# Discussion of "Data Mining Process for Integrated Evaporation Model" by M. E. Keskin, Ö. Terzi, and E. U. Küçüksille

January/February 2009, Vol. 135, No. 1, pp. 39–43.
DOI: 10.1061/(ASCE)0733-9437(2009)135:1(39)

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The authors should be commended for presenting an investigation on the ability and accuracy of the data mining process to estimate daily pan evaporation values for the three lakes (Eğirdir, Kovada, and Karacaören Dam) in Turkey.

First of all, the pan evaporation estimation is the indirect procedure for evaporation from lakes and reservoirs. A water budget of the water body is also used to estimate evaporation losses. Estimation of evaporation from meteorological parameters uses energy-based and/or aerodynamic-based evapotranspiration estimation models (Abtew 2001). On the other hand, the data mining process appears under a multitude of names, which includes knowledge discovery in databases, data or information harvesting, data archeology, functional dependency analysis, knowledge extraction, and data pattern analysis (Bessler et al. 2003). In addition, there exists a large number of definitions for this group of methods. The term "data mining" is used for both the whole process of knowledge discovery and also for the specific algorithms that are used to achieve this. Of several related definitions of data mining one that is most appropriate for real-world applications is given by Fayyad et al. (1996): "Data mining is the nontrivial process of identifying valid, novel, potentially useful, and ultimately understandable patterns in data." In other words, data mining is the search for relationships and global patterns that exists among parameters, but are hidden among the data (Bessler et al. 2003).

The paper under discussion (Keskin et al. 2009) focuses on the following objectives:

1. "To develop pan evaporation models using the data mining process for the three lakes;
2. To form an integrated evaporation model by aggregation of daily pan evaporation of the three lakes; and
3. To use the data mining models, when daily pan evaporation is not measured."

Although the present technique and findings observed through the investigation seem largely understandable, the following points need to be clarified:

1. To estimate daily pan evaporation for each lake and the integrated daily pan evaporation model, the paper presents and compares (in the paper Table 1) five algorithms through the modeling phase of the proposed data mining process: REP Tree, KStar, decision table, artificial neural networks (ANNs), and multilinear regression. Among the proposed five algorithms, for the REP tree model the paper states: "As a result, in comparing the developed models with measured daily pan evaporation values, the REP tree model has better agreement with measured daily pan evaporation than other models." Furthermore, under the subtitle "*REP Tree*" the paper says: "The decision tree tool of REP tree in Weka was employed for formulating the resource access patterns for the considered applications that are common in the target execution environment."

   However, it is not clearly illustrated the expansions or the meanings of the terms "REP tree model" through the above statement. It is useful to concisely define this term in the section where it is first mentioned. Moreover, the software called "Weka" and its origin could be addressed through the text.
2. Through the section "Artificial Neural Networks," the paper presents general information on the ANN procedure. However, this section does not give sufficient information on the ANN topology used in the proposed integrating evaporation estimation model. To implement the proposed ANN procedure, some characteristics or specifications of the ANN architecture should be clearly illustrated. For instance, the number of hidden layers constructed, the number of neurons, the related activation functions etc.
3. Through the section "Multilinear Regression," what are the coefficients $(a, b, c, \ldots)$ of predictors $(X_1, X_2, \ldots X_n)$ to determine the dependent variable $Y$ (e.g., $Y = a + bX_1 + cX_2 + \ldots$)? If the values of coefficients are previously known one can easily apply the regression formulations to reproduce the results.
4. What are the basic assumptions of the present algorithms (especially for the REP tree model, KStar, and decision table)? For instance, the multilinear regression has some well-known assumptions such as linearity, normality, homoscedasticity (Şen et al. 2003).
5. It could be very understandable for the readers to present in a functional and tabular form of the proposed four models [Evaporation rates for Lake Eğirdir (E), Lake Kovada (Ko), Lake Karacaören Dam, (KD) and integrated model (I)]. This type of representation would make it easier to follow each model construction. The representation shown in Table 1 would be preferred.

