Optimal Design of a Special Class of Curvilinear Bottomed Channel Section

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Abstract: Section elements of a new class of curvilinear bottomed channel whose boundary maps along a circle onto the hodograph plane have been investigated in the study. The perimeter of this type of channel always lies between an ellipse and a parabola of the same top width and flow depth. Presented herein are optimal section properties from least area and minimum seepage loss point of view for the curved channel section. The study also addresses the constraints on the channel dimensions and the velocity of flow. A nondimensional parameter approach has been used to simplify the analysis. Design procedures for different cases have been presented to demonstrate the simplicity of the method.

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Introduction

An open channel may be rigid boundary (lined) or mobile boundary (unlined) section. Unlined channels lose a substantial part of the usable water through seepage. Seepage loss results not only in depleted freshwater resources but also causes water logging, salinization, and ground water contamination. Canals are lined for slowing the seepage loss. A perfect lining would prevent all the seepage loss, but canal lining deteriorates with time. Cracks in the lining may develop anywhere on the perimeter due to settlement of the subgrade, weed growth in the canal, construction defects and use of inferior quality lining materials, weathering etc. As a result, seepage from a canal with cracked lining is likely to approach the quantity of seepage from an unlined canal. Therefore, seepage loss should be considered while designing a canal section. Bandini (1966) considered seepage loss in the canal design and compared the economy of lined and unlined canals. Kacimov (1992) obtained optimal trapezoidal and rectangular sections considering seepage and presented results in tabular and graphical form. Chahar (2005b) and Swamee et al. (2000a,b, 2001b, 2002a,b) included seepage loss in objective functions and presented explicit equations for designing optimal sections of polygon channels. Swamee and Kashyap (2001, 2004) suggested equations for minimum seepage loss nonpolygon channels however there are some drawbacks in their method as highlighted by Kacimov (2003).

River beds, unlined canals and irrigation furrows all tend to acquire a curved shape (Mironenko et al. 1984). Therefore, unlined channels can be made more hydraulically stable by initially constructing them in a curved shape. Though curved sections may pose some difficulties in construction and maintenance, a lined curved channel has no sharp angles of stress concentration where cracks may occur, and can be prefabricated in moulded sections. Small curved ditches can be constructed by bulldozers and other types of earth moving equipment (Mironenko et al. 1984). One form of a curved channel is the parabolic shape, which is investigated and designed by Mironenko et al. (1984), Loganathan (1991), Babaeyan-Koopaei et al. (2000), and Chahar (2005a). Exact analytical solutions for seepage from curved channels like semicircular and parabolic shapes are not achievable as their geometries map in curvilinear shapes onto the hodograph and inverse hodograph planes, for which Schwarz-Christoffel transformation is not possible. One possible way forward is an inverse method where the shape of the unknown channel is sought as part of a solution (Ilyinskii and Kacimov 1984). Kozeny (Harr 1962) investigated seepage from a curved channel using Zhukovsky function and found that resultant channel has a trochoid shape. Hunt (1972) presented an approximate solution for seepage from a shallow channel of arbitrary cross section. Verigin (Kovacs 1981) analytically found an approximate solution for a semicircular section in terms of a rapidly converging series. Kacimov (2003) pointed out the mistake in Vergin's solution. Ilyinskii and Kacimov (1984) used an inverse boundary value problem method to find the optimal shape of a curved irrigation channel from the point of view of minimum seepage loss. Using the inverse method, Chahar (2006) obtained an exact solution for seepage from a curved channel whose boundary maps along a circle onto the hodograph plane. The resulting channel shape is non self intersecting and is feasible from a slit (very narrow and deep channel) to a strip (very wide and shallow channel) unlike the Kozeny's trochoid shape, which is self intersecting for top width-to-depth ratios greater than 1.14. This channel shape, also, possesses other interesting and useful properties (Chahar 2006). For that reason, it is pertinent to investigate hydraulic properties of this new class of curved section and also its section dimensions corresponding to the maximum flow (or minimum area) and minimum seepage loss sections.

