Optimal Spacing in an Array of Fully Penetrating Ditches for Subsurface Drainage

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Abstract: A methodology for optimal spacing in an array of ditches fully penetrating into homogeneous and isotropic porous medium of finite depth over an impervious layer is presented. The cost function includes the depth-dependent earthwork cost and the capitalized cost of pumping of drain discharge. Essentially, it is a problem of minimization of a nonlinear objective function of single variable. The input variables consist of rainfall intensity, hydraulic conductivity of the porous medium, width and depth of ditches, earthwork cost, cost of pumps and pumping energy cost, efficiency of pumping unit, and rate of interest. Using nonlinear data fitting method an explicit equation has been proposed for computing the optimal spacing between the ditches.

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Introduction

Subsurface drainage is provided to remove the excess water from a waterlogged area to control salinity, to leach harmful salts out of the soil, and to ensure all weather use of playgrounds, golf courses, race courses, parks, and other amenities (Chahar and Vadodaria 2008). Subsurface drainage system is designed for drainage efficiency, practicability, and economy. Minimization of cost by proper choice of depth and spacing of drains constitutes an important aspect of subsurface drainage (Acharya and Holsambre 1982). Numerous researchers have worked for cost analysis of different types of subsurface drainage systems (Bhattacharya et al. 1977; Bhattacharya and Broughton 1978; Dunford et al. 1984; Boumans and Smedema 1986; Ritzema et al. 2006). Most of the cited work is regarding installation of tile drain and optimizing depth for lateral pipe drains. One of the main drawbacks in the installation of tile/pipe type subsurface drainage system is high initial investment (Upadhyaya and Chauhan 2000). The advantages of open ditches are easy to construct, low initial cost, and ability to carry large quantities of water; while the disadvantages are interference with farming operations, removal of land, regular maintenance, and side slope stability (Luthin 1966). In spite of these limitations, an array of ditches may present an economical method of subsurface drainage. The factors to be considered in the design of subsurface ditch drainage system are spacing and depth of ditches. This technical note presents a method for optimal spacing in an array of ditches fully penetrating into homogeneous and isotropic porous medium of finite depth over an impervious layer for draining a ponded area.

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Drainage of Ponded Area: Drainage Quantity and Spacing

Subsurface drainage of ponded area underlain by an impervious layer by an array of fully penetrating ditches was solved by Kirkham (1950), Fukunda (1957), Warrick and Kirkham (1969), Youngs (1994), and Barua and Tiwari (1995). An extensive solution has been obtained by Chahar and Vadodaria (2008) for drainage from a ponded surface by an array of ditches having spacing $2S$ (m) and water depth y (m) and the ditches fully penetrate into homogeneous and isotropic porous medium of finite depth d (m) over an impervious layer as shown in Fig. 1. The solution is

$$\frac{S}{d}=K\left(\sqrt{\frac{(1-\beta)\alpha}{(1-\alpha)\beta}}\right)/K\left(\sqrt{\frac{(\beta-\alpha)}{(1-\alpha)\beta}}\right) \tag{1}$$

$$\frac{y}{d}=1-\frac{F\left(\sin^{-1}\sqrt{\beta},\sqrt{\frac{(\beta-\alpha)}{(1-\alpha)\beta}}\right)}{K\left(\sqrt{\frac{(\beta-\alpha)}{(1-\alpha)\beta}}\right)} \tag{2}$$

and

$$\frac{q}{kd}=\frac{\sqrt{(1-\alpha)\beta}}{\pi K\left(\sqrt{\frac{(\beta-\alpha)}{(1-\alpha)\beta}}\right)}\int_{\beta}^{1}\frac{\tan^{-1}\sqrt{(t-\alpha)/(1-t)}}{\sqrt{(1-t)(t-\beta)(t-\alpha)t}}dt \tag{3}$$

where q=seepage discharge from one side of the ditch (m^2/s per unit length), so total seepage into the ditch is $2q$; t=dummy variable; α and β=transformation parameters; $K(.)$=complete elliptical integral of first kind; and $F(.,.)$=incomplete elliptical integral of first kind (Byrd and Friedman 1971). Eqs. (1) and (2) can be solved simultaneously for α and β for given values of S, d,

