Optimal Design of Transmission Canal

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Abstract: This paper presents design equations for the least-cost canal sections considering earthwork cost which may vary with depth of excavation, cost of lining, and cost of water lost as seepage and evaporation from irrigation canals of triangular, rectangular, and trapezoidal shapes passing through a stratum underlain by a drainage layer at shallow depth. The optimal design equations are in explicit form and result into optimal dimensions of a canal in single-step computations. Using these least-cost section equations and applying the Fibonacci search method, equations for computation of the optimal subsection length and corresponding cost of a transmission canal have been presented. The optimal design equations along with the tabulated section shape coefficients provide a convenient method for the optimal design of a transmission canal. A step-by-step design procedure for rectangular and trapezoidal canal sections has been presented to demonstrate the simplicity of the method.

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Introduction

Canals continue to be major conveyance systems for delivering water for irrigation. The seepage loss from irrigation canals constitutes a substantial percentage of the usable water (Rohwer and Stout 1948). According to the Bureau of Indian Standards (Bureau 1980) the loss of water by the seepage from unlined canals in India generally varies from 0.3 to 7.0 m^3/s per 10^6 m^2 of wetted surface. The seepage loss from canals is governed by hydraulic conductivity of the subsoils, canal geometry, location of water table relative to the canal, and several other factors ("Controlling" 1967).

Canals are lined to check the seepage. But canal lining deteriorates with time and hence, significant seepage losses continue to occur from a lined canal (Wachyan and Rushton 1987). Therefore, seepage loss must be considered in the design of a canal section. Morel-Seytoux (1964); Bandini (1966); Ilyinsky and Kacimov (1984); Kacimov (1992); Kacimov (1993); Swamee et al. (2000b,c, 2001); and Chahar (2000) presented canal design methods considering seepage loss.

A transmission canal conveys water from the source to a distribution canal. Many times the area to be irrigated lies very far from the source, and hence requires long transmission canals. For example, the Rajasthan canal system has the transmission canal length of 204 km carrying a discharge about 524 m^3/s (Kanwar Sain 1967; Hooja 1993). There is no withdrawal from a transmission canal but it loses water through seepage and evaporation. Chahar (2000) and Swamee et al. (2000c) expressed the quantity of water loss q_w (m^2/s) as seepage q_s (m^2/s) and evaporation q_E (m^2/s) from a unit length of canal as

$$q_w = q_s + q_E = kF_s y + ET \tag{1}$$

where k = hydraulic conductivity of the porous medium (m/s); F_s = seepage function (dimensionless), which is a function of channel geometry and depth to drainage layer below the canal (see the Appendix); y = depth of flow (m); E = evaporation discharge per unit surface area (m/s); and T = width of free surface (m). See Fig. 1 for y and T.

Since a transmission canal loses water through seepage and evaporation, it is not economical to continue the same section throughout the length of a long transmission canal. Instead the transmission canal should be divided into subsections or reaches and the cross section for each of the subsections must be designed separately. This adds cost of transition in between two subsections, but the transition cost is overcome by the reduced cost of the cross section. The reduced cross section not only results in cost saving for earthwork, lining, and water lost, but also requires less cost in land acquisition, construction of bridges, and cross-drainage works.

A method of the optimal design of a transmission canal is not available in the literature. The least-cost canal section design equations considering earthwork cost, cost of lining, and cost of water lost as seepage and evaporation from canals passing through a homogeneous medium are available (Swamee et al. 2000c), but in many alluvial plains the soils are stratified and highly permeable layers of sand and gravel underlie a low permeable top layer of finite depth. The seepage from a canal running through such a stratified strata is much more than those of a canal in a homogeneous seepage layer of very large depth (Muskat 1946; El Nimr 1963; Bruch 1966; Bruch and Street 1967a,b; Chahar 2000). The present paper first presents the least-cost canal section design equations considering earthwork cost, cost of lining, and cost of water lost as seepage and evaporation for canals of triangular, rectangular, and trapezoidal shapes passing through a stratum underlain by a drainage layer at finite

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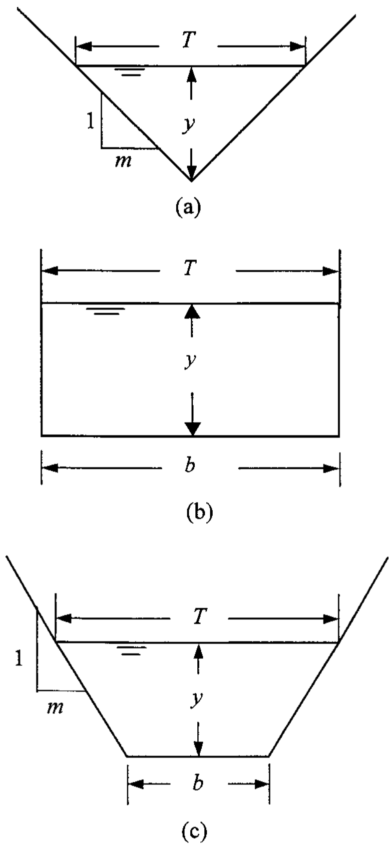

Fig. 1. Canal sections. (a) triangular section; (b) rectangular section; and (c) trapezoidal section

depth. Further, using these least-cost section design equations, optimal transmission canal design equations have been presented.

Requirements for Flow in Canal

A lined canal is designed for the condition of uniform flow. The flow requirements to be taken into account in designing a canal for uniform flow are the channel surface roughness, the minimum permissible velocity, the limiting (maximum permissible) velocity, the freeboard, and the hydraulic efficiency of the canal section. The minimum permissible velocity or the nonsilting velocity is the lowest velocity that will not initiate sedimentation and will not induce the growth of vegetation. Sedimentation and growth of vegetation decrease the carrying capacity and increase the maintenance cost of the canal. In general, an average velocity of 0.6 to 0.9 m/s will prevent sedimentation when the silt load of the flow is low and a velocity of 0.75 m/s is usually sufficient to prevent the growth of vegetation (Chow 1973). Hence, the minimum permissible velocity can be assumed in the range from 0.75 to 0.9 m/s. Higher velocities are desired in rigid boundary canals to reduce costs. However, high velocities may cause scour and erosion of the boundaries. In rigid boundary canals the maximum permissible velocity or the limiting velocity V_L (m/s) that will not cause erosion depends on the channel surface material. Table 1 lists the limiting velocities for different type of channel surface materials (Sharma and Chawla 1975; Bureau 1982; Subramanya 1997).

