Comprehensive Design of Minimum Cost Irrigation Canal Sections

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Abstract: Design of a minimum cost canal section involves minimization of the sum of costs per unit length of the canal, subject to uniform flow condition in the canal. Essentially it is a problem of minimization of a nonlinear objective function subject to a nonlinear equality constraint. In this investigation, the objective function has been expressed as the cost per unit length of the canal for lining, the depth-dependent unit volume earthwork cost, and the cost of water lost as seepage and evaporation losses. A general resistance equation has been used as an equality constraint. Using a nonlinear optimization technique on an augmented function, generalized empirical equations and section shape coefficients have been obtained for the design of minimum cost irrigation canals of triangular, rectangular, and trapezoidal shapes. The optimal dimensions for any shape can be obtained from the proposed equations along with tabulated section shape coefficients. The equation for optimal cost along with the corresponding section shape coefficients is useful during the planning of a canal project. A design example with sensitivity analysis has been included to demonstrate the simplicity of the present method.

INTRODUCTION

Networks of irrigation canals are used to convey, distribute, and apply water to the land. A canal in the network may be a rigid boundary (lined) canal or a mobile boundary (unlined) canal. As the lined canals permit higher average velocities, there is a saving in the cross-sectional area of the canal and land acquisition, with corresponding saving in the cost of excavation and masonry works. Further, a lined canal can be laid on steep slopes to save the cost of earthwork in formation. On the other hand, the smooth surface of lining reduces the friction forces, which enables the canal to be laid on a flatter bed slope, with a corresponding increase in command area and a larger working head for power generation. The maintenance cost of a lined canal is less than that of unlined canals, because the lining ensures protection against bed and bank erosion. The canal lining becomes essential for stability of the canal banks in expansive soils. Continuous seepage from canals may cause serious water logging accompanied with salt accumulation, converting a once fertile land into a huge waste and spoil. One of the reasons for the waterlogging of 12,000 ha of land in the Indira Ghandi Canal command in Rajasthan, India (Hooja et al. 1997), is canal seepage. Lining is provided to check the seepage from a canal. Thus, canals are lined wherever feasible in general and to overcome the likely consequences of seepage and conserve precious water resources in a water-scarce area in particular.

The seepage loss from canals has been estimated for different sets of specific conditions (Harr 1962; Polubarinova-Kochina 1962; Morel-Seytoux 1964; Garg and Chawla 1970; Subramanya et al. 1973; Sharma and Chawla 1979). The analytical form of these solutions, which contain improper integrals and unknown implicit state variables, is not convenient in estimating seepage from the existing canals and in designing canals. Swamee et al. (2000) presented simplified algebraic equations for computing seepage from triangular, rectangular, and trapezoidal canals. The perfect lining would prevent all the seepage loss, but canal lining deteriorates with time. An examination of canals by Wachyan and Ruston (1987) indicated that, even with the greatest care, the lining does not remain perfect and significant seepage losses do occur from a canal even if it is lined. Therefore, seepage loss must be considered in the design of a canal section. Bandini (1966) and Kacimov (1992) initiated canal design methods considering seepage loss. Swamee et al. (2000) gave explicit equations for the design of minimum seepage loss canal sections.

Generally, evaporation loss from a canal is a small fraction of the total loss, but it becomes significant for small capacity long channels running through arid climatic conditions. Hence, care should be taken in the design of such canals to account for evaporation loss along with seepage loss. Several equations for estimating evaporation from a free water surface are available in the literature. Warnaka and Pochop (1988) and Ikebuchi et al. (1988) compared the merits of various equations. Fulford and Sturm (1984) found that mass transfer equations are more convenient and useful for determining evaporation from flowing canals.

