Determination of Length of a Horizontal Drain in Homogeneous Earth Dams

Bhagu R. Chahar[1]

Abstract: An earth dam can be prevented from a seepage failure due to softening of the downstream slope by providing a rock toe or horizontal drainage blanket. Analytical solutions are not available for determining the length of the filtered drainage blanket and downstream slope cover, though graphical solutions are available for them. Explicit equations have been obtained in the present work for calculating the downstream slope cover and the length of the downstream horizontal drain in homogeneous isotropic and anisotropic earth dams. Similar equations have also been obtained for maximum downstream slope cover and minimum and maximum effective length of the filtered drainage. These equations are nonlinear and representative graphs have been plotted for them covering all the practical ranges of the dam geometry. The numerical example demonstrates that the proposed equations are simple to use, hence the designers may find these equations as an additional check to their design by the conventional flownet method.

DOI: 10.1061/(ASCE)0733-9437(2004)130:6(530)

CE Database subject headings: Dams, earth; Dams, embankment; Filters; Horizontal drains; Dam failure; Slope stability.

Introduction

Earth dams are generally built of locally available materials in their natural state with a minimum of processing. Homogeneous earth dams are built whenever only one type of material is economically available. The material must be sufficiently impervious to provide an adequate water barrier and slopes must be relatively flat to make it safe against piping and sloughing. The general design procedure is to make a first estimate on the basis of experience with similar dams and then to modify the estimate as required after conducting a stability analysis except where there is a surplus of material. The United States Department of the Interior Bureau of Reclamation (USBR) and other agencies (Sherard et al. 1967; Sharma 1991) suggested limits for the upstream and the downstream slopes for different types of materials and dams. The upstream slopes of most of the earth dams in actual practice usually vary from 2.0 (horizontal):1 (vertical) to 4:1 and the downstream slopes are generally between 2:1 and 3:1 (USBR 2003). Freeboard depends on the height and action of waves. USBR (2003) recommends normal freeboard about 1.5 to 3 m depending on the fetch. The width of the dam crest is determined by considering the nature of embankment materials, height and importance of structure, possible roadways requirements, and practicability of construction. A majority of dams have the crest widths varying between 5 and 12 m. Empirical relations are given for different ranges of the dam height by USBR (2003), which can be adopted as a guideline.

About 30% of dams had failed due to the seepage failure, viz piping and sloughing (Middlebrooks 1953). Recent comprehensive reviews by Foster et al. (2000a,b) and Fell et al. (2003) show that internal erosion and piping are the main causes of failure and accidents affecting embankment dams; and the proportion of their failures by piping increased from 43% before 1950 to 54% after 1950. The sloughing of the downstream face of a homogeneous earth dam occurs under the steady-state seepage condition due to the softening and weakening of the soil mass when the top flow line or phreatic line intersects it. Regardless of flatness of the downstream slope and impermeability of soil, the phreatic line intersects the downstream face to a height of roughly one-third the depth of water (Justin et al. 1944). It is usual practice to use a modified homogeneous section in which an internal drainage system in the form of a horizontal blanket drain or a rock toe or a combination of the two is provided. The drainage system keeps the phreatic line well within the body of the dam. Horizontal filtered drainage blankets are widely used for dams of moderate height. Lion Lake dike (6.5 m high), Pishkun dikes (13 m high), Stubblefield dam (14.5 m high), Dickinson dam (15 m high), etc. are examples of small homogeneous dams built by USBR (2003). Also, USBR constructed the 50 m high Vega dam, which is one of the highest with a homogenous section and a horizontal downstream drain. Design criteria of filtered drainage can be found in many references (Terzaghi and Peck 1967; Vaughan and Soares 1982; Sherard et al. 1984a,b; Sherard and Dunnigan 1985; Honjo and Veneziano 1989; Sharma 1991).

A graphical solution for estimation of the downstream slope cover for a given length of underdrain and vice versa was given by Casagrande (Harr 1962). In the graphical method, a series of basic parabolas (phreatic lines) corresponding to various lengths of an underdrain are plotted and then the loci of three other curves

[1]Dept. of Civil Engineering, Indian Institute of Technology, New Delhi-110 016, India. E-mail: chahar@civil.iitd.ac.in; chahar_br@yahoo.com

Note. Discussion open until May 1, 2005. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this technical note was submitted for review and possible publication on February 5, 2003; approved on February 23, 2004. This technical note is part of the ***Journal of Irrigation and Drainage Engineering***, Vol. 130, No. 6, December 1, 2004. ©ASCE, ISSN 0733-9437/2004/6-530–536/$18.00.

are drawn (refer to "Example" and its figure). The graphical method is time consuming due to the involvement of the long procedure of plotting. It is, also, subjected to personal skills and judgement. USBR (2003) suggested empirical guidelines for estimating initial length of the horizontal drainage blanket without considering the downstream slope cover. So far, analytical solutions in the form of explicit relations between the filtered drainage length and the downstream slope cover have not been reported in the literature. The present work puts forward explicit solutions for minimum length of the horizontal blanket drain required to keep the phreatic line within the body of the dam by a specified depth and also equations for maximum downstream slope cover and minimum and maximum effective lengths of the downstream filtered drainage system.