Various investigators have presented several versions of uniform flow. Using Darcy-Weisbach friction formula and Colebrook formula (‘‘Friction’’ 1963), Swamee (1994) gave the following general resistance equation:

Table 1. Limiting Velocities for Channel Surfaces

Channel surfaces	Limiting velocity (m/s)
Sandy soil	0.3–0.6
Black cotton soil	0.6–0.9
Muram and hard soil	0.9–1.1
Firm clay and loam	0.9–1.15
Gravel	1.0–1.25
Boulder	1.0–1.5
Disintegrated rock	1.3–1.5
Brunt clay tile	1.5–2.0
Concrete tile	2.0–2.5
Concrete	2.5–3.0
Hard rock	3.0–4.0

$$V=-2.457\sqrt{gRS_0}\ln\left(\frac{\varepsilon}{12R}+\frac{0.221\nu}{R\sqrt{gRS_0}}\right) \tag{2}$$

where V = average flow velocity (m/s); g = gravitational acceleration (m/s^2); R = hydraulic radius (m) defined as the ratio of the flow area A (m^2) to the flow perimeter P (m); S_0 = longitudinal canal bed slope (dimensionless); ε = average roughness height of the canal lining (m); and ν = kinematic viscosity of water (m^2/s).

Ganguillet and Kutter’s, and Bazin’s formulas have largely gone out of use. In the Commonwealth Countries of Independent States (erstwhile USSR) the Pavlovsky formula is used (‘‘Friction’’ 1963). The most commonly used uniform flow formula around the world is the Manning equation due to its simplicity and acceptable degree of accuracy in most practical applications. However, the Manning equation is applicable to fully rough turbulent flow for the following limited bandwidth of relative roughness (Swamee 1994):

$$\varepsilon \geqslant \frac{30\nu}{(gS_0\sqrt{Q})^{0.4}} \tag{3a}$$

$$0.004 \leqslant \frac{\varepsilon}{R} \leqslant 0.04 \tag{3b}$$

where Q = discharge in the canal (m^3/s). On the other hand, Eq. (2) involves physically conceivable parameters ε and ν, and is applicable for hydraulically rough or smooth surfaces or transition in between. Also, Eq. (2) holds good for all types of open channel sections in the following ranges (Hager 1989):

$$10^3 \leqslant \frac{VR}{\nu} \leqslant 10^8 \tag{4a}$$

$$10^{-6} \leqslant \frac{\varepsilon}{R} \leqslant 10^{-3} \tag{4b}$$

The law of conservation of mass gives the following general uniform flow resistance equation in terms of discharge:

$$Q=AV=-2.457A\sqrt{gRS_0}\ln\left(\frac{\varepsilon}{12R}+\frac{0.221\nu}{R\sqrt{gRS_0}}\right) \tag{5}$$

From a hydraulic viewpoint the canal section having the least wetted perimeter for a given flow area has the maximum discharge carrying capacity. However, the best hydraulic section is not necessarily the least-cost section. The least-cost section results from the minimization of canal section cost provided it carries the design discharge (Streeter 1945; Swamee and Bhatia 1972; Guo and Hughes 1984; Flynn and Mariño 1987; Loganathan 1991; Froehlich 1994; Monadjemi 1994; Swamee 1995a,b). Since the least-cost canal section is designed to sustain uniform flow, Eq. (5) provides the required condition as an equality constraint function in the design. Also, depth of flow in the canal becomes normal depth of flow y_n (m).

Unit Length Canal Section Cost

The most general case for the optimal channel design is that which considers cost of earthwork per unit length of canal C_e (\$/m) that varies with depth of the canal, cost of lining per unit length of canal C_L (\$/m), and capitalized cost of water lost as seepage and evaporation per unit length of canal C_w (\$/m). The cost of canal per unit length C (\$/m) was obtained (Chahar 2000; Swamee et al. 2000c) as

$$C = C_e + C_L + C_w = c_e A + c_r A\bar{y} + c_L P + c_{ws} F_s y_n + c_{wE} T \quad (6)$$

where c_e = cost per unit volume of earthwork at ground level (\$/m^3); c_r = increase in the unit excavation cost per unit depth (\$/m^4); c_L = cost per unit surface area of lining (\$/m^2); $c_{ws} = 3.156\times 10^7 c_w k/r$ (\$/m^2); $c_{wE} = 3.156\times 10^7 c_w E/r$ (\$/m^2); c_w = cost per unit volume of water (\$/m^3); r = rate of interest (\$/\$/year); and $\bar{y}$ = depth of centroid of excavated area from the free water surface (m).

It can be seen from Eq. (6) that for given cost factors c_e, c_r, c_L, c_{ws}, and c_{wE} the cost per unit length of the canal is a function of the canal geometry and depth of the drainage layer d (m), since d appears in the seepage function (Appendix). As c_L/c_e, c_e/c_r, c_{ws}/c_e, and c_{wE}/c_e have length dimension, they remain unaffected by the monetary units chosen. These ratios can be obtained for various types of linings, soil strata, and climatic condition by using appropriate unit rates (Swamee 2000c).