**Table 1.** Functional Form, Inputs and Output for the Proposed Four Models Regarded in the Original Paper.

| Model Number | Function | Inputs | Output |
|---|---|---|---|
| 1 | E=f(Ko,KD) | Ko, KD | E |
| 2 | Ko=f(E,KD) | E, KD | Ko |
| 3 | KD=f(E,Ko) | E, Ko | KD |
| 4 | I=f(E,Ko,KD) | E, Ko, KD | I |

## Acknowledgments

The Scientific and Technological Research Council of Turkey (TUBITAK) is acknowledged for supporting the researcher's time at Texas A&M University by the fellowship and grants program (No. 2219).

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# Closure to "Data Mining Process for Integrated Evaporation Model" by M. E. Keskin, Ö. Terzi, and E. U. Küçüksille

January/February 2009, Vol. 135, No. 1, pp. 39–43.
DOI: 10.1061/(ASCE)0733-9437(2009)135:1(39)

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The writers wish to thank Dr. Mehmet Özger and Dr. Gürol Yıldırım for the discussion. The writers found that the questions raised by the discussers are of general type and they are not all related with the paper under discussion. The following points can be made:

1. The Web address of WEKA software is ⟨http://www.cs.waikato.ac.nz/ml/weka⟩. Also, in the guidelines of ASCE, it is indicated that "The abstract should contain the purpose of the work, the scope of the effort, the procedures used to execute the work (if of special interest), major findings, and key conclusions. Do not include jargon, equations, figure callouts, table callouts, or reference citations in the abstract." The REP tree model and WEKA software are not mentioned in the abstract based on the guidelines.
2. Because topology and specifications of ANN are well known by users, they are not provided in our manuscripts. When data-mining-related studies are examined, this case is apparent as can be noted from Bellazzi and Zupan (2008), Goodwin (2003), Hui and Jha (2000), and Li and Shue (2004).
3. The article should be examined entirely by the discussers. MLR is one of the data mining modeling techniques. We believe that giving MLR coefficients in the paper is not necessary. This is because the best evaporation model was obtained from the REP tree model. Also, the developed models may not give appropriate results for regions with different climatological characteristic. For this reason, new models must be developed for other regions. Upon examining literature, this case is apparent: Bellazzi and Zupan (2008), Goodwin (2003), Hui and Jha (2000), and Li and Shue (2004).
4. The basic assumptions for all data mining models can be found in WEKA software. Detailed information can be found at the following Web address: ⟨http://www.cs.waikato.ac.nz/ml/weka⟩. Again, upon examining literature (Zhu 2010); Rocha et al. (2007), basic algorithm assumptions are not generally given.
5. We don't agree with the comments. Since input and output parameters are already explained and given in the paper (see the section "Data Understanding"), it isn't necessary to repeat them.

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# Discussion of "Least-Cost and Most Efficient Channel Cross Sections" by Gerald E. Blackler and James C. Y. Guo

March/April 2009, Vol. 135, No. 2, pp. 248–251.
DOI: 10.1061/(ASCE)0733-9437(2009)135:2(248)

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The authors have formulated a cost function for a trapezoidal section and have used the Lagrangian multiplier method to obtain a bed width to flow depth ratio for a least-cost section. The discusser would like to draw attention to the following points:

1. The review of the literature is incomplete. For example, Swamee et al. (2000a, 2000b, 2001, 2002) developed cost functions more general than the authors' cost function; Chahar and Basu (2009) used a similar cost function (except land acquisition cost) and optimization method for circular and parabolic bed trapezoidal canal sections along with bounds on the canal dimensions to get the most hydraulically efficient and minimum-cost sections; Chahar (2005, 2007), for a parabolic section and a curvilinear section, respectively, adopted similar methodology to obtain the most efficient and

minimum-cost channel dimensions; Froehlich (1994, 2008) and Monadjemi (1994) also used the Lagrangian multiplier method to obtain optimal section properties of a trapezoidal channel; and "Das (2006)" in the paper should be Das (2007).