The current study is an attempt to address design problems in

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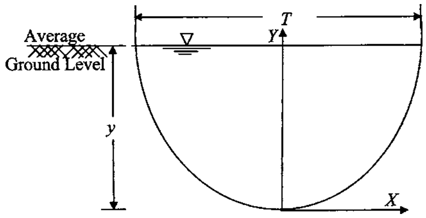

Fig. 1. Geometric elements of a curvilinear bottomed channel

Chahar's (2006) channel of minimum area section and minimum seepage loss section including constraints on the channel dimensions and the flow velocity. Starting from the geometric properties of the new curved section and the uniform flow equation, the optimization method for the minimum area section has been described. The next part proposes design equations for the minimum seepage loss section. The optimization problem has been solved by Lagrange's method of undetermined multipliers as used by several investigators (Monadjemi 1994; Froehlich 1994; and Chahar 2005a) in optimal channel design. The resultant optimal parameters have been obtained in nondimensional form. Constraints on the channel dimensions and the velocity of flow have also been treated. The proposed design equations have been obtained in explicit form through regression analysis. Finally, a step-by-step design procedure with design examples and limitations of the method is presented.

Properties of the Curvilinear Bottomed Section

Geometric Properties

Assuming the origin of the axes at the water surface, and using a conformal mapping technique, Chahar (2006) obtained a relationship for the shape of the perimeter of a curvilinear bottomed channel whose boundary maps along a circle onto hodograph plane. Shifting the origin of the axes to the invert of the channel (Fig. 1), the relationship becomes

$$Y = \frac{y}{2G}\int_0^{\sinh^{-1}\tan(\pi X/T)} \frac{\tau d\tau}{\cosh \tau} \tag{1}$$

where y=flow depth (m); T=top width of the channel at the water surface (m); Y=ordinate; X=abscissa; G=0.915965594... =Catalan's constant; and τ=dummy variable. Eq. (1) represents an approximate semiellipse with the top width as the major axis and twice the depth of flow as the minor axis or vice versa. Actually, it always lies between an ellipse and a parabola and any coordinate is nearly exactly the average of coordinates of the corresponding ellipse and parabola. A semicircle is a special case of a semi-ellipse; consequently at a top width-to-depth ratio equal to 2, this class of curved channel can be approximated by a semicircular channel (Chahar 2006). The flow area $A(\mathrm{m}^2)$ is computed as

$$\begin{aligned} A &= 2\left(\frac{yT}{2} - \int_0^{T/2} Y dX\right) \\ &= yT - \frac{y}{G}\int_0^{T/2}\left(\int_0^{\sinh^{-1}\tan(\pi X/T)} \frac{\tau dX}{\cosh \tau}\right)dX \\ &= 0.7368yT \end{aligned} \tag{2}$$

The wetted perimeter, $P(m)$ is obtained as

$$\begin{aligned} P &= \int ds \\ &= 2\int_0^{T/2} \sqrt{1 + \left(\frac{dY}{dX}\right)^2} dX \\ &= 2\int_0^{T/2} \sqrt{1 + \left(\frac{d}{dX}\left(\frac{y}{2G}\int_0^{\sinh^{-1}\tan(\pi X/T)} \frac{\tau d\tau}{\cosh \tau}\right)\right)^2} dX \end{aligned} \tag{3}$$

This expression for P was evaluated numerically for a wide range of y/T ratio ($10^{-3} \leq y/T \leq 10^3$) and then fitted into a simple algebraic equation

$$P = T\left(1 + \left(\frac{2y}{T}\right)^{5/3}\right)^{3/5} \tag{4}$$

The fitted equation is nearly exact since the maximum error in the equation is only about half percent at $2y/T$=1.5.

Seepage Loss

The seepage discharge per unit length of curvilinear bottomed channel $q_s(\mathrm{m}^2/\mathrm{s})$ passing through a homogeneous isotropic porous medium of infinite extent having water table at very large depth was obtained by Chahar (2006) as

$$q_s = ky\left(\frac{T}{y} + \frac{\pi^2}{4G}\right) \tag{5}$$

where k=hydraulic conductivity (permeability) of porous medium (m/s).

