

DESIGN OF MINIMUM SEEPAGE LOSS CANAL SECTIONS^a

Discussion by Sohan L. Atmapoojya⁴ and Ramesh N. Ingle⁵

The authors are to be congratulated for adopting the concept of minimum seepage loss in designing the optimum canal section. They have used (10) for calculation of resistance in uniform channel flow. Eq. (10) is applicable for partially as well as fully turbulent flows and hence can be considered more general in character. However, Manning's equation is more popular and generally adopted for design of canal section by practicing engineers, due to the fact that the flow in canals is in the fully turbulent region in most cases and engineers are more familiar with the values of Manning's coefficient of roughness n , due to its availability in literature for a wide range of lining materials. Further, it is difficult to estimate the value of ϵ in situations where the canal lining is cracked or has uneven and irregular joints, since this type of information is not available in the literature. Therefore, the discussers are suggesting a simplified procedure for design of canal section following the authors' method and using Manning's equation.

Using Manning's equation, the section factor for computation of uniform flow in an open channel can be written as

$$AR^{2/3} = \frac{Q \cdot n}{\sqrt{S_0}} \quad (22)$$

For a trapezoidal channel section, the section factor can be expressed as

$$AR^{2/3} = \frac{y^{8/3}(b/y + m)^{5/3}}{(b/y + 2\sqrt{1 + m^2})^{2/3}} \quad (23)$$

From (22) and (23), the normal depth of flow can be obtained as

$$y_n = \left(\frac{nQ}{\sqrt{S_0}} \right)^{3/8} \cdot \frac{(b/y_n + 2\sqrt{1 + m^2})^{1/4}}{(b/y_n + m)^{5/8}} \quad (24)$$

From (9), (11), and (24), the seepage loss per unit length of canal in nondimensional form can be expressed as

$$Sq_s = \frac{q_s}{k(nQ/\sqrt{S_0})^{3/8}} = \frac{(b/y_n + 2\sqrt{1 + m^2})^{1/4}}{(b/y_n + m)^{5/8}} \cdot F \quad (25)$$

where F is given by (9)

Eq. (25) expresses seepage loss per unit length of canal in nondimensional form as a function of b/y_n and m , and it is solved by a computer for various values of m in the range 0–3.0, varying b/y_n from 0 to 3.0. The values of Sq_s obtained for different values of m are plotted against b/y_n in Fig. 4.

In the conventional method of designing the canal section, the designer chooses the ratio b/y_n , then arrives at the dimensions of the canal section and checks for its suitability from the consideration of depth and velocity of flow. Fig. 4 can be very conveniently used, as it will give additional information regarding seepage loss for the chosen value of b/y_n . Also, if desired, b/y_n can be chosen corresponding to the minimum value of seepage loss, i.e., the minimum value of Sq_s .

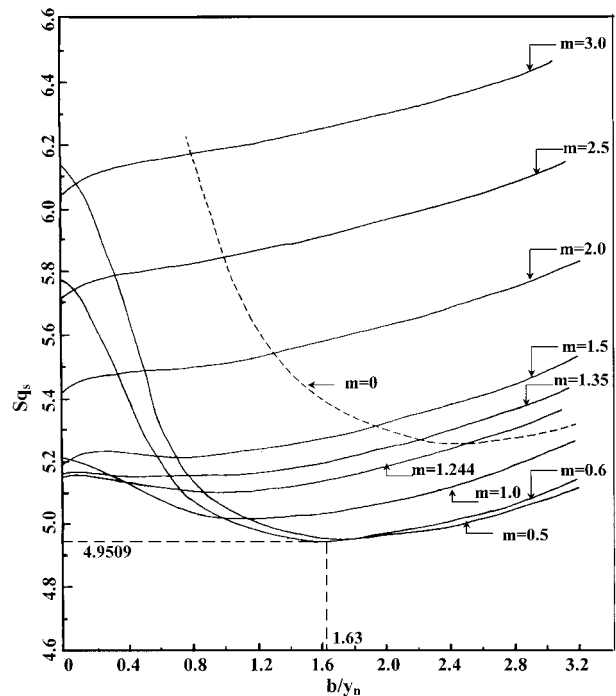


FIG. 4. Variation of Dimensionless Seepage Loss with b/y_n Ratio and Side Slope

The values calculated by the suggested simplified method based on Manning's equation compare very well with the values calculated by the authors. From Fig. 4, the minimum value of Sq_s for a trapezoidal section occurs for $m = 0.6$ and $b/y_n = 1.63$, while according to the authors, it occurs at $m = 0.598$ and $b/y_n = 1.646$. For a triangular canal section, Sq_s is minimum at $m = 1.241$ (1.244 by the authors) and for a rectangular canal section, Sq_s is minimum at $b/y_n = 2.51$ (2.513 by the authors). These values are very close to those of the authors.

