rectangular, and trapezoidal canal sections. Using these equations and the resistance equation for open channel flow (Swamee 1994), minimum seepage loss sections have been obtained for these three canal shapes.

SEEPAGE LOSS

The seepage loss from a canal in a homogeneous and isotropic porous medium, when the water table is at a very large depth, can be expressed as

$$q_s = kyF \tag{1}$$

where q_s = seepage discharge per unit length of canal (m^2/s); k = hydraulic conductivity of the porous medium (m/s); y = depth of water in the canal (m); F = function of channel geometry (dimensionless); and yF = width of seepage flow at the infinity. Hereafter, F will be referred to as the seepage function.

Triangular Section

For a triangular channel, Vedernikov (Harr 1962) gave the following equation for the seepage function:

$$F = \frac{\pi m \int_0^1 (1 - t^2)^{-(0.5+\sigma)} t^{-(1-2\sigma)}\, dt}{\int_0^1 \cos^{-1} t (1 - t^2)^{-(0.5+\sigma)} t^{-(1-2\sigma)}\, dt} \tag{2}$$

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where m = side slope (dimensionless) [see Fig. 1(a)]; $\sigma = 1/\pi \cot^{-1} m$; and t = dummy variable (dimensionless). Using (2), for a given m, F was obtained numerically by Gauss-Chebyshev integration. Repeating the process, F was obtained for a large number of m lying in the range $0 \le m \le 1{,}000$. Using these computations the following equation for F was fitted:

$$F = \{[\pi(4 - \pi)]^{1.3} + (2m)^{1.3}\}^{0.77} \tag{3}$$

Eq. (3) is exact for $m = 0$, and $m = \infty$. Combining (1) and (3), the seepage discharge can be obtained for a triangular section.

Rectangular Section

For a rectangular canal, Morel-Seytoux (1964) gave the following equation for the seepage function:

$$F = \frac{\pi^2}{\displaystyle\int_\alpha^\infty \ln\left\{\frac{2}{1 + \alpha^2}\left[t^2 + \frac{1 - \alpha^2}{2} + \sqrt{(1 + t^2)(t^2 - \alpha^2)}\right]\right\}\frac{dt}{1 + t^2}} \tag{4}$$

where α = state variable given by

$$\frac{b}{y} = \frac{\displaystyle 2\int_0^\alpha \cos^{-1}\left(\frac{2t^2 + 1 - \alpha^2}{1 + \alpha^2}\right)\frac{dt}{1 + t^2}}{\displaystyle\int_\alpha^\infty \ln\left\{\frac{2}{1 + \alpha^2}\left[t^2 + \frac{1 - \alpha^2}{2} + \sqrt{(1 + t^2)(t^2 - \alpha^2)}\right]\right\}\frac{dt}{1 + t^2}} \tag{5}$$

where b = bed width (m) [see Fig. 1(b)]. Using (5) for a given b/y, the state variable α was obtained by a trial-and-error procedure. Furthermore, substituting α in (4) the seepage function was obtained. Repeating this process, F was obtained for a large number of b/y lying in the range $0 \le b/y \le 1{,}000$. Using b/y and F so obtained, the following equation, which is exact at $b/y = 0$ and $b/y = \infty$, was fitted:

$$F = \left\{[\pi(4 - \pi)]^{0.77} + \left(\frac{b}{y}\right)^{0.77}\right\}^{1.3} \tag{6}$$