Dimensionless Parameters

Rigid boundary channels are designed for uniform flow and the most commonly used uniform flow formula is the Manning equation (Chow 1973), which in SI form for discharge $Q(\mathrm{m}^3/\mathrm{s})$ is

$$Q = AV = \frac{1}{n}A\left(\frac{A}{P}\right)^{2/3} S_0^{1/2} \tag{6}$$

where V=mean velocity of uniform flow (m/s); n=Manning's roughness coefficient; and S_0=bed slope of the channel (dimensionless).

Assuming a length scale L

$$L = (Qn/\sqrt{S_0})^{3/8} \tag{7}$$

the following nondimensional variables were defined

$$y_* = y/L \tag{8a}$$

$$P_* = P/L \tag{8b}$$

$$T_* = T/L \tag{8c}$$

$$A_* = A/L^2 \tag{8d}$$

$$q_{s*} = q_s/kL \tag{8e}$$

$$V_* = VL^2/Q \tag{8f}$$

where a variable with a subscript asterisk denotes its nondimensional counterpart. Rewriting Eq. (6) as

Table 1. Comparison of Optimal Parameters in Nondimensional Form

S. number	Nondimensional parameter	Optimal values: Minimum area (classical optimal) curvilinear bottomed section	Minimum seepage loss curvilinear bottomed section: Value	Minimum seepage loss curvilinear bottomed section: Percentage difference	Minimum area parabolic section (Chahar 2005a): Value	Minimum area parabolic section (Chahar 2005a): Percentage difference
1	$(T/y)^*$	2.0000	2.4694	23.470	2.0560	2.800
2	y_*^*	1.0356	0.9341	−9.801	1.0806	4.345
3	T_*^*	2.0712	2.3068	11.375	2.2216	7.261
4	A_*^*	1.5803	1.5876	0.462	1.6004	1.272
5	P_*^*	3.1393	3.1757	1.159	3.2400	3.208
7	R_*^*	0.5034	0.5000	−0.675	0.4940	−1.877
8	V_*^*	0.6328	0.6299	−0.458	0.6249	−1.248
9	$q_{s^*}^*$	4.8609	4.8231	−0.777	—	—

$$nQ/\sqrt{S_0} = A^{5/3}/P^{2/3} \quad (9)$$

and then using relations given by Eqs. (2), (4), and (7)

$$L^{8/3} = y^{10/3}(0.736757T/y)^{5/3}/(y^{2/3}((T/y)^{5/3} + (2)^{5/3})^{2/5}) \quad (10)$$

Solving for y_*

$$y_* = ((T/y)^{5/3} + (2)^{5/3})^{3/20}/(0.736757T/y)^{5/8} \quad (11)$$

Substitution of y from Eq. (11) yielded

$$A_* = ((T/y)^{5/3} + (2)^{5/3})^{3/10}/(0.736757T/y)^{1/4} \quad (12)$$

$$T_* = 1.2104(T/y)^{3/8}((T/y)^{5/3} + (2)^{5/3})^{3/20} \quad (13)$$

$$P_* = ((T/y)^{5/3} + (2)^{5/3})^{3/4}/(0.736757T/y)^{5/8} \quad (14)$$

$$V_* = (0.736757T/y)^{1/4}/((T/y)^{5/3} + (2)^{5/3})^{3/10} \quad (15)$$

$$q_{s^*} = ((T/y) + (\pi^2/4G))((T/y)^{5/3} + (2)^{5/3})^{3/20}/(0.736757T/y)^{5/8} \quad (16)$$

All these nondimensional parameters are only a function of the ratio T/y. Once T/y is known, any other parameter can be calculated.

