Extension of Vedernikov's Graph for Seepage from Canals

by Bhagu R. Chahar[1]

Abstract

In this investigation, using previously derived equations by Vedernikov and Morel-Seytoux, closed-form solutions have been obtained to compute the seepage from a slit and a strip. Also, a graphical solution as an extension of Vedernikov's graph has been presented for computing quantity of seepage from triangular, rectangular, and trapezoidal canals. The solution replaces approximately the cumbersome evaluation of improper integrals with unknown implicit transformation variables.

Introduction

Canals continue to be major conveyance systems for delivering water for irrigation. The seepage losses from irrigation canals constitute a substantial percentage of the usable water. By the time the water reaches the field, it has been estimated that, depending upon the site-specific conditions, the seepage losses may be on the order of 45% of the water supplied at the head of the canal (Sharma and Chawla 1975). According to the Indian Standard (IS:9452 1980) the loss of water by the seepage from unlined canals in India generally varies from 0.3 to 7.0 m^3/s per 10^6 m^2 of wetted surface. The transit losses are more accentuated in alluvial canals. Sometimes a higher rate of seepage is desired for artificial ground water recharge. On the other hand, seepage losses not only lead to depletion of fresh water resources but also cause water logging, salinization, and ground water contamination. Therefore, computation of the seepage loss from canals is an important aspect for sustainable management of land and water resources. Canals are often lined to reduce the seepage. A perfect lining checks all the seepage but the canal lining deteriorates with time and becomes ineffective in controlling the seepage. Since an increase in the hydraulic conductivity of the lining reduces the resistance against the seepage drastically, a well-maintained canal with a 99% perfect lining (hydraulic conductivity of lining being 0.01 times of subsoil medium) checks the seepage only about 30% to 40% of the time (Wachyan and Rushton 1987). Because the efficiency of the lined channel decreases rapidly with deterioration of the lining, the seepage from a deteriorated or cracked lined channel may approach the seepage from an unlined canal.

The seepage from a canal is governed by the horizontal and the vertical hydraulic conductivities of the subsoil, canal geometry, hydraulic gradient between the canal and the aquifer underneath and its fluctuations with time, and the initial and boundary conditions. The seepage from canals was estimated for different sets of specific conditions (Harr 1962; Polubarinova-Kochina 1962; Morel-Seytoux 1964; Garg and Chawla 1970; Subramanya et al. 1973; Sharma and Chawla 1979). Vedernikov (1934, 1936, 1937, 1939), Harr (1962), Morel-Seytoux (1961), and Polubarinova-Kochina (1962) presented an exact mathematical solution to unconfined steady-state seepage from a trapezoidal canal in a homogeneous isotropic porous medium of large depth. The solution was obtained using inversion of hodograph and conformal mapping technique. A triangular canal is a particular case of the trapezoidal canal. The seepage from a rectangular canal cannot be computed from the analytical solution given by Vedernikov for the trapezoidal canal. Morel-Seytoux (1964) obtained an analytical solution for a rectangular section using conformal mapping and Green-Neumann functions. The use of these analytical solutions, which contain improper integrals and unknown implicit transformation variables, is not convenient in estimating the quantity of seepage. Swamee et al. (2000) simplified these analytical methods. The graphical solution, such as Vedernikov's graph (Harr 1962), provides another simpler alternative to analytical methods, but such a graph is not available for rectangular canals and trapezoidal canals with side slope steeper than 1:1. In the present study Vedernikov's graph has been extended for computation of the seepage from triangular, rectangular, and trapezoidal canals.

Quantity of Seepage from Canals

The unconfined steady-state seepage from a canal in a homogeneous and isotropic porous medium of infinite extent, when the water table is at a very large depth (Figure 1), was expressed by Vedernikov (Harr 1962) as

