Discussion of "Seepage through a Levee" by G. C. Mishra and A. K. Singh

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The authors have used the method of fragments for computing seepage through a levee. The proposed method has been claimed to be more accurate than Kozeny's solution and simpler than the exhaustive solution by Numerov (1942). The discusser would like to draw attention to the following points:

1. The point G on the phreatic line (Fig. 1of the Technical Note) divides the wet portion of the dam into two fragments, and the point G has been taken vertically below the point M as d_1= distance of line MGS from the origin. Actually the point G must be at the point of inflection, which depends on the upstream slope β and h_w/L ratio. With an increase in this ratio for a fixed β, the point of inflection shifts toward the point D; and for large values, it disappears. Similarly, as β increases, the point of inflection again shifts towards the point D for a fixed h_w/L ratio and for the value $\pi/2$, it coincides with D. For a given dam geometry (m_1, m_2, T, h_L), the h_w/L ratio changes with a change in freeboard ($F_B=h_L-h_w$) or filter length ($l=b_L-L$); therefore, it is unlikely to lie vertically below the point M.
2. For the lower part of the fragment I, the curvilinear streamlines (BF_1) may not be replaced by straight streamlines (BF) of almost equivalent length normal to the upstream face for smaller β or h_w, since the point F may reach or cross point H, resulting in overestimation of the stagnation zone.
3. A solution should preferably be a function of independent parameters. The authors' solutions contain d_1, but d_1 is a dependent parameter given by $d_1=T+m_2h_L-l$.
4. The downstream vertical cover over the phreatic line (Kozeny's basic parabola) was established by choosing a different coordinate system and minimizing a function. The resultant equation includes h_1, which in turn requires solving an implicit equation involving numerical integration. In fact, $\Delta Y_{\min}$ is the vertical distance between the downstream face and the line parallel to it and is tangent to the phreatic line. Because of the properties of a parabola and a straight line, the point of tangency can easily be shown to be at $[0.5(1-m_2^2)y_0, m_2y_0]$. Therefore

$$\Delta Y_{\min}=\frac{1}{2m_2}(2l-y_0(1+m_2^2)) \tag{1}$$

Since the point of tangency shifts upstream with an increase in the h_w/L ratio, $\Delta Y_{\min}$ also increases until

$$0.5(m_2^2-1)y_0 \leqslant m_2h_L-l \tag{2}$$

or the point of tangency falls vertically below the point N (Chahar 2005; Srivastava 2006), where it attains a maximum value. Thereafter, in the range

$$T+m_2h_L-l \geqslant (m_2^2-1)\frac{y_0}{2} \geqslant m_2h_L-l \tag{3}$$

it decreases to

$$\Delta Y_{\min}=h_L-m_2y_0 \tag{4}$$

5. Harr (1962) described a graphical method to find a cover over the phreatic line normal to the downstream face (ΔY_n). Chahar (2004) proposed algebraic equations for and used properties of a parabola and straight line, as follows:

$$\Delta Y_n=\frac{m_2}{\sqrt{1+m_2^2}}\Delta Y_{\min}=\frac{1}{2\sqrt{1+m_2^2}}(2l-y_0(1+m_2^2)) \tag{5}$$

This relationship can be used to find the minimum length of the filter ($l_{\min}$), which is sufficient to keep the phreatic line just within the levee body (i.e., $\Delta Y_n=0$)

$$l_{\min}=0.5y_0(1+m_2^2) \tag{6}$$

Eqs. (1) to (6) involve y_0, which depends on h_1. The complexity in this dependence can be avoided by adopting y_0, as suggested by Casagrande's (Harr 1962)

$$y_0=\sqrt{[(m_1+m_2)h_L-0.7m_1h_w+T-l]^2+h_w^2} - [(m_1+m_2)h_L-0.7m_1h_w+T-l] \tag{7}$$