Optimal Channel Section

Classical Optimal Section

The classical optimal section is the hydraulic section that has the maximum flow velocity or the minimum flow area and wetted perimeter for a specified discharge and channel bed slope. Mathematically, it could be stated as

$$\text{Minimize } A = A(y,T) \quad (17a)$$

$$\text{subject to } \phi = Q - \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}S_0^{1/2} = \phi(A,P) = \phi(y,T) = 0 \quad (17b)$$

As the curvilinear bottomed channel is completely described by two independent variables y and T, applying Lagrange's method of undetermined multipliers (Monadjemi 1994; Chahar 2005a)

$$\frac{\partial A}{\partial y}\frac{\partial \phi}{\partial T} = \frac{\partial A}{\partial T}\frac{\partial \phi}{\partial y} \quad (18)$$

resulting in

$$\frac{\partial A}{\partial y}\frac{\partial P}{\partial T} = \frac{\partial A}{\partial T}\frac{\partial P}{\partial y} \quad \text{or} \quad (T/y)^* = 2 \quad (19)$$

where a superscript asterisk denotes optimum value. This shows that the minimum area section results when the curved channel acquires an approximate semicircular section shape.

Minimum Seepage Loss Section

The minimum seepage loss section design method can be stated as

$$\text{Minimize } q_s = ky\left(\frac{T}{y} + \frac{\pi^2}{4G}\right) \quad (20)$$

subject to the condition set in Eq. (17*b*). Consequently similar to Eq. (18)

$$\frac{\partial q_s}{\partial y}\frac{\partial \phi}{\partial T} = \frac{\partial q_s}{\partial T}\frac{\partial \phi}{\partial y} \quad (21)$$

which after some algebra simplifies to

$$5\frac{\pi^2}{8G}\left(\frac{2y}{T}\right)^{8/3} - 3\left(\frac{2y}{T}\right)^{5/3} + 3\frac{\pi^2}{8G}\frac{2y}{T} - 5 = 0 \quad (22)$$

with solution

$$\frac{2y}{T} = 0.8099 \quad \text{or} \quad (T/y)^* = 2.4694 \quad (23)$$

Thus, for the identical depths of flow, the minimum seepage loss section is 23.47% wider than the minimum area section.

Nondimensional Optimal Parameters

Substituting the value of $(T/y)^*$ from Eq. (19) for minimum area section and from Eq. (23) for minimum seepage loss section in Eqs. (11)–(16) respectively, yield nondimensional optimal values of the various parameters as listed in Table 1. A review of Table 1 shows that the optima near the minimum area point, as well as near the minimum seepage loss point, are not very sensitive, as there is a 23.47% change in the control variable (T/y) between the two optimal solutions but only 0.4619% change in the channel area and −0.7776% change in the seepage loss over that range. Table 1 also lists nondimensional optimal parameters for a minimum area parabolic section (Chahar 2005a). A comparison between the two classical optimal sections reveals that the curvilinear bottomed section has greater hydraulic radius (more hydraulic efficiency) and less area (more economy) than the parabolic section. Also, a parabolic section is more efficient and economical than a rounded corner triangular section, which in turn is more efficient and economical compared to a trapezoidal section (Monadjemi 1994).

Constraints on Channel Dimensions and Velocity

Thus far in the analysis it has been assumed that the depth and the top width are unconstrained. However, in practice, limits are encountered on y (i.e., $y \le y_L$) due to existence of unfavourable strata and/or ground water at a shallow depth and on T (i.e., $T \le T_L$) due to restrictions on the span of bridges and cross drainage works, or on the width of right of way etc. The variables with subscript L denote their limiting values. The constraints on channel dimension become effective only if $y_L <$ unconstrained y^*, or $T_L <$ unconstrained T^*. Otherwise the prescribed constraint on a particular parameter will be nonbinding, and hence the parameter as well as the remaining parameters will attain their unconstrained optimal values.

The imposition of constraints y_L or T_L restrains the curved section from achieving the most preferred T/y value for that discharge and is forced to attain the T/y value as permitted by the limiting y_L or T_L. For example, if y_L is binding (i.e., $y_L <$ unconstrained y^*), the optimal value of depth of flow equals its constraint (i.e., $y^* = y_L$) and the top width must be large enough to permit the design discharge to pass through the optimal area. For any known y, determination of T is not straight forward since it is a function of T/y given by Eq. (13) and T/y is implicitly related to y [Eq. (11)]. A simple explicit relation based on regression of Eq. (11) is

$$\log(T/y)^* = 2.2589 - 3.2175 y_*^* + 1.5788 y_*^{*2} - 0.2923 y_*^{*3} \quad (24)$$