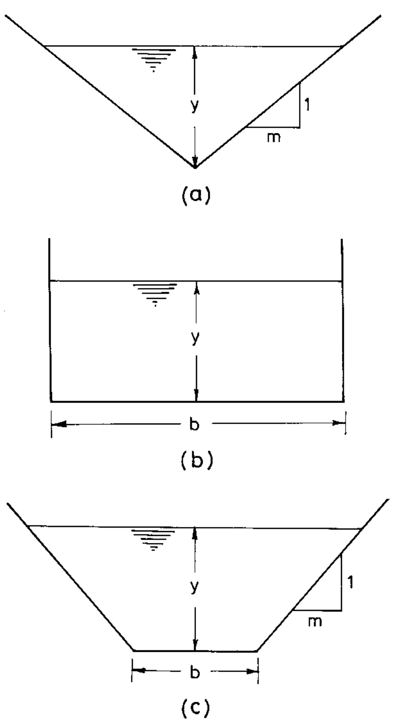

FIG. 1. Canal Sections: (a) Triangular Section; (b) Rectangular Section; (c) Trapezoidal Section

Trapezoidal Section

For a trapezoidal canal [see Fig. 1(c)], Vedernikov (Harr 1962) gave the following equation for the seepage function:

$$F = \frac{\displaystyle \pi m \int_\beta^1 (1 - t^2)^{-(0.5+\sigma)}(t^2 - \beta^2)^{-(1-\sigma)} t\, dt}{\displaystyle \int_\beta^1 \cos^{-1} t (1 - t^2)^{-(0.5+\sigma)}(t^2 - \beta^2)^{-(1-\sigma)} t\, dt} \tag{7}$$

where β = state variable given by

$$\frac{b}{y} = \frac{\displaystyle 2\sqrt{1 + m^2}\int_0^\beta \sin^{-1} t (1 - t^2)^{-(0.5+\sigma)}(\beta^2 - t^2)^{-(1-\sigma)} t\, dt}{\displaystyle \int_\beta^1 \cos^{-1} t (1 - t^2)^{-(0.5+\sigma)}(t^2 - \beta^2)^{-(1-\sigma)} t\, dt} \tag{8}$$

Using a process similar to that described for a rectangular canal, F was obtained for a large number of m and b/y lying in the ranges of $0 \le m \le 1{,}000$ and $0 \le b/y \le 1{,}000$. Using m, b/y, and F so obtained, the following equation (which is exact at $m = 0$, $b/y = 0$; $m = 0$, $b/y = \infty$; $m = \infty$, $b/y = 0$; and $m = \infty$, $b/y = \infty$) was fitted:

$$F = \left(\{[\pi(4 - \pi)]^{1.3} + (2m)^{1.3}\}^{(0.77+0.462m)/(1.3+0.6m)} + \left(\frac{b}{y}\right)^{(1+0.6m)/(1.3+0.6m)}\right)^{(1.3+0.6m)/(1+0.6m)} \tag{9}$$

Eq. (9) supplements Vedernikov's graphs for computation of seepage for trapezoidal canals frequently used with steeper side slopes (i.e., $m < 1$).

Fig. 2 depicts the errors involved in (9). A perusal of Fig. 2 shows the maximum error as 1.8% for the triangular section ($b = 0$). For the rectangular section ($m = 0$), the maximum error is within 1%. The involved error in the practical range is <0.9% for the triangular section ($0.5 \le m \le 2.5$), 0.5% for the rectangular section ($0.5 \le b/y \le 10$), and 1.4% for the trapezoidal section ($0.5 \le m \le 5$ and $0.5 \le b/y \le 10$).

RESISTANCE EQUATION

A rigid boundary irrigation canal is designed by using the uniform flow resistance equation. The most commonly used uniform flow resistance formula is the Manning equation (Chow 1973), which is applicable for rough turbulent flow and in a limited band-width of relative roughness (Christensen 1984). Relaxing these restrictions, Swamee (1994) gave the following resistance equation:

$$Q = -2.457A\sqrt{gRS_0}\ln\left(\frac{\varepsilon}{12R} + \frac{0.221\nu}{R\sqrt{gRS_0}}\right) \tag{10}$$

where Q = canal discharge (m^3/s); A = flow area (m^2); g = gravitational acceleration (varying between 9.780 m/s^2 at the equator to 9.832 m/s^2 at the poles); $R = A/P$, where P (m) is wetted perimeter; S_0 = longitudinal canal bed slope (dimensionless); ε = average roughness height of the canal lining (m); and ν = kinematic viscosity of water (m^2/s). Similar to the case of the resistance equation for pipe flow, (10) involves physically conceivable parameters ε and ν.