Methodology

Problem Formulation

A transmission canal of length L_c (m) was divided into N subsections of lengths $x_1, x_2, x_3, \ldots, x_N$ (m). Hence, $N-1$ number of transitions were required in the transmission canal. The cost of a transmission canal consists of the cost of subsections and the cost of transitions. Letting the cost of each transition C_T (monetary units, \$) be the same for all the transitions, and assuming canal geometry and depth to the drainage layer remains constant throughout each subsection, the overall cost of the transmission canal C_o (monetary units, \$) would be

$$\begin{aligned} C_o &= \sum_{i=1}^{N} C_i x_i + (N-1)C_T \\ &= \sum_{i=1}^{N} (c_e A_i + c_r A_i \bar{y}_i + c_L P_i + c_{ws} y_{ni} F_{si} + c_{wE} T_i) x_i \\ &\quad + (N-1)C_T \end{aligned} \quad (7)$$

Therefore, the problem of the least-cost design of a transmission canal became

$$\text{Minimize} \quad C_o = \sum_{i=1}^{N} (c_e A_i + c_r A_i \bar{y}_i + c_L P_i + c_{ws} y_{ni} F_{si} + c_{wE} T_i) x_i + (N-1)C_T \quad (8)$$

Subject to

$$Q_i + 2.457 A_i \sqrt{g R_i S_{0i}} \ln\left(\frac{\varepsilon_i}{12 R_i} + \frac{0.221 \nu_i}{R_i \sqrt{g R_i S_{0i}}} \right) = 0 \quad \text{for all } i \quad (9a)$$

$$Q_{i+1} - Q_i + (k_i y_{ni} F_{si} + E_i T_i) x_i = 0 \quad \text{for all } i \quad (9b)$$

$$\sum_{i=1}^{N} x_i - L_c = 0 \quad (9c)$$

where the index i indicates a subsection.

The first constraint (9*a*) imposes the condition of uniform flow in each subsection, while, the second constraint (9*b*) satisfies the continuity in discharge from one section to the next section, and the third constraint (9*c*) is obvious. The optimization problem stated by Eqs. (8) and (9*a*)–(9*c*) is some sort of a dynamic programming. The problem is complicated due to unknown number of subsections N, i.e., number of unknown constraints. This optimization problem was expressed in dimensionless form and then solved.

Nondimensionalization

Assuming S_0, ε, ν, k, and E constant for all the subsections and using a length scale λ (m) as

$$\lambda_i = (Q_i/\sqrt{g S_0})^{0.4} \quad (10)$$

and $\lambda = \lambda_1$; the following dimensionless parameters were obtained:

$$\lambda_{i*} = \lambda_i/\lambda \quad (11a)$$

$$\varepsilon_* = \varepsilon/\lambda \quad (11b)$$

$$\nu_* = \nu\lambda/Q \quad (11c)$$

$$C_{o*} = C_o/(c_e \lambda^2 L_c) \quad (11d)$$

$$c_{r*} = c_r \lambda/c_e \quad (11e)$$

$$c_{L*} = c_L/(c_e \lambda) \quad (11f)$$

$$c_{ws*} = c_{ws}/(c_e \lambda) \quad (11g)$$

$$c_{wE*} = c_{wE}/(c_e \lambda) \quad (11h)$$

$$c_{T*} = C_T/(c_e \lambda^2 L_c) \quad (11i)$$

$$A_{i*} = A_i/\lambda_i^2 \quad (11j)$$

$$\bar{y}_{i*} = \bar{y}_i/\lambda_i \quad (11k)$$

$$P_{i*} = P_i/\lambda_i \quad (11l)$$

$$y_{ni*} = y_{ni}/\lambda_i \quad (11m)$$

$$T_{i*} = T_i/\lambda_i \quad (11n)$$

$$R_{i*} = R_i/\lambda \quad (11o)$$

$$x_{i*} = x_i/L_c \quad (11p)$$

$$k_* = k L_c/\sqrt{g S_0 \lambda^3} \quad (11q)$$

$$E_* = E/k \quad (11r)$$

where variables with subscript * denotes corresponding nondimensional parameter. Using Eqs. (11*a*)–(11*r*) in Eqs. (8) and (9*a*)–(9*c*), the optimization problem in nondimensional form reduced to

Minimize

$$C_{o*}=\sum_{i=1}^{N}(A_{i*}\lambda_{i*}^2+c_{r*}A_{i*}\overline{y}_{i*}\lambda_{i*}^3+c_{L*}P_{i*}\lambda_{i*}+c_{ws*}y_{ni*}F_{si}\lambda_{i*}+c_{wE*}T_{i*}\lambda_{i*})x_{i*}+(N-1)c_{T*} \quad (12)$$

Subject to

$$\Phi_i=1+2.457A_{i*}\sqrt{R_{i*}}\ln\left(\frac{\varepsilon_*}{12R_{i*}\lambda_{i*}}+\frac{0.221\nu_*}{R_{i*}^{1.5}\lambda_{i*}^{1.5}}\right)=0 \quad \text{for all } i \quad (13a)$$

$$\lambda_{(i+1)*}^{2.5}-\lambda_{i*}^{2.5}+k_*(y_{ni*}F_{si}+E_*T_{i*})x_{i*}=0 \quad \text{for all } i \quad (13b)$$

$$\sum_{i=1}^{N}x_{i*}-1=0 \quad (13c)$$

This dimensionless optimization problem was simplified to an optimization problem with one variable N. This was achieved by providing the least-cost section for each subsection of the transmission canal, and considering the length of each subsection to be equal.

Least-Cost Section Design Equations for Each Subsection

The cost of each subsection must be minimum to arrive at the least-cost transmission canal. The least-cost canal section for a particular subsection of the transmission canal could be obtained from Eq. (8), without the transition cost, subject to Eq. (9*a*) and dropping the index i from them. The optimization problem in dimensionless form became

$$\text{Minimize} \quad C_*=A_*+c_{r*}A_*\overline{y}_*+c_{L*}P_*+c_{ws*}F_sy_{n*}+c_{wE*}T_* \quad (14)$$

$$\text{Subject to} \quad \Phi=1+2.457A_*\sqrt{R_*}\ln\left(\frac{\varepsilon_*}{12R_*}+\frac{0.221\nu_*}{R_*^{1.5}}\right)=0 \quad (15)$$

This nonlinear optimization problem with equality constraint was numerically solved for triangular, rectangular, and trapezoidal channel sections (Fig. 1) using the procedure similar to Swamee et al. (2000a,b,c), and Chahar (2000) in the following ranges of the input variables:

$$10^{-6}\leqslant\varepsilon_*\leqslant10^{-3} \quad (16a)$$

$$10^{-7}\leqslant\nu_*\leqslant10^{-5} \quad (16b)$$

$$0\leqslant c_{L*}\leqslant\infty \quad (16c)$$

$$0\leqslant c_{ws*}\leqslant\infty \quad (16d)$$

$$0\leqslant c_{r*}\leqslant1.0 \text{ or } 0\leqslant c_{r*}\leqslant50c_{L*} \quad (16e)$$

$$0\leqslant c_{wE*}\leqslant0.2 \text{ or } 0\leqslant c_{wE*}\leqslant c_{ws*} \quad (16f)$$

$$d_*\geqslant0.25 \quad (16g)$$

where $d_*=d/\lambda$. If c_{L*} or c_{r*} or c_{wE*} is not equal to zero, a lower value of d_* may be used in Eq. (16*g*) depending upon the value of c_{L*}, c_{r*}, c_{wE*}, etc.

