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Design of Minimum Earthwork Cost Canal Sections

PRABHATA K. SWAMEE[1], GOVINDA C. MISHRA[2] and BHAGU R. CHAHAR[3]
[1] *Univ. of Roorkee, Roorkee 247 667, India, e-mail: swamifce@mail.rurkiu.ernet.in;*
[2] *Water Resour. Devlpmnt. Training Ctr., Univ. of Roorkee, Roorkee 247 667, India, e-mail: gcmdrfwt@rurkiu.ernet.in;*
[3] *M.B.M. Engrg. College, J.N.V. Univ., Jodhpur 342 011, India, e-mail: chahar_br@yahoo.com*

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Abstract. Though the minimum area section is generally adopted for canals, it is not the least earthwork cost section as it does not involve the cost of earthwork which varies with the excavation depth. On account of complexities of analysis, explicit design equations for minimum earthwork cost canal sections has not available yet. In this investigation explicit equations and section shape coefficients for the design variables of minimum earthwork cost canal sections for triangular, rectangular, trapezoidal, and circular shapes have been obtained by applying non-linear optimization technique. Application of the proposed design equations along with the tabulated section shape coefficients results directly into the optimal dimensions and corresponding cost of a least earthwork cost canal section without going through the conventional trial and error method of canal design.

Key words: canal design, earthwork, minimum earthwork cost, optimal canal design, optimal section

Notation

The following symbols are used in this paper:

A	flow area of canal [m^2];
a	flow area up to elevation η [m^2];
b	bed width of canal [m];
C_e	earthwork cost per unit length of canal [\$/m];
c_e	unit cost of earthwork at ground level [\$/m^3];
c_r	additional cost of excavation per unit depth [\$/m^4];
D	diameter of canal [m];
g	gravitational acceleration [m s^{-2}];
k_{fs}	section shape coefficients corresponding to subscripts *f* and *s*;
L	length scale [m];
m	side slope of canal;
P	flow perimeter of canal [m];
p	penalty parameter;

Q discharge [m^3 s^{-1}];

R hydraulic radius [m];

r_{fs} section shape exponents corresponding to subscripts f and s;

S_0 bed slope of canal;

s_{fs} section shape exponents corresponding to subscripts f and s;

t_{fs} section shape coefficients corresponding to subscripts f and s;

V average velocity [m s^{-}];

V_L limiting velocity [m s^{-1}];

y_n normal depth of flow in canal [m];

$\overline{y}$ depth of the centroid of area from the ground surface [m];

ε average roughness height of canal surface [m];

η vertical coordinate [m];

ξ half width of canal at ordinate η [m];

λ length scale [m];

ν kinematic viscosity [m^2 s^{-1}];

Φ equality constraint function;

Ψ auxiliary cost function;

φ angle at centre of a circular canal made by wetted perimeter [radian]; and

$ dollar [monetary unit].

Subscript

c cost;

b bed width;

D diameter;

e earthwork;

m side slope;

r depth dependent earthwork case;

y normal depth; and

$*$ non dimensional.

Superscript

* optimal.

1. Introduction

An open channel is used to convey water for irrigation, power generation, industrial and domestic uses. An open channel functioning as an irrigation canal may be a rigid boundary canal or a mobile boundary canal. A canal passing through hard/firm strata may be kept unlined but it is designed as a rigid boundary channel. A rigid boundary canal is designed for uniform flow formula considering hydraulic efficiency, practicability, and economy (Streeter, 1945). Chow (1973) and French (1994) have listed various properties of the most hydraulically efficient sections. Swamee and Bhatia (1972) expressed all the channel dimensions in term of a length scale comprising independent design variables and developed curves for the optimal design of trapezoidal, rounded bottom and rounded corner sections. Sakhuja *et al*. (1984) pointed out that a rounded corner section or a rounded bottom section is more economical than the conventional trapezoidal section. Guo and Hughes (1984) found that a channel narrower than the hydraulically best section results in minimum excavation when free board is taken into consideration. Flynn and Marimno (1987) suggested a non-dimensional shape parameterizing approach for choosing optimal cross-section of canals. Loganathan (1991) presented optimal design procedure for parabolic canals accounting for freeboard and bounds on canal dimensions and velocity. Froehlich (1994) obtained width and depth constrained best trapezoidal section using Lagrange's method of undetermined multipliers. Monadjemi (1994) and Swamee (1995b) have performed a comprehensive investigation for the optimal dimensions for various canal shapes.