For comparing values calculated by the authors in the illustrative examples, the discussers have applied (24) and (25). The equivalent values of n are calculated by the discussers corresponding to $\epsilon = 1$ mm, $\nu = 1.1 \times 10^{-6}$ m²/s, and $g = 9.79$ m/s². The values obtained for example 1 are: $n = 0.01439$, $b = 7.889$ m (7.902 m by the authors), $y_n = 3.143$ (3.145 m by the authors), and $q_s = 2.0154 \times 10^{-5}$ m²/s (2.018 $\times 10^{-5}$ m²/s by the authors); and for example 2, $n = 0.01539$, $b = 12.985$ m (13.055 m by the authors), $y_n = 7.966$ m (7.929 m by the authors), and $q_s = 4.614 \times 10^{-5}$ m²/s (4.606 $\times 10^{-5}$ m²/s by the authors).

Discussion by A. R. Kacimov⁶

In their paper, the authors solved an optimization problem for triangular, rectangular, and trapezoidal canals with seepage losses q_s from unlined unclogged bottoms as an objective function, canal discharge Q as an integral restriction, and geometrical parameters of the cross section as control variables. The authors considered one possible seepage regime [condition A' from Bouwer (1978), with phreatic surfaces tending to vertical asymptotes] and used the known Vedernikov formulas (Polubarinova-Kochina 1977) for q_s . Their attempt to theoretically design canals with minimal losses and to save water can be commended. However, some points should be amended.

First, it is incorrect to say that "no work has been done on

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the minimum seepage loss canal sections.” The problem of minimization of q_s at a given cross-sectional area A was solved in the class of arbitrary canal cross sections in a seminal paper by Preissmann (1957). Later, the technique of Preissmann (1957) and Morel-Seytoux (1964) was extensively used for design of optimal soil channels, drains, ditches, dams, and other hydrotechnic constructions (e.g., Ilyinsky and Kacimov 1984, 1991, 1992; Kacimov 1993, 1997; Kasimov and Tartakovsky 1993; Ilyinsky et al. 1998). In particular, optimization in more general regimes [a water bearing formation at a finite depth, i.e., condition A of Bouwer (1978) and capillary fringe flow according to the Vedernikov model] was done. Similar analytical shaping procedures were elaborated in the design of shark-skin surfaces in viscous flows (Kacimov 1994), high-speed aircraft (Elizarov et al. 1997), cooling surfaces conducting heat (Kacimov and Obnosov 1997), and electrical capacitors (Kacimov 2001), among other objects, where the process is governed by the Laplace equation. In canal design, Kacimov (1985, 1992) solved the problem of optimal shaping in three particular classes (triangular, rectangular, and trapezoidal sections) with the same criterion (q_s) as the authors and different constraints (Q , A , or hydraulic radius R). It was shown that all these problems have unique global minima, which occurred to be: (1) robust to small deviations of the canal shape from the optimum in the corresponding class; (2) close to each other in the value of q_s ; and (3) close both in the loss values and geometrically (in the y/L ratio, where L = canal width at the water surface) to the absolute minimum determined by Preissmann (1957). These results conform to the general theory of optimal forms (Pironneau 1984). Note that Kacimov (1985, 1992) used the exact Vedernikov formula for q_s .

Second, with modern computer algebra tools (Bahder 1995), exact calculation of the integrals approximated by the authors is a routine. In what follows, we shall focus on triangular canals only, for which the upper integral in (2) is $I_1(\alpha) = \Gamma[0.5 - \alpha/\pi] \Gamma[\alpha/\pi] / (2\sqrt{\pi})$, where $\alpha = \sigma\pi$ and Γ = gamma function. The second integral, $I_2(\alpha)$, in (2) can be calculated by separation of singularities, similarly to Hunt (1983), and by further implementation of Mathematica numerical integration without any dubious parameter fitting, as in (3). The authors are correct that (3) gives a good approximation at $\alpha > \pi/4$. However, at smaller α , (3) is very poor, as Fig. 5 shows. There, curve (1) corresponds to the graph $\mu(\alpha)$ plotted according to (2), curve 2 is plotted according to (3), and the nondimensional parameter μ is defined as $\mu = q_s / (k\sqrt{A})$. When the optimization problem is stated, it is unknown whether the optimum fits the range $\alpha > \pi/4$; hence, (3) should not be used.

