
Optimal Parabolic Section with Freeboard

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ABSTRACT:

Optimal design equations for a parabolic channel section with freeboard are presented in this paper. The design equations are in explicit form and result in optimal dimensions of a channel in single step computations. These have been obtained after applying the Lagrange method of undetermined multipliers. A non-dimensional parameter approach has been used to simplify the design and a set of graphs for non-dimensional parameters are presented as an alternative for design. Design procedures for discharge dependent as well as depth dependent freeboard cases have been presented to demonstrate the simplicity of the method.

1. INTRODUCTION

An open channel is a conduit for flow which has a free surface. Open channels are used to transport liquids from source to destination for water supply, irrigation, industrial, power generation etc. purposes. Artificial channels are usually designed with section of regular geometric shapes. River beds, unlined canals and irrigation furrows all tend to approximate a stable parabolic shape (Miraneko et al 1984). Therefore, unlined canals can be made more hydraulically stable by initially constructing them in a parabolic shape. Since the channel side slopes along the cross section are always less than the maximum allowable side slope at the water surface, parabolic channels are physically more stable (Miraneko et al 1984, Babaeyan-Koopaei et al 2000, Babaeyan-Koopaei 2001). A lined parabolic channel has no sharp angles of stress concentration where cracks may occur, and can be prefabricated in moulded sections. Small parabolic ditches can be constructed by bulldozers and other types of earth moving equipment (Miraneko et al 1984). The round-bottom triangle is an approximation of the parabola; it is a form usually created by excavation with shovels.

Irrigation channels are lined for several purposes (Swamee et al 2000a,b). Lined channels are designed for uniform flow considering hydraulic efficiency, practicability, and economy (Streeter 1945). The factors to be considered in the design are: the material forming the channel surface, which determines the roughness coefficient; the minimum permissible velocity, to avoid deposition of silt or debris; the limiting velocity, to avoid erosion of the channel surface; the topography of the channel route, which fixes the channel bed slope; and the efficiency of the channel section, which indicates how much the section is hydraulically and/or economically efficient (Chow 1973). Some shapes are more efficient than others. From a hydraulic viewpoint the channel section having the least wetted perimeter for a given flow area has the maximum discharge carrying capacity; such a section is known as the best hydraulic section. The best hydraulic section has the maximum flow velocity or the

minimum flow area and wetted perimeter for a given discharge and channel bed slope. Monadjemi (1994) and others (Swamee 1995, Froehlich 1994, Atmapoojya and Ingle 2001, 2003, Chahar 2007) presented a fundamental approach for determining the best hydraulic section based on Lagrange's method of undetermined multipliers. Recently, Godara (2003) and Chahar (2005) presented optimal design procedure for a parabolic channel section but the freeboard has not been considered therein. However, freeboard is required as a safety factor against any number of unforeseen circumstances which might cause the water surface to be higher than expected. Consequently, freeboard can not be ignored in the design of a parabolic channel. Novman (2003) and the present paper address the same.

2. REQUIREMENTS FOR FLOW IN A CHANNEL

The flow requirements to be taken into account in designing a channel for uniform flow are the channel surface roughness, the minimum permissible velocity, the limiting velocity, the freeboard, and the hydraulic efficiency of the channel section.

Uniform Flow

Since rigid boundary channels are designed for uniform flow, the most commonly used uniform flow formula around the world is the Manning equation (Chow 1973) due to its simplicity and acceptable degree of accuracy in most of practical applications.

$$V = \frac{1}{n} R^{2/3} S_f^{1/2} = \frac{1}{n} R^{2/3} S_o^{1/2} \quad (1)$$

where V = mean velocity of uniform flow (m/s); R = hydraulic radius (m), defined as the ratio of flow area A (m) to the flow perimeter P (m); n = Manning's roughness coefficient; S_f = energy slope (dimensionless); and S_o = bed slope of the channel (dimensionless). For uniform flow $S_f = S_o$. In the Manning's formula all the terms except n can be directly measured. The roughness coefficient is a parameter

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representing the integrated effects of the channel cross-sectional resistance. The selection of a value of n is subjective, based on experience and engineering judgement. Chow (1973) lists values of n for different conditions of a channel.