2. The presented cost function for a trapezoidal section includes the land acquisition, lining, and excavation costs. Swamee et al. (2000b, 2002), in contrast, incorporated depth-dependent excavation cost, lining cost, and capitalized cost of water lost as seepage and evaporation losses in the cost function for triangular, rectangular, and trapezoidal sections. The width of the land acquisition is equal to the top width of the channel plus the width of the banks and right of way. Thus the land acquisition cost is a function of the top width plus a fixed cost. Since the evaporation loss is a linear function of the top width, the effect of the land acquisition cost on the optimal dimensions is similar to the cost parameter $c_{\mathrm{wE}}$ (Swamee et al. 2000b, 2002).
3. The authors' regressed equation is for only $b/y$ involving $c_2/c_3$, whereas the design equations for the minimum-cost trapezoidal channel section proposed by Swamee et al. (2000b, 2002) are more general.
4. Swamee et al. (2000a, 2000b, 2001, 2002) developed a cost function as well as an optimization procedure in dimensionless form and highlighted the advantage that this method is independent of monetary units chosen and so can be applied to other regions. Thus, the equations in the paper are not new in terms of allowing this approach to be transferred to other regions when the local cost data are available.
5. The note concludes that the least-cost channel section tends to be deeper if the land cost is much higher than the lining cost, and when cost factor, $R$, vanishes the difference between the most efficient and least-cost channel cross sections becomes negligible. It is obvious and has already been pointed out by Swamee et al. (2000b, 2002) that with increase in $c_{\mathrm{wE}}$, the channel section becomes deeper and narrower than the minimum area (most efficient) section; on the other hand, with increase in $c_{\mathrm{e}}$ and/or $c_{\mathrm{L}}$, the channel section approaches the corresponding minimum area section.

The discusser feels that the inclusion of these comments would make the work comprehensive and more useful to the scientific community.

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## Closure to "Least-Cost and Most Efficient Channel Cross Sections" by Gerald E. Blackler and James C. Y. Guo


March/April 2009, Vol. 135, No. 2, pp. 248–251.
DOI: 10.1061/(ASCE)0733-9437(2009)135:2(248)



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The author would like to thank the discusser for the comments. As the discusser has stated, the literature review in the original paper did not include certain articles that would've contributed to its references. Specifically, Swamee et al. (2000a,b, 2001, 2002) have produced similar studies, and Froehlich (1994, 2008) and Monadjemi (1994) incorporated a Lagrangian multiplier method that produced an optimal section of a trapezoidal channel. It is also noted that the reference to Das's paper should read Das (2007).

However, the cost factor $R$ developed in the original paper still stands as a simple and easy method of designing a least-cost trapezoidal channel section. It is also noted that the designer can use $R$ for rectangular channel sections by setting the side slopes ($z$) to zero. Parabolic sections were avoided by the authors because they are rarely constructed in the region where the original research was prepared. Parabolic sections are generally impractical in the case of a concrete-lined section because it is difficult to construct a parabolic concrete form. A parabolic section can also be difficult to construct if soil conditions are not adequate.

It is true that the cost of land in the original paper is similar to the evaporation loss function presented in Swamee et al. (2000b, 2002). However, it is not directly related as the cost of land varies from the channel top width as a function of the freeboard depth and side slope. If a large freeboard depth is specified at mild side slopes then the required land acquisition can be substantially different from the top width of the hydraulic surface and thus the parameter from an evaporation loss function could vary.

As presented in the original paper, the ratio of $c_2/c_3$ is used to create $R$ because other ratios are less sensitive. Although more general applications can be found elsewhere (Swamee et al. 2000b, 2002), the logarithmic nature of $R$ creates a direct solution to the optimal trapezoidal section. This direct solution, either by chart or logarithmic equation, is easy for the practicing engineer or designer to apply.

As pointed out by the discusser, dimensionless cost ratios for open channel construction cost and design parameters are not new and were presented by Swamee et al. (2000a,b, 2001, 2002). However, dimensionless charts and equations are the best methods to apply to costs because costs can vary heavily depending on region and other economic factors. Although the method presented is not "new," it is applicable to the original research and parallel to common engineering design and practice.