Development of Analytical Solutions

The position of the phreatic line influences the stability of the earth dam because of potential piping due to excessive exit gradient and sloughing due to the softening and weakening of the soil mass as if it touches the downstream slope or intersects it. When the dam embankment is homogeneous or when the downstream zone is of questionable permeability, a horizontal drainage blanket is provided to keep the phreatic line well within the dam body, to allow adequate embankment and foundation drainage, and to eliminate piping from the foundation and the embankment. As the dams are made of fine-grained soil, saturation may occur due to the capillary rise above the phreatic surface so it is necessary to account for capillary rise while calculating the minimum length of the downstream filtered drainage. Though the suction head in the soil matrix above the phreatic surface within the dam body due to capillary rise generally improves the stability of the downstream slope, once the capillary fringe intersects the downstream slope the pressure changes from negative (suction) to atmospheric and the downstream face may become a seepage face leading to its failure. Hence the phreatic line should not intersect the downstream slope and it should be a distance greater than capillary rise below the sloping face so that the chances of the sloughing or piping may be nullified.

The amount of water seeping through and under an earth dam, together with the location of the phreatic surface, can be estimated by using the theory of flow through porous media (Harr 1962; Polubarinova-Kochina 1962). Steady-state seepage through a porous media is governed by the Laplace equation. An analytical solution of the Laplace equation for seepage through a homogeneous earth dam with a downstream drainage may be obtained (Polubarinova-Kochina 1962) but the actual numerical values can only be evaluated from lengthy and difficult numerical integrations. Approximate solutions of such problems by numerical methods (Zienkiewicz et al. 1966; Finn 1967; Taylor and Brown 1967; Jepson 1968, 1969; Neuman and Witherspoon 1970, etc.) have gained importance due to easy availability of high speed digital computers along with specialized softwares. However, generalized solutions in the functional form are not possible through numerical methods as they result only in a problem-specific particular solution. Forchheimer and Richardson (Todd 1995) obtained approximate solutions of the Laplace equation and independently developed a powerful graphical (flownet) method. This method is widely used for the earth dams after Casagrande's work (1937) because of its simplicity. Kozeny (Harr 1962) obtained a solution for the problem of seepage through a homogeneous earth dam with a parabolic upstream face resting on an

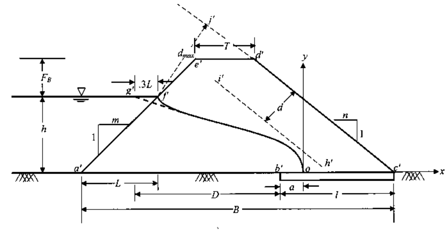

Fig. 1. Definition sketch

impervious base and having a horizontal downstream drainage filter using mapping functions. Casagrande (Harr 1962) extended this solution for a linear upstream-faced homogeneous earth dam on an impervious base and with a downstream drainage.

Similar to Casagrande (Harr 1962), consider a linear upstream-faced homogeneous earth dam $a'b'c'd'e'f'a'$ on an impervious base $a'c'$ and with a horizontal downstream filtered drainage $b'c'$ of length l as shown in Fig. 1. The upstream-face of the dam section $a'e'$ has slope $m:1$ (horizontal:vertical), while the downstream face $c'd'$ has $n:1$. The free surface (phreatic line) follows a parabola (called Kozeny's basic parabola) passing through the apex o and g' (0.3 L from the intersection of water level with the upstream slope line) with its focus at the upstream edge of the filter b' (Punmia et al. 1994; Singh and Chowdhary 1994). To develop a relation for the downstream slope cover over the phreatic line the geometrical properties of a parabola and a straight line (Sastry 1994) were used. The equation of the parabola og', when the origin was taken at the apex of the base parabola with the axes as shown in Fig. 1, became

$$y^2 = -4ax \tag{1}$$

Substituting the coordinates of the point $g'[h,-(D+a)]$ and simplifying

$$a = (\sqrt{D^2+h^2} - D)/2 \tag{2}$$

hence

$$y^2 = -2(\sqrt{D^2+h^2} - D)x \tag{3}$$

The downstream slope line $c'd'$ has slope $M=-1/n$ and intersects the x axis at the point

$$x = l - a = l - (\sqrt{D^2+h^2} - D)/2 \tag{4}$$

so its equation took the form

$$y = -\frac{x}{n} + \frac{1}{n}[l - (\sqrt{D^2+h^2} - D)/2] \tag{5}$$

To determine the distance of the phreatic line from the downstream sloping face, which is known as the downstream slope cover, line $h'i'$ was drawn parallel to the downstream slope at a distance d apart (see Fig. 1). The equation of the line $h'i'$ parallel to the line $c'd'$ at a distance d apart would be

$$y = -\frac{x}{n} + \frac{1}{n}[l - (\sqrt{D^2+h^2} - D)/2 - d\sqrt{1+n^2}] \tag{6}$$