Carrying Capacity

The discharge carrying capacity of a channel depends upon the flow area and flow velocity. The law of conservation of mass gives the uniform flow rate or discharge Q (m^3/s) in a channel as

$$Q = AV \quad (2)$$

Combining Eqs. (1) and (2) the following general flow resistance equation in terms of discharge is obtained:

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2} = \frac{A^{5/3}}{nP^{2/3}} S_0^{1/2} \quad (3)$$

Thus, the discharge carrying capacity of a channel section increases with increase in the hydraulic radius or area and with a decrease in the wetted perimeter. The best hydraulic section has the maximum discharge carrying capacity for a given flow area and channel bed slope and conversely it has the maximum flow velocity or the minimum flow area and wetted perimeter for a given discharge and channel bed slope.

Permissible Velocities

The minimum permissible velocity or non-silting velocity is the lowest velocity that will not initiate sedimentation and will not allow the growth of vegetation. Sedimentation and growth of vegetation decrease the carrying capacity and increase the maintenance cost of a canal. In general, an average velocity of 0.6 to 0.9 m/s will prevent sedimentation when the silt load of the flow is low and a velocity of 0.75 m/s is usually sufficient to prevent the growth of vegetation (Chow 1973). Hence, the minimum permissible velocity can be assumed in the range from 0.75 to 0.9 m/s.

Higher velocities are desired in rigid boundary channels to reduce costs. However, high velocities may cause scour and erosion of the boundaries. In rigid boundary channels the maximum permissible velocity or the limiting velocity V_L (m/s) that will not cause erosion depends on the lining material. Swamee et al (2001, 2002) and Subramanya (1997) list limiting velocities for different type of linings.

Freeboard

Freeboard is provided to prevent waves or fluctuations in water surface from overtopping the banks. The freeboard depends upon a number of factors, such as size of channel, velocity of flow, depth of flow, curvature of alignment, condition of storm and drain water inflow, fluctuations in water level due to operation of flow regulating structures, and wind action.

Discharge Dependent Freeboard: The discharge dependent freeboard is constant once the design discharge is known. Indian Standard (IS: 10430: 1982) recommends the following discharge dependent freeboards:

Table 1 Freeboard for Different Discharges

Discharge (m^3/s)	< 0.06	0.06 - 1.0	1.0 - 5.0	5.0 - 10.0	> 10.0
Freeboard (m)	0.1 - 0.15	0.3	0.50	0.60	0.75

Depth Dependent Freeboard: In opposition to discharge-dependent freeboard, one may also consider depth-dependent freeboard, which will be a function of depth y even when the discharge is given. The freeboard based on discharge alone is not quite satisfactory because it remains same for shallow and deep channels for a given discharge. A depth dependent freeboard is more general in that, in addition to accounting for the design discharge through the design normal depth, it does differentiate between shallow and deep channels. A recommended form of depth depended board is (Chow 1973)

$$F = \epsilon y^\delta \quad (4)$$

With general value, $\delta = 0.5$, and ϵ ranges from 0.6735 for $Q = 0.57 m^3/s$ (20 cusecs) to 0.8723 for $Q \geq 85 m^3/s$ (3000 cusecs). However there seems to be lack of information on the variation of ϵ between these limits (Longnathan 1992). The variation of ϵ between these limits is assumed to be linear in this study. To ascertain the value of ϵ for different values of Q between above limits the following equation could be used (Novman 2003):

$$\epsilon = 0.6722 + 2.3569 \times 10^{-3} Q \quad (5)$$

3. GEOMETRIC PROPERTIES OF A PARABOLIC SECTION

A parabolic channel (Fig 1) is typically characterized as (6)

$$Y = aX^2$$

where, Y = ordinate; X = abscissa; a = parameter.