Third, the authors did not state clearly the optimization problem. In particular, the control variable is not specified and

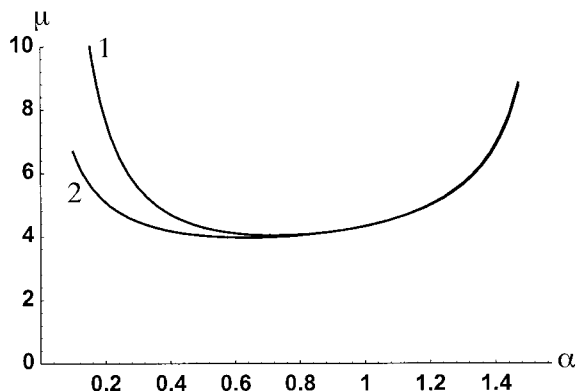


FIG. 5. Nondimensional Seepage Losses from Triangular Canal Calculated According to Vedernikov's Formula (Curve 1) and Its Approximation (Curve 2)

the optimization algorithm is not described properly. The authors report only that “a large number of optimal sections were obtained.” Were these optima obtained for different ε_* and v_* in (18a,b)? With which grid size? Were the optima found local? Do they converge to a global one?

Fourth, let us show that the authors' problem is reduced to finding a minimum of a function of one variable, i.e., to a computer algebra routine. Factually, the only difference from Kacimov (1985, 1992) is (10) for Q . For a triangular canal, the Manning formula gives $Q = \kappa L^{3/3} \tan \alpha / (4 \cos^{3/3} \alpha)$, where κ is a constant that is independent on the shape (α , L) and optimization is reduced to determination of a minimum of $\eta(\alpha)$, where $\eta = q_s / [k(Q/\kappa)^{3/8}]$ (Kasimov 1985, 1992). Fig. 6, curve 1 illustrates $\eta(\alpha)$ with a minimum at $\alpha = 0.736 = 42.2^\circ$.

Eq. (10) contains a logarithmic term, which makes impossible direct formulation of an explicit function as $\eta(\alpha)$, and optimization requires one additional step. We introduce the nondimensional variables $(L_n, \varepsilon_n) = (L, \varepsilon) / a^{2/5}$, $q_n = q_s / (ka^{2/5})$, and $v_n = v / (\sqrt{gS_0 a^{3/5}})$, where $a = Q_0 / \sqrt{gS_0}$ and Q_0 = specified canal discharge (we further omit the subindex n). Hence, we have to minimize

$$q = \frac{\pi}{2} L \frac{I_1(\alpha)}{I_2(\alpha)} \quad (26)$$

with the constraint

$$\frac{2.457}{8} \tan \alpha \sqrt{\sin \alpha} L^{2.5} \ln \left[\frac{\varepsilon}{3L \sin \alpha} + 4^{3/2} \frac{0.221v}{L^{1.5} \sin^{3/2} \alpha} \right] + 1 = 0 \quad (27)$$

Eq. (27) is a nonlinear equation with respect to α and L , which should be solved to determine $L = L(\alpha)$ (or vice versa) at fixed ε and v . Then, $L(\alpha)$ should be substituted into (26), which becomes a function of one variable, $q(\alpha)$, as was the case in Kacimov (1985). We solved (27) by Mathematica (Bahder 1995). Fig. 7 shows the graph $L(\alpha)$ plotted at dimensional values of $\varepsilon = 10^{-3}$ m, $S_0 = 4 \times 10^{-2}$ (upper curve), $\varepsilon = 10^{-5}$ m, $S_0 = 4 \times 10^{-4}$ (the lowest curve), and $\varepsilon = 10^{-3}$ m, $S_0 = 4 \times 10^{-4}$ (intermediate curve). As seen from the graphs, the influence of the bed roughness and the topographical slope are small. Next, we substituted the roots of (27) into (26) and found the minimum. Curve 2 in Fig. 6 shows $q(\alpha)$ at $\varepsilon = 10^{-3}$ m, $S_0 = 4 \times 10^{-4}$, $Q = 50$ m³/s (parameters taken from the authors' example 1). Curve 2 behaves similarly to curve 1; i.e., it exhibits a unique global minimum $q = 1.399$ at $\alpha = 0.73 = 42^\circ$. Thus, we arrived at the same optimal canal section as through the Manning formula.