OPTIMIZATION ALGORITHM

The canal design pertains to the condition of uniform flow throughout its length. Thus, it is sufficient to consider the unit length of the canal, in which the depth of flow is the normal depth y_n. In such a case, the seepage loss is determined by

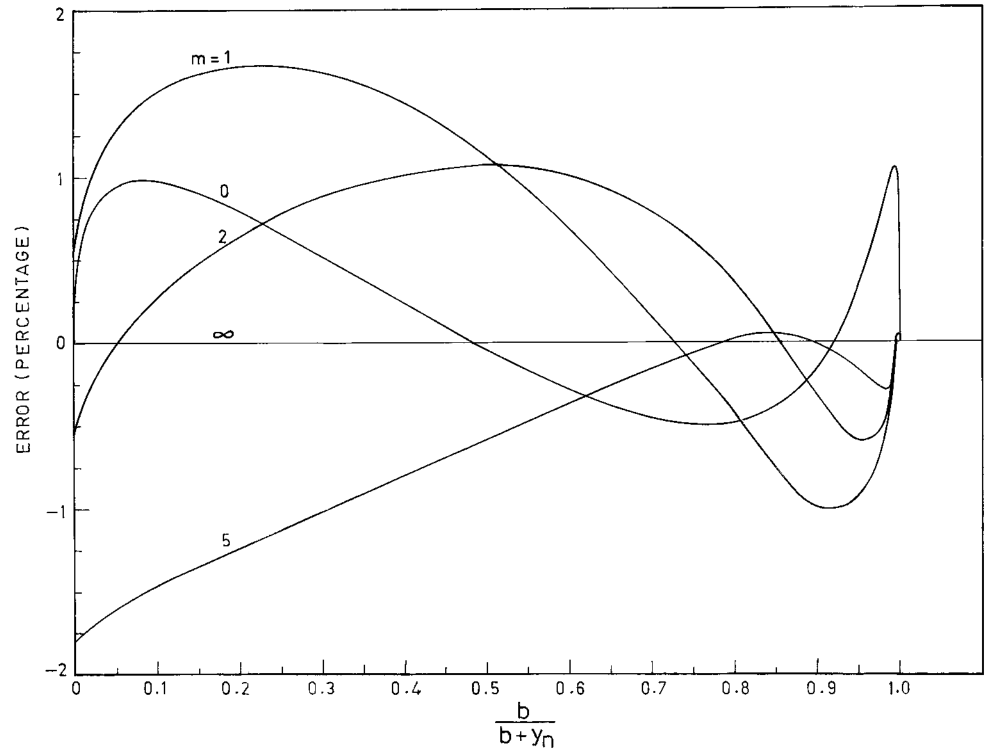

FIG. 2. Error Diagram

replacing y by y_n in (1). Thus, the problem of determination of the shape of the minimum seepage loss canal section was reduced to

minimize

$$q_s = ky_nF \tag{11}$$

subject to

$$\phi = 2.457A\sqrt{gRS_0}\,\ln\left(\frac{\varepsilon}{12R} + \frac{0.221\nu}{R\sqrt{gRS_0}}\right) + Q = 0 \tag{12}$$

where ϕ = equality constraint function. The constrained optimization problem [(11) and (12)] was solved by minimizing the augmented function ψ given by

$$\psi = q_s + p\phi^2 \tag{13}$$

where p = penalty parameter. Adopting small p, (13) was minimized using the grid search algorithm. Increasing p fivefold, the minimization was carried through various cycles until the optimum stabilized.