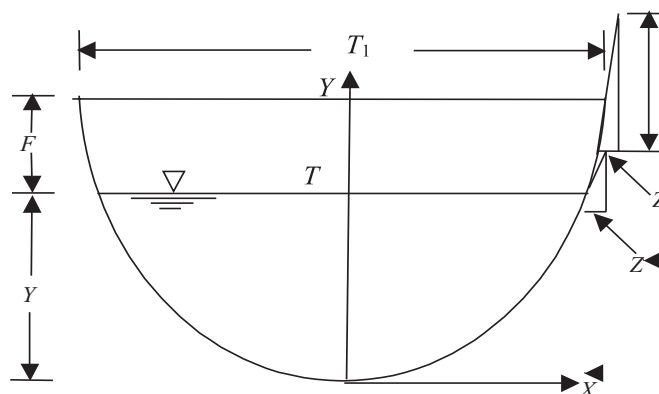


Fig. 1: A Parabolic Channel Section with Freeboard.

The total area A_t (m^2) is computed as

$$A_t = 2 \left[(y+F) \frac{T_1}{2} - \int_0^{T_1/2} Y dX \right] = \frac{8}{3} (y+F)^2 z_1 = \frac{8z}{3} y^{1/2} (y+F)^{3/2} \quad (7)$$

where y = depth of flow (m); F = freeboard (m); z = side slope at water level ($Y = y$); z_1 = side slope at $Y = y + F$ given by

$$z_1 = z \sqrt{y/(y+F)} \quad (8)$$

and T_1 = top width of the channel (m) at $Y = y + F$ which is equal to

$$T_1 = 4z_1(y+F) \quad (9)$$

The wetted perimeter P (m) is obtained by integrating length ds of the parabola given by

$$P = \int ds = \sqrt{(dX)^2 + (dY)^2} = 2yz^2 \left[\frac{1}{z} \sqrt{1 + \frac{1}{z^2}} + \ln \left(\frac{1}{z} + \sqrt{1 + \frac{1}{z^2}} \right) \right] \quad (10)$$

Eqs. (9) and (7) give the top width and total excavation area respectively considering free board (as shown in Fig. 1). Without free board ($F = 0$) these expressions give top width at water level T (m) as

$$T = 4yz \quad (11)$$

and flow area A (m^2)

$$A = \frac{8}{3} y^2 z \quad (12)$$

respectively. Substituting z in terms of T in Eq (10) we get

$$P = \frac{T}{2} \left[\sqrt{1 + \left(\frac{4y}{T} \right)^2} + \frac{T}{4y} \ln \left(\frac{4y}{T} + \sqrt{1 + \left(\frac{4y}{T} \right)^2} \right) \right] \quad (13)$$

When $0 < 4y/T \leq 1$ or $z \geq 1$ Eqs. (10) and (13) can be satisfactorily approximated (Chow 1973) to

$$P = T + \frac{8y^2}{3T} = y \left(4z + \frac{2}{3z} \right) = y f_{1z} \quad (14)$$

where

$$f_{1z} = 4z + \frac{2}{3z} = \frac{2}{3z} (6z^2 + 1) \quad (16)$$

Non-dimensionalisation of Parameters

Assuming a length scale L (m)

$$L = \left(Qn / \sqrt{S_0} \right)^{3/8} \quad (17)$$

the following non-dimensional variables can be defined

$$y_* = y/L; \quad P_* = P/L; \quad T_* = T/L; \quad A_* = A/L^2; \quad V_* = VL^2/Q \quad (18a-e)$$

Using Eqs. (3), (12), (14), and (17)

$$L^{8/3} = \left(\frac{8}{3} y^2 z \right)^{5/3} / (y f_{1z})^{2/3} \quad (19)$$