Analysis of large numbers of optimal sections so obtained in the above ranges resulted to general empirical equations in explicit form for the optimal side slope m, bed width b (m) [see Fig. 1 for m and b], normal depth and cost per unit length of canal as follows:

$$m^*=k_{me}\frac{c_eL+k_{mr}c_rL^2+k_{mL}c_L+k_{ms1}(1+k_{md}L/d)L_sc_w}{c_eL+k_{mL}c_L+k_{ms2}L_sc_w+k_{mE}L_Ec_w} \quad (17a)$$

$$b^*=k_{be}\frac{c_eL+k_{br}c_rL^2+k_{bL}c_L+k_{bs1}(1+k_{bd}L/d)L_sc_w}{c_eL+k_{bL}c_L+k_{bs2}L_sc_w+k_{bE}L_Ec_w}L \quad (17b)$$

$$y_n^*=k_{ye}\frac{c_eL+k_{yL}c_L+k_{ys2}L_sc_w+k_{yE}L_Ec_w}{c_eL+k_{yr}c_rL^2+k_{yL}c_L+k_{ys1}(1+k_{yd}L/d)L_sc_w}L \quad (17c)$$

$$C^*=k_{cr}c_rL^3+k_{ce}c_eL^2+k_{cL}c_LL+k_{cs}(1+k_{cd}L/d)L_sc_wL+k_{cE}L_Ec_wL \quad (17d)$$

where superscript * indicates optimality; k_{fs} = section shape coefficients in which the first subscripts m, b, y, and c denote side slope, bed width, normal depth and cost, respectively, and the second subscripts e, r, L, s, d, and E denote depth independent earthwork, additional earthwork cost because of canal depth, lining, seepage, depth to drainage layer, and evaporation, respectively; $L_s=c_{ws}/c_w$ = length scale (m) for seepage; $L_E=c_{wE}/c_w$ = length scale (m) for evaporation; and L = length scale (m), given by

$$L=\lambda(\varepsilon_*+8\nu_*)^{0.04} \quad (18)$$

Table 2 lists the section shape coefficients appeared in Eq. (17). A perusal of Eq. (17) indicates that for $c_r=0$ and $c_w=0$, the optimal section is the minimum area section (Swamee 1995a)

$$m^*=k_{me} \quad (19a)$$

$$b^*=k_{be}L \quad (19b)$$

$$y_n^*=k_{ye}L \quad (19c)$$

$$C^*=k_{ce}c_eL^2 \quad (19d)$$

On the other hand, when the cost of water is not a limiting factor $c_w=0$, Eq. (17) reduces to design equations for minimum cost lined section (Swamee 2000a)

$$m^*=k_{me}\frac{c_eL+k_{mr}c_rL^2+k_{mL}c_L}{c_eL+k_{mL}c_L} \quad (20a)$$

$$b^*=k_{be}\frac{c_eL+k_{br}c_rL^2+k_{bL}c_L}{c_eL+k_{bL}c_L}L \quad (20b)$$

$$y_n^*=k_{ye}\frac{c_eL+k_{yL}c_L}{c_eL+k_{yr}c_rL^2+k_{yL}c_L}L \quad (20c)$$

$$C^*=k_{cr}c_rL^3+k_{ce}c_eL^2+k_{cL}c_LL \quad (20d)$$

Furthermore, as $d\to\infty$, Eqs. (17) become optimal design equations for canal with the drainage layer at a large depth (Swamee 2000c); while as other cost factors become negligibly small in comparison to L_s c_w and $d\to\infty$, Eqs. (17) convert into the minimum seepage loss section design equations (Swamee 2000b)

$$m^*=k_{me}\frac{k_{ms1}}{k_{ms2}}=k_{ms} \quad (21a)$$

$$b^*=k_{be}\frac{k_{bs1}}{k_{bs2}}L=k_{bs}L \quad (21b)$$

Table 2. Section Shape Coefficients for Least-Cost Canal Section

Entity	Section shape coefficients	Section shape		
		Triangular	Rectangular	Trapezoidal
Side slope	k_{me}	1.000		0.57735
	k_{mL}	15.049		14.277
	k_{mr}	0.3039		0.2163
	k_{ms}	1.24450		0.59836
	k_{ms1}	16.756		23.494
	k_{ms2}	13.441		22.668
	k_{md}	0.1249		0.1235
	k_{mE}	8.3307		32.189
Bed width	k_{be}		0.71136	0.43407
	k_{bL}		15.028	14.243
	k_{br}		0.3201	0.3484
	k_{bs}		0.79857	0.54458
	k_{bs1}		18.283	18.086
	k_{bs2}		16.286	14.416
	k_{bd}		0.1111	0.1302
	k_{bE}		5.5434	0.2878
Normal depth	k_{ye}	0.50301	0.35568	0.37592
	k_{yL}	15.039	15.023	14.227
	k_{yr}	0.1397	0.3066	0.2233
	k_{ys}	0.45177	0.31779	0.33090
	k_{ys1}	16.245	18.737	16.910
	k_{ys2}	14.590	16.741	14.885
	k_{yd}	0.0539	0.0964	0.1122
	k_{yE}	4.0357	5.6244	4.0362
Cost	k_{ce}	0.25302	0.25302	0.24476
	k_{cL}	1.4249	1.4240	1.3037
	k_{cr}	0.03965	0.03961	0.03723
	k_{cs} or k_{qs}	2.0015	2.0399	1.9227
	k_{cd}	0.1423	0.1337	0.1609
	k_{cE}	0.9888	0.6856	0.8194

$$y_n^* = k_{ye}\frac{k_{ys2}}{k_{ys1}}L = k_{ys}L \quad (21c)$$

$$C^* = k_{cs}L_s c_w L \quad \text{or} \quad q_s^* = k_{qs}kL \quad (21d)$$

where k_{qs} = section shape coefficient for seepage discharge, which is numerically equal to k_{cs}.