This parallel line will be tangential to the parabola of the phreatic line if it satisfies the condition

$$a = MC \tag{7}$$

where a is defined by Eq. (2) for the basic parabola; M=slope of the line $h'i'=-1/n$; and C=y-axis intercept of the line $h'i'$ given by

$$C = [l - (\sqrt{D^2 + h^2} - D)/2 - d\sqrt{1 + n^2}]/n \tag{8}$$

Substituting the corresponding values, Eq. (7) became

$$-(\sqrt{D^2 + h^2} - D)/2 = -\frac{1}{n} \times [l - (\sqrt{D^2 + h^2} - D)/2 - d\sqrt{1 + n^2}]/n \tag{9}$$

which further reduced to

$$l - (1 + n^2)(\sqrt{D^2 + h^2} - D)/2 = d\sqrt{1 + n^2} \tag{10}$$

Therefore

$$d = \frac{l}{\sqrt{1 + n^2}} - \frac{\sqrt{1 + n^2}}{2}(\sqrt{D^2 + h^2} - D) \tag{11}$$

From the geometric properties of the dam section and basic parabola (see Fig. 1)

$$D = B - l - 0.7mh \tag{12}$$

and

$$B = [(h + F_B) \times (m + n) + T] \tag{13}$$

so

$$D = 0.3mh + nh + F_B(m + n) + T - l \tag{14}$$

and hence Eq. (11) resulted to

$$d = \frac{l}{\sqrt{1 + n^2}} - \frac{\sqrt{1 + n^2}}{2}\{\sqrt{[0.3mh + nh + F_B(m + n) + T - l]^2 + h^2} - [0.3mh + nh + F_B(m + n) + T - l]\} \tag{15}$$

Eq. (15) is an explicit relation for the downstream slope cover. The relation was converted into dimensionless form by defining

$$d_* = \frac{d}{h} \tag{16a}$$

$$F_{B*} = \frac{F_B}{h} \tag{16b}$$

$$T_* = \frac{T}{h} \tag{16c}$$

$$l_* = \frac{l}{h} \tag{16d}$$

Dividing Eq. (15) by h and thereafter substituting Eqs. (16*a*)–(16*d*) became

$$d_* = \frac{l_*}{\sqrt{1 + n^2}} - \frac{\sqrt{1 + n^2}}{2}\{\sqrt{[0.3m + n + F_{B*}(m + n) + T_* - l_*]^2 + 1} - [0.3m + n + F_{B*}(m + n) + T_* - l_*]\} \tag{17}$$

The subscript asterisk denotes the corresponding nondimensional parameter.

Maximum Cover

With increase in the length of the filtered drainage blanket the corresponding downstream slope cover to the phreatic line also increases until it reaches the point of intersection of the upstream slope with the line of water level f', as the phreatic line cannot shift inside the dam body beyond this point. This will be the maximum value for d and hereafter will be called maximum downstream slope cover over the phreatic line $d_{\max}$. To calculate the maximum thickness of the cover a perpendicular $f'j'$ from the point of intersection of water level and the upstream slope line f' was drawn on the downstream slope line $c'd'j'$. The length of the perpendicular from the point $f'[-(a+D-0.3L), h]$ or $[-[D+(\sqrt{D^2+h^2}-D)/2-0.3mh], h]$ to the line $c'd'j'$ [Eq. (5)] became

$$d_{\max} = \frac{n}{\sqrt{1 + n^2}}\left\{\frac{1}{n}[nh + F_B(m + n) + T - l + (\sqrt{D^2 + h^2} - D)/2] - h + \frac{1}{n}[l - (\sqrt{D^2 + h^2} - D)/2]\right\} \tag{18}$$

Further simplifying

$$d_{\max} = \frac{[F_B(m + n) + T]}{\sqrt{1 + n^2}} \tag{19}$$

Dividing Eq. (18) by h and defining

$$d_{\max*} = d_{\max}/h \tag{20}$$

resulted in the nondimensional form for the maximum cover as

$$d_{\max*} = \frac{[F_{B*}(m + n) + T_*]}{\sqrt{1 + n^2}} \tag{21}$$

Eq. (19) or Eq. (21) shows that $d_{\max}$ is independent of the filter length and is a function of the dam geometry only. Furthermore, $d_{\max}$ is a linear function of the free board, the top width, and the upstream slope while it is a nonlinear function of the downstream slope.