The author would like to thank the reviewer for pointing out research that is parallel to that outlined in the original paper. This discussion and closure will clarify to the scientific community that many other studies on optimal channel design are available. The author would also like the discusser and the scientific community to view the original paper more as a practical application to open channel design than as an academic exercise. Although it is not explicitly stated, the original paper was produced to appeal to the engineer designer or practitioner in an attempt to make the application of $R$ simple and effective enough that it will be used in open channel design.

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# Discussion of "Generalized Analytical Solutions for Groundwater Head in a Horizontal Aquifer in the Presence of Subsurface Drains" by S. K. Singh


May/June 2009, Vol. 135, No. 3, pp. 295–302.
DOI: 10.1061/(ASCE)IR.1943-4774.0000069



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This discussion aims to enhance the generality of the analytical solutions presented by the author. The discussion will focus on two essential components relating to the drainage problem. The first one concerns the initial condition, which describes the shape of the water table at time $t=0$ and the second concerns the form of the equation that describes the recharge rate $R(t)$.

### Initial Water Table Profile

The analytical solutions derived by Singh (2009) are based on the assumption that the water table is initially flat. However, this situation seldom represents reality, particularly in fields with already-established subsurface drainage systems. Tapp and Moody (Dumm 1964) observed that the initial water table profile encountered in the field was more often parabolic than flat. For the description of the initial water table shape, the U.S. Bureau of

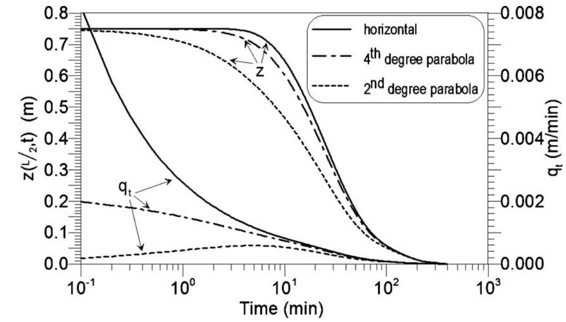


**Fig. 1.** Influence of initial condition on falling water table and drain discharge

Reclamation after extensive experimentation recommended the use of a fourth-degree parabola. In accordance to the variables used in Eq. (37) of the original paper, the initial condition is written

$$z = 8z_0\left(\frac{x}{L} - 3\left(\frac{x}{L}\right)^2 + 4\left(\frac{x}{L}\right)^3 - 2\left(\frac{x}{L}\right)^4\right) \quad (1)$$

Besides the Eq. (1), as initial condition may be used the solution of steady state form of the Boussinesq equation for a constant recharge rate (Karamouzis and Teloglou 1993). In this case, the initial water table profile is a second-degree parabola and can be written as

$$z = 4z_0\left(\frac{x}{L} - \left(\frac{x}{L}\right)^2\right) \quad (2)$$

Singh et al. (2006) obtained analytical solutions to describe falling water tables between drains using as an initial condition, among others, another type of parabola expressed as

$$z = 16z_0\left(\frac{x}{L} - \left(\frac{x}{L}\right)^2\right)^2 \quad (3)$$

Combining Eqs. (1) and (3) with a weighting factor λ, results in a more general form of an equation for the description of initial water table profile

$$z = z_0\left[\{\lambda\}\left(1 - \left(1 - \frac{2x}{L}\right)^4\right) + \{1-\lambda\}16\left(\frac{x}{L} - \left(\frac{x}{L}\right)^2\right)^2\right], \quad 0 \leqslant \lambda \leqslant 1 \quad (4)$$

For $\lambda = 1$, 0.5, and 0.0, Eq. (4) becomes equivalent to Eqs. (1)–(3), respectively.