OPTIMAL SECTION SHAPES

Considering the length scale λ as

$$\lambda = [Q^2/(gS_0)]^{0.2} \tag{14}$$

the following nondimensional variables were defined:

$$y_{n*} = y_n/\lambda;\quad \varepsilon_* = \varepsilon/\lambda;\quad \nu_* = \nu\lambda/Q \tag{15a–c}$$

$$q_{s*} = q_s/(k\lambda);\quad \phi_* = \phi/Q \tag{15d,e}$$

For a triangular section, (11) and (12), in nondimensional form, were reduced to

minimize

$$q_{s*} = y_{n*}\{[\pi(4 - \pi)]^{1.3} + (2m)^{1.3}\}^{0.77} \tag{16}$$

subject to

$$\phi_* = 1.737\,\frac{m^{1.5}y_{n*}^{2.5}}{(1 + m^2)^{0.25}}\,\ln\left(\frac{\varepsilon_*(1 + m^2)^{0.5}}{6my_{n*}} + 0.625\nu_*\,\frac{(1 + m^2)^{0.75}}{(my_{n*})^{1.5}}\right) + 1 = 0 \tag{17}$$

Using the optimization algorithm on a nondimensional augmented function for a number of values of ε_*, and ν_*, varying in the following ranges:

$$10^{-6} \le \varepsilon_* \le 10^{-3};\quad 10^{-7} \le \nu_* \le 10^{-5} \tag{18a,b}$$

a large number of optimal sections were obtained. Making use of these optimal sections and adopting Swamee's procedure (Swamee 1995), the following empirical equations were derived:

$$m^* = 1.244;\quad y_n^* = 0.452L;\quad q_s^* = 2.001kL \tag{19a–c}$$

where the superscript asterisk (*) indicates optimality, and

$$L = \lambda(\varepsilon_* + 8\nu_*)^{0.04} \tag{20}$$

Following the above procedure for rectangular and trapezoidal sections, the generalized optimal dimensions for all three canal sections were expressed as

$$b^* = k_bL;\quad y_n^* = k_yL;\quad A^* = k_AL^2 \tag{21a–c}$$

$$V^* = k_vQL^{-2};\quad q_s^* = k_qkL \tag{21d,e}$$

where V = average flow velocity in the canal (m/s); and k_b, k_y, k_A, k_v, and k_q = section shape coefficients for bed width, normal depth, cross-sectional area, average velocity, and seepage loss, respectively. Table 1 lists the optimal section shape coefficients. A perusal of Table 1 reveals that as compared to the optimal triangular and rectangular sections, the seepage loss as well as the cross-sectional area is minimum for the optimal trapezoidal section. Optimal cross-sectional areas for triangular and rectangular canals are found to be equal; however, the rectangular section has a slightly higher seepage loss than the triangular section.

For a given set of data, the use of (20) and (21), along with

TABLE 1. Properties of Optimal Canal Sections

Section shape (1)	Side slope m (2)	Section-Shape Coefficients				
		k_b (3)	k_y (4)	k_A (5)	k_v (6)	k_q (7)
Triangular	1.244	0.000	0.452	0.254	3.937	2.001
Rectangular	0.000	0.799	0.318	0.254	3.937	2.040
Trapezoidal	0.598	0.545	0.331	0.246	4.070	1.923

TABLE 2. Limiting Velocities

Lining material (1)	Limiting velocity (m/s) (2)
Boulder	1.0–1.5
Brunt clay tile	1.5–2.0
Concrete tile	2.0–2.5
Concrete	2.5–3.0

Table 1, results in the optimal canal section. For this section, (21*e*) and (21*d*) can be used to obtain the quantity of the seepage loss and the average flow velocity, respectively. The average flow velocity should be greater than the nonsilting velocity but less than the limiting velocity V_L. The limiting velocity depends on the lining material as given in Table 2 (Sharma and Chawla 1975). If V is greater than V_L, a superior lining material should be selected.

DESIGN EXAMPLES

Example 1

Design a minimum seepage loss concrete-lined rectangular canal section for carrying a discharge of 50 m^3/s on a longitudinal slope of 0.0004.