Rigid boundary canals are designed for uniform flow considering hydraulic efficiency, practicability, and economy (Streeter 1945). The factors to be considered in the design are: (1) the kind of material forming the channel surface, which determines the roughness coefficient; (2) the minimum permissible velocity, to avoid deposition of silt or debris; (3) the limiting velocity, to avoid erosion of the channel surface; (4) the topography of the channel route, which fixes the channel bed slope; and (5) the efficiency of the channel section, which indicates how much the section is hydraulically and/or economically efficient (Chow 1973). Chow (1973) and French (1994) have listed various properties of the most hydraulically efficient sections. Swamee and Bhatia (1972) expressed all the channel dimensions in term of a length scale comprising independent design variables and developed curves for the optimal design of trapezoidal, rounded bottom, and rounded corner sections. The best hydraulic section has the minimum flow area and flow perimeter for a given discharge but not necessarily the most economical section. Where the water surface is below the bank tops or when free board is considered, a channel narrower than the hydraulically best section results in minimum excavation (Guo and Hughes 1984). Monadjemi (1994) and Swamee (1995b) have done a comprehensive investigation for the optimal dimensions for various canal shapes. Canal cost and practicability requirements have been combined into an objective function and constraints in obtaining the optimal channel section by several investigators (Trout 1982; Flynn and Marimno 1987; Imam et al. 1991; Loganathan 1991; Froehlich 1994). Manning's equation as a uniform flow equation was used in the optimal channel design by the

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Note. Discussion open until March 1, 2001. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on December 7, 1999. This paper is part of the ***Journal of Irrigation and Drainage Engineering***, Vol. 126, No. 5, September/October, 2000. ©ASCE, ISSN 0733-9437/00/0010-0322–0327/$8.00 + $.50 per page. Paper No. 22151.

above investigators. However, Manning's equation is applicable only for rough turbulent flow and in a limited bandwidth of relative roughness (Christensen 1984). A more general resistance equation based on roughness height was used in the optimal design of irrigation canals by Swamee (1995a) and Swamee et al. (2000).

A network of canals represents a major cost item in an irrigation project, and the economy of the canal network is vital. The maximum economy is achieved by minimizing the cost of the canals. Design of minimum cost irrigation canals involves minimization of the sum of the earthwork cost, which varies with canal depth, cost of lining, and cost of water lost as seepage and evaporation subject to uniform flow condition in the canal. Such a minimum cost canal design problem results in a nonlinear objective function and a nonlinear equality constraint, making the problem hard to solve analytically. In this paper, generalized empirical equations and section shape coefficients for the design of minimum cost irrigation canal sections have been obtained for triangular, rectangular, and trapezoidal canals.

COST FUNCTION

The objective function consists of the cost per unit length of the canal. This includes the depth-dependent earthwork cost, the cost of lining, and the cost of water lost as seepage and evaporation. In the study, the earthwork and lining costs have been considered for the flow section only.

Earthwork Cost

Considering the earthwork for the flow section, the earthwork cost C_e (\$/m) was given by

$$C_e = c_e A + c_r A\bar{y} \quad (1)$$

where c_e = cost per unit volume of earthwork at ground level (\$/m^3); c_r = increase in the unit excavation cost per unit depth (\$/m^4); A = flow area (m^2); and $\bar{y}$ = depth of centroid of area from the free water surface.

Lining Cost

Considering the cost per unit surface area of lining c_L (\$/m^2) as independent of the depth of placement, the cost of lining C_L (\$/m) was expressed as

$$C_L = c_L P \quad (2)$$

where P = flow perimeter (m).

Cost of Water Lost as Seepage and Evaporation

The seepage loss from a canal in a homogeneous and isotropic porous medium when the water table is at very large depth was written by Swamee et al. (2000) as

$$q_s = k y_n F_s \quad (3)$$

where q_s = water loss as seepage per unit length of canal (m^2/s); k = hydraulic conductivity of the porous medium (m/s); y_n = normal depth of water in the canal (m); and F_s = seepage function (dimensionless), which is a function of the channel geometry (Appendix I).

On the other hand, the evaporation loss was written as

$$q_E = ET \quad (4)$$

where q_E = water loss as evaporation per unit length of canal (m^2/s); T = width of free surface (m) (Fig. 1); and E = evaporation discharge per unit surface area (m/s). In the mass transfer equation, E is a function of wind velocity over the evaporating surface, water surface temperature, air temperature, and relative humidity of the air above the water surface.