Manning's equation as a uniform flow equation was used in the optimal channel design by the above investigators. However, Manning's equation is applicable for rough turbulent flow, and in a limited bandwidth of relative roughness (Anonymous, 1963; Christensen, 1984). A more general resistance equation based on roughness height was used in the optimal design of irrigation canals by Swamee (1995a). All the above-referred optimal sections pertain to minimum area section. Thus the review of literature reveals that considerable work has been reported on the design of minimum area cross section using Manning's equation. Practically no work has been reported on explicit design equation for the least earthwork cost canal sections using general resistance equation. Swamee *et al*. (2000a-c) obtained explicit design equations for minimum seepage loss canal sections and minimum cost lined (with or without cost of water lost as seepage and evaporation) canal sections.

Design of a minimum earthwork cost canal section involves minimization of the earthwork cost, which depends on the excavation depth subject to uniform flow condition in the canal. This optimal design problem results to a non-linear objective function and a non-linear equality constraint making the problem hard to solve analytically. In this paper, generalized empirical equations for design of minimum earthwork cost canal sections have been obtained for triangular, rectangular, trapezoidal, and circular canals.

Table I. Limiting Velocities

Channel surface material	Limiting velocity (m s^{-1})
(1)	(2)
Sandy soil	0.3–0.6
Black cotton soil	0.6–0.9
Muram and hard soil	0.9–1.10
Firm clay and loam	0.9–1.15
Gravel	1.20
Boulder	1.50
Disintegrated rock	1.50
Hard rock	4.0

2. Theory

2.1. FLOW REQUIREMENTS

A rigid boundary canal is designed for the condition of uniform flow. The flow requirements to be taken into account in designing a canal for uniform flow are the channel surface roughness, the minimum permissible velocity, the limiting (maximum permissible) velocity, the freeboard, and the hydraulic efficiency of the canal section. The minimum permissible velocity or the nonsilting velocity is the lowest velocity that will not initiate sedimentation and will not induce the growth of vegetation. Sedimentation and growth of vegetation decrease the carrying capacity and increase the maintenance cost of the canal. In general, an average velocity of 0.6 to 0.9 m s^{-1} will prevent sedimentation when the silt load of the flow is low and a velocity of 0.75 m s^{-1} is usually sufficient to prevent the growth of vegetation (Chow, 1973). Hence, the minimum permissible velocity can be assumed in the range from 0.75 to 0.9 m s^{-1}. Higher velocities are desired in rigid boundary canals to reduce costs. However, high velocities may cause scour and erosion of the boundaries. In rigid boundary canals the maximum permissible velocity or the limiting velocity V_L (m s^{-1}) that will not cause erosion depends on the channel surface material. Table I lists the limiting velocities for different channel surface materials (Subramanya, 1997).

Various investigators have presented several versions of uniform flow equation. Using Darcy-Weisbach friction formula and Colebrook formula (Anonymous, 1963), Swamee (1994) gave the following general resistance equation:

$$V = -2.457\sqrt{gRS_0}\ln\left(\frac{\varepsilon}{12R} + \frac{0.221\nu}{R\sqrt{gRS_0}}\right) \quad (1)$$

where V = average flow velocity (m s^{-1}); g = gravitational acceleration (m s^{-2}); R = hydraulic radius (m) defined as the ratio of the flow area A (m^2) to the flow perimeter P (m); S_0 = longitudinal canal bed slope (dimensionless); ε = average roughness height of the canal surface (m); and ν = kinematic viscosity of water ($m^2\ s^{-1}$).