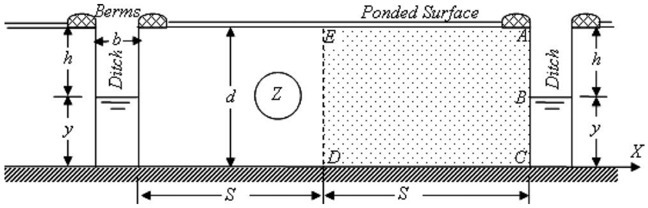

Fig. 1. Drainage of ponded surface by an array of fully penetrating ditches

and y and then Eq. (3) yields the drain discharge. Alternatively the following approximate but explicit relation without involving the transformation parameters (Chahar and Vadodaria 2008) can be used to compute the drain discharge

$$\frac{q}{kd} = \frac{2}{\pi(4-\pi)}\left[\frac{(2S/d)^{3.44}}{(2S/d)^{3.44}+4.3}\right]^{(0.28-y/4d)/(1-2y/3d)}[1-(y/d)^2]^{0.7} \tag{4}$$

In the design problem (fixing appropriate spacing between the ditches) Eqs. (2) and (3) have to be solved for α and β for given values of q, d, and y and then Eq. (1) gives the drain spacing. Instead, Eq. (4) can be manipulated to get the spacing directly as

$$\frac{2S}{d} = 1.528\left(\left\{\frac{2[1-(y/d)^2]^{0.7}}{\pi(4-\pi)q/kd}\right\}^{(1-2y/3d)/(0.28-y/4d)} - 1\right)^{-0.29} \tag{5}$$

Cost Function

The cost of installation of drainage system by an array of ditches depends on the costs per unit length and the number of ditches per unit area. Ditch being an earthen structure, the basic cost will be its excavation cost. Also the drained water into the ditch has to be pumped out. Therefore, the pumping cost is to be taken into account which includes the cost of the pumps and their operating and maintenance costs. In the drainage system, the number of ditches varies inversely with the spacing. With the increase in spacing, the number of ditches decreases resulting in decrease in excavation cost but increase in pumping cost.

The cost of earthwork depends on the volume and depth of cut, the strata to be excavated, and the distance of haulage if required in transporting the soil materials (Swamee et al. 2001). Similar to Swamee et al. (2000,2001), the earthwork cost C_e (monetary unit per unit length, e.g., \$/m) for the ditch section as shown in Fig. 1, can be expressed as

$$C_e = c_e A + c_r A d/2 \tag{6}$$

where c_e=cost per unit volume of excavation at ground level (\$/m^3); c_r=the additional cost per unit volume of excavation per unit depth (\$/m^4); $A=bd$=ditch area (m^2); b=width of the ditch (m); and d=depth of ditch from the ground surface (m). In the second part of Eq. (6), Swamee et al. (2001) assumed that the cost per unit volume of excavation is linear function of the depth of excavation.

The major factors that influence the pumping cost depends on the volume of water to be pumped, weight density of the water, hydraulic head, efficiency of the pump, and fuel cost (Moradi-Jalal et al. 2003; Sharma and Swamee 2006; Purohit 2007). Among the pumping cost components, the cost of the pumping units is considered in the cost function, where as the cost of pump house and operating staff are independent of the design variables, and they may be considered as fixed costs (Swamee 2001). As the fixed costs do not affect the cost optimization, they are excluded from the cost function. Thus, the cost of pump units (C_{pu}) and the pumping energy cost (C_{pE}) including the annual repair and maintenance can be expressed (Swamee 2001) as

$$C_p = C_{pu} + C_{pE} = k_p\frac{\gamma(2q)h}{\eta} + \frac{8.76R_E\gamma(2q)h}{\eta r} \tag{7}$$

where k_p=proportionality constant or per watt cost of the pump including maintenance and replacement (\$/kW); γ=weight density of the fluid (N/m^3); $h=d-y$=pumping head (m); and η =combined efficiency of the pump and the prime mover; R_E =the cost of the electricity per kilowatt-hour (\$/kW.h); and r =the rate of interest expressed as dollars per dollars per year (\$/\$/year).

Adding the cost of earthwork and the cost of pumping, the cost of a single ditch per unit length C_s (\$/m) becomes

$$C_s = C_e + C_p = c_e A + c_r A\frac{d}{2} + k_p\frac{\gamma(2q)h}{\eta} + \frac{8.76R_E\gamma(2q)h}{\eta r} \tag{8}$$