Solving for y

$$y = \left(\frac{3}{8z} \right)^{5/8} L f_z^{1/4} \quad ; \quad \text{or} \quad y_* = \left(\frac{3}{8z} \right)^{5/8} f_z^{1/4} \quad (20a,b)$$

Substituting y from Eq. (20a) in Eqs. (11), (12), (14), and (1), Chahar (2005) obtained

$$T_* = 4 \left(\frac{3}{8} \right)^{5/8} z^{3/8} f_z^{1/4} \quad ; \quad A_* = \left(\frac{3}{8z} \right)^{1/4} f_z^{1/2} \quad (21a,b)$$

$$P_* = \left(\frac{3}{8z} \right)^{5/8} f_z^{5/4} \quad ; \quad V_* = \left(\frac{8z}{3} \right)^{1/4} \frac{1}{f_z^{1/2}} \quad (21c,d)$$

Non-dimensional expressions for the depth of flow Eq. (20b), the flow area Eq. (21a), the top width Eq. (21b), the wetted perimeter Eq. (21c), and the uniform velocity Eq. (21d) are only a function of the side slope z . Graphical representation of the above equations could be used to obtain the values of these non-dimensional parameters for a wide range of z . Using equations, a set of graphs have been plotted for variations in y_* , T_* , P_* , and V_* with z as shown in Fig. 2.

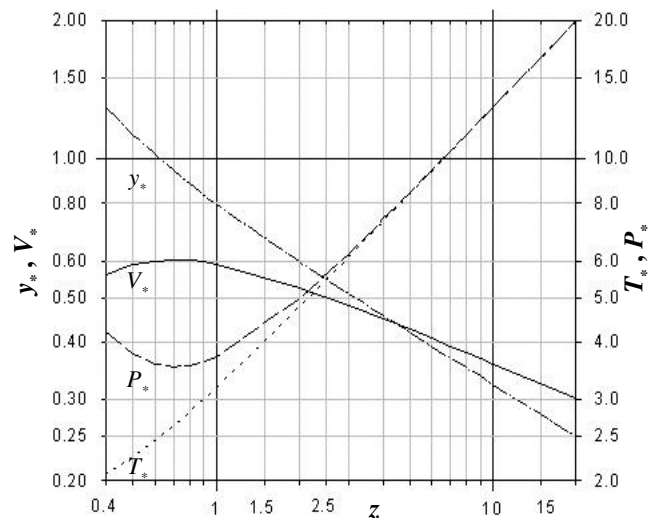


Fig. 2: Variations in y_* , T_* , P_* and V_* with z

4. OPTIMAL CHANNEL SECTION

To pass the design flow in any channel section with given roughness coefficient and longitudinal slope there theoretically exists an infinite number of combinations of section variables. The best hydraulic section has the maximum flow velocity or the minimum flow area and wetted perimeter for a given discharge and channel bed slope. To design a parabolic channel with minimum cross section to convey a given discharge with freeboard, the

solution lies in minimizing total area subject to total flow (Novman 2003) i.e.

$$\text{Minimize } A_t = \frac{8z}{3} y^{1/2} (y + F)^{3/2} \quad (22)$$

$$\text{Subject to } \phi = Q - \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S_0^{1/2} = \phi(A, P) = \phi(y, z) = 0 \quad (23)$$

A parabolic section has two independent geometrical variables i.e. depth of flow and side slope (y and z). Thus total excavation area A_t and equality constraint ϕ can be expressed as $A_t = A_t(y, z)$ and $\phi = \phi(y, z)$. Since a parabolic channel is completely described by two independent variables y and z , applying Lagrange's method of undetermined multipliers (Kreyszig 2001)

$$\frac{\partial A_t}{\partial y} + \lambda \frac{\partial \phi}{\partial y} = 0 \quad (24a)$$

$$\frac{\partial A_t}{\partial z} + \lambda \frac{\partial \phi}{\partial z} = 0 \quad (24b)$$

$$\phi = \phi(y, z) = 0 \quad (24c)$$

Elimination of λ between Eqs. (24a) and (24b) results in

$$\frac{\partial A_t}{\partial z} \frac{\partial \phi}{\partial y} = \frac{\partial A_t}{\partial y} \frac{\partial \phi}{\partial z} \quad (25)$$