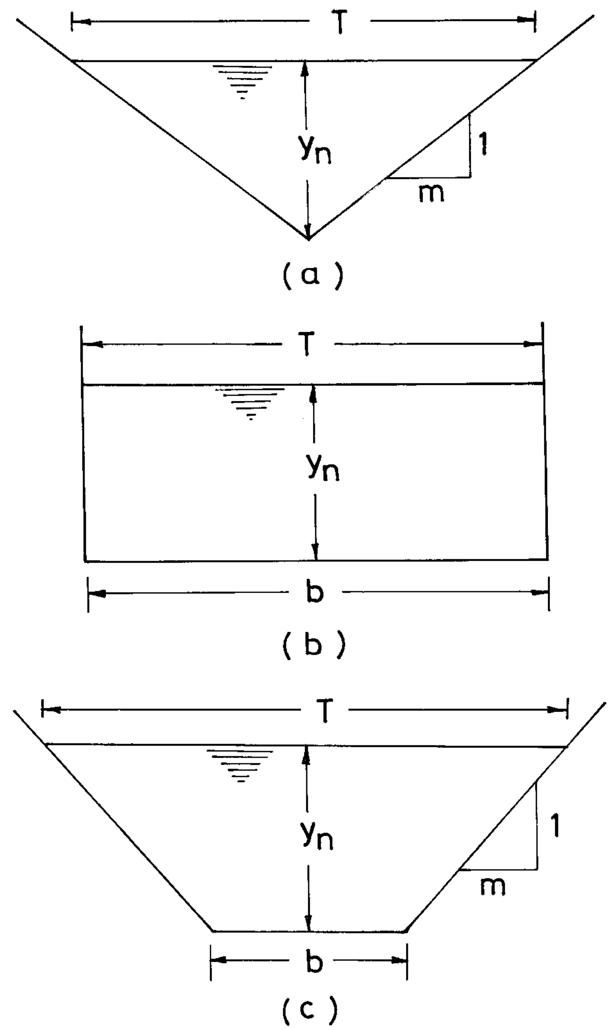

FIG. 1. Canal Sections: (a) Triangular Section; (b) Rectangular Section; (c) Trapezoidal Section

Adding (3) and (4), the water loss per unit length of canal q_w (m^2/s) became

$$q_w = k y_n F_s + ET \quad (5)$$

Assuming a very long, useful life of the canal, the capitalized cost of water lost C_w (\$/m) was expressed as

$$C_w = \frac{3.156 \times 10^7 c_w}{r}(k y_n F_s + ET) \quad (6)$$

where r = rate of interest (\$/\$/year); and c_w = cost per unit volume of water (\$/m^3). The volumetric cost of water may be different for seepage loss and evaporation loss, depending upon the side effects caused by the seepage loss. Eq. (6) may be rewritten in the following form:

$$C_w = c_{ws} y_n F + c_{wE} T \quad (7)$$

where

$$c_{ws} = 3.156 \times 10^7 k c_w / r \quad (8)$$

and

$$c_{wE} = 3.156 \times 10^7 E c_w / r \quad (9)$$

Eq. (7) is also applicable for a short life of the canal and/or for different unit costs of water for seepage loss and evaporation loss; however, (6), (8), and (9) must be modified.

TABLE 1. Lining and Earthwork Cost Coefficients

Types of strata (1)	c_L/c_e (m) CONCRETE TILE LINING With LDPE Film 100μ (2)	200μ (3)	Without film (4)	BRICK TILE LINING With LDPE Film 100μ (5)	200μ (6)	Without film (7)	BRUNT CLAY TILE LINING With LDPE Film 100μ (8)	200μ (9)	Without film (10)	c_e/c_r (m) (11)
Ordinary soil	12.75	13.02	12.24	6.39	6.67	5.88	6.08	6.35	5.57	6.96
Hard soil	10.00	10.22	9.60	5.01	5.23	4.62	4.77	4.99	3.37	8.86
Impure lime nodules	8.90	9.10	8.55	4.47	4.66	4.11	4.25	4.44	3.89	9.96
Dry shoal with shingle	6.56	6.71	6.30	3.29	3.43	3.03	3.13	3.27	2.86	13.50
Slush and lahel	6.40	6.54	6.14	3.21	3.35	2.95	3.05	3.19	2.79	13.86

Note: LPDE = low density polyethylene.

Unit Length Canal Cost

Adding (1), (2), and (7), the cost of canal per unit length C (\$/m) was obtained as

$$C = C_e + C_L + C_w = c_e A + c_r A\bar{y} + c_L P + c_{ws} F_s y_n + c_{wE} T \quad (10)$$

As c_L/c_e, c_e/c_r, c_{ws}/c_e, and c_{wE}/c_e have length dimensions, they remain unaffected by the monetary unit chosen. These ratios can be obtained for various types of linings, soil strata, and climatic condition by using appropriate unit rates. Using *Schedule* (1997) and ''U.P.'' (1992), the c_L/c_e and c_e/c_r ratios were obtained as listed in Table 1.