Inverse Solution

Eq. (15) or Eq. (17) can be used to determine the downstream cover on the phreatic line for an existing earth dam. In the design stage, the designer is interested in fixing the length of a horizontal downstream drain for a specified cover over the phreatic line. For practical problems of design, it is more useful to have an explicit equation for length of the horizontal blanket drain. Herein explicit relations for the drain length with variable downstream slope cover and dam geometry were developed. Rewriting Eq. (11) in the following form

$$\sqrt{D^2 + h^2} = \frac{2l}{(1 + n^2)} - \frac{2d}{\sqrt{1 + n^2}} + D \tag{22}$$

Squaring both the sides and rearranging

$$\frac{4l^2}{(1 + n^2)^2} + \frac{4d^2}{(1 + n^2)} - \frac{8dl}{(1 + n^2)^{3/2}} - h^2 - \frac{4dD}{\sqrt{1 + n^2}} + \frac{4lD}{(1 + n^2)} = 0 \tag{23}$$

Putting the value of D from Eq. (14) and simplifying

$$n^2l^2 - l\{d\sqrt{1 + n^2}(n^2 - 1) + (1 + n^2) \times [0.3mh + nh + F_B(m + n) + T]\} + \{d(1 + n^2)^{3/2} \times [0.3mh + nh + F_B(m + n) + T] - d^2(1 + n^2) + 0.25(1 + n^2)^2h^2\} = 0 \tag{24}$$

Solving the quadratic equation and retaining minus term as the significant root

$$l = \frac{1+n^2}{2n^2}\left(0.3mh + nh + F_B(m+n) + T + \frac{n^2-1}{\sqrt{1+n^2}}d - \sqrt{[0.3mh + nh + F_B(m+n) + T - d\sqrt{1+n^2}]^2 - n^2h^2}\right) \quad (25)$$

Eq. (25) was expressed into nondimensional form as

$$l_* = \frac{1+n^2}{2n^2}\left(0.3m + n + F_{B*}(m+n) + T_* + \frac{n^2-1}{\sqrt{1+n^2}}d_* - \sqrt{[0.3m + n + F_{B*}(m+n) + T_* - d_*\sqrt{1+n^2}]^2 - n^2}\right) \quad (26)$$

Minimum and Maximum Effective Length of Drain

The minimum length of the downstream blanket drain $l_{\min}$, which can keep the phreatic line just within the body of dam, can be calculated by putting the value of the downstream slope cover equal to zero in Eq. (25) or Eq. (26). Substituting $d=0$ in Eq. (25) and $d_*=0$ in Eq. (26) yielded, respectively

$$l_{\min} = \frac{1+n^2}{2n^2}\{0.3mh + nh + F_B(m+n) + T - \sqrt{[0.3mh + nh + F_B(m+n) + T]^2 - n^2h^2}\} \quad (27)$$

$$l_{\min *} = \frac{1+n^2}{2n^2}\{0.3m + n + F_{B*}(m+n) + T_* - \sqrt{[0.3m + n + F_{B*}(m+n) + T_*]^2 - n^2}\} \quad (28)$$

Eq. (28) is the nondimensional form for the minimum length of the downstream filtered drainage system. The length of the horizontal downstream blanket drainage should not be less than the above-mentioned length; otherwise, the seepage line will intersect the downstream slope resulting in a seepage (saturated) face, which may cause sloughing and/or piping failure.

As the specified downstream slope cover increases the required length of the downstream blanket drain also increases. The specified downstream slope cover cannot be increased beyond the maximum cover $d_{\max}$. The maximum effective length of the filtered drainage system $l_{\max}$ corresponding to $d_{\max}$ was obtained by combining Eqs. (21) and (26)

$$l_{\max *} = F_{B*}(m+n) + T_* + \frac{1+n^2}{2n^2}[0.3m + n - \sqrt{(0.3m+n)^2 - n^2}] \quad (29)$$

Eq. (29) is the nondimensional expression for maximum effective length of the horizontal downstream blanket drainage. If the length of the filtered blanket drainage is increased beyond this value it is ineffective in increasing the downstream slope cover and it reduces the length of the path of seepage leading to increased quantity of seepage. A minimum length filter blanket is desirable because filters are expensive to construct. Thus the designed length of filter or drain must be between $l_{\min}$ and $l_{\max}$.

Anisotropic Conditions

The method of placement and compaction in earth fills is such that stratifications are generally built into embankments. Generally tamping or sheepfoot rollers are used to compact the fine grained earthfill materials in construction of an earth dam. As rollers compact the lifts of earthfill, the horizontal permeability tends to be larger than the vertical. As the difference increases, the effectiveness of the horizontal drains in lowering the free surface decreases, hence an extra length is required to achieve the same efficiency. Ratios of the horizontal permeability k_x to the vertical permeability k_y in compacted fills tend to be even larger than those of 2 to 10 in normally consolidated sedimentary clays (Lambe and Whitman 2000). Such an anisotropic section has to be transformed into an isotropic one and then the Laplace equation can be solved graphically. The analytical method presented here is applicable for a transformed isotropic dam section where the Laplace equation is valid. For an anisotropic section using a scale ratio λ as