Fifth, the authors interpret their optimization as applicable to lined canals with liner defects through which water leaks. This is incorrect, because leakage through cracks, holes, punctures, and other liner flaws depends not on the shape of the

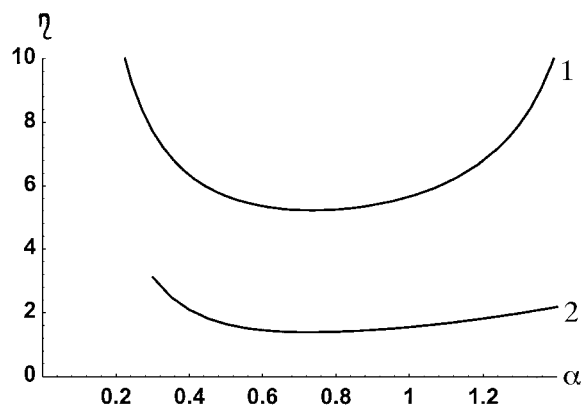


FIG. 6. Nondimensional Seepage Losses Calculated According to Manning's Formula (Curve 1) and Eq. (10) (Curve 2)

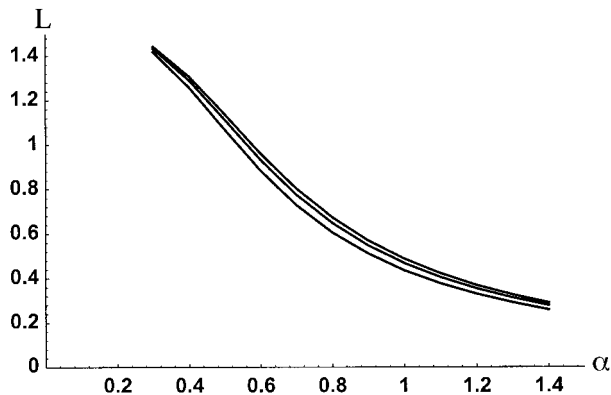


FIG. 7. Nondimensional Canal Width as Function of α at Different Roughness Factors and Topographical Slopes

cross section but on the size of the defect, the height of water directly above the defect, the conductivity and capillarity of the porous medium, and the drainage conditions in the near-defect soil as thoroughly studied by Faure (1979) [see the recent review by Giroud et al. (1997)].

From our experience of optimization of canal shapes with q_s as a criterion, we can conclude that the designer should not go far away from the shape found by Preissmann while small deviations from the optimum are not crucial. Since canal sections should satisfy multiple criteria [some of them are incorporated into seepage minimization by Kacimov (1992)] and real water conservation can only be reached by isolation of water from the soil, i.e., lining and proper liner maintenance, the optimal shape can be determined by any reasonable formula for the discharge (though the Manning formula gives the fastest answer) and the Vedernikov exact formulas for seepage losses.

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Closure by Prabhata K. Swamee,⁷ Govinda C. Mishra,⁸ and Bhagu R. Chahar⁹

The writers are thankful to the discussers for their interest in the paper. Their replies to Kacimov's comments are given below:

1. The comprehensive literature review provided by Kacimov adds to the perspective of the paper. The present investigation assumes that the seepage characteristics are similar to an unlined canal passing through a homogeneous medium of large extent. Hence, the investigation is not applicable where seepage is affected by the lining material. The authors mean to say that no work is available on explicit equations for designing minimum seepage loss canal sections. The work of Preissmann (1957) pertains to minimization of q_s for a given cross-sectional area A . The cross-sectional area is not an independent variable known a priori. In a canal section design problem, Q , S_0 , e , and ν are the independent variables and the cross-sectional elements are the dependent variables. Thus, being dependent on cross-sectional elements, A is an unknown entity. The work of Morel-Seytoux (1964) pertains to determination of seepage loss for a given cross section. As Ilyinsky and Kacimov (1984, 1991, 1992), Ilyinsky et al. (1998), Kacimov (1993, 1997), and Kacimov and Tartakovsky (1993) extensively use the direct problem of seepage loss determination, these works do not pertain to the problem solved in the present paper.
2. Numerical evaluation of the integral with known parameters gives a number that is devoid of physical insight. On the other hand, Swamee et al. (2000) give an equation in which the controlling parameters appear explicitly. As suggested by Kacimov, the integrals of (2) can be evaluated explicitly and, instead of (3), an exact equation can be obtained. It can be tested that, besides being simple, (3) is close to such an exact equation. Thus, the parameters appearing in (3) are not dubious. These parameters have been obtained by minimizing the average error using the steepest descent method (Burley 1974). However, while dealing with (4) and (5), things are not straightforward. That is, even if the integrals involved in (4) are expressible in exact form, the inversion of (5) expressing α in terms of b/y is difficult; furthermore, this expression has to be substituted in (4). The resulting equation will still be more involved in expressing the seepage loss. The seepage function has been obtained in a very much simplified form containing b/y for a rectangular canal [(6)]. The same optimization process obtains the parameter appearing in (6). The same procedure was