Optimization Algorithm

The spacing between the ditches in the drainage system should be such that it could drain out the rain water with minimum cost. Let the ponded area to be drained is of length L (m) and width W (m). Therefore the number of ditches in the area will be $W/(2S+b)$ and the total discharge Q (m^3/s) across the drainage area will be product of $2q$, length of ditches and total number of ditches given by

$$Q = 2qL\left(\frac{W}{2S+b}\right) \tag{9}$$

Thus

$$\frac{q}{kd} = \frac{Q(2S/d + b/d)}{2WLk} = \frac{1}{2}\frac{i}{k}(2S/d + b/d) \tag{10}$$

since the rainfall intensity i (m/s)=Q/WL. Consequently, the total cost of the ditch drainage system in the area turns out to be

$$C_t = C_s LW/(2S+b) \tag{11}$$

Substituting and converting into nondimensional form

$$C_{t^*} = (1+0.5c_{r^*})\frac{b/d}{2S/d + b/d} + c_{p^*}(1-y/d) \tag{12}$$

where

$$C_{t^*} = \frac{C_t}{c_e LWd} \tag{13a}$$

$$c_{r^*} = \frac{c_r d}{c_e} \tag{13b}$$

$$c_{p^*} = i\frac{\gamma}{\eta}(k_p + 8.76R_E/r)/c_e \tag{13c}$$

in which subscript * denotes the corresponding nondimensional parameter. It is evident from Eqs. (1)–(3) that y/d is function of q/kd and S/d. Therefore, y/d is a function of S/d for a known value of q/kd [from Eq. (10)]. Thus, the total cost given by Eq.

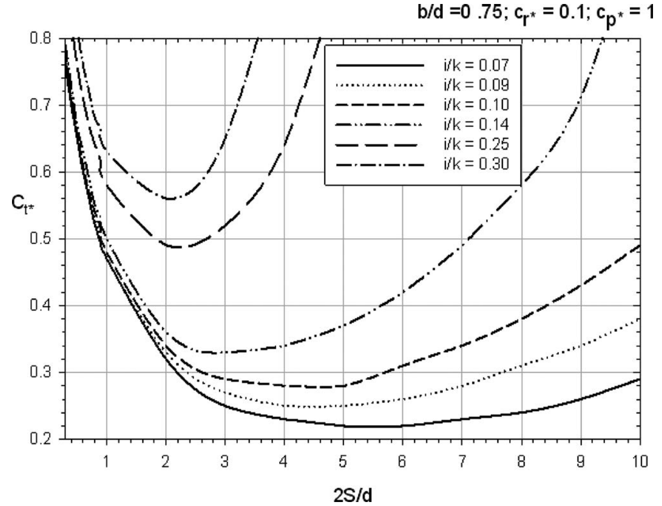

Fig. 2. Variation in the cost with ditch spacing

(12) is function of single variable S/d for given parameters i/k, b/d, c_{r^*}, and c_{p^*} as shown in Fig. 2. The unit excavation costs have been compiled from Schedule of Rates (2007) and the pump parameters have been ascertained from market surveys of branded pumps of various heads and capacities. Thereafter, the ranges of c_{r^*} and c_{p^*} have been worked out.

Consequently, the problem of determination of optimal ditch spacing can be expressed as

$$\text{Minimize } \; C_{t^*} = (1 + 0.5c_{r^*})\frac{b/d}{2S/d + b/d} + c_{p^*}(1 - y/d) \quad (14)$$

which is one-dimensional nonlinear optimization problem in variable S/d.

A MATLAB program (MATLAB 2007) has been written for the solution of the minimization problem stated in Eq. (14) comprising the following main steps: (1) read input data i.e., d, b, k, i, c_e, c_r, k_p, R_E, η, γ, and r; (2) compute b/d, i/k, c_{r^*} [Eq. (13*b*)], and c_{p^*} [Eq. (13*c*)]; (3) assume an initial value of S/d and find q/kd from Eq. (10); (4) solve Eqs. (1) and (3) simultaneously for the transformation parameters α and β; (5) determine y/d from Eq. (2); (6) finally calculate the cost using Eq. (14); and (7) repeat Steps (3)–(6) adopting an optimization function (*fminsearch* of MATLAB) to get minimum cost and hence optimal spacing corresponding to input parameters of Steps (1) and (2).

Explicit Relation for Optimal Spacing

The program has been run for different set of c_{r^*} (0.01–1); c_{p^*} (0.1–2.0); b/d (0.15–0.75); and i/k (0.02–0.5) to generate optimal spacing between the ditches. A representative graph for the optimal spacing for a selected set of parameters is shown in Fig. 3. Further Gauss-Newton method of nonlinear least-squares data fitting has been used on the above generated optimal values, which resulted into the following explicit equation

$$\frac{2S^*}{d} = 0.957(k/i)^{2/3}(b/d)^{1/3}\left(\frac{1 + 0.175c_{r^*}}{c_{p^*}^{2/5}}\right) \quad (15)$$

where superscript * denotes the optimal value. On substituting c_{r^*} and c_{p^*} from Eqs. (13*b*) and (13*c*) in the Eq. (15), the resulted equation is