Gangullit and Kutter's (Garrett, 1948), and Bazin's (Rouse and Ince, 1963) formulae have largely gone out of use. In the Commonwealth Countries of Independent States (erstwhile USSR) the Pavlovsky formula (Anonymous, 1963) is used. The most commonly used uniform flow formula around the world is the Manning equation (Chow, 1973) due to its simplicity and acceptable degree of accuracy in most of practical applications. However, Manning equation is applicable to fully rough turbulent flow for the following limited bandwidth of relative roughness (Swamee, 1994):

$$\varepsilon \geq \frac{30\nu}{(gS_o\sqrt{Q})^{0.4}}; \quad \text{and} \quad 0.004 \leq \frac{\varepsilon}{R} \leq 0.04 \tag{2a,b}$$

where Q = discharge ($m^3\ s^{-1}$). On the other hand, Equation (1) is applicable for hydraulically rough or smooth surfaces or transition in between. It holds good for all types of open channel sections in the following ranges (Swamee, 1994):

$$10^3 \geq \frac{VR}{\nu} \leq 10^8; \quad \text{and} \quad 10^{-6} \geq \frac{\varepsilon}{R} \leq 10^{-3} \tag{3a,b}$$

Similar to the case of resistance equation for pipe flow, the general resistance equation for the uniform flow in a canal involves physically conceivable parameters ε and ν.

The discharge carrying capacity of a canal depends upon the flow area and flow velocity. The law of conservation of mass gives the uniform flow rate or discharge in a canal as

$$Q = AV \tag{4}$$

Combining Equations (1) and (4) the following general flow resistance equation in terms of discharge was obtained:

$$Q = -2.457A\sqrt{gRS_0}\ln\left(\frac{\varepsilon}{12R} + \frac{0.221\nu}{R\sqrt{gRS_0}}\right) \tag{5}$$

Since a rigid boundary canal section is designed to sustain uniform flow, (5) provides the required condition as an equality constraint function in the design.

2.2. COST STRUCTURE

Earthwork in the form of cutting and/or filling along the canal alignment is required for providing canal flow area. Earthwork cost is the major cost item for a canal passing through hard/firm strata, where lining may not be required. Some times canals are lined with low cost lining materials, in which case cost of the earthwork

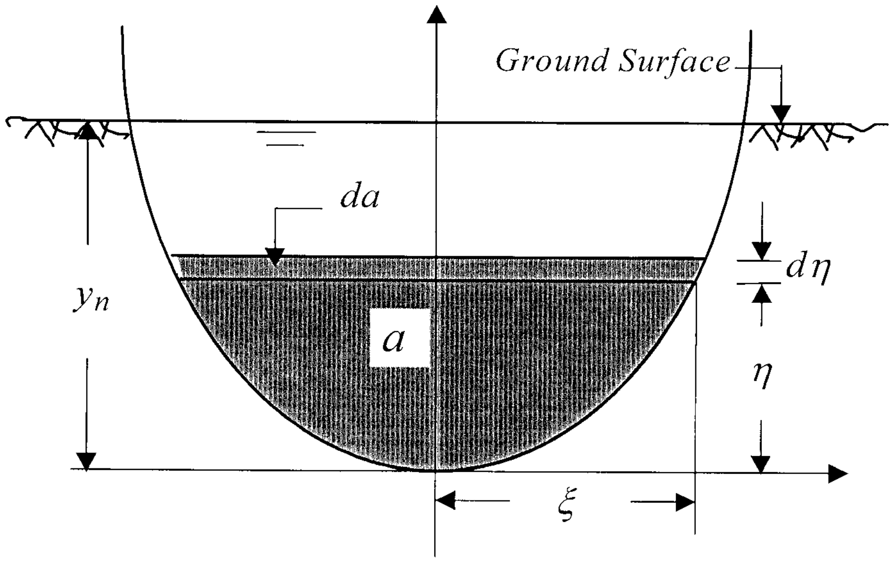


Figure 1. Definition sketch for earthwork cost.