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used to obtain the seepage function (9) for a trapezoidal canal for a given b/y and side slope m . Note that no parameter other than canal dimensions appears in (9). Since Fig. 5 contains α on both axes, it can not be interpreted in terms of error in seepage loss. As per Fig. 6, for a small value of $\alpha = 0.197$ ($m = 5$), the error in seepage loss is less than 2%, which is small enough for all practical purposes. No canals are constructed for smaller than this value of α .

Conceiving a proper functional form and minimizing errors obtain (3). The proper functional form was arrived at (Chahar 2000) by obtaining the asymptotic solutions for slit ($m = 0$ or $\sigma = 0.5$) and strip ($m = \infty$ or $\sigma \rightarrow 0$). For a slit

$$F = \pi^2/(4G) \approx \pi(4 - \pi) \quad (28)$$

where $G = 0.91596 \dots =$ Catalan's constant (Spiegel 1990). On the other hand, for a strip

$$F = 2m + 16G/\pi^2 \approx 2m \quad (29)$$

Eq. (3) is, thus, a generalization of (28) and (29), where, contrary to the observations made by Kasimov, the errors are contained in the stated limits.

3. The explicit statement of the problem is provided by (11) and (12). For different sections, (11) and (12) assume different forms. Thus, the problem is to find the section elements for a given Q , S_0 , ϵ , and ν . Since the problem is unimodal, it has only one optimum. Yes, a large number of sections for different ϵ_* and ν_* were generated. These ϵ_* and ν_* were varied in logarithmic cycles.
4. As stated by Kasimov, the problem defined by (11) and (12) can be solved numerically. Unless put into functional form, the numerical solution itself can not give any insight regarding the functional dependence of a design variable on the input variables. Eqs. (20) and (21a-e), along with Table 1, provide such an insight.

As per recommendations of ASCE's Task Force Committee on Open Channels (1963), the resistance equation for open channels contains a logarithmic term. The Manning's equation term $R^{1/6}$ is an approximation of the logarithm in the range $0.0025 \leq \epsilon/R \leq 0.025$ (Christensen 1984). No wonder for a triangular channel the solution $m = 1.104$ using Manning's equation is different from the present solution $m = 1.244$ given in Table 1.

5. The design always considers the worst condition and not any other intermediate condition. In this case, for the canal design, the worst case considered is that the canal lining is uniformly damaged so that it acts as an unlined canal. Moreover, an analytical solution to the seepage problem considering nonhomogeneity and complex

TABLE 3. Properties of Optimal Canal Sections for Manning's Equation

Section shape	Side slope (m)	Section-Shape Coefficients				
		k_b	K_v	k_A	k_v	k_q
Triangular	1.241	0.000	1.168	1.692	0.591	5.161
Rectangular	0.000	2.054	0.821	1.687	0.592	5.260
Trapezoidal	0.599	1.396	0.854	1.629	0.614	4.951

boundary conditions is not possible. For a trapezoidal canal, bed width b and side slope m are the decision variables. Fig. 3 shows the family of curves for seepage loss. It can be seen that, for different shapes, the question of small variation in an optimal value does not arise. All three cases—triangular, rectangular and trapezoidal—give distinct optima that are considerably separated.

The optimization is based on Vedernikov's seepage loss equation and the dimensionally consistent resistant equation, for which solutions were not available earlier. Manning's equation, being dimensionally inhomogeneous and applicable in a narrow range of relative roughness, does not relate to real situations.

Atmapoojya and Ingle provide an alternate solution using Manning's equation. The writers have also used Manning's equation at an earlier stage of their work and have obtained the optimal section shape coefficients (Table 3). Due to the limitations and shortcomings of Manning's equation and the availability of a general resistance equation, the writers excluded these results from their paper. However, using coefficients of Table 3, the same set of design equations (21a-e) can be obtained. In this case, the length scale L was modified to

$$L = \left(\frac{Qn}{\sqrt{S_0}} \right)^{3/8} \quad (30)$$

where $n =$ Manning's roughness coefficient.

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