$$q_s = ky(A + W/y) \tag{1}$$

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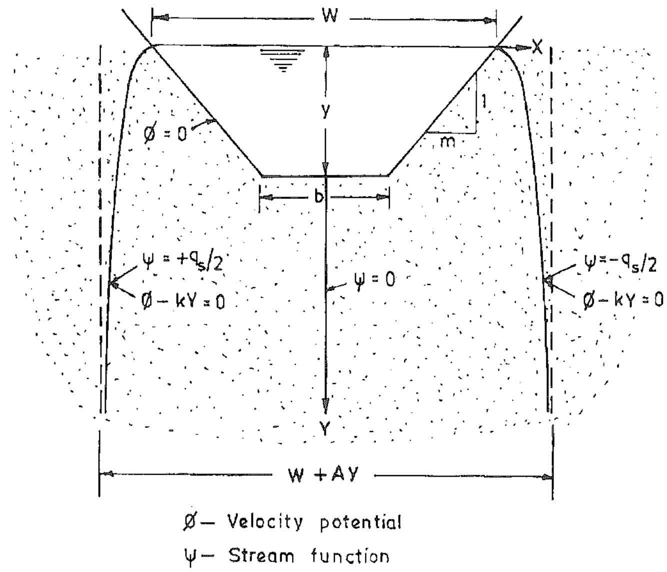

Figure 1. Flow field and boundary conditions for seepage from a canal.

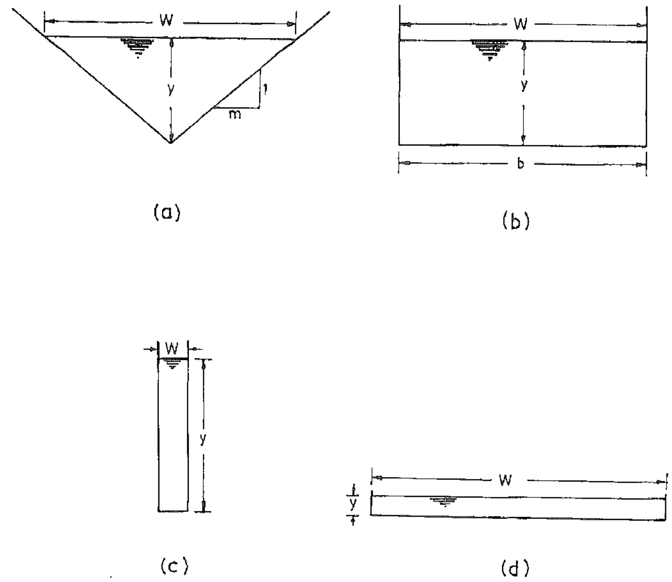

Figure 2. Canal section shapes: (a) triangular section; (b) rectangular section; (c) slit section; and (d) strip section.

where q_s = quantity of seepage per unit length of canal (m^2/s); k = hydraulic conductivity of the porous medium (m/s); y = maximum depth of water in the canal (m); W = width of the canal at the water surface (m); and A = seepage parameter (dimensionless), which is a function of channel geometry. Figure 1 and Figure 2 (a through d) show W and y for trapezoidal, triangular, rectangular, slit, and strip canal sections.

Triangular Section

For a triangular channel (Figure 2a) Vedernikov (Morel-Seytoux 1961; Harr 1962) gave the following equation for the seepage parameter:

$$A = \frac{2m\int_0^1 \sin^{-1} t(1-t^2)^{-(0.5+\sigma)}\, t^{-(1-2\sigma)}dt}{\int_0^1 \cos^{-1} t(1-t^2)^{-(0.5+\sigma)}\, t^{-(1-2\sigma)}\, dt} \qquad (2)$$

where m = side slope (dimensionless) (Figure 2 a); $\sigma = \frac{1}{\pi}\cot^{-1} m$; and t = dummy variable (dimensionless).

Rectangular Section

For a rectangular canal (Figure 2b) Morel-Seytoux (1961, 1964) presented the following equation for the seepage parameter:

$$A = \frac{\pi^2 - 2\int_0^{\alpha} \cos^{-1}\left(\frac{2t^2+1-\alpha^2}{1+\alpha^2}\right)\frac{dt}{1+t^2}}{2\int_{\alpha}^{\infty} \ln\left[\frac{\sqrt{1+t^2}+\sqrt{t^2-\alpha^2}}{\sqrt{1+\alpha^2}}\right]\frac{dt}{1+t^2}} \qquad (3)$$

where α = transformation variable given by

$$\frac{b}{y} = \frac{\int_0^{\alpha} \cos^{-1}\left(\frac{2t^2+1-\alpha^2}{1+\alpha^2}\right)\frac{dt}{1+t^2}}{\int_{\alpha}^{\infty} \ln\left[\frac{\sqrt{1+t^2}+\sqrt{t^2-\alpha^2}}{\sqrt{1+\alpha^2}}\right]\frac{dt}{1+t^2}} \qquad (4)$$

where b = bed width (m) (Figure 2b).