is more significant than the cost of lining. The cost of earthwork depends on the volume and depth of cut and fill. It also depends on the strata to be excavated and the distance of haulage if required in transporting the soil materials (Nichols, 1959; Singh, 1976). The cost function consists of earthwork cost of unit length of the canal. For a canal section with the normal water surface at the average ground level as shown in Figure 1, the earthwork cost C_e (monetary unit per unit length, e.g. \$/m) was given by

$$C_e = c_e A + c_r \int_{Area} (y_n - \eta) da = c_e A + c_r \int_0^{yn} (y_n - \eta) 2\xi d\eta \tag{6}$$

where c_e = cost per unit volume of earthwork at ground level (\$ m^{-3}); c_r = the additional cost per unit volume of excavation per unit depth (\$ m^{-4}); η = vertical ordinate; ξ = half the width of excavation at the ordinate η; $d\eta$ = incremental vertical ordinate; y_n = normal depth of flow (m); a = excavation area (m^2) up to height η; and da = i ncremental excavation area (m^2). It was assumed in Equation (6) that the cost per unit volume of excavation is linear function of the depth of excavation. Integrating Equation (6) by parts resulted to

$$C_e = c_e A + c_r \mid a(y_n - \eta) \mid_o^{y_n} + c_r \int_0^{y_n} a d\eta = c_e A + c_r A \overline{y} \tag{7}$$

where $\overline{y}$ = depth (m) of the centroid of the area of excavation from the ground surface. Table II lists A and for triangular, rectangular, trapezoidal, and circular canal sections. The cost function Equation (7) can be extended to include other canal costs (Chahar, 2000).

Table II. Geometrical properties of canal sections

Section shape	Geometric Elements	
	Area of flow A	Depth of centroid of area $\overline{y}$
(1)	(2)	(3)
Triangular	my_n^2	$y_n/3$
Rectangular	by_n	$y_n/2$
Trapezoidal	$(b+my_n)y_n$	$\frac{y_n}{6}\left(\frac{3b+2my_n}{b+my_n}\right)$
Circular	$\frac{D^2}{8}(\vartheta - \sin\vartheta)$	$\frac{D}{6}\left(\frac{6\sin\vartheta/2-3\cos\vartheta/2-2\sin^3\vartheta/2}{\vartheta-\sin\vartheta}\right)$
where $\vartheta = 2\cos^{-1}(1 - 2y_n/D)$		

2.3. OPTIMIZATION ALGORITHM

From hydraulic viewpoint the canal section having the least wetted perimeter for a given flow area has the maximum discharge carrying capacity; such a section is known as the best hydraulic section. The best hydraulic section has the maximum flow velocity or the minimum flow area and wetted perimeter for a given discharge and canal bed slope. However, the best hydraulic section is not the least earthwork cost section. The least earthwork cost section results from the minimization of earthwork cost per unit length of canal provided it carries the design discharge. Thus the problem of determination of the least earthwork cost canal section was reduced to

$$\text{Minimize } C_e = c_e A + c_r A\overline{y} \tag{8}$$

$$\text{subject to } Q + 2.457A\sqrt{gRS_0}\ln\left(\frac{\varepsilon}{12R} + \frac{0.221\nu}{R\sqrt{gRS_0}}\right) = 0 \tag{9}$$