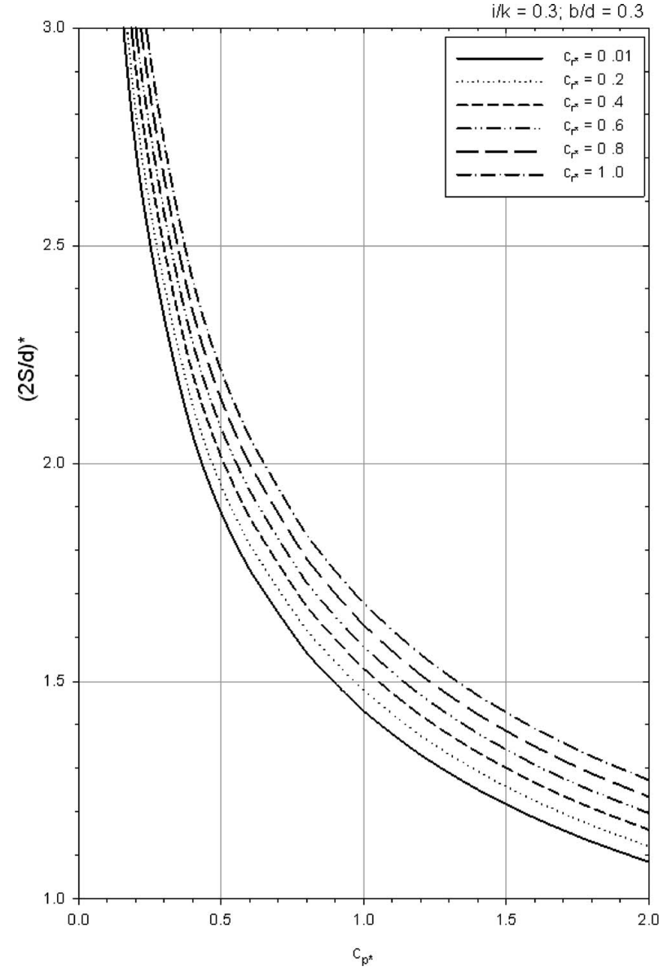

Fig. 3. Changes in optimal spacing with cost and input parameters

$$\frac{2S^*}{d} = 0.957(k/i)^{2/3}(b/d)^{1/3}\frac{1 + 0.175(c_r d/c_e)}{\left(i\frac{\gamma}{\eta}(k_p + 8.76R_E/r)/c_e\right)^{2/5}} \quad (16)$$

The errors in the fitted equation in the above mentioned ranges of parameters are within 7%. The higher errors occur at the combinations of extreme ends of ranges of parameters.

Examples

Case 1

Consider drainage of a ponded area underlain by an impervious layer at 3 m by fully penetrating parallel ditches of width 0.45 m. The porous medium of the area is of ordinary soil for which hydraulic conductivity $= 10^{-6}$ m/sec; $c_e = 0.5\,\$/\text{m}^3$; and $c_e/c_r = 7.5$ m. The pumping cost parameters are: $\eta = 0.75$; $R_E = 0.15$ \$/kW.h; and $k_p = 0.3$. Assume the rainfall intensity $= 1.5$ mm/hr, weight density of water $= 9810$ N/m^3 and rate of interest $r = 0.04$.

For the given data $i/k = 0.4167$; $b/d = 0.15$; $c_{r^*} = 0.4$; and $c_{p^*} = 0.361$. Using Eq. (15) $(2S/d)^* = 1.4659$, hence the optimal spacing between the ditches $= 4.40$ m. The actual optimal spacing from the optimization program based on Eq. (14) is 4.586 m, so the error in the computed value using explicit equation is 4.1%.

Case 2

Consider another case in which hydraulic conductivity $=3 \times 10^{-6}$ m/sec; $c_e/c_r=6.667$ m; $R_E=0.125$ \$/kW.h; the rainfall intensity $=1.25$ mm/hr; and other conditions are identical to the Case 1. For this case $i/k=0.1157$; $b/d=0.15$; $c_{r*}=0.45$; and $c_{p*}=0.2514$ and hence $(2S/d)^*=4.0116$ from Eq. (15). Therefore the optimal spacing $=12.035$ m, while the actual optimal spacing $=12.514$ m indicating 3.83% error.