Similarly, if T_L is binding (i.e., $T_L <$ unconstrained T^*), the optimal value of top width equals its constraint (i.e., $T^* = T_L$) and the depth of flow must be large enough to permit the design discharge. Solving Eq. (13) for T/y using curve fitting

$$\log(T/y)^* = -1.4414 + 1.3319 T_*^* - 0.2865 T_*^{*2} + 0.0234 T_*^{*3} \quad (25)$$

Moreover, the design of a channel section must take into account the minimum permissible (nonsilting) velocity ($V_{\min}$) and the maximum permissible (nonscouring or limiting) velocity (V_L). Swamee et al. (2001a,b) lists limiting velocities for different type of linings. To consider this in the design, restrictions can be imposed on the velocity of flow (i.e., $V_{\min} \le V < V_L$). A minimum area section yields the greatest flow velocity $V_{\max}$ for a given discharge, which also implies that the specification of a velocity larger than the velocity in a classical optimal section (i.e., $V_L > V_{\max}$) will result in infeasibility for that discharge. Therefore, constraints on velocity becomes effective if $V_L <$ unconstrained V^*. If the velocity is binding (i.e., $V_L <$ unconstrained optimal velocity) then $V^* = V_L$ and there will be two candidate cross sections, since the flow area increases in either direction (i.e., for increasing and decreasing values of T/y) from the optimal value, i.e., $T/y = 2.0$. Solving Eq. (15) for T/y using regression yielded

$$\log(T/y)^* = -166.284 + 866.114 V_*^* - 1510.869 V_*^{*2} + 881.927 V_*^{*3} \quad (26)$$

for $T/y < 2.0$ and

$$\log(T/y)^* = 90.40 - 476.311 V_*^* + 849.767 V_*^{*2} - 508.771 V_*^{*3} \quad (27)$$

for $T/y > 2.0$. As both the sections have the same flow area, the wider of the two sections (i.e., $T/y > 2.0$) is preferred if an increase in earthwork cost with depth of excavation is the primary interest, whereas the narrower section (i.e., $T/y < 2.0$) is selected if such a cost is ignored and freeboard is to be provided. However the wider section allows less seepage loss than the corresponding narrower section.

Regressed equations (24)–(27) have correlation coefficients nearly unity ($r^2 > 0.99$). Therefore these equations give nearly exact solutions for the T/y ratio. Once the T/y ratio is fixed based on the appropriate binding case, other parameters can easily be determined by using Eqs. (11)–(16).

Fixed Area Minimum Seepage Loss Section

Sometimes channels are designed for minimum seepage loss for a fixed area (Kacimov 1992; Ilyinskii and Kacimov 1984). Such channels are useful as watering ditches (with standing water) in an irrigation system. The minimization problem for this case becomes: Eq. (20), subject to

$$\phi(y,T) = A - 0.736757 yT = 0 \quad (28)$$

This revised equality constraint yields

$$(T/y)^* = \frac{\pi^2}{4G} = 2.6938 \quad (29)$$

and hence the corresponding optimal parameters are

$$q_s^* = \frac{\pi^2}{2G} ky = 5.3875 ky \quad (30a)$$

$$A^* = 1.9847 y^2 \quad (30b)$$

$$P^* = 3.5831 y \quad (30c)$$

These parameters for the earlier minimum seepage loss section are $5.1632ky$, $1.8193y^2$, and $3.3996y$, respectively, which show that the seepage, the area and the perimeter for the current optimal section are larger than the previous minimum seepage loss section for the identical water depths.

Optimal Design Procedure, Examples, and Discussion

Design Procedure

Based on the presented equations, a rigid boundary optimal curved channel section can be designed by adopting the following steps

1. For a given set of data (Q and S_0) and chosen n, find L using Eq. (7).

2. Select an appropriate optimal T/y ratio and the corresponding nondimensional optimal parameters for a particular optimization case (minimum area or minimum seepage loss section) from Table 1. If there are no constraints on channel dimension or velocity, then skip the next step.
3. Using L and given constraints on "channel dimension" or velocity, find the corresponding nondimensional parameter. Compare this nondimensional parameter with the corresponding nondimensional optimal parameter as determined in the previous step. If it happen to be a nonbinding parameter, then go to the next step, otherwise adopt it as an optimal parameter and determine T/y via Eqs. (24)–(27) depending upon the binding case and further compute the remaining parameters in nondimensional form using Eqs. (11)–(16).
4. Use of L and nondimensional parameters yield the corresponding parameters for the optimal curved channel. For velocity unconstrained case, the average velocity should be less than the limiting velocity V_L. If not, then redesign the section with revised bed slope or surface roughness.