Partial derivatives of ϕ with respect to z and y are

$$\frac{\partial \phi}{\partial z} = \frac{S_0^{1/2} A^{2/3}}{3n P^{5/3}} \left(2A \frac{\partial P}{\partial z} - 5P \frac{\partial A}{\partial z} \right) \quad (26a)$$

$$\frac{\partial \phi}{\partial y} = \frac{S_0^{1/2} A^{2/3}}{3n P^{5/3}} \left(2A \frac{\partial P}{\partial y} - 5P \frac{\partial A}{\partial y} \right) \quad (26b)$$

Therefore

$$\frac{\partial A_t / \partial z}{\partial A_t / \partial y} = \frac{\partial \phi / \partial z}{\partial \phi / \partial y} = \frac{5P \frac{\partial A}{\partial z} - 2A \frac{\partial P}{\partial z}}{5P \frac{\partial A}{\partial y} - 2A \frac{\partial P}{\partial y}} \quad (27)$$

Taking partial derivatives of A and P with respect to z and y

$$\frac{\partial A}{\partial z} = \frac{8}{3} y^2 \quad (28a)$$

$$\frac{\partial A}{\partial y} = \frac{16}{3} yz \quad (28b)$$

$$\frac{\partial P}{\partial y} = f_{1z} = 4z + \frac{2}{3z} = \frac{2}{3z} (6z^2 + 1) \quad (28c)$$

$$\frac{\partial P}{\partial z} = y \frac{2}{3z^2} (6z^2 - 1) = \frac{y}{z} f_{2z} \quad (28d)$$

where

$$f_{2z} = 4z - \frac{2}{3z} = \frac{2}{3z} (6z^2 - 1) \quad (28e)$$

Using these values in Eq. (27)

$$\frac{\partial A_t / \partial z}{\partial A_t / \partial y} = \frac{y(5f_{1z} - 2f_{2z})}{z(8f_{1z})} \quad (29)$$

Discharge Dependent Freeboard

Taking partial derivatives of A_t [using Eq. (7)] with respect to z and y

$$\frac{\partial A_t}{\partial z} = \frac{8}{3} y^{1/2} (y + F)^{3/2} \quad (30a)$$

$$\frac{\partial A_t}{\partial y} = \frac{4}{3} z \sqrt{\frac{y + F}{y}} (4y + F) \quad (30b)$$

Combining Eqs (29) and (30), we get;

$$\frac{(y + F)}{(4y + F)} = \frac{5f_{1z} - 2f_{2z}}{16f_{1z}} \quad (31)$$

Simplifying

$$F = 4y \frac{f_{1z} - 2f_{2z}}{11f_{1z} + 2f_{2z}} \quad (32a)$$

Substituting y from Eq. (20a)

$$F_* = 4 \left(\frac{3}{8z} \right)^{5/8} f_{1z}^{1/4} \frac{f_{1z} - 2f_{2z}}{11f_{1z} + 2f_{2z}} \quad (32b)$$