Using Eq. (7) in Eq. (5) and nondimensionalizing it yields the following:

$$\Delta Y_{n^*}=\frac{l_*}{\sqrt{1+m_2^2}}-\frac{\sqrt{1+m_2^2}}{2}\times\begin{pmatrix}\sqrt{((m_1+m_2)h_{L^*}-0.7m_1+T_*-l_*)^2+1}\\-((m_1+m_2)h_{L^*}-0.7m_1+T_*-l_*)\end{pmatrix} \tag{8}$$

where the subscript $_*$ denotes a nondimensional parameter. Eq. (8) can be used to determine the downstream cover in an existing levee. For practical problems of design, having an explicit equation for the length of a filter for a specified downstream cover is more useful. Chahar (2004) presented the following relationship for this purpose:

$$l_*=\frac{1+m_2^2}{2m_2^2}\times\begin{pmatrix}(m_1+m_2)h_{L^*}-0.7m_1+T_*+\dfrac{m_2^2-1}{\sqrt{1+m_2^2}}\Delta Y_{n^*}-\\\sqrt{((m_1+m_2)h_{L^*}-0.7m_1+T_*-\Delta Y_{n^*}\sqrt{1+m_2^2})^2-m_2^2}\end{pmatrix} \tag{9}$$

Eq. (9) with $\Delta Y_n=0$ yields

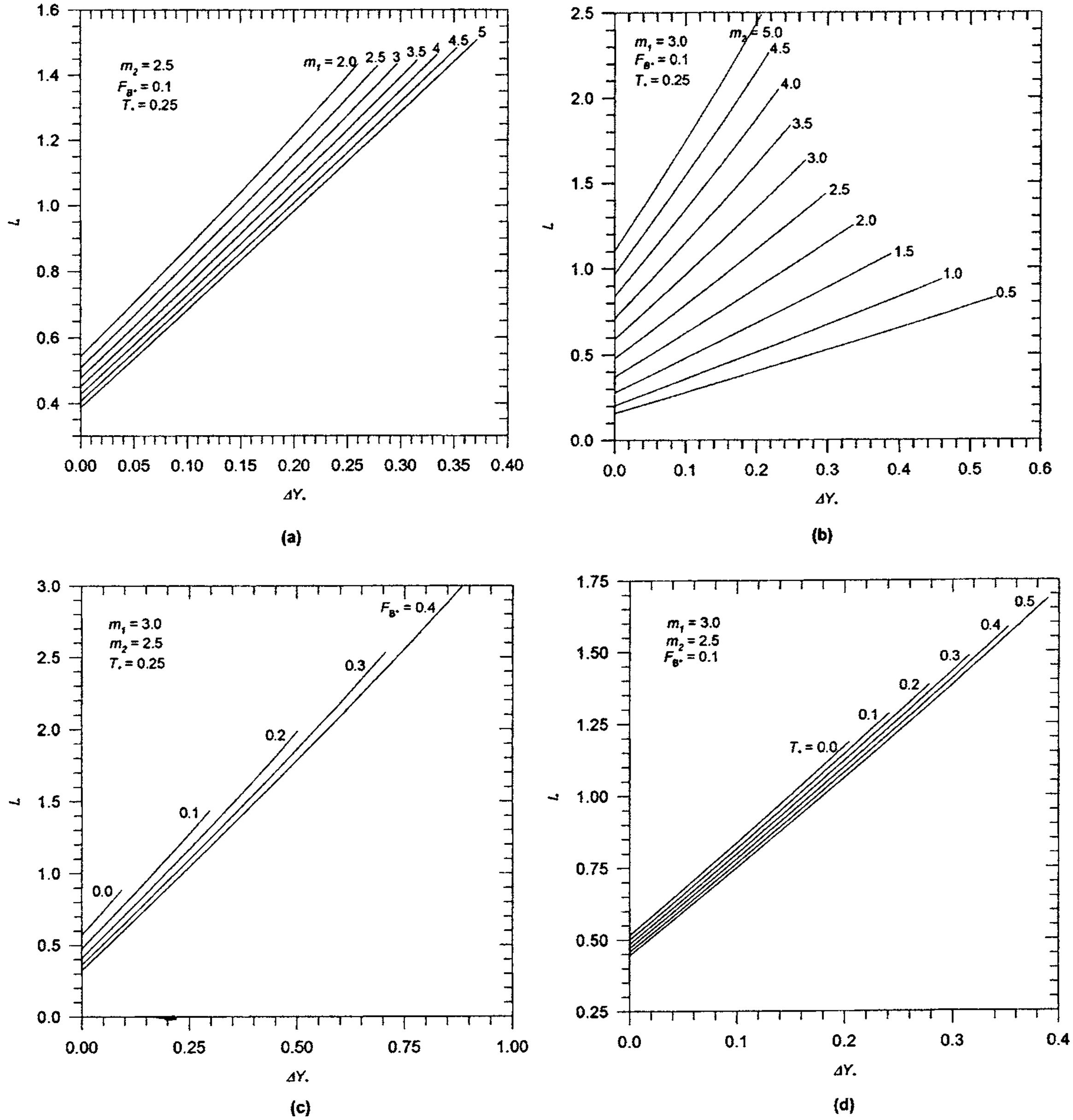

Fig. 1. Variations in the filter length with the downstream slope cover: (a) for different upstream slopes; (b) for different downstream slopes; (c) for different freeboards; and (d) for different top widths