$$\lambda = \sqrt{k_y/k_x} \quad (30)$$

geometric elements of the transformed section became

$$m_T = \lambda m \quad (31a)$$

$$n_T = \lambda n \quad (31b)$$

$$T_T = \lambda T \quad (31c)$$

$$l_T = \lambda l \quad (31d)$$

$$F_{BT} = F_B \quad (31e)$$

$$h_T = h \quad (31f)$$

$$d_T = \lambda d\sqrt{\frac{1+n^2}{1+\lambda^2 n^2}} \quad (31g)$$

where the subscript T denotes the corresponding geometric element of the transformed section. Eq. (17) or Eq. (26) with the modified geometric elements (31*a*)–(31*g*) may be used to estimate the downstream slope cover or the filtered drainage length for the transformed section. Reverse transformation results in the downstream slope cover or the drain length for the actual dam section. Alternatively, the following relations estimate the downstream slope cover or the length of the downstream blanket filter directly without going through the transformation steps

$$d_* = \frac{l_*}{\sqrt{1+n^2}} - \frac{1+\lambda^2 n^2}{2\sqrt{1+n^2}}\{\sqrt{[0.3m + n + F_{B*}(m+n) + T_* - l_*]^2 + \lambda^{-2}} - [0.3m + n + F_{B*}(m+n) + T_* - l_*]\} \quad (32)$$

$$d_{\max *} = \frac{[F_{B*}(m+n) + T_*]}{\sqrt{1+n^2}} \quad (33)$$

$$l_* = \frac{1+\lambda^2 n^2}{2\lambda^2 n^2}\left(0.3m + n + F_{B*}(m+n) + T_* + \frac{\lambda^2 n^2 - 1}{1+\lambda^2 n^2}\sqrt{1+n^2}d_* - \sqrt{[0.3m + n + F_{B*}(m+n) + T_* - d_*\sqrt{1+\lambda^2 n^2}]^2 - n^2}\right) \quad (34)$$

$$l_{\min *} = \frac{1+\lambda^2 n^2}{2\lambda^2 n^2}\{0.3m + n + F_{B*}(m+n) + T_* - \sqrt{[0.3m + n + F_{B*}(m+n) + T_*]^2 - n^2}\} \quad (35)$$

$$l_{\max*} = F_{B*}(m+n) + T_* + \frac{1+\lambda^2 n^2}{2\lambda^2 n^2}\left[0.3m + n - \sqrt{(0.3m+n)^2 - n^2}\right] \tag{36}$$

Eq. (33) shows that maximum downstream slope cover is unaffected by the stratification.

Foundation Conditions

In the derivation of relations, a dam section resting on an impervious foundation has been assumed, in which the phreatic line follows the Kozney's basic parabola. Generally the foundation is somewhat pervious and a horizontal blanket drain is placed on the surface of the foundation under the downstream section of the dam to intercept seepage passing through the foundation that could cause hydraulic pressure under the downstream section of the dam. The proposed method is applicable for a homogeneous earth dam with a horizontal filtered drainage placed under the downstream section and resting on any type of foundation, where the phreatic surface within the dam body follows a parabolic shape with its focus lying at the upstream edge of the horizontal drain. Most earth dams are designed with a cutoff extending through the upper layers of the foundation more vulnerable to seepage. The cutoff should preferably be just upstream of the starting point of the horizontal drain and its location can be fixed after calculating the length of the drain. Under such cases any foundation seepage that gets past the cutoff can be intercepted by the drain.

Discussion and Example

The derived relations are functions of the dam geometry, the length of the horizontal drain, the downstream slope cover, and the ratio of horizontal-to-vertical permeability. These relations are independent of the permeability of the dam material but k_x/k_y. The reason lies in the fact that the position of the phreatic line in steady-state seepage depends only on the geometry of the section and it acquires an identical position for soils of different permeabilities but of the same ratio of horizontal-to-vertical permeability. Eq. (25) or Eq. (26) can be used to determine the length of the horizontal downstream blanket drain for a given dam geometry and specified downstream slope cover. The downstream slope cover depends on the capillarity of the dam material, and should be more than the height of capillary rise in soil forming the dam body. Polubarinova-Kochina (1962) lists typical values of the height of capillary rise in some soils. Minimum and maximum effective lengths of filter drain are independent of d; hence for a given dam geometry maximum downstream slope cover as well as minimum and maximum effective lengths of the filtered drainage system are fixed. The variations in the drainage filter length with respect to different downstream slope cover and geometrical parameters of the dam are plotted in Figs. 2(a–d). The plots are representative of selected values of dimensionless parameters (d_*, l_*, T_*, F_{B*}, m, and n) and one can draw more plots for different set of theses dimensionless parameters. The starting points on the vertical axis shows the minimum effective length of the filter or drain while the terminating points show the coordinates corresponding to maximum downstream slope cover and maximum effective length of the filter. Eqs. (17) and (26) show that the downstream cover and the filter length have a nonlinear relationship with each other as well as with the top width, free board,