Examples

Example 1: Design an optimal curved channel to carry a discharge of 50 m^3/s on a longitudinal bed slope of 0.001 passing through a porous medium of hydraulic conductivity $=5\times10^{-6}$ m/s.

Solution: Assuming float finished concrete lining with a Manning's roughness coefficient=0.015. From Eq. (7), $L=(50\times0.015/\sqrt{0.001})^{3/8}=3.2783$ m.

Minimum Area Section: Using Table 1: $(T/y)^*=2.0$; $y_*^*=1.0356$; $P_*^*=3.1393$; $T_*^*=2.0712$; $A_*^*=1.5803$; $V_*^*=0.6328$; and $q_{s,*}^*=4.8609$. Therefore, $y^*=1.0356\times3.2783=3.3950$ m; $P^*=3.1393\times3.2783=10.2916$ m; $T^*=2.0712\times3.2783=6.7900$ m; $A^*=1.5803\times(3.2783)^2=16.9839$ m^2; $q_s^*=4.8609\times3.2783\times5\times10^{-6}=7.96774\times10^{-5}$ m^2/s; and $V^*=0.6328\times50/(3.2783)^2=2.9440$ m/s. Alternatively $V=Q/A=50/16.9839=2.9440$ m/s, which is less than the limiting velocity for a concrete lining (4.0 m/s).

Using the same data and Table 1 for the parabolic section, the optimal parameters are, $y^*=3.5424$ m; $P^*=10.6218$ m; $T^*=7.2831$ m; $A^*=17.1995$ m^2, and $V^*=2.9071$ m/s. Thus the curvilinear bottomed section is more hydraulically efficient and more economical than the parabolic section.

Minimum Seepage Loss Section: Using nondimensional optimal parameters from Table 1 corresponding to $(T/y)^*=2.4694$ (minimum seepage loss section), $y^*=3.0623$ m; $P^*=10.4109$ m; $T^*=7.5624$ m; $A^*=17.0623$ m^2; $q_s^*=7.90578\times10^{-5}$ m^2/s; and $V^*=2.9305$ m/s.

Fixed Area Minimum Seepage Loss Section: From Eq. (29), $(T/y)^*=2.6938$. Adopting $y=3.0623$ m (identical to the previous case), $T^*=8.2492$ m. Using value of y in Eqs. (30*a*)–(30*c*), $q_s^*=8.2491\times10^{-5}$ m^2/s; $A^*=18.6119$ m^2; and $P^*=10.9725$ m.

Example 2: Redesign an optimal curved channel for the data as stated in Example 1 if (1) maximum possible depth of flow =2.0 m; (2) top width restricted to 6.0 m; and (3) permissible velocity =2.5 m/s.

Solution: **Case (a)**: As $y_L=2.0$ m, $y_{L^*}=2.0/3.2783=0.6101$, which is less than the unconstrained optimal value of 0.9341. Hence it is binding and optimal for the present case ($y_*^*=0.6101$). From Eq. (24) the value of T/y corresponding to this value equals 6.5643. For $(T/y)^*=6.5643$, Eqs. (8*a*)–(8*f*) and (11)–(16) yield $T^*=13.1141$ m; $P^*=14.1712$ m; $A^*=19.3019$ m^2; $q_s^*=9.2477\times10^{-5}$ m^2/s; and $V^*=2.5904$ m/s.