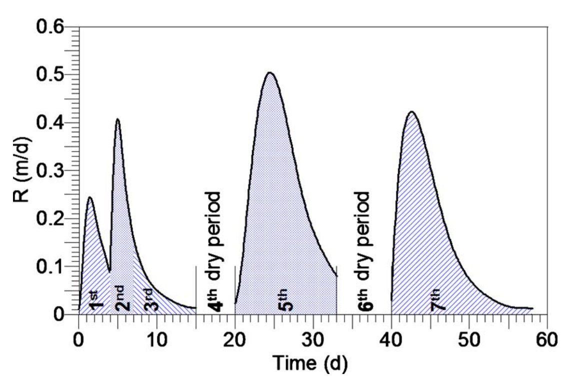


**Fig. 2.** Intermittent recharge rate hydrograph

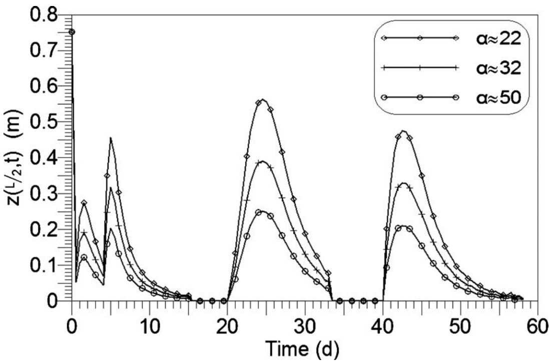


**Fig. 3.** Water table fluctuation due to intermittent recharge rate

## Time-Varying Recharge Rate

In nature, the recharge rate to the water table occurs in an irregular pattern. To describe mathematically this temporally varying recharge, a polynomial form is suggested by the author. However, this formula may accurately describe the recharge rate after only an isolated rainfall event or irrigation application. In the circumstances where there is a need to simulate the water table fluctuation or drain discharge variation in response to an intermittent recharge, a more general form of $R(t)$ should be defined. For this, it is proposed the subdivision of the recharge rate hydrograph into smaller piecewise continuous segments, each of which may be approximated by a polynomial of p-order. Hence, the rate of recharge may be described by the following general equation

$$R(t)=\begin{cases}\displaystyle\sum_{m=0}^{p_j} a_{j,m}t^m, & t_j \leqslant t \leqslant t_{j+1}, \quad (j=1,2,3\ldots k-1)\\ \displaystyle\sum_{m=0}^{p_k} a_{k,m}t^m, & t \geqslant t_k\end{cases} \qquad (5)$$

where $a_{j,m}$=the coefficients of the polynomial at segment $j$ of the recharge rate curve; $m$=an integer denoting the power of $t$ [dimensionless]; $P$=maximum degree of the polynomial that is best fitted at each segment; and $k$=maximum number of segments composing the recharge rate hydrograph at time $t$.

## Generalized Analytical Solution

### *Large Saturated Thickness*

Applying the finite Fourier sine transform (Carslaw and Jaeger 1959) to Eq. (37) and associated boundary conditions Eqs. (39) and (40) of the original paper, results the following ordinary differential equation

$$\frac{d\bar{z}_s}{dt}+\left(\frac{n^2\pi^2\beta}{L^2}+\frac{c}{S_y}\right)\bar{z}_s=\frac{L[1-\cos(n\pi)]}{n\pi S_y}R(t) \qquad (6)$$

where $\bar{z}_s$ is the transformation of $z(x,t)$

The solution of Eq. (6) subject to the transform of Eq. (4), is readily obtained as

$$\bar{z}_s=\frac{16z_0L[24(1-2\lambda)+n^2\pi^2(5\lambda-2)][1-\cos(n\pi)]}{n^5\pi^5}e^{-(n^2\pi^2\beta/L^2+c/S_y)t}+\frac{L[1-\cos(n\pi)]}{n\pi S_y}\int_0^t R(t')e^{-(n^2\pi^2\beta/L^2+c/S_y)(t-t')}dt' \qquad (7)$$

where $t'$ is a dummy variable of integration.

The inverse finite Fourier sine transformation (Carslaw and Jaeger 1959) of Eq. (7), taking into account the recharge rate function $R(t)$ given by Eq. (5), leads to the physical solution

$$z(x,t)=\frac{64z_0}{\pi^5}\sum_{n=0}^{\infty}\frac{[24(1-2\lambda)+(2n+1)^2\pi^2(5\lambda-2)]}{(2n+1)^5}e^{-[(2n+1)^2\alpha+c/S_y]t}\sin\frac{(2n+1)\pi x}{L}+\frac{4}{\pi S_y}\sum_{n=0}^{\infty}\frac{e^{-[(2n+1)^2\alpha+c/S_y]t}}{(2n+1)}\sin\frac{(2n+1)\pi x}{L}\left\{\sum_{j=1}^{k-1}\int_{t_j}^{t_{j+1}}\sum_{m=0}^{p_j}a_{j,m}t'^m e^{[(2n+1)^2\alpha+c/S_y]t'}dt'+\int_{t_k}^{t}\sum_{m=0}^{p_k}a_{k,m}t'^m e^{[(2n+1)^2\alpha+c/S_y]t'}dt'\right\} \qquad (8)$$

where α is defined by the author (characterize the reaction factor of a drainage system, Smedema et al. 2004)