$$1_{\min} = \frac{1+m_2^2}{2m_2^2}\Big((m_1+m_2)h_L - 0.7m_1h_w + T - \sqrt{((m_1+m_2)h_L - 0.7m_1h_w + T)^2 - m_2^2h_w^2}\Big) \quad (10)$$

6. Kozeny's solution is for a parabolic upstream face. After Casagrande's correction, it becomes applicable for an upstream sloping face. Hence, the authors should not compare their method (in Fig. 2 and the related discussion) with that of Kozeny. Since approximations are involved both in the proposed method and in Casagrande's method, either method may give a better result than the other, depending on the set of parameters (m_1, m_2, T, h_L, h_w, and l). Therefore, both methods should be compared with Numerov's exact solution for wide practical ranges of m_1, m_2, T_*, F_{B^*}, and l_* to determine the one that is better for various ranges.

7. Similar to Chahar (2004), a set of plots (more informative than the authors' Fig. 3) is presented in Fig. 1. The plots are based on Eq. (9) for variations in the filter length with respect to downstream slope cover and different geometric parameters (T_*, F_{B^*}, m_1, and m_2) of the dam of levee. Although Eqs. (8) and (9) present nonlinear relationships among these parameters, Fig. 1 reveals that the relationships between ΔY_n and l for a given set of m_1, T, F_B, and m_2 are almost linear. Furthermore, ΔY_n or l also varies almost linearly with m_1, F_B, or T but varies nonlinearly with m_2. Actually, the influence of m_1, F_B, or T is limited in fixing the minimum-length of the filter (Chahar 2004). Also, Fig. 1(a–c) shows that with an increase in top width, freeboard, or upstream slope (i.e., flattening of the upstream slope), the required filter length reduces for a given value of the downstream slope cover. The reverse happens with a change in m_2, as

indicated in Fig. 1(b). Further, ΔY_n or l is less affected by the change in the top width, freeboard, or upstream slope, whereas they are more sensitive to change in the downstream slope.

8. It is less common to find a zoned levee with fragment I and fragment II of different hydraulic conductivities. A more practical case is a stratified levee section. The method of placement and compaction in earth fills is such that stratifications are generally built into levees. Rollers are generally used to compact the earth fill materials; and rollers compact the lifts of earth fill. The horizontal permeability tends to be larger than the vertical ones. As the difference increases, the effectiveness of the filter in lowering the phreatic surface decreases; hence, an extra length is required to achieve the same efficiency. Such an anisotropic section has to be transformed into an isotropic one by using a scale ratio $\lambda=\sqrt{k_y/k_x}$ to compute seepage, however the following relations can be used directly to estimate ΔY_n or l:

$$\Delta Y_{n^*} = \frac{l_*}{\sqrt{1+m_2^2}} - \frac{1+\lambda^2 m_2^2}{2\sqrt{1+m_2^2}} \times \left(\begin{array}{l} \sqrt{((m_1+m_2)h_{L^*} - 0.7m_1 + T_* - l_*)^2 + \lambda^{-2}} \\ -((m_1+m_2)h_{L^*} - 0.7m_1 + T_* - l_*) \end{array} \right) \quad (11)$$

$$l_* = \frac{1+\lambda^2 m_2^2}{2\lambda^2 m_2^2} \times \left(\begin{array}{l} (m_1+m_2)h_{L^*} - 0.7m_1 + T_* + \dfrac{\lambda^2 m_2^2 - 1}{1+\lambda^2 m_2^2}\sqrt{1+m_2^2}\Delta Y_{n^*} - \\ \sqrt{((m_1+m_2)h_{L^*} - 0.7m_1 + T_* - \Delta Y_{n^*}\sqrt{1+\lambda^2 m_2^2})^2 - m_2^2} \end{array} \right) \quad (12)$$

$$l_{\min^*} = \frac{1+\lambda^2 m_2^2}{2\lambda^2 m_2^2}\Big((m_1+m_2)h_{L^*} - 0.7m_1 + T_* - \sqrt{((m_1+m_2)h_{L^*} - 0.7m_1 + T_*)^2 - m_2^2}\Big) \quad (13)$$

The discusser believes that including these comments would make the work comprehensive and more useful to the scientific community.

