SOLUTION FOR A STREAM DEPLETION PROBLEM

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ABSTRACT: River recharge is one of the important factors affecting the yield of nearby wells. The mathematical complexities of the existing methods in computation of rate and volume of stream depletion can be very much simplified through numerical methods without loss of much accuracy. In the present study, simple equations for stream depletion rate and total volume of stream flow depleted are presented. The example illustrates the relative simplicity of the proposed equations.

INTRODUCTION

The discharge of a well near a stream is derived from storage in the aquifer as well as from infiltration from the stream. Determination of the stream contribution in the pumped water is important to ascertain the quality and quantity of the water that may be withdrawn through wells over long periods of time without critically depleting the available ground-water supply. Since stream depletion also affects the stream stage and flow in the stream, its determination is important from the point of view of the legal rights of downstream users.

The analytical determination of the rate of stream depletion was first presented by Theis (1941). Jeffords (1945) ascertained the river recharge and found the stream contribution through a chemical composition test of pumped water. Glover and Balmer (1954) expressed the rate of stream depletion as a complementary error function. The method of Glover and Balmer (1954) was later generalized by Hantush (1955, 1964a, 1965). De Wiest (1963) analyzed steady-state stream aquifer interaction for different boundary conditions. Simplified equations for rate and total volume of stream depletion presented in this study are useful approximations based on the exact analytical solutions developed by Glover and Balmer (1954) and Hantush (1964a).

STREAM CONTRIBUTION ESTIMATION

Existing Methods

The fraction of stream contribution rate P_r in the total uniform discharge Q (m^3/s) from a well that is steadily discharging from a homogeneous, uniform, isotropic aquifer hydraulically connected to a fully penetrating stream was expressed by Theis (1941) as

$$P_r = \frac{q}{Q} = \frac{2}{\pi} \int_0^{\pi/2} \exp(-\alpha \sec^2 u)\, du \tag{1}$$

where q = discharge contributed by stream (m^3/s); $\alpha = a^2S/(4Tt)$ (dimensionless); a = effective distance between the pumping well and the recharging stream (m); S = aquifer storage coefficient (dimensionless); T = aquifer transmissivity (m^2/s); t = time since the start of pumping (s); and u = dummy variable. Furthermore, (1) is based on the assumption that the river stage does not change with time, which is valid when the flow in the river is much more than the quantity contributed by the river to pumping.

Eq. (1) was expressed as a complementary error function by Glover and Balmer (1954) under confined conditions or unconfined conditions for ratios of drawdown to saturated thickness of up to 0.25 when the storage coefficient remains constant:

$$P_r = erfc(\sqrt{\alpha}) \tag{2}$$

The total volume V (m^3), by which the stream is depleted by a continuously pumping well, can be found by

$$V = \int_0^t q\, dt \tag{3}$$

The fraction of stream contribution volume P_v in the total volume of water pumped (i.e., Qt) from the well was given by Hantush (1964a,b) as

$$P_v = \frac{V}{Qt} = 4i^2erfc(\sqrt{\alpha}) \tag{4}$$

where $i^2erfc(\cdot)$ = second repeated integral of the error function. Eq. (4), as shown by Hantush (1964a), is valid for confined conditions or unconfined conditions when the ratio of drawdown to saturated thickness is less than 0.5. The values of the repeated integrals of the error functions are available in tabular form in Abramowitz and Stegun (1970).

Simplified Equations

The existing methods to compute the rate of stream depletion are not convenient, because they require computation of the complementary error function, interpolation from tables of probability integrals (Glover 1978), or numerical integration of (1). The integral involved in (1) was obtained numerically by Gauss-Chebyshev integration. The resultant values were fitted to the following equation:

$$P_r = \left\{\left[1 + 2\sqrt{\frac{\alpha}{\pi}}\right]^{2.75} + [e^{\alpha}\sqrt{\pi\alpha}(1 + (130\alpha e^{\alpha})^{-0.21})]^{2.75}\right\}^{-0.364} \tag{5}$$

The fractions of stream contribution volume were obtained numerically for $\alpha < 0.25$. For α ranging from 0.25 to 25, the values of P_v were interpolated from the table of repeated integrals of the error function (Abramowitz and Stegun 1970). The resulting values were fitted to the following equation, which avoids the use of both tables and numerical integration:

$$P_v = \left\{\left[1 + 4\sqrt{\frac{\alpha}{\pi}}\right]^{2.6} + [\alpha e^{\alpha}\sqrt{\pi\alpha}(1 + 2.655\alpha^{-0.97})]^{2.6}\right\}^{-0.385} \tag{6}$$

Eqs. (5) and (6) are exact when $\alpha \to 0$ or $\alpha \to \infty$. The maximum error involved in (5) is 2.1%, occurring at $\alpha = 16$. On the other hand, the maximum error in use of (6) is 0.7%, which occurs at $\alpha = 1.6$. The behavior of (5) and (6) is shown in Fig. 1 for a wide range of α.