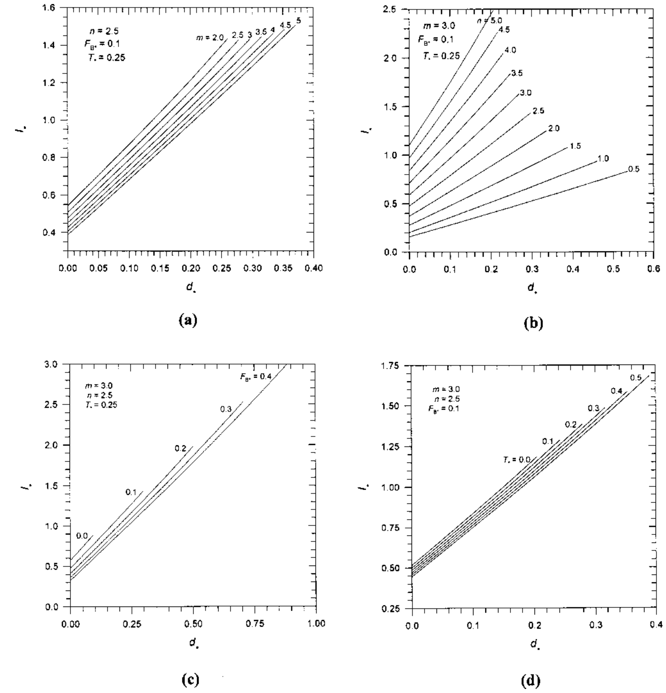

Fig. 2. Variations in the filter length with the downstream slope cover (a) for different upstream slope; (b) for different downstream slope; (c) for different free board; and (d) for different top width

upstream slope, and downstream slope. A perusal of plots [Figs. 2(a–d)] reveals that the relationships between d and l for a given set of m, T, F_B, and n are almost linear. Furthermore, the plots in Figs. 2(a, c, and d) are nearly parallel for different m, F_B, and T, respectively, showing that d or l also vary almost linearly with m, F_B, and T. On the other hand, graphs in Fig. 2(b) for different n are not parallel to each other indicating that d or l is a nonlinear function of n. The underlying reason may be understood by the careful study of Eq. (26). In the practical ranges, the term $(d_*\sqrt{1+n^2})$ becomes small in comparison to the term $[0.3m+n+F_{B*}(m+n)+T_*]$. After dropping the term $(d_*\sqrt{1+n^2})$, Eqs. (26) and (28) gave

$$l_* \approx l_{\min*} + \frac{\sqrt{1+n^2}(n^2-1)}{2n^2}d_* \tag{37}$$

Fig. 2, roughly, depicts the behavior of Eq. (37). It can be seen that the influence of m, F_B, and T are limited in fixing minimum length of the drainage filter. The length of the filter more than minimum length is not affected by m, F_B, and T for a specified n and d. Also, Figs. 2(a, c, and d) show that with increase in (flattening of) the upstream slope, top width, or free board, the required filter length reduces for a given value of the downstream slope cover. The reverse happens with a change in n [see Fig. 2(b)]. Further, it can be seen that d or l is less affected by the change in the top width, free board, or upstream slope while they are more sensitive to the change in the downstream slope.

For an anisotropic dam section, the behavior of Eq. (34) can be plotted similar to Fig. 2 but with an additional parameter λ. Eq. (33) shows that maximum downstream slope cover is unaffected by the stratification. Similar to Eq. (37), Eq. (34) can be approximated to

$$l_* \approx l_{\min*} + \frac{\sqrt{1+n^2}(\lambda^2 n^2-1)}{2\lambda^2 n^2}d_* \tag{38}$$