Optimal Cost and Length of Each Subsection

The use of Eqs. (17) for designing each subsection automatically satisfies the constraint (13*a*). The optimization problem was further simplified by assuming the length of each subsection of the transmission canal to be same, i.e.,

$$x_i = x = L_c/N \quad \text{or} \quad x_{i*} = x_* = 1/N \quad \text{for all } i \quad (22)$$

Thus, two constraints were eliminated and the problem became

Table 3. Coefficients for a Transmission Canal

Entity	Coefficients	Section shape		
		Triangular	Rectangular	Trapezoidal
Transition	k_{NT}	14.00	14.00	15.65
	k_{Tc}	0.450	0.458	0.433
	k_{Tc1}	251.0	247.0	272.0
Earthwork	k_{Nr}	0.15	0.20	0.25
	k_{Tr}	0.215	0.22	0.19
	k_{Tr1}	0.18	0.20	0.17
Lining	k_{NL}	3.00	3.00	2.18
	k_{TL}	2.00	2.00	1.93
	k_{TL1}	4.25	4.30	4.10
Seepage	k_{Nd0}	0.05	0.14	0.15
	k_{Ns}	4.00	4.20	4.10
	k_{Nd}	0.17	0.20	0.21
	k_{Td0}	0.10	0.12	0.15
	k_{Ts}	4.00	4.19	3.45
	k_{Td}	0.24	0.17	0.16
	k_{Ts1}	8.45	8.60	7.202
Evaporation	k_{NE0}	0.47	0.35	0.40
	k_{NE}	2.35	1.56	2.34
	k_{TE0}	0.55	0.37	0.48
	k_{TE}	0.35	0.20	0.97
	k_{TE1}	0.25	0.00	1.56

Minimize

$$C_{o*} = \frac{1}{N}\sum_{i=1}^{N}(A_{i*}\lambda_{i*}^2 + c_{r*}A_{i*}\overline{y}_{i*}\lambda_{i*}^3 + c_{L*}P_{i*}\lambda_{i*} + c_{ws*}y_{ni*}F_{si}\lambda_{i*} + c_{wE*}T_{i*}\lambda_{i*}) + (N-1)c_{T*} \quad (23)$$

Subject to

$$\lambda_{(i+1)*}^{2.5} - \lambda_{i*}^{2.5} + k_*(y_{ni*}F_{si} + E_*T_{i*})/N = 0$$

$$\text{for all } i \quad (24)$$

Once the optimal dimensions of a section are fixed using Eqs. (17), the discharge or λ for the next section becomes a function of *N* satisfying Eq. (24). Now the optimization problem stated by Eqs. (23) and (24) is left with finding the minimum of Eq. (23) for only one unknown *N*. By applying Fibonacci search (Bazaraa and Shetty 1979) on triangular, rectangular, and trapezoidal canal sections, a large number of optimal *N* were obtained for a number of input variables varying in the ranges

$$10^{-5} \leq c_{T*} \leq \infty; \quad 0 \leq k_* \leq 1.0 \quad (25)$$

and the ranges of the other input variables (ε^*, ν^*, c_{L*}, c_{r*}, c_{ws*}, c_{wE*}, E_*, and d_*) being the same as used in developing Eqs. (17). Analysis of the optimal data so obtained resulted to the following equation for optimal number of subsections in the transmission canal for all the three canal shapes:

$$N^* = L_c\sqrt{\left(\frac{k + k_{Nd0}kL/d + k_{NE0}E}{\sqrt{gLS_0}}\right)\left(\frac{c_eL + k_{Nr}c_rL^2 + k_{NL}c_L + k_{Ns}(1 + k_{Nd}L/d)L_sc_w + k_{NE}L_Ec_w}{k_{NT}C_T}\right)} \quad (26)$$

where the subscripts N, and T in section shape coefficients denote number of subsections, and transition, respectively. The value obtained from Eq. (26) to be rounded to nearest integer. If the optimal number of subsections happens to be zero or one then no transition is required and assume $N^*=1$. The optimal length of the each subsection x^* of the transmission canal was given by

$$x^*=L_c/N^* \tag{27}$$

Further analysis of the optimal costs resulted to an empirical equation for the minimum cost of the transmission canal as given below

$$C_o^*=C^*L_c\left[1+\left(\frac{k_{Tc}L_c}{L}\,\frac{(k+k_{Td0}kL/d+k_{TE0}E)}{\sqrt{gLS_0}}\right)^{1.28}\right.$$
$$\left.\times\left(\frac{c_eL^2+k_{Tr}c_rL^3+k_{TL}c_LL+k_{Ts}(1+k_{Td}L/d)L_sc_wL+k_{TE}L_Ec_wL}{k_{Tc1}C_T/L_c+c_eL^2+k_{Tr1}c_rL^3+k_{TL1}c_LL+k_{Ts1}L_sc_wL+k_{TE1}L_Ec_wL}\right)\right]^{-1} \tag{28}$$

For a large depth of the drainage layer or water table and negligible evaporation loss, Eqs. (27) and (28) reduce to the following simplified expressions:

$$x^*=\sqrt{\left(\frac{\sqrt{gLS_0}}{k}\right)\left(\frac{k_{NT}C_T}{c_eL+k_{Nr}c_rL^2+k_{NL}c_L+k_{Ns}L_sc_w}\right)} \tag{29}$$

$$C_o^*=C^*L_c\left[1+\left(\frac{k_{Tc}L_c}{L}\,\frac{k}{\sqrt{gLS_0}}\right)^{1.28}\right.$$
$$\left.\times\left(\frac{c_eL^2+k_{Tr}c_rL^3+k_{TL}c_LL+k_{Ts}L_sc_wL}{k_{Tc1}C_T/L_c+c_eL^2+k_{Tr}c_rL^3+k_{TL1}c_LL+k_{Ts}L_sc_wL}\right)\right]^{-1} \tag{30}$$

Similarly other cases can be reduced from Eqs. (26) and (28). Coefficients for transmission canal appearing in Eqs. (26) and (28) are listed in Table 3.