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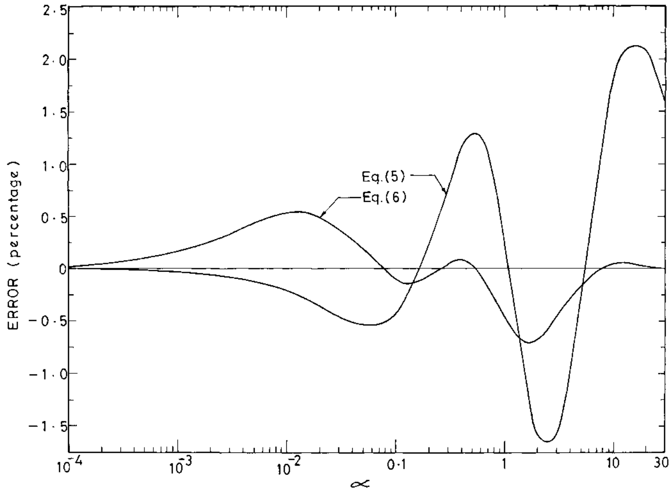

FIG. 1. Error Diagram

A large α means early pumping, a high storage coefficient, low transmissivity, and a large distance from well to river. At the beginning of pumping, the aquifer contributes to the well discharge. For a higher storage coefficient and lower transmissivity, most of the water is taken from the aquifer storage. Also, when the distance from well to river is large, it is the aquifer that contributes more to the pumping. For a large α, the contribution of the stream in the pumping becomes insignificant. For example, for $\alpha \geq 25$, $P_r \leq 1.537 \times 10^{-12}$ and $P_v \leq 5.612 \times 10^{-14}$. The significant rate of stream depletion $P_r \geq 0.05$ corresponds to $\alpha \leq 1.9$. In this range, the error involved in (5) is less than 1.5%. On the other hand, the significant fraction of stream depletion volume $P_v \geq 0.05$ corresponds to $\alpha \leq 1.07$. For this range, the maximum error involved in (6) is 0.55%.

TABLE 1. Comparison between Traditional and Proposed Methods of Computation of P_r and P_v for a = 804.67 m and T/S = 6.9677 × 10^{-2} m^2/s

Time (month) (1)	$\alpha \times 10^{-2}$ (2)	FRACTION OF STREAM DEPLETION			
		Rate, P_r		Volume, P_v	
		Glover (1978) (3)	Eq. (5) (4)	Interpolated[a] (5)	Eq. (6) (6)
0	∞	0	0	0	0
6	14.938	0.5875	0.5839	0.3851	0.3832
12	7.4691	0.7008	0.6955	0.5201	0.5173
18	4.9794	0.7542	0.7482	0.5903	0.5887
24	3.7346	0.7858	0.7807	0.6342	0.6350
30	2.9877	0.8083	0.8032	0.6691	0.6684
36	2.4897	0.8250	0.8200	0.6948	0.6938
42	2.1340	0.8375	0.8332	0.7148	0.7141
48	1.8673	0.8475	0.8438	0.7309	0.7307
54	1.6598	0.8567	0.8527	0.7443	0.7447
60	1.4938	0.8633	0.8602	0.7556	0.7567

[a] Table of repeated integrals of error function (Abramowitz and Stegun 1970).

EXAMPLE

Using the data of a well near a stream (a = 804.67 m; T/S = 6.9677 × 10^{-2} m^2/s) as used by Glover (1978), the fractions of stream depletion rate and volume were calculated from the simplified equations, as shown in Table 1. The fractional rate of stream depletion was compared with those reported by Glover (1978), while the fraction of stream volume depleted was compared with the interpolated values from the table of repeated integrals of the error function (Abramowitz and Stegun 1970). The results compared very well.

CONCLUSIONS

The infiltration of stream water into an aquifer is significant because of its effect on the quality and quantity of pumped water. Locations of well sites and design of wells are decided according to the stream contribution to the pumping. The existing methods to estimate stream depletion have been simplified through numerical methods. In the simplified method, interpolation from tables of complementary error functions for the rate of stream depletion and the second repeated integral of the error function for total volume of stream depletion is not required, nor is numerical integration necessary.

APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

a = effective distance between well and stream (m);
$erfc(\cdot)$ = complementary error function;
$i^2erfc(\cdot)$ = second repeated integral of error function;
P_r = fractional stream depletion rate;
P_v = fractional stream depletion volume;
Q = discharge of well (m^3/s);
q = discharge contribution from stream (m^3/s);
S = aquifer storage coefficient (dimensionless);
T = aquifer transmissivity (m^2/s);
t = time since start of pumping (s); and
V = volume of stream depletion (m^3).