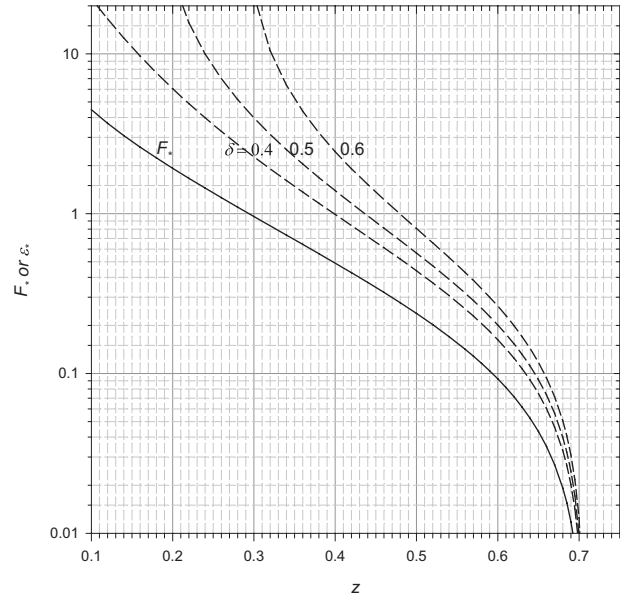


Fig. 3: Variation in F_* and e_* with z .

where $F_* = F/L$. Actually, F_* is independent parameter and z is dependent parameter, but explicit relation for z is possible so explicit relationship for F_* as in Eq. (32b) has been obtained. To make calculation possible for z corresponding to given F_* a graph for Eq. (32b) has been plotted for a wide range of side slope z as shown in Fig. 3. The graph is handy in reading out the optimal value of the side slope for chosen freeboard. Further, a simple explicit

algebraic equation for optimal side slope has been proposed, which is nearly exact to Eq. (32b)

$$z^* = 1/\left(\sqrt{2} + 2F^*\right) \quad (33)$$

where superscript * denotes optimum value.

Depth Dependent Free-Board

Taking partial derivatives of A_{t*} [using Eq. (7)] with respect to y keeping in view that the freeboard is a function of y and given by $F = \varepsilon y^\delta$ we get

$$\frac{\partial A_t}{\partial y} = \frac{4z}{3} \sqrt{(y+F)/y} (4y + F(1+3\delta)) \quad (34)$$

where

$$\frac{\partial F}{\partial y} = \varepsilon \delta y^{\delta-1} = \frac{\delta F}{y} \quad (35)$$

The other values being the same as that of discharge dependent freeboard case. Substituting these values in Eq. (29) and simplifying, we get

$$\frac{(y+F)}{(4y+F(1+3d))} = \frac{5f_{1z} - 2f_{2z}}{16f_{1z}} \quad (36)$$

Solving for F

$$F = \varepsilon y^\delta = 4y \frac{f_{1z} - 2f_{2z}}{16f_{1z} - (1+3d)(5f_{1z} - 2f_{2z})} \quad (37a)$$

Thus

$$\varepsilon = 4y^{1-\delta} \frac{f_{1z} - 2f_{2z}}{16f_{1z} - (1+3d)(5f_{1z} - 2f_{2z})} \quad (37b)$$

Substituting y from Eq. (20a) and solving

$$\varepsilon_* = 4 \left(\frac{3}{8z}\right)^{5(1-\delta)/8} f_{1z}^{(1-\delta)/4} \frac{f_{1z} - 2f_{2z}}{16f_{1z} - (1+3d)(5f_{1z} - 2f_{2z})} \quad (38)$$

where

$$\varepsilon_* = \varepsilon/L^{1-\delta} \quad (39)$$

Fig. 3 also plots Eq. (38) for values of ε_* for three values of δ (i.e. $\delta = 0.4, 0.5$ and 0.6) for wide range of z . Similar to discharge dependent freeboard case a simple explicit algebraic equation for optimal side slope has been proposed, which is nearly exact to Eq. (38) with $\delta = 0.5$

$$z^* = \frac{1}{\sqrt{2}} - \varepsilon_* / (1.795 + 2\varepsilon_*) \quad (40)$$

Without Freeboard

It is evident from Fig. 3 that the value of F^* or ε_* approaches zero at $z = 1/\sqrt{2}$. This can be verified by putting F^* equal to zero in Eq. (33b) or ε_* equal to zero in Eq. (38) and solving for optimal z without considering freeboard

$$f_{1z} = 2f_{2z} \Rightarrow 4z + \frac{2}{3z} = 2\left(4z - \frac{2}{3z}\right) \Rightarrow z^* = 1/\sqrt{2} \quad (41)$$