Trapezoidal Section

For a trapezoidal canal (Figure 1), Vedernikov (Morel-Seytoux 1961; Harr 1962) expressed the seepage parameter as the following:

$$A = \frac{2\cos(\pi\sigma)\int_{\beta}^{1} \frac{t\cos^{-1} t\, dt}{(1-t^2)^{(0.5+\sigma)}\,(t^2-\beta^2)^{(1-\sigma)}} - 2\int_0^{\beta}\frac{t\sin^{-1} t\, dt}{(1-t^2)^{(0.5+\sigma)}\,(\beta^2-t^2)^{(1-\sigma)}}}{\sin(\pi\sigma)\int_{\beta}^{1} \cos^{-1} t\,(1-t^2)^{-(0.5+\sigma)}\,(t^2-\beta^2)^{-(1-\sigma)}\, tdt} \qquad (5)$$

where β = transformation variable given by

$$\frac{b}{y} = \frac{2\int_0^{\beta} \sin^{-1} t(1-t^2)^{-(0.5+\sigma)}\,(\beta^2-t^2)^{-(1-\sigma)}\, tdt}{\sin(\pi\sigma)\int_{\beta}^{1} \cos^{-1} t(1-t^2)^{-(0.5+\sigma)}\,(t^2-\beta^2)^{-(1-\sigma)}\, tdt} \qquad (6)$$

Slit

A narrow and deep channel (Figure 2c) can be assumed as a slit. Solution for the seepage from a slit forms a particular case of the solutions given for triangular, rectangular, and trapezoidal canals. For a slit, width at water surface approaches zero, i.e., $W/y \to 0$. This means $m \to 0$ for a triangular section; $b/y \to 0$ for a rectangular channel; and both $m \to 0$ and $b/y \to 0$ for a trapezoidal canal. Thus, Equation 1 for the seepage from a slit became

$$q_s = Aky \qquad (7)$$

Evaluation of Equation 2 with limit $m \to 0$ yields (Chahar 2000)

$$A = \frac{\pi^2}{4G} = 2.69377 \qquad (8)$$

where G = Catalan's constant = 0.915965594 . . . (Spiegel 1990).

Equation 8 was approximated, within 0.11% error, by the following simple expression:

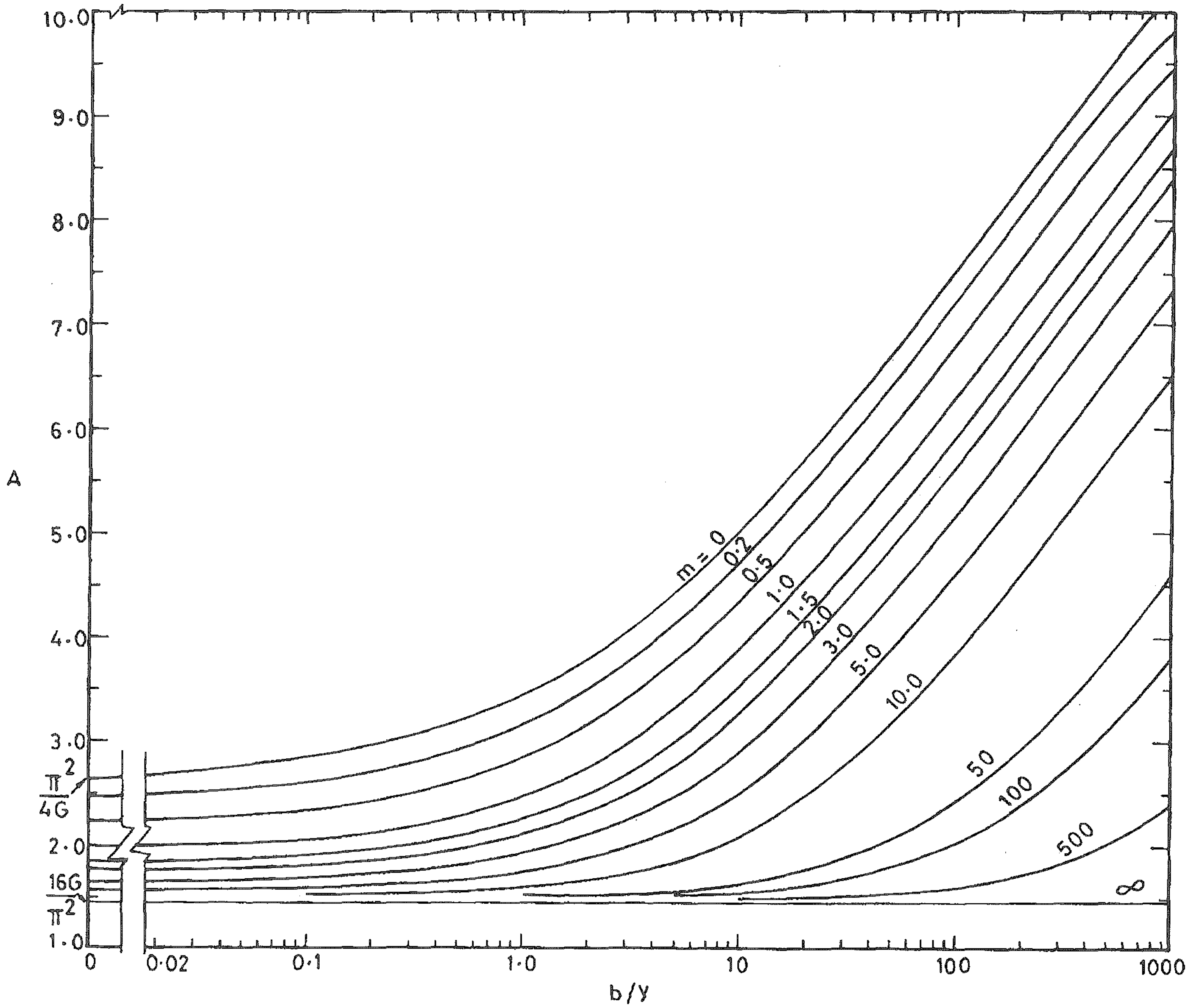

Figure 3. Extended Vedernikov's graph.

$$A \approx \pi(4 - \pi) \approx 2.69676 \tag{9}$$

The same result was obtained from Equation 3 for a rectangular section with limit $b/y \to 0$ and from Equation 5 for a trapezoidal channel with limit $m \to 0$ and $b/y \to 0$.

Strip

A strip is a reverse case of a slit, i.e., for a strip $W/y \to \infty$. This happens for a wide and shallow channel (Figure 2d). Solution of Equation 2 with limit $m \to \infty$ gave

$$A = \frac{16G}{\pi^2} \approx \frac{4}{\pi(4 - \pi)} \approx 1.48326 \tag{10}$$

However, the seepage parameter A becomes negligible in comparison to W/y, when $W/y \to \infty$. Therefore, the seepage from a strip was given by

$$q_s = kW \tag{11}$$

Equation 11 shows that the strip model, as a limiting case as $W \to \infty$ that ponded thickness is infinitesimal, becomes simply Darcy's law.

Extension of Vedernikov's Graph

The seepage parameter A was obtained numerically for a triangular section using Equation 2, for a given m by Romberg integration (Press et al. 1996). Repeating the process, A was obtained for a large number of m lying in the range $0 \le m < \infty$. For a rectangular channel, the transformation variable α was obtained by a trial-and-error procedure using Equation 4 for a known b/y. Further, substituting α in Equation 3, the seepage parameter was obtained. Repeating this process, A was obtained for a large number of b/y values lying in the range $0 \le b/y < \infty$. Adopting the similar procedure and using Equations 5 and 6, A was obtained for a trapezoidal section in the ranges $0 \le m < \infty$ and $0 \le b/y < \infty$. A perusal of the seepage parameters so obtained for rectangular and trapezoidal canals revealed that the values of A for different m attain a maximum and then asymptotically decrease to reach at $A = 16G/\pi^2$. An analysis also was carried to show that the maximum value of the seepage parameter ($A_{max} = 16G = 14.65545$) occurs for the rectangular section at $b/y = 3.3208 \times 10^5$.