CONSTRAINT FUNCTION

Swamee (1994) gave the following resistance equation:

$$V = -2.457\sqrt{gRS_0}\ \ln\left(\frac{\varepsilon}{12R} + \frac{0.221\nu}{R\sqrt{gRS_0}}\right) \quad (11)$$

where V = average flow velocity (m/s); g = gravitational acceleration (m/s^2); R = hydraulic radius (m), defined as the ratio of the flow area to the flow perimeter; S_0 = longitudinal canal bed slope (dimensionless); ε = average roughness height of the canal lining (m); and ν = kinematic viscosity of water (m^2/s). Using the continuity equation and (11), the discharge Q (m^3/s) was obtained as

$$Q = AV = -2.457A\sqrt{gRS_0}\ \ln\left(\frac{\varepsilon}{12R} + \frac{0.221\nu}{R\sqrt{gRS_0}}\right) \quad (12)$$

Since a canal is designed to sustain uniform flow, (12) provides the required condition as an equality constraint function in the design.

OPTIMIZATION ALGORITHM

The problem of determination of optimal canal section shape was reduced to

$$\text{Minimize}\quad C = c_e A + c_r A\bar{y} + c_L P + c_{ws} F_s y_n + c_{wE} T \quad (13)$$

$$\text{Subject to}\quad Q + 2.457A\sqrt{gRS_0}\ \ln\left(\frac{\varepsilon}{12R} + \frac{0.221\nu}{R\sqrt{gRS_0}}\right) = 0 \quad (14)$$

Adopting a length scale λ (m) as

$$\lambda = (Q/\sqrt{gS_0})^{0.4} \quad (15)$$

the following nondimensional variables were obtained:

$$\varepsilon_* = \varepsilon/\lambda;\quad \nu_* = \nu\lambda/Q;\quad C_* = C/(c_e\lambda^2) \quad (16a–c)$$

$$c_{L*} = c_L/(c_e\lambda);\quad c_{r*} = c_r\lambda/c_e;\quad c_{ws*} = c_{ws}/(c_e\lambda) \quad (16d–f)$$

$$c_{wE*} = c_{wE}/(c_e\lambda);\quad y_{n*} = y_n/\lambda;\quad \bar{y}_* = \bar{y}/\lambda \quad (16g–i)$$

$$P_* = P/\lambda;\quad T_* = T/\lambda;\quad A_* = A/\lambda^2;\quad R_* = R/\lambda \quad (16j–m)$$

where subscript * denotes the corresponding nondimensional parameter.

Using (13), (14), and (16), the problem of determination of optimal canal section shape in nondimensional form was reduced to

$$\text{Minimize}\quad C_* = A_* + c_{r*}A_*\bar{y}_* + c_{L*}P_* + c_{ws*}F_s y_{n*} + c_{wE*}T_* \quad (17)$$

$$\text{Subject to}\quad \Phi = 1 + 2.457A_*\sqrt{R_*}\ \ln\left(\frac{\varepsilon_*}{12R_*} + \frac{0.221\nu_*}{R_*^{1.5}}\right) = 0 \quad (18)$$

where Φ = equality constraint function.

This constrained optimization problem was converted into an unconstrained optimization problem by forming an augmented function Ψ given by

$$\Psi = C_* + p\Phi^2 \quad (19)$$

where p = a penalty parameter.

Adopting a small p initially, (19) was minimized using Powel's conjugate direction search method (Bazaara and Shetty 1979) to find the design variables. Increasing p tenfold, the minimization was carried through various cycles until the optimization results stabilized.

OPTIMAL DESIGN EQUATIONS

The optimization algorithm was applied on triangular, rectangular, and trapezoidal canal sections (Fig. 1c) for a number of input variables varying in the ranges

$$10^{-6} \le \varepsilon_* \le 10^{-3};\quad 10^{-7} \le \nu_* \le 10^{-5} \quad (20a,b)$$

$$0 \le c_{L*} \le \infty;\quad 0 \le c_{ws*} \le \infty \quad (20c,d)$$

$$0 \le c_{r*} \le 1.0 \quad \text{or} \quad 0 \le c_{r*} \le 50c_{L*} \quad (20e)$$

$$0 \le c_{wE*} \le 0.2 \quad \text{or} \quad 0 \le c_{wE*} \le c_{ws*} \quad (20f)$$