It is obvious from Eq. (8) that the first term represents the influence of initial water table profile whereas the second term shows the effect of recharge rate on the water table elevation.

Evaluating the integrals in Eq. (8), the general solution takes the form

$$z(x,t)=\frac{64z_0}{\pi^5}\sum_{n=0}^{\infty}\frac{[24(1-2\lambda)+(2n+1)^2\pi^2(5\lambda-2)]}{(2n+1)^5}e^{-[(2n+1)^2\alpha+c/S_y]t}\sin\frac{(2n+1)\pi x}{L}+\frac{4}{\pi S_y}\sum_{n=0}^{\infty}\left\{\sum_{j=1}^{k-1}\sum_{m=0}^{p_j}\frac{a_{j,m}IE_{j,m}}{(2n+1)[(2n+1)^2\alpha+c/S_y]}\right.$$

**Table 1.** Polynomial Coefficients for the Description of an Intermittent Recharge Hydrograph

| $j$ | $a_{j,0}$ | $a_{j,1}$ | $a_{j,2}$ | $a_{j,3}$ | $a_{j,4}$ | $a_{j,5}$ | $a_{j,6}$ | $a_{j,7}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.011446318 | 0.045495229 | 0.558815227 | −0.607479901 | 0.253240587 | −0.047965635 | 0.003446985 | 0.0 |
| 2 | −34.50798138 | 124.9969232 | −116.6454116 | 50.5392227 | −11.90468071 | 1.577548758 | −0.110896764 | 0.003225734 |
| 3 | 1.317329603 | −0.018245761 | −0.105379847 | 0.02234219 | −0.001989678 | 8.38118E−05 | −1.37321E−06 | 0.0 |
| 4 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 5 | 2073.277032 | −464.3628688 | 42.84720994 | −2.086508023 | 0.056613789 | −8.12348E−04 | 4.82015E−06 | 0.0 |
| 6 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 7 | −11079.555 | 1296.405358 | −63.02442636 | 1.629922362 | −0.02365578876 | 1.827143217E−04 | −5.868425338E−07 | 0 |

$$+\sum_{m=0}^{p_k}\frac{a_{k,m}I\mathrm{E}_{k,m}}{(2n+1)[(2n+1)^2\alpha+c/S_y]}\Bigg\}\sin\frac{(2n+1)\pi x}{L} \quad (9)$$

$$I\mathrm{E}_{j,0}=e^{-[(2n+1)^2\alpha+c/S_y](t-t_{j+1})}-e^{-[(2n+1)^2\alpha+c/S_y](t-t_j)} \quad (10)$$

$$I\mathrm{E}_{j,m}=t_{j+1}^m e^{-[(2n+1)^2\alpha+c/S_y](t-t_{j+1})}-t_j^m e^{-[(2n+1)^2\alpha+c/S_y](t-t_j)} - \frac{m}{[(2n+1)^2\alpha+c/S_y]}I\mathrm{E}_{j,m-1} \quad (11)$$

$$I\mathrm{E}_{k,0}=1-e^{-[(2n+1)^2\alpha+c/S_y](t-t_k)} \quad (12)$$

$$I\mathrm{E}_{k,m}=t^m-t_k^m e^{-[(2n+1)^2\alpha+c/S_y](t-t_k)}-\frac{m}{[(2n+1)^2\alpha+c/S_y]}I\mathrm{E}_{k,m-1} \quad (13)$$

This solution describes the water table fluctuations between drains in response of an intermittent recharge and in the presence of evapotranspiration. In the case where $k=1$, the second term of Eq. (9) reduces to that of Eq. (45) of the original paper.