Design Steps

For the design, $g = 9.79$ m/s^2; $\nu = 1.1 \times 10^{-6}$ m^2/s (water at 20°C); and $\varepsilon = 1$ mm are adopted.

Using (14), $\lambda = 14.488$ m; using (15*b*), $\varepsilon_* = 6.902 \times 10^{-5}$; using (15*c*), $\nu_* = 3.187 \times 10^{-7}$; and using (20), $L = 9.890$ m.

Using Table 1 the section shape coefficients are $k_b = 0.799$; $k_y = 0.318$; $k_A = 0.254$; $k_v = 3.937$; and $k_q = 2.040$.

Using (21*a*–*d*), $b^* = 0.799 \times 9.890 = 7.902$ m; $y_n^* = 0.318 \times 9.890 = 3.145$ m; $A^* = 0.254 \times 9.890^2 = 24.844$ m^2; and $V^* = 3.937 \times 50/9.890^2 = 2.013$ m/s, which is within the permissible limit (Table 2).

Assuming that the lining is cracked and $k = 10^{-6}$ m/s; (21*e*) results in the seepage loss $q_s = 2.040 \times 10^{-6} \times 9.890 = 2.018 \times 10^{-5}$ m^2/s.

Example 2

Design a trapezoidal canal section for $Q = 250$ m^3/s and $S_0 = 0.0001$.

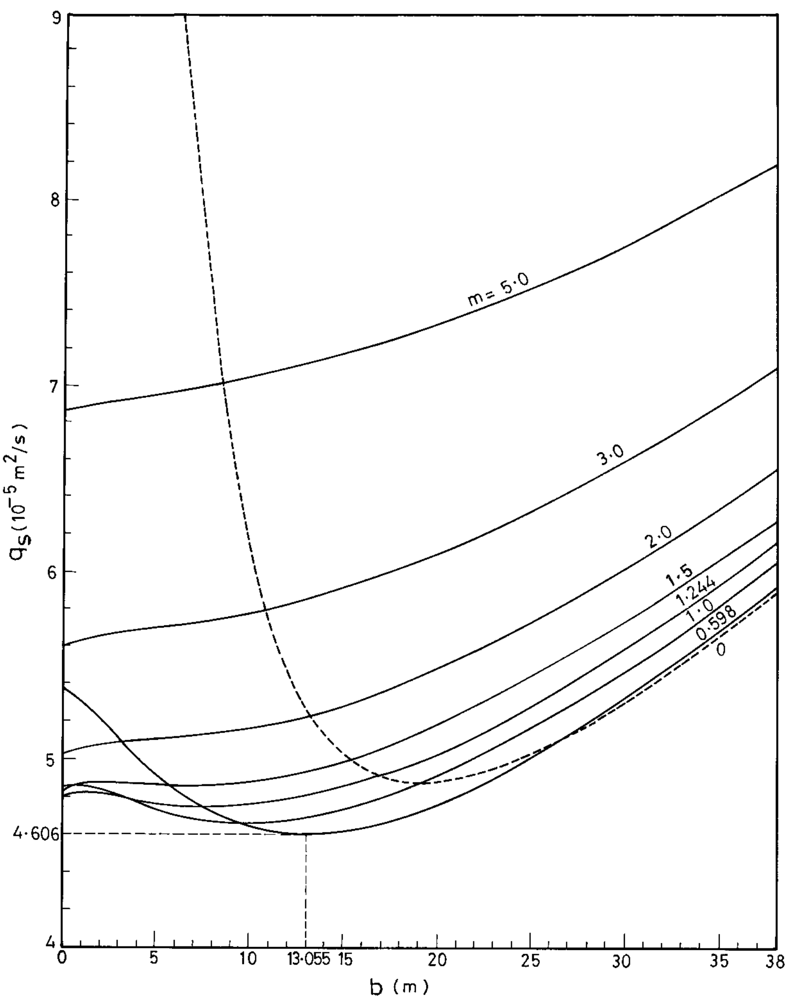

FIG. 3. Variation of Seepage Loss with Bed Width and Side Slope

Design Steps

Following the steps similar to a rectangular section, λ = 36.393 m; $\varepsilon_* = 2.748 \times 10^{-5}$; $\nu_* = 1.601 \times 10^{-7}$; and L = 23.954 m.