Analysis of a large number of optimal sections so obtained for triangular, rectangular, and trapezoidal canal sections revealed that the dimensionless optimal design variables are linear functions of the dimensionless costs in the ranges indicated in (20*a*–*f*). Further analysis of the optimal sections in the above ranges resulted in the following generalized empirical equations in explicit form for all three types of canal sections:

$$m^* = k_{meo}\frac{c_eL + k_{mr}c_rL^2 + k_{mL}c_L + k_{ms1}c_{ws}}{c_eL + k_{mL}c_L + k_{ms2}c_{ws} + k_{mE}c_{wE}} \quad (21a)$$

$$b^* = k_{beo}\frac{c_eL + k_{br}c_rL^2 + k_{bL}c_L + k_{bs1}c_{ws}}{c_eL + k_{bL}c_L + k_{bs2}c_{ws} + k_{bE}c_{wE}}L \quad (21b)$$

$$y_n^* = k_{yeo}\frac{c_eL + k_{yL}c_L + k_{ys2}c_{ws} + k_{yE}c_{wE}}{c_eL + k_{yr}c_rL^2 + k_{yL}c_L + k_{ys1}c_{ws}}L \quad (21c)$$

$$C^* = k_{cr}c_rL^3 + k_{ceo}c_eL^2 + k_{cL}c_LL + k_{cs}c_{ws}L + k_{cE}c_{wE}L \quad (22)$$

where superscript * indicates optimality; m = side slope of the canal section [Fig. 1(a and c)]; b = bed width for rectangular and trapezoidal channels [Fig. 1(b and c)]; k_{fs} = section shape coefficients in which the first subscripts m, b, y, and c denote side slope, bed width, normal depth, and cost, respectively, and the second subscripts eo, r, L, s, and E denote depth-independent earthwork (minimum area), additional earthwork cost because of canal depth, lining, seepage, and evaporation, respectively; and L = length scale (m), given by

$$L = \lambda(\varepsilon_* + 8\nu_*)^{0.04} \quad (23)$$

Table 2 lists the section shape coefficients. A perusal of (21) with Table 2 indicates that, for $c_r = 0$, $c_{ws} = 0$, and $c_{wE} = 0$, the optimal section is the minimum area section. However, with increase in c_r, the canal section becomes wider and shallower, while the reverse is the case with increase in c_{wE}. On the other hand, with increase in c_e and/or c_L, the canal section approaches the corresponding minimum area section, while with increase in c_{ws}, it approaches the minimum seepage loss section (Swamee et al. 2000).

For a given set of data, a canal can be designed by minimizing (13) subjected to constraint (14). This requires a considerable amount of numerical work. Alternatively, using (21) along with Table 2, the optimal section can be obtained in a single-step computation. For the designed section, the average flow velocity V can be obtained by (12). This velocity should be greater than the nonsilting velocity but less than the limiting velocity V_L. The limiting velocity depends on the lining material (Bureau 1982). Table 3 lists the limiting velocities for different types of linings (Sharma and Chawla 1975). If V is greater than V_L, a superior lining material should be selected.

TABLE 2. Properties of Optimal Canal Sections

Entity (1)	Section shape coefficients (2)	Section Shape		
		Triangular (3)	Rectangular (4)	Trapezoidal (5)
Side slope	k_{meo}	1.000		0.577
	k_{mr}	0.304		0.216
	k_{mL}	15.049		14.277
	k_{ms1}	16.756		23.494
	k_{ms2}	13.441		22.668
	k_{mE}	8.331		32.189
Bed width	k_{beo}		0.711	0.434
	k_{br}		0.320	0.348
	k_{bL}		15.028	14.243
	k_{bs1}		18.283	18.086
	k_{bs2}		16.286	14.416
	k_{bE}		5.543	0.288
Normal depth	k_{yeo}	0.503	0.356	0.376
	k_{yr}	0.140	0.307	0.223
	k_{yL}	15.039	15.023	14.227
	k_{y1}	16.245	18.737	16.910
	k_{ys2}	14.590	16.741	14.885
	k_{yE}	4.036	5.624	4.036
Cost	k_{ceo}	0.253	0.253	0.245
	k_{cr}	0.040	0.040	0.037
	k_{cL}	1.425	1.424	1.303
	k_{cs}	2.002	2.040	1.923
	k_{cE}	0.989	0.686	0.820