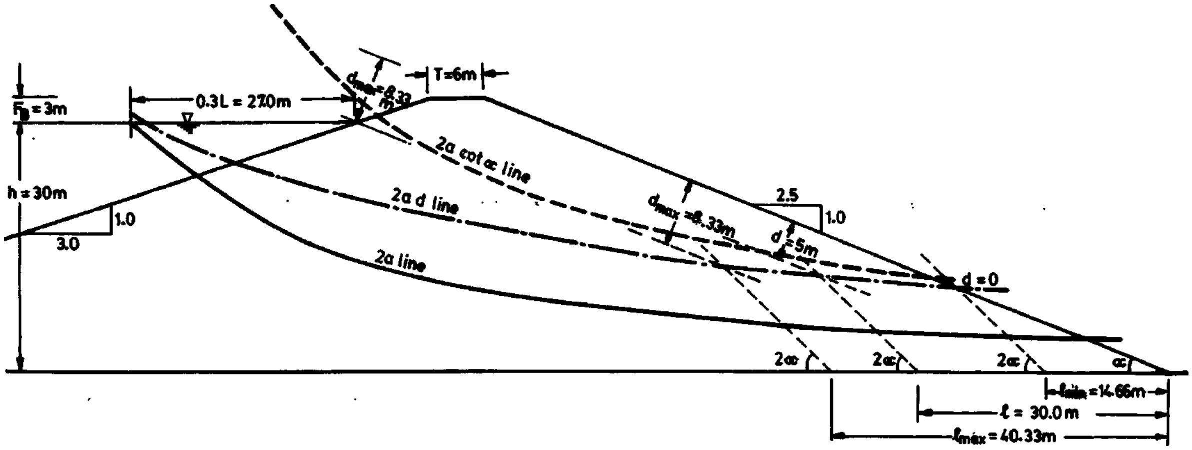

Fig. 3. Graphical solution by Casagrande's method

In the derivations of the proposed equations it has been assumed that the horizontal filter drain functions properly. This implies that the filter drain remains unchoked, i.e., all water seeping through the dam section is intercepted by the filter drain and then it is drained downstream of the dam section in such a way that the water level is always within the filter drain. Under such conditions, the filter drain acts as zero equipotential line and the phreatic line follows Kozeny's basic parabola with its focus coinciding with the upstream edge of the filter drain. Performance of a filtered drain depends on the capacity of the drain, hydraulic conductivity contrast between dam material and drainage material, type of foundation, and the dam geometry. If the filter drain gets choked due to improper design, the proposed equations should not be applied.

Example

Determine the length of a filter cum drain in a 33 m high homogeneous earth dam for a downstream slope cover over the phreatic line equal to 5.0 m. Assume the top width = 6.0 m, the free board = 3.0 m, the upstream slope equal to 3:1, and the downstream slope equal to 2.5:1.

Since the total height of the dam = 33 m and the freeboard = 3.0 m, hence $h = 30.0$ m. Using Eq. (16*a*)–(16*c*), $d_* = 0.167$; $F_{B*} = 0.1$; and $T_* = 0.2$. Use of Eq. (19) yields maximum downstream slope cover that can be achieved = 8.356 m. Using Eqs. (28) and (29), $l_{\min *} = 0.4857$ and $l_{\max *} = 1.3855$, respectively, which in turn gives the minimum effective length of filter required to keep the phreatic line just within the dam body = 14.573 m and maximum effective length of filter = 41.564 m, respectively. Finally making use of Eq. (26) results in $l_* = 1.0125$ and hence the required length of filter to keep the phreatic line 5 m below the downstream slope = 30.37 m.

Adopting the graphical solution as given by Casagrande (see Fig. 3), $l_{\min} = 14.66$ m, $l = 30.0$ m, and $l_{\max} = 40.33$ m were obtained. This shows that the proposed equations replicate the results of Casagrande's graphical method.

If the dam is assumed to be an anisotropic section having the horizontal permeability four times greater than the vertical permeability, then $\lambda = 0.5$. Using Eqs. (34)–(36); $l_* = 1.048$, $l_{\min *} = 0.6868$, and $l_{\max *} = 1.6484$, respectively, which result into l = 31.44 m, $l_{\min} = 20.60$ m, and $l_{\max} = 49.45$ m. The comparison of results reveals that the required length of the drain to keep the phreatic line by a specified distance in a dam section in which the horizontal permeability is higher than the vertical permeability is more than the isotropic section.

Conclusions

Using the geometrical properties of the dam section and algebraic analysis, explicit equations can be obtained for the downstream slope cover and the length of the downstream drainage filter. Similar equations can also be obtained for maximum downstream slope cover and minimum and maximum effective length of the drain. The relations between the filter length and the downstream slope cover for a different set of upstream slope, top width, free board, or downstream slope are nonlinear, but these can be approximated by a linear relationship. With increase in (flattening of) the upstream slope, top width, or free board the required filter length reduces for a given value of the downstream slope cover, while it increases with increase in the downstream slope. Further, the downstream slope cover or the length of the horizontal drain is less affected by the change in the top width, free board, or upstream slope, while they are more sensitive with the change in the downstream slope. Also, maximum downstream slope cover as well as minimum and maximum effective lengths of the filtered drainage system are fixed for a given dam geometry. Presented equations have been nondimensionalized and are very simple to be used in determination of the horizontal downstream drainage filter length in a given dam section for a specified downstream slope cover. The method is simple, straightforward, and does not involve personal skills and judgement, hence it is convenient to use for a new designer. Expert designers continue to develop flownet as a basic and important requirement for a safe dam design; still the proposed equations may be handy to them in reducing the number of trial flownets by providing a better initial section or for cross-checking their design.

Acknowledgment

The writer would like to thank the anonymous reviewers for their insightful review and constructive suggestions, which resulted in a significant improvement of the manuscript.

Notation

The following symbols are used in this technical note:

a = focal length of parabola, i.e., filter length intercepting seepage (m);
$a', b', c', \ldots$ = points;
B = base width of dam (m);
C = y-axis intercept of a line (m);
D = horizontal distance from edge of drain (focus of parabola) to modified entry point (m);
d = downstream slope cover to phreatic line (m);
F_B = freeboard (m);
h = water head in dam (m);
k_x = horizontal permeability of dam material (m/s);
k_y = vertical permeability of dam material (m/s);
L = horizontal projection (mh) of upstream face up to water level (m);
l = length of the horizontal blanket filter or drain (m);
M = slope of a line (dimensionless);
m = upstream face slope (dimensionless);
n = downstream face slope (dimensionless);
T = top width of dam (m);
x = horizontal axis;
y = vertical axis;
$\alpha = \cot n$ = angle of downstream face with the horizontal (dimensionless); and
λ = scale ratio, i.e., square root of the ratio of horizontal to vertical permeability (dimensionless).