References

Chahar, B. R. (2004). “Determination of length of a horizontal drain in homogeneous earth dams.” *J. Irrig. Drain. Eng.*, 130(6), 530–536.

Chahar, B. R. (2005). “Closure to ‘Discussion on determination of length of a horizontal drain in homogeneous earth dams’ by E. H. Chahar.” *J. Irrig. Drain. Eng.*, 132(1), 89–90.

Harr, M. E. (1962). *Groundwater and seepage*, Mc-Graw Hill, New York.

Numerov, S. N. (1942). “Solution of problem of seepage without surface of seepage and without evaporation or infiltration of water from free surface.” *P M M*, 6.

Srivastava, R. (2006). “Discussion of ‘Determination of length of a horizontal drain in homogeneous earth dams’ by E. H. Chahar.” *J. Irrig. Drain. Eng.*, 132(1), 88–89.

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1. In the paper, the method of fragments suggested by Pavlovsky (see Harr 1962, p. 55) and Kozeny’s analytic solution (see Harr 1962, pp. 64–66) were adopted to analyze seepage through a levee with a blanket drain. To analyze unconfined seepage through an earth dam without a blanket drain, Pavlovsky has chosen a vertical line through point M for decomposing the flow domain into fragments. Point G is the intersection of the vertical line MGS and the phreatic line; it is not necessarily the point of inflection, which is unknown a priori. The point that separates the convex part of a continuous curve from the concave part is called the point of inflection of the curve (Piskunov 1974, p. 178). At the point of inflection on the phreatic line, the velocity is at a minimum. Developing the point of inflection on the phreatic line depends on the upstream slope angle, the distance of the filter from the upstream toe, and upstream pondage depth (Poluborinova Kochina 1962, pp. 42–46). It can be located by solving the unconfined seepage problem by using inversion of hodograph and conformal mapping (Mishra 2004). The point of inflection on the phreatic line is noticed in Numerov’s analytical solution to the Reimann-Hilbert problem (earth dam on impervious base with toe filter) (see Harr 1962, p. 223).

The analytical solution derived by Kozeny is the basis of the present paper. The flow domain of Fragment II is similar to that considered by Kozeny. The phreatic line is defined by $\phi=-ky$, where $\phi=$ velocity potential function. The locus of the phreatic line as derived by Kozeny is $y^2-y_0^2+2y_0x=0$. At any point on the phreatic line, the components of velocity are $u=\partial\phi/\partial x=-k\,\partial y/\partial x=ky_0/y$; $v=\partial\phi/\partial y=-k$. The square of the resultant velocity is $u^2+v^2=k^2(1+y_0^2/y^2)$. The velocity at a point on the phreatic line is at a minimum where y is at a maximum, that is, at the intersection of the parabolic equipotential boundary $\phi=-kh_1$ and the phreatic line. Point G therefore happens to be the point of minimum velocity. This statement is only true if the resistance of the part of earth dam between the upstream sloping face and the Kozeny parabola $\phi=-kh_1$ has been neglected. The point of inflection for the levee section is otherwise unknown.

The method of fragments is an approximate method and is much simpler than the rigorous hodograph method and Numerov’s analytical method. In the method of fragments, streamlines and equipotential lines form the boundary of fragments. The continuity equation is satisfied in each of the fragments. By choosing an equipotential line passing through the point of inflection as the fragment boundary, no significant improvement can be made in the result. Moreover, if the point of inflection coincides with point D, which is a point on the upstream boundary and if the downstream equipotential boundary of Fragment I is to be chosen to pass through the point of inflection according to the discusser's suggestion, it will be incorrect to choose the two different equipotential boundaries of Fragment I, i.e., $\phi=-kh_w$ and $\phi=-kh_1$ to pass through a single point D. Thus, the suggestion that point G must be the point of inflection (if it exists on the phreatic line) is not sacrosanct. Moreover, the point of inflection is unknown a priori.