Also, the discharge of the drains per unit drained area is given by the following equation

$$q_t=2\frac{K}{L}\left(h\frac{\partial h}{\partial x}\right)_{x=0} \quad (14)$$

Combining Eq. (9) with Eq. (14), results

$$q_t=\frac{128KDz_0}{\pi^4L^2}\sum_{n=0}^{\infty}\frac{[24(1-2\lambda)+(2n+1)^2\pi^2(5\lambda-2)]}{(2n+1)^4} e^{-[(2n+1)^2\alpha+c/S_y]t}+\frac{8\beta}{L^2}\sum_{n=0}^{\infty}\Bigg\{\sum_{j=1}^{k-1}\sum_{m=0}^{p_j}\frac{a_{j,m}I\mathrm{E}_{j,m}}{[(2n+1)^2\alpha+c/S_y]} +\sum_{m=0}^{p_k}\frac{a_{k,m}I\mathrm{E}_{k,m}}{[(2n+1)^2\alpha+c/S_y]}\Bigg\} \quad (15)$$

Similar solutions can also be derived for the case of small saturated thickness.

Application of Eqs. (9) and (15) with $\lambda=0.5$, 1, and Eq. (45) of the original paper, may illustrate the effect of the initial water table profile on the rate of water table drop and consequently on the drain discharge. By using the aquifer data example given by the author, the falling water table curves as well as the drain discharge $q_t$ are depicted in Fig. 1.

To show the ability of Eq. (9) to predict the water table height in response to an intermittent nature of recharge rate, the data of illustrative example given by the author are used. The recharge rate hydrograph shown in Fig. 2 is approximated by seven polynomials and the corresponding coefficients are given in Table 1. Fig. 3 depicts the water table fluctuation due to an intermittent recharge, for different values of reaction factor $\alpha$.

## References


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# Closure to "Generalized Analytical Solutions for Groundwater Head in a Horizontal Aquifer in the Presence of Subsurface Drains" by S. K. Singh


May/June 2009, Vol. 135, No. 3, pp. 295–302.
DOI: 10.1061/(ASCE)IR.1943-4774.0000069



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The writer is grateful to the discusser for highlighting certain aspects of the paper that could be considered for further generalizing the analytical solutions obtained by the writer. These aspects include the considerations for a nonhorizontal (nonflat) initial groundwater head and multiple isolated recharge events each represented by a polynomial function as proposed in the original paper. These aspects can be easily be incorporated using the approach and solutions suggested by writer provided the condition for the initial groundwater head is assumed the same in the direction along the length of the parallel subsurface drains. The readers may prefer to check the discusser's solution before using it. The integrals appearing in Eq. (8) of the discussion were first evaluated by the writer in the original paper. The generalized expressions for calculating the flow to drains have been obtained by Singh (2010). I regret that Table 1 of the paper involves computational errors due to a bug (related to convergence) in the relevant computer program. These errors are reflected in the values listed under the table subheads pertaining to small saturated thicknesses and are corrected in the errata given herein.

## Errata

On page 300, in Table 1: The values given for small saturated thicknesses (for both cases, i.e., recharge, and recharge with ET) should be read as given below in Table 1.

## References


Singh, S. K. (2010). "Generalized analytical solutions for groundwater head in an inclined horizontal aquifer in the presence of subsurface drains." *J. Irrig. Drain. Eng.*, 136(3), 194–203.