Adopting a length scale λ (m) as

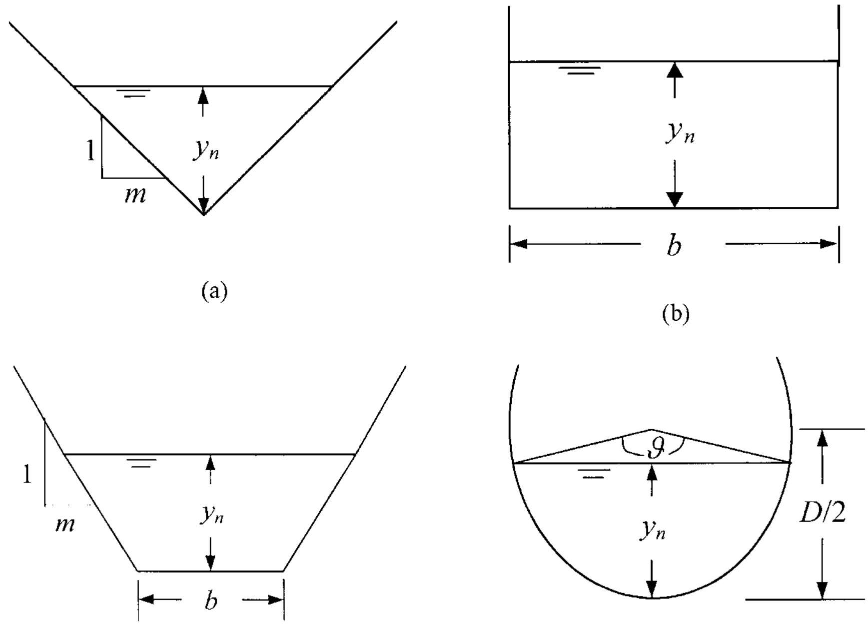
$$\lambda = \left(Q/\sqrt{gS_0}\right)^{0.4} \tag{10}$$

the following non dimensional variables were defined:

$$\varepsilon_* = \varepsilon/\lambda; \quad \nu_* = \nu\lambda/Q; \quad C_{e*} = C_e/(C_e\lambda^2); \quad c_{r*} = c_r\lambda/c_e \tag{11a–d}$$

$$A_* = A/\lambda^2; \quad \overline{y}_* = \overline{y}/\lambda; \quad P_* = P/\lambda; \quad R_* = R/\lambda \tag{11e–h}$$

where variables with subscript $*$ denotes corresponding non-dimensional parameter.



(c) (d)

Figure 2. Canal Sections: (a) Triangular Section (b) Rectangular Section (c) Trapezoidal Section (d) Circular Section.

Using Equations (8), (9), and (11) the problem of determination of optimal canal section shape in non-dimensional form was reduced to

$$\text{Minimize } C_{e*} = A_* + c_{r*}A_*\bar{y}_* \tag{12}$$

$$\text{subject to } \Phi = 1 + 2.457A_*\sqrt{R_*}\ln\left(\frac{\varepsilon_*}{12R_*} + \frac{0.221\nu_*}{R_*^{1.5}}\right) = 0 \tag{13}$$

where Φ = equality constraint function.

This constrained optimization problem was converted into an unconstrained optimization problem using penalty function (Fox 1971). The auxiliary cost function Ψ for the unconstrained optimization problem was expressed as

$$\Psi = C_{e*} + p\Phi^2 \tag{14}$$

where p = a penalty parameter.

Adopting small p initially, Equation (14) was minimized using Powel's conjugate direction search method (Avriel, 1976; Himmelblau, 1972) to find the design variables. Increasing p ten-fold, the minimization was carried through various cycles till the optimization results stabilized. The optimization technique can be extended to compromise other cost functions (Chahar, 2000; Swamee *et al.*, 2000a-c).

2.4. OPTIMAL SECTION SHAPES

A large number of optimal sections were obtained by applying the optimization algorithm on triangular, rectangular, trapezoidal, and circular canal sections for input variables ε_*, ν_*, and c_{r*} varying in the ranges

$$10^{-6} \leq \varepsilon_* \leq 10^{-3}; \quad 10^{-7} \leq \nu_* \leq 10^{-5}; \quad 0 \leq c_{r*} \leq 100 \qquad \text{(15a–c)}$$