The section shape coefficients from Table 1 are $m^* = 0.598$; $k_b = 0.545$; $k_y = 0.331$; $k_A = 0.246$; $k_v = 4.070$; and $k_q = 1.923$.

Using (21a–d), $b^* = 13.055$ m; $y_n^* = 7.929$ m; $A^* = 141.153$ m^2; and $V^* = 1.773$ m/s, which is safe.

Using (21e) with $k = 10^{-6}$ m/s, $q_s = 4.606 \times 10^{-5}$ m^2/s.

Sensitivity of Optimal Design

For b ranging from 0 to 40 m and m ranging from 0 to 5, the normal depths were obtained using (10). Furthermore, seepage losses were calculated by (1). Fig. 3 shows the variation of q_s with b and m. It can be seen that the seepage loss from a trapezoidal section with side slope of 0.598 and bed width of 13.055 m is the global minimum. Furthermore, the optimum is less sensitive to the increase in bed width and more sensitive otherwise. This trend of sensitivity continues for $0 < m < 1.5$. For $m \geq 1.5$ the optimum shifts to $b = 0$ (triangular section). However, as seen in Fig. 3 the optimum for a rectangular section ($m = 0$) is highly sensitive to a decrease in bed width.

CONCLUSIONS

Simplified functions in terms of canal geometry have been given for computing seepage losses from triangular, rectangular, and trapezoidal canals. These functions, which replace accurately the cumbersome evaluation of improper integrals with unknown implicit state variables, have been obtained using previously derived equations by Vedernikov and Morel-Seytoux. The seepage function for a trapezoidal section supplements Vedernikov's graphs for computation of seepage. The section shape coefficients for all three canal shapes have been obtained to facilitate design of the minimum seepage loss canals. Seepage from a triangular canal is minimum for m = 1.244. A rectangular channel with a ratio of bed width to normal depth = 2.513 has minimum seepage. Among the optimal sections, the optimal trapezoidal section (m = 0.598 and bed width to normal depth ratio = 1.646) loses the least seepage. The design examples have demonstrated the relative simplicity of the method. The sensitivity analysis for the trapezoidal canal section design has revealed that the optimum is less sensitive to the increase in bed width and more sensitive otherwise.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = flow area (m^2);
b = bed width (m);
F = seepage function (dimensionless);
g = gravitational acceleration (m/s^2);
L = length scale (m);
k = hydraulic conductivity (m/s);
k_A = section shape coefficient for area (dimensionless);
k_b = section shape coefficient for bed width (dimensionless);
k_q = section shape coefficient for seepage loss (dimensionless);
k_v = section shape coefficient for velocity (dimensionless);
k_y = section shape coefficient for normal depth (dimensionless);
m = side slope (dimensionless);
p = penalty parameter (dimensionless);
Q = discharge (m^3/s);
q_s = seepage discharge per unit length of canal (m^2/s);
R = hydraulic radius (m);
S_0 = bed slope (dimensionless);
t = dummy variable (dimensionless);
V = average velocity (m/s);
V_L = limiting velocity (m/s);
y = water depth in channel (m);
y_n = normal depth (m);
α = state variable (dimensionless);
β = state variable (dimensionless);
ε = roughness height (m);
λ = length scale (m);
ν = kinematic viscosity (m^2/s);
ϕ = equality constraint (m^3/s); and
ψ = augmented function.

Subscript

$*$ = nondimensional.

Superscript

$*$ = optimal.