TABLE 3. Limiting Velocities

Lining material (1)	Limiting velocity (m/s) (2)
Boulder	1.0–1.5
Brunt clay tile	1.5–2.0
Concrete tile	2.0–2.5
Concrete	2.5–3.0

Eq. (22) gives the optimal cost per unit length of the canal. Dividing a canal alignment into various reaches with constant Q, S_0, ε, ν, c_e, c_L, c_r, c_{ws}, and c_{wE}, the total canal cost C_t($) can be worked out by using (22). Thus

$$C_t = \sum_{i=1}^{n}(k_{cr}c_{ri}L_i^3 + k_{ceo}c_{ei}L_i^2 + k_{cL}c_{Li}L_i + k_{cs}c_{wsi}L_i + k_{cE}c_{wEi}L_i)x_i \quad (24)$$

where n = number of reaches; and x_i = length of canal in the ith reach. Considering various alignments, the minimum cost alignment can be finalized. Further, by changing the cross sections, the coefficients k_{ceo}, k_{cr}, k_{cL}, k_{cs}, and k_{cE} occurring in (24) can be altered. Thus, the canal shape yielding minimum C_t can be found. Similarly, by trying various types of linings having different roughnesses, one may arrive at the appropriate lining. Thus, (24) can be used at the planning stage of a water resources project.

DESIGN EXAMPLE

Design a concrete-lined trapezoidal canal section to carry a discharge of 250 m^3/s on a longitudinal bed slope of 0.0001. The canal passes through a stratum of ordinary soil for which c_e/c_r = 7 m and c_{ws}/c_e = 10 m. Further, it is proposed to provide concrete lining with c_L/c_e = 12 m. The climatic condition of the canal area is such that c_{wE}/c_e = 2 m.

Design Steps

For the design g = 9.79 m/s^2; $\nu = 1.1 \times 10^{-6}$ m^2/s (water at 20°C); and ε = 1 mm (concrete lining) are adopted.

Using (15), λ = 36.393 m; from (16a), $\varepsilon_* = 2.748 \times 10^{-5}$; (16$b$) yields $\nu_* = 1.601 \times 10^{-7}$; and (23) yields L = 23.954 m. Using (16d–g), $c_{L*} = 0.330$; $c_{r*} = 5.200$; $c_{ws*} = 0.275$; and $c_{wE*} = 0.055$, all of which are well within range of (20).

For a trapezoidal section, Table 2 gives the following section shape coefficients: for side slope, $k_{meo} = 0.577$, $k_{mr} = 0.216$, $k_{mL} = 14.277$, $k_{ms1} = 23.494$, $k_{ms2} = 22.668$, and $k_{mE} = 32.189$; for bed width, $k_{beo} = 0.434$, $k_{br} = 0.348$, $k_{bL} = 14.243$, $k_{bs1} = 18.086$, $k_{bs2} = 14.416$, and $k_{bE} = 0.288$; for normal depth, $k_{yeo} = 0.376$, $k_{yr} = 0.223$, $k_{yL} = 14.227$, $k_{ys1} = 16.910$, $k_{ys2} = 14.886$, and $k_{yE} = 4.036$; and for cost, $k_{ceo} = 0.245$, $k_{cr} = 0.037$, $k_{cL} = 1.303$, $k_{cs} = 1.923$, and $k_{cE} = 0.819$.

Using (21) with these coefficients: $m^* = 0.532$; b^* = 12.38 m; and y_n^* = 8.29 m. Further, (22) gives the earthwork cost per meter $C_e = 140.58c_e + 72.65c_e = 213.23c_e$; the lining cost per meter $C_L = 374.54c_e$; and the cost of water lost per meter $C_w = 460.64c_e + 39.24c_e = 499.88c_e$. Thus, the canal cost per meter = $C_e + C_L + C_w = 1{,}087.7c_e$. The cost of lining and water lost as seepage share the major portion of the total cost. These dimensions yield $A = y_n(b + my_n)$ = 139.19 m^2. Thus, V = 250.0/139.19 = 1.796 m/s, which is within the permissible limit (Table 3).