Analysis of these optimal sections (Chahar, 2000) resulted to general empirical equations for the optimal side slope m, bed width b (m), normal depth y_n (m), diameter D (m), and earthwork cost per unit length of canal. See Figure 2 for definition of m, b, y_n, and D. The following general equations applicable to all the four canal shapes were obtained for the optimal dimensions and corresponding cost of a least earthwork cost section:

$$m^* = k_m \left[1 + t_{mr}\,(c_r L/c_e)^{r_{mr}}\right]^{s_{mr}} \qquad \text{(16a)}$$

$$y_n^* = k_{ye} \left[1 + t_{yr}(c_r L/c_e)^{r_{yr}}\right]^{-s_{yr}} L \qquad \text{(16b)}$$

$$b^* = k_{be} \left[1 + t_{br}(c_r L/c_e)^{r_{br}}\right] s_{br} L \qquad \text{(16c)}$$

$$D^* = k_{De} \left[1 + t_{Dr}(c_r L/c_e)^{r_{Dr}}\right]^{s_{Dr}} L \qquad \text{(16d)}$$

$$C_e^* = k_{ce} \left[1 + t_{cr}(c_r L/c_e)^{r_{cr}}\right]^{s_{cr}} c_e L^2 \qquad \text{(16e)}$$

where superscript * indicates optimality; k_{fs} and t_{fs} = section shape coefficients; r_{fs} and s_{fs} = section shape exponents; and L = length scale (m) given by Swamee (1995a) as

$$L = \lambda(\varepsilon_* + 8\nu_*)^{0.04} \qquad \text{(17)}$$

Equation (17) shows that L is a lumped response to design discharge, canal bed slope, roughness properties of canal surface and fluid resistance property.

The first subscript *m, b, D, y*, and *c* appearing in section shape coefficients and exponents denote side slope, bed width, diameter, normal depth, and cost respectively; and the second subscript *e* and *r* denote depth independent earthwork (minimum area) case and depth dependent earthwork case respectively. The optimal section shape coefficients and exponents are listed in Table III.

The behavior of optimal design Equations (16a-d) was plotted in Figure 3, which shows that side slope and bed width of the optimal section increase and normal depth of the optimal section decreases with increase in additional cost of excavation with canal depth. Accordingly, the optimal section is wider and shallower than the minimum area section. The optimal dimensions are very sensitive for $c_r L/c_e > 2$. For $c_r = 0$, Equation (16) reduces to the following equations for the minimum area section as obtained by Swamee (1995a):

$$m^* = k_{me}; \quad b^* = k_{be}L; \quad y_n^* = k_{ye}L; \quad D^* = k_{De}L; \quad C_e^* = k_{ce}c_e L^2 \qquad \text{(18)}$$

Table III. Coefficients and exponents for minimum earthwork cost sections

Entity	Coefficients or exponents	Section shape			
		Triangular	Rectangular	Trapezoidal	Circular
(1)	(2)	(3)	(4)	(5)	(6)
	k_{me}	1.0000		0.57735	
Side	t_{mr}	0.2572		10.000	
Slope	r_{mr}	0.8613		1.2586	
	s_{mr}	1.1525		0.08069	
	k_{be} or k_{De}		0.71136	0.43407	0.78065
Bed Width	t_{br} or t_{Dr}		0.1122	0.3195	0.07023
or	r_{br} or r_{Dr}		0.7143	0.9342	0.9014
Diameter	s_{br} or S_{Dr}		2.3759	1.0712	3.2330
	k_{ye}	0.50301	0.35568	0.37592	0.39032
Normal	t_{yr}	0.02797	0.2550	0.2088	0.0161
Depth	r_{yr}	0.7127	0.7422	0.8123	0.7196
	s_{yr}	4.3600	1.0375	1.0280	6.0000
	k_{ce}	0.25302	0.25302	0.24476	0.23932
Cost	t_{cr}	0.2451	0.2234	0.2277	0.2124
	r_{cr}	1.0637	0.9233	0.9544	0.9744
	s_{cr}	0.6212	0.7107	0.6855	0.7440

The design equations for the least earthwork cost section can be reduced by dropping lining cost from design equations for a minimum cost lined section (Swamee *et al.*, 2000b) or by dropping lining cost and cost of water from design equations for a minimum cost general section (Swamee *et al.*, 2000c). Equations so obtained are linear functions of the additional earthwork cost on account of excavation depth. Also, such equations are applicable for a limited range of the additional earthwork cost i.e., $c_r* \leq 1$. Only a small capacity canal may satisfy this range. c_r* ≥ 1 for most of the canals. For higher values of c_r, the optimal design parameters are highly non-linear functions as shown in Figure 3 and hence, require non-linear equations (16a-e) for designing the least earthwork cost section.