Application

Design a 200-km-long transmission canal for carrying a discharge of 250 m^3/s on a longitudinal bed slope of 0.0001. The canal passes through a stratum of ordinary soil for which $c_e/c_r=7$ m, $k=5\times10^{-6}$ m/s, and depth of drainage layer = 7.5 m. Further, it is proposed to provide concrete lining with $c_L/c_e=12$ m. The climatic condition of the canal area is such that the maximum evaporation loss was estimated as 2.0×10^{-7} m/s (17.28 mm/day). Take rate of interest = 0.04\$/\$/year, $c_w/c_e=1.267\times10^{-2}$, and $c_T/c_e=4\times10^5$.

Single Section Design of Transmission Canal

For the design $g=9.79$ m/s^2; $\nu=1.007\times10^{-6}$ m^2/s (water at 20°C); and $\varepsilon=1$ mm (smooth finished concrete lining) have been adopted.

Using Eq. (10), $\lambda=36.393$ m; Eq. (11c) $\varepsilon^*=2.748\times10^{-5}$; Eq. (11$d$) $\nu^*=1.466\times10^{-7}$; and Eq. (18) $L=23.950$ m.

Minimum Cost General Section

Using $c_{ws}=3.156\times10^7/kc_w/r$, and $c_{wE}=3.156\times10^7Ec_w/r$ gave $c_{ws}=3945.0c_w$, and $c_{wE}=157.8c_w$, respectively. Thus, $L_s=3945.0$ m; and $L_E=157.8$ m. Furthermore, using Eqs. (11f)–(11i) $c_{r*}=5.200$; $c_{L*}=0.330$; $c_{ws*}=1.373$; $c_{wE*}=0.055$; and $d_*=0.2$, which are well within application range (16) of design Eqs. (17).

For a rectangular section Table 2 gives section shape coefficients: for bed width $k_{be}=0.71136$, $k_{br}=0.3201$, $k_{bL}=15.028$, $k_{bs1}=18.283$, $k_{bs2}=16.286$, $k_{bd}=0.1111$, and $k_{bE}=5.5434$; for normal depth $k_{ye}=0.35568$, $k_{yr}=0.3066$, $k_{yL}=15.023$, $k_{ys1}=18.737$, $k_{ys2}=16.741$, $k_{yd}=0.0964$, and $k_{yE}=5.6244$; and for cost $k_{ce}=0.25302$, $k_{cr}=0.03961$, $k_{cL}=1.4240$, $k_{cs}=2.0399$, $k_{cd}=0.1337$, and $k_{cE}=0.6856$.

Using Eqs. (17b) and (17c) with these coefficients: $b^*=24.306$ m; and $y_n^*=6.164$ m. Further, Eq. (17d) gives: the canal cost per meter $C^*=77.74c_e+145.13c_e+409.26c_e+3485.72c_e+32.84c_e=4150.69c_e$. These dimensions yield $A=b\ y_n=149.82$ m^2. Thus, $V=250.0/149.82=1.669$ m/s, which is within the permissible limit (Table 1).

For a trapezoidal section, the corresponding parameters are: $m^*=0.767$; $b^*=17.065$ m; $y_n^*=6.264$ m; $A=137.0$ m^2; and $C^*=4112.83c_e$.

In the absence of a drainage layer or when a drainage depth is at a very large depth the optimal canal dimensions would be: (a) for a rectangular section $b^*=18.940$ m, and $y_n^*=7.688$ m; and (b) for a trapezoidal section $m^*=0.575$, $b^*=12.795$ m, and $y_n^*=8.055$ m.

Minimum Cost Lined Section

If cost of the water is not taken into account, the canal design parameters, using Eq. (20) along with Table 2 for a rectangular section, are: $b^*=19.225$ m; $y_n^*=7.586$ m; and $C^*=632.13c_e$. On the other hand, a trapezoidal section with: $m^*=0.630$; $b^*=11.920$ m; $y_n^*=8.23$ m; and $C^*=588.14c_e$ will result into a minimum cost lined section.

Minimum Seepage Loss Section

Assuming the drainage layer at a large depth and using Eqs. (21a)–(21c) along with Table 2, the canal dimensions for a rectangular section are: $b^*=19.126$ m; and $y_n^*=7.611$ m; and for a trapezoidal section are: $m^*=0.598$; $b^*=13.045$ m; and $y_n^*=7.926$ m.