Case (b): If the top width is restricted to 6.0 m, then $T_{L^*}=6.0/3.2783=1.8302<2.0712$; which is binding and hence $T_*^*=1.8302$. Corresponding to this value, $T/y=1.5188$ from Eq. (25). For $(T/y)^*=1.5188$, $y^*=3.9112$ m; $P^*=10.4950$ m; $A^*=17.1171$ m^2; $q_s^*=8.238\times10^{-5}$ m^2/s; and $V^*=2.9212$ m/s.

Case (c): If the permissible velocity $V_L=2.5$ m/s, then $V_*=0.5374$, which is less than the unconstrained optimal value of 0.6299 and hence optimal for the present case. Corresponding to $V_*^*=0.5374$, two values of $(T/y)^*$ (0.5059 and 7.5969) are possible from Eqs. (26) and (27). Considering $(T/y)^*=0.5059$, corresponding design parameters calculated from the appropriate equations are: $y^*=7.3296$ m; $T^*=3.7079$ m; $P^*=15.5318$ m; $A^*=20.0$ m^2; and $q_s^*=1.172\times10^{-4}$ m^2/s. Similarly for the wider section $(T/y)^*$ is equal to 7.5969 and the corresponding design parameters are: $y^*=1.8837$ m; $T^*=14.3108$ m; $P^*=15.220$ m; $A^*=19.9$ m^2; and $q_s^*=9.6925\times10^{-5}$ m^2/s. Hence the seepage loss for a wider section is less than its counterpart.

Discussion

To simplify the design problem, freeboard has not been included in the optimization scheme. The required freeboard may be provided in the optimal section or optimal section may be designed for an increased discharge leaving a margin for the freeboard. However, such a section deviates from the true optimal section. Nevertheless, the current method can easily be extended for designing an optimal channel section considering freeboard. Also, the current design method applies only to the design of lined (i.e., rigid boundary) channels. For unlined (erodible) channels, the principle of tractive force must be used to determine an efficient section. The seepage loss Eq. (5) was obtained assuming a homogeneous isotropic porous medium of infinite extent having water table at very large depth. Hence the design method is not applicable for a channel passing through a porous medium having impervious layer or water table or drainage layer at shallow depth, because the quantity of seepage would be different. Further, the lining is assumed as cracked and cracks may develop anywhere on the perimeter. The seeping water through the cracks may saturate the soil along the channel boundary and hence the seepage pattern for a cracked lined channel may likely to become similar to an unlined channel. If the soil along the channel perimeter were unsaturated, the method should not be used. Moreover, the optimal design problem can be solved numerically (using numerical integration along with EXCEL or other optimization software). However, numerical methods result only in a numerical value as problem specific particular solution, whereas the presented method is a general solution in the functional form and the each optimal design parameter can be computed with a hand calculator in a single step.

Conclusions

The geometrical parameters of a class of curvilinear bottomed channel can be described in terms of two independent parameters, top width T and depth of flow y. Lagrange's undetermined multiplier method can be used in obtaining optimal parameters corresponding to minimum area section as well as minimum seepage loss section. For the minimum area section, $T/y=2$, whereas for a minimum seepage loss section, $T/y=2.4694$. On the other hand, the minimum seepage loss occurs at $T/y=2.6938$ for a fixed area channel. The optima near minimum area point as well as near minimum seepage loss point are not very sensitive. The design method can be further simplified in dimensionless form and all the parameters become explicit functions of a single variable T/y. Once the T/y ratio is fixed, other parameters can easily be determined from explicit expressions. A regression analysis can be used to deal with the constraints on channel dimension or velocity of flow. The presented curvilinear bottomed channel section is more efficient than parabolic, rounded corner triangular and trapezoidal sections.

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Notation

The following symbols have been used in this technical note:

A = flow area of channel (m^2);
G = 0.915965594…=Catalan's constant (—);
k = hydraulic conductivity (m/s);
L = length scale (m);
n = Manning's roughness factor (—);
P = flow perimeter of channel (m);
Q = discharge in channel (m^3/s);
q_s = seepage discharge from per unit length of channel (m^2/s);
S_0 = bed slope of channel (—);
T = top width at free water surface (m);
V = average velocity (m/s);
X = horizontal axis;
Y = vertical axis;
y = normal depth of flow in channel (m);
τ = dummy variable; and
ϕ = equality constraint (—).

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