A least earthwork cost section can be designed by an However, this method requires lot of programming and computing, and involves all the associated problems of a non-linear optimization with a non-linear equality constraint. On the other hand, the present method results to a least earthwork cost canal section in single step computations. For a given set of data, the use of Equations (16) and (17), along with Table III yields the optimal dimensions of a canal section. For this section the average flow velocity V can be obtained by Equation (4). This velocity should be greater than the non-silting velocity but less than the limiting velocity V_L.

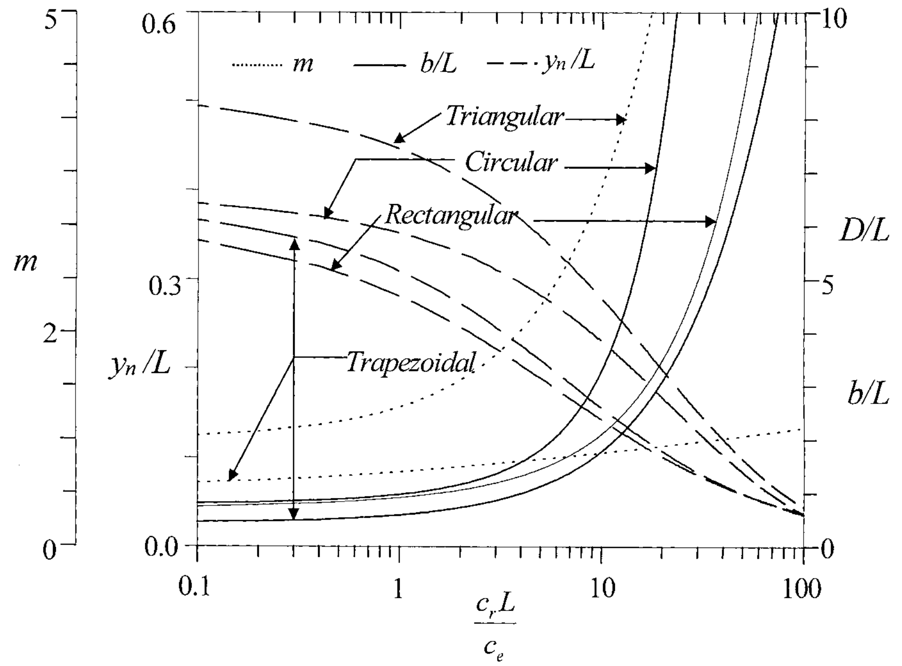


Figure 3. Variations in optimal dimensions with earthwork cost.

If V is greater than V_L, use a canal route with a smaller bed slope, adopt a flatter bed slope with falls, build two smaller canals, provide a channel surface material having higher limiting velocity, or opt for a non-optimal section.

3. Applications

(i) Design a trapezoidal canal section for carrying a discharge of 250 m^3 s^{-1} on a longitudinal slope of 0.0001. The canal passes through a hard stratum for which c_e/c_r = 9 m. For the design g = 9.79 m s^{-2}; ν = 1.007 × 10^{-6} m^2 s^{-1} (water at 20 °C); and ε = 4 mm (rough surface, see Anonymous, 1963) have been adopted.

Using Equation (10), λ = 36.393 m; Equation (11a) ε_* = 1.099 × 10^{-4}; Equation (11b) ν_* = 1.466 × 10^{-7}; Equation (11d) $c_r{}^*$ = 4.044; and Equation (17) L = 25.284 m.

For a trapezoidal section, Table III gives section shape coefficients: for side slope k_{me} = 0.57735, t_{mr} = 10.00, r_{mr} = 1.2586, and s_{mr} = 0.08069; for bed width k_{be} = 0.43407, t_{br} = 0.3195, r_{br} = 0.9342, and s_{br} = 1.0712; for normal depth k_{ye} = 0.37592, t_{yr} = 0.2088, r_{yr} = 0.8123, and s_{yr} =1.0280; and for cost k_{ce} = 0.24476, t_{cr} = 0.2277, r_{cr} = 0.9544, and s_{cr} = 0.6855.