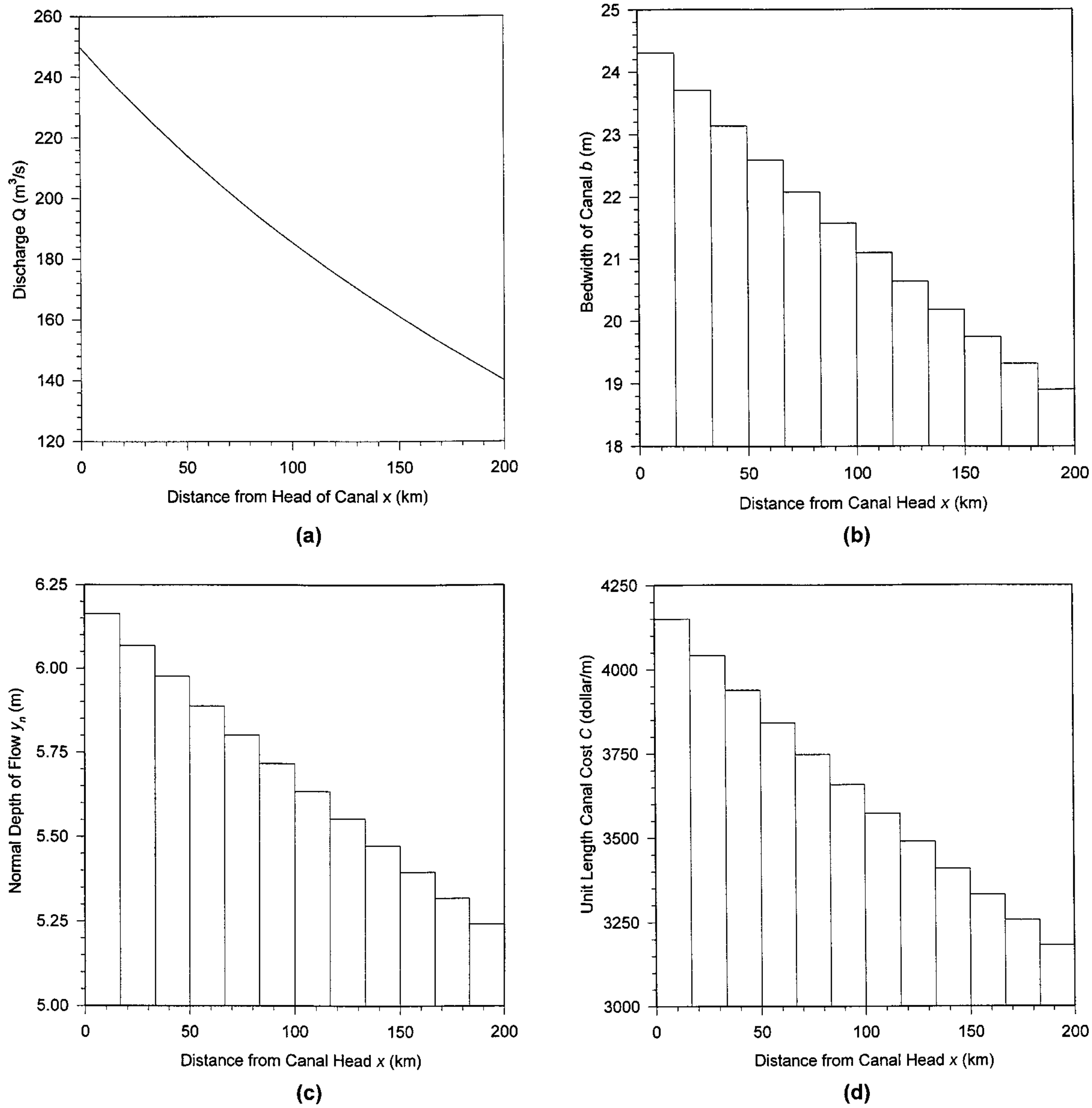

Fig. 2. Variation in canal section: (a) discharge; (b) bed width; (c) normal depth; and (d) cost per unit length

Variable Section Design of Transmission Canal

For a transmission canal of a rectangular shape the uniform section properties without transition are $b^*=24.306$ m; $y_n^*=6.164$ m; and $C^*=4149.57c_e$. Thus, the overall cost of the single section transmission canal $C_o=CL_c=4149.57L_cc_e$.

Using Eq. (11*j*) $c_{T*}=1.8875\times10^{-3}$; and (11r) $k_*=0.1455$, which are within limit indicated by Eq. (16).

Optimal Subsection Length and Cost

For a rectangular section Table 3 gives section shape coefficients for number of transitions in a transmission canal corresponding to cost of: transition $k_{NT}=14.00$; earthwork $k_{Nr}=0.20$; lining $k_{NL}=3.00$; seepage $k_{Nd0}=0.14$, $k_{Ns}=4.20$, and $k_{Nd}=0.20$; and evaporation $k_{NE0}=0.35$, and $k_{NE}=1.56$. Using these coefficients in Eq. (26) results: $N^*=12$, therefore, the optimal length of each subsection of the transmission canal $=200/12=16.667$ km.

Using cost coefficients for a rectangular transmission canal in Eq. (28) yields: the overall cost $C_o^*=0.9286CL_c=3853.29L_cc_e$; which is 92.86% of a uniform section transmission canal.

Optimal Section Dimensions for Second Subsection

Making use of design variables for the first subsection for the rectangular transmission canal, i.e., $b=24.306$ m, $y_n=6.164$ m, and $d=7.5$ m; in Eq. (32) the seepage function $F_s=24.7476$. Using Eq. (1), the water loss per unit length of the transmission canal $=5\times10^{-6}\times24.7476\times6.164+2.0\times10^{-7}\times24.306=7.67582\times10^{-4}$ m^2/s. Thus, the total quantity of water lost in 16.667 km length of the first subsection of the transmission canal $=16{,}666.667\times7.67582\times10^{-4}=12.793$ m^3/s. Hence, the discharge available in the second subsection of the transmission canal $=250.0-12.793=237.207$ m^3/s. Therefore, in the second subsection: $\lambda=35.636$ m; $\varepsilon_*=2.806\times10^{-5}$; $\nu_*=1.513\times10^{-7}$; and $L=23.472$ m. Using Eqs. (17*b*) and (17*c*) along with section shape coefficients from Table 2 for a rectangular section: $b^*=23.702$ m; $y_n^*=6.068$ m; and $C^*=4041.25c_e$.

Likewise the dimensions and cost per unit length of the canal for the successive subsections were calculated and their variations along the length of the transition canal are shown in Figs. 2(a–d).

Salient Features

Generalized explicit equations have been developed for design of least-cost canal section considering cost of earthwork and lining, and capitalized cost of water lost as seepage and evaporation. Other costs such as cost of acquiring land, cost of roads on banks, cost of side right-of-ways, etc., have not been included in the cost function considering them as fixed costs (independent of canal dimensions). These costs may be considered in an extended cost function. Using the optimal design equations along with the tabulated section shape coefficients, the optimal design variables for a canal can be obtained. These optimal design equations and coefficients have been obtained by analyzing a very large number of optimal sections resulted from application of optimization procedure in the wide application ranges of input variables. The analysis consists of conceiving an appropriate functional form and then minimizing errors between the optimal values and the computed values from the conceived function with coefficients.

The suggested equations are applicable for all the regular canal shapes. The section shape coefficients to be used in designing an optimal canal section and in designing a transmission canal have been obtained for triangular, rectangular, and trapezoidal canals. The method can be extended to find the coefficients in the optimal design equations for other shapes such as circular section, parabolic section, rounded corner trapezoidal section, etc., if the corresponding seepage functions are developed.

Direct optimization procedures may be adopted for the optimal design of irrigation canal sections and for the transmission canal but they are of limited use and require considerable amount of programming and computation. On the other hand, using the optimal design equations along with the tabulated section shape coefficients, the optimal design variables of a canal can be obtained in single-step computations. Design equations for a transmission canal have been developed with assumption of uniform flow, but in reality the flow in a transmission canal is gradually varied flow. For a short length of a transmission canal the difference in canal section dimensions with uniform flow assumption is negligible in comparison to the approximation involved due to assumptions made in other parameters.