A general Vedernikov type graph was plotted to facilitate computation of quantity of seepage from triangular, rectangular, and trapezoidal canals. In Figure 3 A was plotted on the ordinate axis corresponding to various b/y ratios on the abscissa axis for a family of curves for different m. The ordinate axis (b/y = 0) represents A for a triangular section, while the curve for m = 0 gives A for a rectangular channel section. The intersection point of the curve m = 0 with the ordinate axis gives A (= 2.69377) for a slit, while the line parallel to the abscissa axis and passing through $16G/\pi^2$ represents A (= 1.48491) for a strip. The curve for the rectangular canal envelops all the seepage parameter values for the triangular and the trapezoidal canal sections as shown in Figure 3. The seepage from a rectangular canal or a trapezoidal canal with b/y > 1000 can be estimated, with less than 1% error, by assuming a strip. On the other hand, a canal with b/y < 0.02 can be considered a triangular canal for the seepage computation purpose.

Discussion and Examples

The present method is based on the solutions given by Vedernikov and Morel-Seytoux for the steady seepage from a canal in a homogeneous and isotropic porous medium of a large extent in which the water table is at very large depth. At the beginning of the canal operation and due to fluctuations in the shallow water table the seepage loss from the canal is unsteady. The steady state can be assumed for the depth of the water table greater than W + 3y after long operation of the canal. A horizontally stratified medium having horizontal hydraulic conductivity much more than the vertical hydraulic conductivity can be transformed into a fictitious isotropic medium. The transformed canal dimensions and equivalent hydraulic conductivity should be used to compute the seepage discharge from the canal with the present method. The existence of the water table or impervious boundary or drainage layer at shallow depth limits the use of the method. If such depth, however, is greater than W + 3y, then its effect on the seepage becomes practically insignificant and the present method can safely be used.

For example: Determine the quantity of seepage from (a) a trapezoidal canal with b = 5 m, y = 2 m, and side slope = 2; (b) a rectangular canal with b = 10 m, and y = 2 m; and (c) a triangular canal with y = 2 m, and side slope = 2. The porous medium below the canal is homogeneous and isotropic and the water table is at a large depth.

(a) Trapezoidal canal section: W = b + 2my = 13 m; b/y = 2.5; and W/y = 6.5. For m = 2 and b/y = 2.5, Figure 3 gives $A \approx 2.4$. Also, for m = 2 and W/y = 6.5, $A \approx 2.4$ from Vedernikov's graph (Harr 1962). Using Equation 1, $q_s \approx 2\,(2.4 + 6.5)\,k \approx 17.8\ k\ m^3/s$ per meter length of the canal.

(b) Rectangular canal: b/y = W/y = 5. Figure 3 gives $A \approx 4.4$ for m = 0 and b/y = 5. Using Equation 1, the quantity of seepage $\approx 18.8\ k\ m^3/s$ per meter length of the canal. For b/y = 5, $q_s/k\sqrt{\text{Area}} \approx 4.25$ from the graph plotted by Morel-Seytoux (1964); hence $q_s \approx 4.25\sqrt{10 \times 2}\ k \approx 19.0\ k\ m^2/s$.

(c) Triangular canal section: W = 2my = 8 m; b/y = 0; and W/y = 4. $A \approx 1.8$ for m = 2 and b/y = 0 from Figure 3. $A \approx 1.8$ for m = 2, also, from Vedernikov's graph for a triangular channel (Harr 1962). Therefore, the quantity of seepage per meter length of the triangular canal $\approx 2\,(1.8 + 4)\,k \approx 11.6\ k\ m^3/s$.

Conclusions

Exact analytical solutions for the seepage from a slit and a strip have been presented. Further, a graphical solution as an extension of Vedernikov's graph has been given for computing seepage from canals. Using the single set of graphs, the quantity of seepage from triangular, rectangular, and trapezoidal canals of any side slope can be computed easily.

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