**Table 1.** Values of $z$ (in m) at Different Times and Locations

| $x/L$ | 10 min | 30 min | 60 min | 100 min | 200 min | 300 min |
|---|---|---|---|---|---|---|
| | | | Considering recharge; small saturated thickness | | | |
| 0.1 | $0.437\times10^{+0}$ | $0.277\times10^{+0}$ | $0.213\times10^{+0}$ | $0.173\times10^{+0}$ | $0.940\times10^{-1}$ | $0.488\times10^{-1}$ |
| 0.2 | $0.676\times10^{+0}$ | $0.496\times10^{+0}$ | $0.393\times10^{+0}$ | $0.318\times10^{+0}$ | $0.175\times10^{+0}$ | $0.910\times10^{-1}$ |
| 0.3 | $0.750\times10^{+0}$ | $0.640\times10^{+0}$ | $0.528\times10^{+0}$ | $0.429\times10^{+0}$ | $0.238\times10^{+0}$ | $0.124\times10^{+0}$ |
| 0.4 | $0.763\times10^{+0}$ | $0.716\times10^{+0}$ | $0.612\times10^{+0}$ | $0.498\times10^{+0}$ | $0.278\times10^{+0}$ | $0.145\times10^{+0}$ |
| 0.5 | $0.764\times10^{+0}$ | $0.738\times10^{+0}$ | $0.640\times10^{+0}$ | $0.521\times10^{+0}$ | $0.290\times10^{+0}$ | $0.152\times10^{+0}$ |
| 0.6 | $0.763\times10^{+0}$ | $0.716\times10^{+0}$ | $0.612\times10^{+0}$ | $0.498\times10^{+0}$ | $0.278\times10^{+0}$ | $0.145\times10^{+0}$ |
| 0.7 | $0.750\times10^{+0}$ | $0.640\times10^{+0}$ | $0.528\times10^{+0}$ | $0.429\times10^{+0}$ | $0.238\times10^{+0}$ | $0.123\times10^{+0}$ |
| 0.8 | $0.676\times10^{+0}$ | $0.496\times10^{+0}$ | $0.393\times10^{+0}$ | $0.318\times10^{+0}$ | $0.176\times10^{+0}$ | $0.921\times10^{-1}$ |
| 0.9 | $0.437\times10^{+0}$ | $0.277\times10^{+0}$ | $0.213\times10^{+0}$ | $0.173\times10^{+0}$ | $0.940\times10^{-1}$ | $0.489\times10^{-1}$ |
| | | | Considering recharge and ET; small saturated thickness | | | |
| 0.1 | $0.437\times10^{+0}$ | $0.276\times10^{+0}$ | $0.212\times10^{+0}$ | $0.171\times10^{+0}$ | $0.925\times10-1$ | $0.477\times10^{-1}$ |
| 0.2 | $0.675\times10^{+0}$ | $0.495\times10^{+0}$ | $0.391\times10^{+0}$ | $0.316\times10^{+0}$ | $0.173\times10^{+0}$ | $0.894\times10^{-1}$ |
| 0.3 | $0.750\times10^{+0}$ | $0.638\times10^{+0}$ | $0.526\times10^{+0}$ | $0.425\times10^{+0}$ | $0.234\times10^{+0}$ | $0.122\times10^{+0}$ |
| 0.4 | $0.762\times10^{+0}$ | $0.714\times10^{+0}$ | $0.608\times10^{+0}$ | $0.493\times10^{+0}$ | $0.273\times10^{+0}$ | $0.141\times10^{+0}$ |
| 0.5 | $0.764\times10^{+0}$ | $0.736\times10^{+0}$ | $0.636\times10^{+0}$ | $0.516\times10^{+0}$ | $0.286\times10^{+0}$ | $0.149\times10^{+0}$ |
| 0.6 | $0.762\times10^{+0}$ | $0.714\times10^{+0}$ | $0.608\times10^{+0}$ | $0.493\times10^{+0}$ | $0.274\times10^{+0}$ | $0.142\times10^{+0}$ |
| 0.7 | $0.750\times10^{+0}$ | $0.638\times10^{+0}$ | $0.526\times10^{+0}$ | $0.425\times10^{+0}$ | $0.235\times10^{+0}$ | $0.121\times10^{+0}$ |
| 0.8 | $0.675\times10^{+0}$ | $0.495\times10^{+0}$ | $0.391\times10^{+0}$ | $0.316\times10^{+0}$ | $0.172\times10^{+0}$ | $0.895\times10^{-1}$ |
| 0.9 | $0.437\times10^{+0}$ | $0.276\times10^{+0}$ | $0.212\times10^{+0}$ | $0.171\times10^{+0}$ | $0.925\times10-1$ | $0.480\times10^{-1}$ |