Eq. (26) along with Eq. (27) shows that the optimal subsection length of the transmission canal is independent of the length of the transmission canal. Further, x^* increases with increase in C_T and depth of the drainage layer, while it decreases with increase in hydraulic conductivity of the porous medium, rate of evaporation, earthwork cost, lining cost, and cost of water as expected. The optimal cost of the transmission canal is less sensitive to the optimal number of subsections. Though the optimal design of the transmission canal is obtained with assumption of equal cost for each of the transitions and of equal length for each of the subsections of the transmission canal, the method can be extended for unequal cost of transitions and unequal length of subsections. The assumption of equal length of subsection in obtaining the optimal length of a subsection from Eq. (27) can be relaxed by obtaining a new x^* each time for the remaining section of the transmission canal.

Conclusions

Generalized explicit equations have been presented for the optimal design of a transmission canal. Using the optimal design equations along with the tabulated section shape coefficients, the optimal design variables for a canal can be obtained.

The suggested equations are applicable for all the regular canal shapes. The section shape coefficients to be used in designing a transmission canal and in designing optimal canal sections have been presented for triangular, rectangular, and trapezoidal canals. The method can be extended to find the coefficients in the optimal design equations for other shapes such as circular section, parabolic section, rounded corner trapezoidal section, etc., if the corresponding seepage functions are developed.

A direct optimization procedure may be adopted for the optimal design of irrigation canal sections and for the transmission canal but they require considerable amount of programming and computation. Using the optimal design equations along with the tabulated section shape coefficients, the optimal design variables of a canal can be obtained in single-step computations. The present method can be extended in developing equations for the optimal design of a transmission canal having unequal cost of transitions and unequal length of subsections.

Appendix: Seepage Functions

Chahar (2000) and Swamee et al. (2001) gave the following algebraic equations for the seepage function for triangular, rectangular, and trapezoidal canal sections underlain by a drainage layer at depth d:

Triangular Section

$$F_s=\left\{\left(\frac{1.81m^{1.18}+2.1}{(d/y-1)^{0.26}}\right)^{9.35}+[(4\pi-\pi^2)^{1.3}+(2m)^{1.3}]^{7.2}\right\}^{0.107} \tag{31}$$

Rectangular Section

$$F_s=\left\{\left(\frac{2.5(b/y)^{0.84}+0.45}{(d/y-1)^{0.69}}\right)^{2.38}+[(4\pi-\pi^2)^{0.77}+(b/y)^{0.77}]^{3.094}\right\}^{0.42} \tag{32}$$

Trapezoidal Section

$$F_s=\left\{\left[1.81[m^{1.3}+1.432(b/y)^{0.93}]^{0.9}+\frac{b+100my}{2.22b+47.62my+1.57bm^5}\right]^{p_1}\left(\frac{d}{y}-1\right)^{-p_1p_2}+([(4\pi-\pi^2)^{1.3}+(2m)^{1.3}]^{0.77p_3}+(b/y)^{p_3})^{p_1/p_3}\right\}^{1/p_1} \tag{33}$$

where exponents

$$p_1=\frac{2.38b+7.48my}{b+0.8my};$$

$$p_2=\frac{0.318b+0.26my}{0.461b+my};\quad p_3=\frac{1+0.6m}{1.3+0.6m}$$

For uniform flow, y is replaced by y_n. As $d\to\infty$, Eqs. (31) to (33) become a function of canal geometry only and reduce to the seepage functions for canals passing through a homogeneous porous medium of very large depth (Swamee et al. 2000b).

Notation

The following symbols have been used in this paper:

A = flow area of canal [m^2];
b = bed width of canal [m];
C = cost per unit length of canal [$/m];
C_e = cost of earthwork per unit length of canal [$/m];
C_L = cost of lining per unit length of canal [$/m];
C_o = overall cost of transmission canal [$];
C_T = cost of each transition canal [$];
C_w = capitalized cost of water lost per unit length of canal [$/m];
c_e = cost per unit volume of earthwork at ground level [$/m^3];
c_L = cost per unit surface area of lining [$/m^2];
c_r = increase in unit excavation cost per unit depth [$/m^4];
c_w = cost per unit volume of water [$/m^3];
c_{wE} = $3.156\times10^7 E c_w / r$ [$/m^2];
c_{ws} = $3.156\times10^7 k c_w / r$ [$/m^2];
d = depth of drainage layer/aquifer [m];
E = evaporation discharge per unit free surface area [m/s];
F_s = seepage function [dimensionless];
g = gravitational acceleration [m/s^2];
i = index [dimensionless];
k = hydraulic conductivity [m/s];
k_{fs} = section shape coefficients for subscripts f and s [dimensionless];
L = length scale [m];
L_c = length of transmission canal [m];
L_E = length scale for evaporation [m];
L_s = length scale for seepage [m];
m = side slope of canal [dimensionless];
N = number of subsections in transmission canal [dimensionless];
P = flow perimeter of canal [m];
p_1, p_2, p_3 = exponents [dimensionless];
Q = discharge [m^3/s];
q_E = evaporation loss per unit length of canal [m^2/s];
q_s = seepage loss per unit length of canal [m^2/s];
q_w = water loss per unit length of canal [m^2/s];
R = hydraulic radius [m];
r = rate of interest [$/$/year];
S_0 = bed slope of canal [dimensionless];
T = width of free surface [m];
V = average velocity [m/s];
V_L = limiting velocity [m/s];
x_i = length of transmission canal in ith subsection [m];
y = depth of flow in canal [m];
y_n = normal depth of flow in canal [m];
ε = average roughness height of canal lining [m];
λ = length scale [m];
ν = kinematic viscosity [m^2/s]; and
Φ = equality constraint [dimensionless].

Subscripts

b = bed width;
c = cost;
d = seepage with drainage layer;
E = evaporation;
e = earthwork;
L = lining;
m = side slope;
N = number of subsections;
o = overall cost;
r = depth dependent (additional) excavation;
s = seepage;
T = transition;
w = water;
y = normal depth; and
$*$ = nondimensional.

Superscript

$*$ = optimal.

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