Sensitivity of Optimal Design

For bed width ranging from 0 to 40 m and side slope ranging from 0 to 5, the normal depths were calculated using (12),

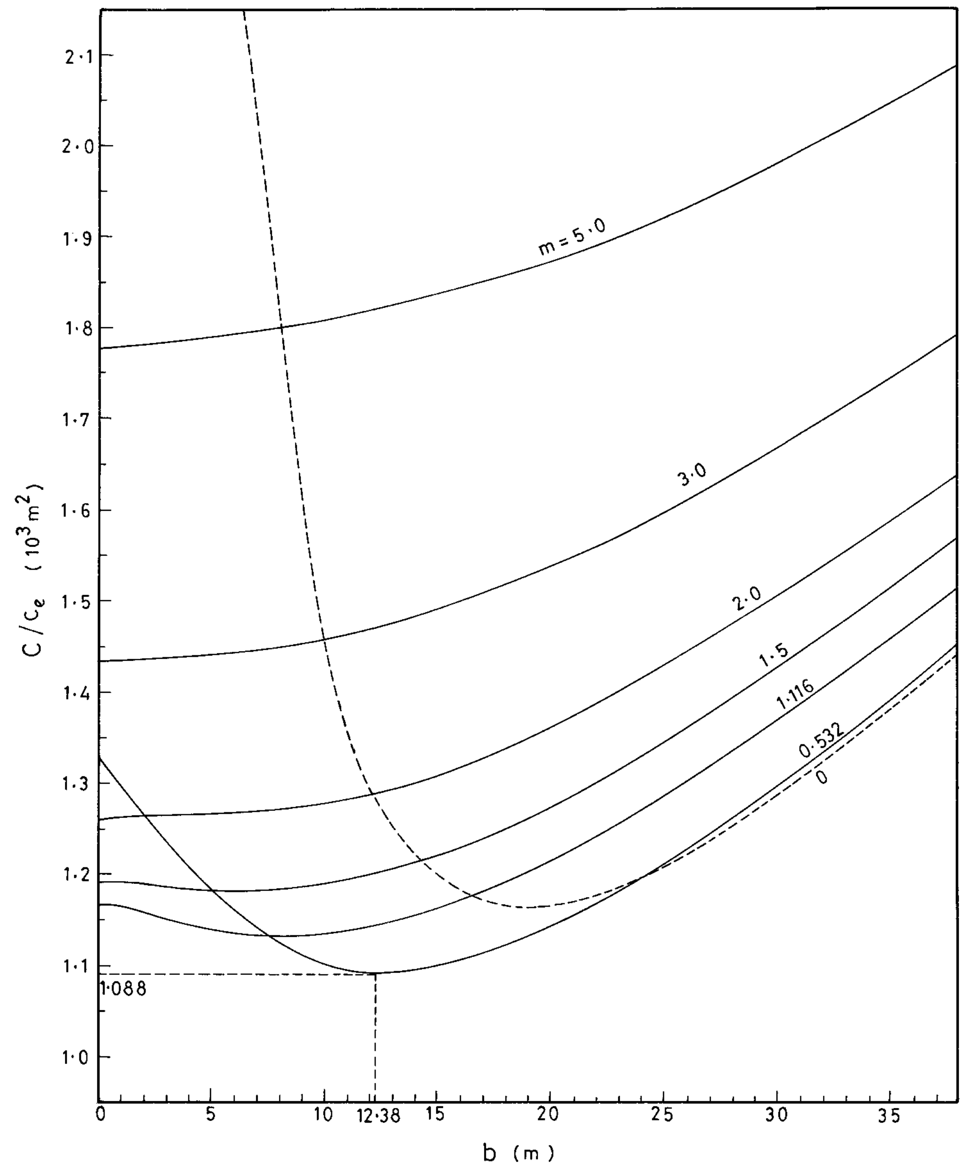

FIG. 2. Sensitivity Analysis of Optimal Design

and costs were obtained using (10). Fig. 2 shows the variation of cost with b and m. The cost per meter length of the canal is least $C = 1{,}087.7c_e$ for a trapezoidal section with side slope 0.532 and bed width 12.38 m. It can be seen that the optimum is less sensitive to increase in bed width and more sensitive to decrease in b. The optimum for the rectangular section ($m = 0$) occurs at $b = 18.62$ m ($C = 1{,}156.5c_e$), and the cost is highly sensitive to decrease in bed width, as seen in Fig. 2. For the same data, the triangular section ($b = 0$) has a minimum cost ($C = 1{,}160.3c_e$) at side slope 1.116. Fig. 2 also shows that, for the design data, the rectangular section is more economical for $b > 24$ m, while the triangular section is more economical for higher side slopes ($m > 2$), and the trapezoidal section is the most economical section otherwise.