Discussion and Limitations

Waterlogging and salinization are the result of over irrigation and lack of natural surface and subsurface drainage. The groundwater systems in irrigated areas are often insufficient to remove excess water and thus engineered drains are necessary to prevent waterlogging and salinization. An array of fully penetrating ditches is one method of subsurface drainage. The cost of such drainage system includes the depth-dependent earthwork cost and the capitalized cost of pumping of drain discharge. Here it has been assumed that the irrigated land is flat and hence pumping of drain water is required. There will be no pumping cost if the drain outfall is above ground level leading to gravity flow. Also, the cost of land used by ditches has not been considered in the study; however this cost can easily be included in the cost function. The cost of land is a linear function of the width of ditch, thus its effect on the optimal spacing would be similar to the width of ditch, which has already been incorporated in Eq. (16).

The present solution is developed with an assumption that the ponded water will initially be drained out by surface field drains. At later stage for small ponded water depth, the subsurface drainage by ditches will take place at steady state. Also, the present solution is obtained for fully penetrating ditches with vertical sides i.e., for rectangular shape of ditch. Hence, there may be a stability issue of vertical sides of ditches. The slope stability problem may be tackled by refilling the ditches with boulders, gravel or other free draining material. For partially penetrating ditches, a similar procedure may be adopted but that will necessitate a different analytical solution in place of Eqs. (1)–(3). Furthermore, the present solution is more suitable for green pastures, orchard plantations, forest swamps, and other amenities, where wider spacing between the ditches is required and the difficulty in farming operation and land removal from cultivation is not much of significance. Moreover, the solution obtained is applicable for homogeneous medium but it may be used to layered media once it is transformed into an equivalent homogeneous medium.

Fig. 3 or Eq. (15) shows that the optimal spacing between the ditches increases with increase in c_{r*} and decrease in c_{p*}, thus the earthwork cost and the pumping cost have opposing effect on the optimal spacing. Also, the optimal spacing between ditches is found to be very small for higher value of intensity of rainfall and lower value of hydraulic conductivity. For $i/k>0.5$ the required spacing between the ditches is very small and hence the subsurface drainage alone may not be sufficient to drain out the high rainfall, and thus a part of the rainfall should be drained out by surface drainage.

The proposed explicit equations [Eqs. (5) and (16)] are simple algebraic equations which can conveniently be used to compute spacing between arrays of fully penetrating ditches. Eq. (5) gives nearly exact solution and Eq. (16) gives close approximations in the practical ranges of c_{r*} (0.01–1); c_{p*} (0.1–2.0); b/d (0.15–0.75); and i/k (0.02–0.5) as involved errors are within 7%.

Conclusion

Exact analytical solution for steady subsurface drainage from a ponded surface by fully penetrating array of ditches is in form of parametric equations containing improper integrals. Computations of the drainage discharge for given ditch spacing and vice-versa involve simultaneous solution of these parametric equations containing improper integrals which can only be evaluated by numerical integration. Eq. (5) overcomes this and directly yields the spacing between ditches to drain out a given quantity q at water depth y in the ditches. Furthermore, fixation of the optimal spacing between the ditches requires amalgamation of cost parameters and an optimization scheme in the above mentioned solution procedure, which is even more complicated. Using a nonlinear data fitting technique, an explicit equation has been proposed for determining spacing between the ditches for the design of minimum cost ditch drainage system. The simplified explicit algebraic equation results in an answer for optimal spacing between fully penetrating array of ditches in a single step computation, thus avoids numerical computation of improper integrals, simultaneous solution of nonlinear equations and implementation of optimization technique.

Notation

The following symbols are used in this technical note:

A = ditch area [m^2];
b = with of ditch [m];
C_e = cost of earthwork per unit length of canal [\$/m];
C_p = pumping plant cost [\$/m];
C_{pE} = pumping energy cost [\$/m];
C_{pu} = cost of pumping unit [\$/m];
C_s = cost of a single ditch per unit length [\$/m];
C_t = total cost of the ditch drainage system [\$];
c_e = cost per unit volume of excavation at ground level [\$/m^3];
c_r = increase in unit excavation cost per unit depth [\$/m^4];
d = depth of impervious layer [m];
$F(.,.)$ = incomplete elliptical integral of the first kind [dimensionless];
h = pumping head [m];
i = rainfall intensity [m/s];
$K(.)$ = complete elliptical integral of the first kind [dimensionless];
k = hydraulic conductivity [m/s];
k_p = pump coefficient[\$/w]
L = length of ponded area [m];
Q = total discharge [m^3/s];
q = seepage discharge from one side of ditch per unit length [m^2/s];
R_E = unit electrical energy cost [\$/kw.h];
r = interest rate [\$/\$/year];
S = spacing (half) between ditches [m];
t = dummy variable [dimensionless];
W = width of ponded area [m];
y = water depth in ditch [m];
α, β = transformation or accessory parameters [dimensionless];
γ = weight density of water [N/m^3];
\$ = monetary unit;

* = nondimensional; and
* = optimal.

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