Substituting the value of z^* from Eq. (41) in the Eqs. (20b) and (21a-d), respectively, have yielded the non-dimensional optimal values of other parameters as

$$y_*^* = 0.93748; \quad A_*^* = 1.65721; \quad T_*^* = 2.65159 \quad (42a-c)$$

$$P_*^* = 3.53546; \quad V_*^* = 0.60342 \quad (42d,e)$$

5. OPTIMAL DESIGN PROCEDURE AND EXAMPLE

The optimal parabolic channel section with freeboard can be designed by adopting the following steps:

- (i) Choose n for a particular type of lining.
- (ii) For a given set of data (Q , and S_o) and chosen n find L using Eq. (17).
- (iii) For given Q , read value of F from Table 1 for discharge dependent freeboard. For the value of ε in case of depth dependent freeboard Eq. (4) could be used. Then obtain F_* or ε_* (depending on the case) with the help of L .
- (iv) Use of the appropriate optimal design equation for a discharge dependent freeboard or depth dependent freeboard case, results in the optimal channel side slope. Alternatively, read out the value of z corresponding F_* or ε_* from Fig 3.
- (v) Using the optimal side slope the remaining geometrical parameters in non-dimensional form can be obtained with help of Eqs. (20b) and (21b-d) or using Fig 2.
- (vi) Use of L and non-dimensional parameters yield corresponding parameters for the optimal parabolic channel.
- (vii) For the designed section the average flow velocity V can be obtained by Eq. (2) i.e. $V = Q/A$ or with the help of Fig. 2 or using Eq. (21d). This velocity should be greater than the non-silting velocity but less than the limiting velocity V_L .
- (viii) If V is greater than V_L , redesign the section with revised bed slope or surface roughness.
- (ix) Finally total area and side slope and top width at freeboard level can be calculated by Eqs. (7) – (9) respectively.

Example

Design an optimal parabolic channel to carry a discharge of $75 \text{ m}^3/\text{s}$ on a longitudinal bed slope of 0.001 .

Solution: Assuming concrete lining, Manning's roughness coefficient = 0.015 (Chow 1973). From Eq. (17) $L = \left(75 \times .02 / \sqrt{0.001}\right)^{3/8} = 3.8166 \text{ m}$.

Discharge dependent Freeboard: From Table 1 value of freeboard for given discharge ($75 \text{ m}^3/\text{s}$) equals 0.75 m . Thus $F_* = 0.75/4.2514 = 0.1965$. From Eq. (33) the corresponding value of z is 0.55 (0.53 from Fig 3). Using

Eqs. (20b) to (21d) (or alternatively from Fig 2) $y^* = 1.0698$, $P^* = 3.6502$; $T^* = 2.3536$; $A^* = 1.6785$; and $V^* = 0.5958$; therefore $y^* = 1.0698 \times 3.8166 = 4.083$ m; $P^* = 3.6502 \times 3.8166 = 13.931$ m; $T^* = 2.3536 \times 3.8166 = 8.983$ m; $A^* = 1.6785 \times (3.8166)^2 = 24.450$ m²; and $V^* = 0.5958 \times 75 / (3.8166)^2 = 2.3777$ m/s. Alternatively $V = Q/A = 75/24.450 = 3.07$ m/s, which is less than the limiting velocity for a concrete lining i.e. 4.0 m/s (Chahar 2001). Substituting value of y^* , F and z^* in Eq. (8) we get $z_1^* = 0.506$; and from Eq. (9) $T_1^* = 9.772$ m; and A_1^* from Eq. (7) equals 31.517 m².