2. If the method of fragments is applied to compute seepage from a levee, it would overestimate the stagnant zone. The overestimation would diminish if the downstream equipotential boundary of Fragment I is chosen to be nearer to the upstream reservoir boundary. The flow lines enter perpendicular to the upstream reservoir boundary. Therefore, a normal from point H to line AM is the upper limiting boundary of the stagnant zone. The streamline, which originates at the intersection of the normal from point H to line AM, merges tangentially with the impervious base of the levee. The length of this streamline is more than the length of the normal. Therefore, replacing the actual length of the streamline by the length of the normal while computing seepage quantity [Eq. (8a)] rectifies underestimation of seepage (consequent to overestimation of the stagnation zone). The focus of the study is not on the stagnant zone. The stagnant zone can be found by drawing a flow net, as shown in Fig. 7.

3. In the seepage problem discussed in the paper, h_1 is the dependent variable and is unknown a priori. After solving h_1, the other dependent variables [ordinate of phreatic line at the origin (y_0), quantity of seepage (ky_0), required length of filter ($y_0/2$), locus of the phreatic line, and distance between the downstream sloping face and the phreatic line] can be found by using Kozeny's solution. Flow in Fragments I and II has been analyzed without any reference to the top width of the dam and downstream slope. In Kozeny's analytic solution, the variables T, m_2, h_L, and l do not appear. The dependent variables y_0, quantity of seepage (q), required length of filter ($y_0/2$), and the locus of the phreatic line are governed by distance d_1, which is an independent variable. If a slope failure occurs in the downstream side, such that the slip surface does not touch the phreatic line but cuts part of the top width of the levee and removes the downstream toe, none of the dependent variables is affected. Hence, T, m_2, h_L, and l, which define the section of the levee, are not independent variables, as envisaged by the discusser. When the variables T, m_2, h_L, and l are chosen as state variables, d_1 becomes a decision variable.One can interchange the *decision* variable with one of the state variables in accordance with the geometry of the structure.

   Let us consider a case in which only the required length of the filter (equal to $y_0/2$) is to be provided in the structure. For such case, the suggestion made by the discusser becomes inapplicable. The discusser has suggested choosing l as an independent variable. If the geometrical distance l can be used as an independent variable, one can select d_1 as an independent variable.

4. Kozeny has derived the slope at any point (x,y) of the free surface to be $-y_0/y$. After solving h_1 from Eq. (13), one can find y_0 from Eq. (11). The vertical distance between the phreaticline and the downstream sloping face is at minimum at that point where the slope of the tangent to the parabola is equal to slope of the downstream face, that is, where $-y_0/y=-1/m_2$ or $y=m_2y_0$. From Eq. (16), the corresponding x-coordinate is

$$x=\frac{y_0^2-(m_2y_0)^2}{2y_0}=0.5y_0(1-m_2^2).$$

If l is the distance of the toe from the origin, then

$$\frac{m_2y_0+\Delta y_{\min}}{l-0.5y_0(1-m_2^2)}=\frac{1}{m_2}$$

and

$$\Delta y_{\min}=\frac{l-0.5y_0(1+m_2^2)}{m_2}.$$

From Eq. (22)

$$[\Delta Y]_{\min}=h_L-\frac{d_1-T}{m_2}-\frac{1+m_2^2}{2m_2}(\sqrt{d_1^2+h_1^2}-d_1)$$
$$=h_L-\frac{d_1-T}{m_2}-\frac{0.5(1+m_2^2)}{m_2}y_0$$
$$=\frac{m_2h_L+T-d_1-0.5(1+m_2^2)y_0}{m_2}$$
$$=\frac{l-0.5(1+m_2^2)y_0}{m_2}$$

   The discusser has derived the same expression. In the present paper, the property of the Kozeny parabola was missed. Instead, a generalized usual procedure was adopted to find $\Delta Y_{\min}$. The general method is applicable to a homogeneous levee with downstream berms.