Subscripts

max = maximum;
min = minimum;
T = transformed; and
* = nondimensional.

References

Casagrande, A. (1937). "Seepage through dams." *J. New England Water Works Association*, reprinted in *Contribution to soil mechanics 1925-1940*, Boston Society of Civil Engineers, Boston, 1940.

Fell, R., Wan, C. F., Cyganiewicz, J., and Foster, M. (2003). "Time for development of internal erosion and piping in embankment dams." *J. Geotech. Geoenviron. Eng.*, 129(4), 307–314.

Finn, W. D. Liam. (1967). "Finite element analysis of seepage through dams." *J. Soil Mech. Found. Div.*, 93(6), 41–48.

Foster, M., Fell, R., and Spannagle, M. (2000a). "The statistics of embankment dam failures and accidents." *Can. Geotech. J.*, 37(5), 1000–1024.

Foster, M., Fell, R., and Spannagle, M. (2000b). "A method for assessing the relative likelihood of failure of embankment dams by piping." *Can. Geotech. J.*, 37(5), 1025–1061.

Harr, M. E. (1962). *Groundwater and seepage*. McGraw Hill, New York.

Honjo, Y., and Veneziano, D. (1989). "Improved filter criterion for cohesionless soils." *J. Geotech. Eng.*, 115(1), 75–94.

Jepson, R. W. (1968). "Seepage through dams in the complex potential plane." *J. Irrig. Drain. Eng.*, 94(1), 23–39.

Jepson, R. W. (1969). "Free surface flow through heterogeneous porous media." *J. Hydraul. Div., Am. Soc. Civ. Eng.*, 95(1), 363–381.

Justin, J. D., Hinds, J., and Creager, W. P. (1944). *Engineering for dams: Earth, rockfill, steel and timber dams*, Vol. III, Wiley, New York.

Lambe, T. W., and Whitman, R. V. (2000). *Soil mechanics*, Wiley, Singapore.

Middlebrooks, T. A. (1953). "Earth dam practice in the United States." *Trans. Am. Soc. Civ. Eng.*, 118, 697–722.

Neuman, S. P., and Witherspoon, P. A. (1970). "Finite element method of analysing steady seepage with a free surface." *Water Resour. Res.*, 6(3), 889–897.

Polubarinova-Kochina, P. Ya. (1962). *Theory of ground water movement*, Princeton University Press, Princeton, N.J.

Punmia, B. C., Jain, A. K., and Jain, A. K. (1994). *Soil mechanics and foundations*, Laxmi, New Delhi.

Sastry, S. S. (1994). *Engineering mathematics*, Vol. 1, Prentice Hall of India, New Delhi.

Sharma, H. D. (1991). *Embankment dams*, Oxford&IBH, New Delhi.

Sherard, J. L., and Dunnigan, L. P. (1985). "Filter and leakage control in embankment dams." *Proc., Symp. Seepage and Leakage from Dams and Impoundments*, ASCE, New York, 1–30.

Sherard, J. L., Dunnigan, L. P., and Talbot, J. R. (1984a). "Basic properties of sand and gravel filters." *J. Geotech. Eng.*, 110(6), 684–700.

Sherard, J. L., Dunnigan, L. P., and Talbot, J. R. (1984b). "Filters for silts and clays." *J. Geotech. Eng.*, 110(6), 701–718.

Sherard, J. L., Woodward, R. J., Gizinski, S. F., and Clvenger, W. A. (1967). *Earth and earth rock dams*, Wiley, New York.

Singh, A., and Chowdhary, G. R. (1994). *Soil engineering in theory and practice: Part 1. Fundamentals and general principles*, CBS, New Delhi.

Taylor, R. L., and Brown, C. B. (1967). "Darcy flow solutions with a free surface." *J. Hydraul. Div., Am. Soc. Civ. Eng.*, 93(2), 25–34.

Terzaghi, K., and Peck, R. B. (1967). *Theoretical soil mechanics*, Wiley, New York.

Todd, D. K. (1995). *Groundwater hydrology*, Willey, New York.

United States Bureau of Reclamation (USBR). (2003). *Design of small dams*, Oxford & IBH, New Delhi.

Vaughan, P. R., and Soares, H. F. (1982). "Design of filters for clay cores of dams." *J. Geotech. Eng. Div., Am. Soc. Civ. Eng.*, 108(1), 17–31.

Zienkiewicz, O., Mayer, P., and Cheung, Y. K. (1966). "Solution of anisotropic seepage by finite elements." *J. Eng. Mech. Div.*, 92(1), 111–120.