With these coefficients Equations (16a-c) yield: m^* = 0.774; b^* = 21.073 m; and $y_n{}^*$ = 6.338 m. Further, Equation (16e) gives: earthwork cost per meter length of canal $C_e{}^*$ = 216.90 c_e. Adopt m = 0.8; b = 21.0 m; and y_n = 6.35 m. These dimensions result to $A = y_n\ (b + my_n) = 165.608$ m^2. Thus, $V = 250/165.608 = 1.509$ m s^{-1} which is within permissible limit (Table II).

On the other hand, the corresponding design values obtained for the depth independent earthwork cost (minimum area) section using Equation (18) are $m^* = 0.57735$; $b^* = 10.975$ m; $y_n{}^* = 9.505$ m; $A = 156.470$ m^2; and $V = 1.598$ m s^{-1}. The canal section cost with these dimensions comes out $C_e{}^* = 229.91c_e$. The design reveals that there is 92% increase in the bed width, 33.3% decrease in the normal depth and 34% increase in the side slope of the canal section, and 5.7% decrease in the canal cost when the earthwork cost as a function of depth is considered. It can be seen that the canal dimensions are more sensitive than the canal section cost to change in earthwork cost with depth of excavation.

(ii) Design rectangular, triangular, and circular canal sections for $Q = 50$ m^3 s^{-1}, $S_o = 0.0002$, and $c_e/c_r = 9$ m. For the given data: $\lambda = 16.643$ m; $\varepsilon_* = 2.403 \times 10^{-4}$; $\nu_* = 3.352 \times 10^{-7}$; $c_r* = 1.849$; and $L = 11.930$ m.

Using the section shape coefficients and exponents for a rectangular section from Table III and design Equations (16b,c,e): $b^* = 11.519$ m; $y_n{}^* = 3.196$ m; and $C_e{}^* = 43.15c_e$. Thus, $A = 36.815$ m^2; and $V = 1.358$ m s^{-1}, which is safe. The corresponding design values for minimum area section, using Equation (18), are $b^* = 8.487$ m; $y_n{}^* = 4.243$ m; $C_e{}^* = 44.50c_e$; $A = 36.01$ m^2; and $V = 1.389$ m s^{-1}.

For a triangular canal section the optimal design parameters are: $m^* = 1.387$; $y_n{}^* = 5.183$ m; $C_e{}^* = 43.01c_e$; $A = 37.260$ m^2; and $V = 1.342$ m s^{-1}. On the other hand, the optimal parameters for a trapezoidal section for the present design case are: $m^* = 0.719$; $y_n{}^* = 3.529$ m; $b^* = 7.515$ m; $C_e{}^* = 41.65c_e$; and $A = 35.475$ m^2. Using coefficients and exponents from Table III in design Equations (16b,d,e) for an optimal circular section: $y_n{}^* = 4.142$ m; $D^* = 12.325$ m; and $C_e{}^* = 40.92c_e$. Using relations for φ and A from Table II, $\varphi = 2.4733$ radians and $A = 32.978$ m^2.

Comparisons of costs between optimal sections show that an optimal circular section has least earthwork cost. A minimum earthwork cost trapezoidal section is more economical than minimum earthwork cost triangular and rectangular sections. Between optimal triangular and rectangular canals, the former canal is marginal economical than the latter canal.

4. Conclusions

Explicit design equations and section shape coefficients have been presented for the design of minimum earthwork cost canals of triangular, rectangular, trapezoidal, and circular shapes. These equations and coefficients have been obtained by applying non-linear optimization technique. Using the optimal design equations along with the tabulated section shape coefficients, the optimal dimensions of a canal and the corresponding cost can be obtained in single step computations. The method avoids the use of non-linear optimization technique in designing a least earthwork cost canal section. The optimal design equations show that on account of additional cost of excavation with canal depth the optimal section is wider and shallower than the minimum area section.

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