CONCLUSIONS

Explicit design equations and section shape coefficients have been presented for the design of minimum cost irrigation canals of triangular, rectangular, and trapezoidal shapes. These equations and coefficients have been obtained by applying nonlinear optimization technique. Using the optimal design equations along with the tabulated section shape coefficients, the optimal dimensions of a canal and the corresponding cost can be obtained. The method avoids the trial-and-error method of canal design and overcomes the complexity of minimum cost design of canals by a constrained nonlinear optimization technique. The optimal design equations show that the optimal section becomes wider and shallower than the minimum area section due to the additional cost of excavation with canal depth, while the reverse is the case due to cost of water lost as evaporation. On the other hand, for increased lining cost and/or excavation cost at the ground level, the optimal canal section approaches the minimum area section, while for increased cost of water lost as seepage, it approaches the minimum seepage loss section. A useful extension of the method in the planning of a canal project has also been presented.

APPENDIX I. SEEPAGE FUNCTION

Swamee et al. (2000) gave the following simple algebraic equations for the seepage functions for triangular, rectangular, and trapezoidal canal sections.

Triangular section:

$$F_s = [(4\pi - \pi^2)^{1.3} + (2m)^{1.3}]^{0.77} \tag{25a}$$

Rectangular section:

$$F_s = [(4\pi - \pi^2)^{0.77} + (b/y_n)^{0.77}]^{1.3} \tag{25b}$$

Trapezoidal section:

$$F_s = \left\{\left[(4\pi - \pi^2)^{1.3} + (2m)^{1.3}\right]^{(0.77+0.462m)/(1.3+0.6m)} + (b/y_n)^{(1+0.6m)/(1.3+0.6m)}\right\}^{(1.3+0.6m)/(1+0.6m)} \quad (25c)$$

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APPENDIX III. NOTATION

The following symbols are used in this paper:

A = flow area of canal (m^2);
b = bed width of canal (m);
C = cost per unit length of canal (\$/m);
C_e = cost of earthwork per unit length of canal (\$/m);
C_L = cost of lining per unit length of canal (\$/m);
C_t = cost of canal (\$);
C_w = capitalized cost of water lost per unit length of canal (\$/m);
c_e = cost per unit volume of earthwork at ground level (\$/m^3);
c_L = cost per unit surface area of lining (\$/m^2);
c_r = increase in unit excavation cost per unit depth (\$/m^4);
c_w = cost per unit volume of water (\$/m^3);
c_{wE} = $3.156 \times 10^7 Ec_w/r$ (\$/m^2);
c_{ws} = $3.156 \times 10^7 kc_w/r$ (\$/m^2);
E = evaporation discharge per unit free surface area (m/s);
F_s = seepage function (dimensionless);
g = gravitational acceleration (m/s^2);
k = hydraulic conductivity (m/s);
k_{fs} = section shape coefficients for subscripts f and s (dimensionless);
L = length scale (m);
m = side slope of canal (dimensionless);
n = number of canal reaches (dimensionless);
P = flow perimeter of canal (m);
p = penalty parameter (dimensionless);
Q = discharge (m^3/s);
q_E = evaporation loss per unit length of canal (m^2/s);
q_s = seepage loss per unit length of canal (m^2/s);
q_w = water loss per unit length of canal (m^2/s);
R = hydraulic radius (m);
r = rate of interest (\$/\$/year);
S_0 = bed slope of canal (dimensionless);
T = width of free surface (m);
V = average velocity (m/s);
V_L = limiting velocity (m/s);
x_i = length of canal in ith reach (m);
y_n = normal depth of flow in canal (m);
ε = average roughness height of canal lining (m);
λ = length scale (m);
ν = kinematic viscosity (m^2/s);
Φ = equality constraint (dimensionless);
Ψ = augmented function (dimensionless); and
\$ = monetary unit.

Subscripts

b = bed width;
c = cost;
E = evaporation;
e = earthwork;
eo = depth-independent earthwork (area minimization) case;
L = lining;
m = side slope;
r = depth-dependent (additional) excavation;
s = seepage;
w = water;
y = normal depth; and
$*$ = nondimensional.

Superscript

$*$ = optimal.