   Let us consider a levee section with $m_2=1$. For such a section, the location of the minimum vertical distance between the parabola and the downstream sloping face is $(0,y_0)$, and $\Delta Y_{\min}=l-y_0$. It does not depend on height h_L, as stated by the discusser [Eq. (4) in the discussion]. We consider two sections. For both the sections, let $m_1=m_2=1$, let the distance of the origin from the downstream toe (l) be the same, and let the upstream pondage depths (h_w) be the same. The heights h_L and consequently the top widths T for both the sections are different. For both sections, the loci of the parabolas are same. The point of minimum and the minimum distance between the parabola and the downstream sloping face do not change. Thus, the expression for $\Delta Y_{\min}=h_L-m_2y_0$ [Eq. (4) in the discussion] derived by the discusser is incorrect.

5. The discusser has defined the minimum filter length as the length that is sufficient to keep the phreatic line just within the levee body, which implies that the downstream sloping face touches the phreatic line. Such a situation would lead to sloughing because of capillary action. Therefore, the parameter $l_{\min}$ is not an appropriate parameter. The distance $l_{\min}$ locates the position of the blanket drain. It should not be regarded as the minimum width of the blanket drain. The minimum required width of a blanket drain that would keep the phreatic line just within the levee body is $0.5y_0$, where y_0 corresponds to $l_{\min}$. A blanket drain of minimum width is a strip drain.

   The capillary saturation above the phreatic line should not touch the downstream sloping face. In other words, $\Delta y_{\min}$ needs to be less than the capillary saturation. The location of the filter is determined by using this condition. The required length and location of the filter should be directly related to the capillary saturation. Instead, the discusser has suggested this condition indirectly by choosing the normal to the phreatic line at the point of minimum vertical distance between the phreatic line and the downstream sloping face.

6. The result obtained by the method of fragments has been compared with Numerov's result for a limited case, and the result obtained by the approximate method of fragments has been shown to be very close to the result obtained by Numerov's method. As suggested by the discusser, the comparison should be made for a wide range of levee sections. Estimation by the method of fragments will be closer to the true value than the one computed by using Casagrande's correction because Casagrande's method is a semiempirical one. While applying Casagrande's correction to the flow domain, that is, by appending 0.3Δ at the upstream face, the stagnant zone is eliminated and the flow net configuration corresponding to the original flow domain is changed. While applying Pavlovsky's method of fragments to solve the seepage problem, equipotentials and streamlines have been selected as boundaries of the fragment. Therefore, in the method of fragments, the characteristics of the flow domain are conserved. The comparison of results by the method of fragments with that by Kozeny's solution has been presented to focus on resistance offered by the flow domain between the upstream face and Kozeny's parabola.
7. The set of graphs presented in Fig. 3 focus on the influence of capillary saturation in deciding the appropriate location of the blanket drain. The results presented in Fig. 3 have been computed by using the method of fragments. For the reason stated in the previous paragraph, applying the method of fragments is more logical than Casagrande's semiempirical correction for accounting for the resistance of soil between the upstream sloping face and Kozeny's parabolic surface.
8. As stated by Sherard et al. (1963, p. 25) one of the principal disadvantages of horizontal drainage blankets occurs because earth dam embankments tend to be stratified and consequently more pervious in the horizontal direction than in the vertical. Occasionally, horizontal layers, much more impervious than the average material are constructed into the embankment, so that in spite of a horizontal drain, the seepage water may flow horizontally on top of a relative impervious layer and discharge on the face of the downstream slope. To overcome the problems, inclined or vertical chimney drains are constructed. Since nonhomogeneity and anisotropy characteristics are exhibited simultaneously, the derivation made by the discusser has very limited use. Such a complex problem is to be solved by numerical methods.

Conclusion

The minimum length of a blanket drain and its location in a homogeneous levee are governed by capillary saturation as well as the geometry of the levee section. The clue to this study is based on Terzaghi's observation on capillary saturation. The authors thank the discusser for providing an opportunity to further highlight the usefulness of the paper.

References

Harr, M. E. (1962). *Ground water and seepage*, McGraw-Hill, New York, 315.

Mishra, N. K. (2004). "Analysis of steady flow through a homogeneous earth dam with horizontal toe drain." MS thesis, Dept. of Civil Engineering, IIT Roorkee, Uttaranchal, India.

Piskunov, N. (1974). *Differential and integral calculus*, Mir Publishers.

Polubarinova-Kochina, P. Ya. (1962). *Theory of ground water movement*. J. M. R. De Wiest translator, Princeton University Press, Princeton, N.J.

Sherard, J. L., Woodward, R. J., Gizienski, S. F., and Clevenger, W. A. (1963). *Earth and earth-rock dams*, Wiley, New York.