Depth dependent Freeboard: Using Eq. (5) for $Q = 75$ m³/s, $\varepsilon = 0.8489$ m^{1/2}, hence non-dimensional value from Eq. (39) and $d = 0.5$ becomes $\varepsilon_s = 0.4345$. Value of z corresponding to this value from Eq. (40) is 0.544 (0.53 from Fig 3). Following steps with $z = 0.544$ similar to the discharge dependent freeboard, $y^* = 4.108$ m; $P^* = 13.973$ m; $T^* = 8.939$ m; $A^* = 24.481$ m; $V^* = 3.064$ m/s. Using Eq. (4) $F = 0.8489(4.108)^{0.5} = 1.72$ m. Therefore, $z_1 = 0.457$; $T_1^* = 10.647$ m; and $A_1^* = 41.392$ m².

CONCLUSIONS

Using Lagrange's method of undetermined multiplier method, optimal design of parabolic channel section considering freeboard can be done. Presented equations are simple and in explicit algebraic form, which are convenient to use in the optimal design of a parabolic channel considering freeboard either discharge dependent or depth dependent. The method results in channel dimensions in single step computations. Alternatively, the presented graphs can be used as a ready reckoner. The optimal section with freeboard is narrower and deeper than the best hydraulic section without freeboard.

REFERENCES

Atmapoojya, S.L. and R.N. Ingle, Optimal design for a canal section, *J. Indian Water Works Association*, 51 – 54, 2001.
Atmapoojya, S.L. and R.N. Ingle, The optimal canal section with consideration of freeboard, *J. Institution of Engineers (India)*, 83 (CV4),

268 – 272, 2003.

Babaeyan-Koopaei, K. Dimensionless curves for normal depth calculations in canal sections, *J. Irrig. and Drain. Engrg., ASCE*, 127(6), 386-389, 2001.

Babaeyan-Koopaei, K., E.M. Valentine and D.C. Swailes, Optimal design of parabolic bottomed triangle canals, *J. Irrig. and Drain. Engrg., ASCE*, 126(6), 408-411, 2000.

Bureau of Indian Standards, Criteria for design of lined canals and guidelines for selection of type of lining, *IS:10430*, Bureau of Indian Standards, New Delhi, 1982.

Chahar, B.R., Optimal Design of Parabolic Canals, *J. Irrig. and Drain. Engrg., ASCE*, 131(6), 546 – 554, 2005.

Chahar, B.R., Optimal Design of a special class of curvilinear bottomed channel section, *J. Hyd. Engrg., ASCE*, 133(4), In Press, 2007.

Chow, V.T., *Open channel hydraulics*. McGraw-Hill, New York, 1973.

Froehlich, D.C., Width and depth constrained best trapezoidal section, *J. Irrig. and Drain. Engrg., ASCE*, 120(4), 828-835, 1994.

Godara, R., Optimal Design of Parabolic channel section, *M.E. Thesis*, Dept. of Civil Engg., JNV University, Jodhpur, 2003.

Kreyszig, E., *Advanced engineering mathematics*, John Wiley & sons (ASIA) pvt. Ltd. Singapore, 2001.

Loganathan, G.V., Optimal design of parabolic canals, *J. Irrig. and Drain. Engrg., ASCE*, 117(5), 716-735, 1991.

Mironenko, A.P., L.S. Willardson and S.A. Jenab, Parabolic canal design and analysis, *J. Irrig. and Drain. Engrg., ASCE*, 110(2), 241-246, 1984.

Monadjemi, P., General formulation of best hydraulic channel section, *J. Irrig. and Drain. Engrg., ASCE*, 120(1), 27-35, 1994.

Novman, A., Optimal Parabolic channel section with freeboard, *M.E. Thesis*, Dept. of Civil Engg., JNV University, Jodhpur, 2003.

Streeter, V. L., Economical canal cross sections, *Trans. ASCE*, 110, 421-430, 1945.

Swamee, P.K., Discussion on 'General formulation of best hydraulic channel section' by P. Monadjemi, *J. Irrig. and Drain. Engrg., ASCE*, 121(2), 222, 1995.

Swamee, P.K., G.C. Mishra and B.R. Chahar, Minimum cost design of lined canal sections, *J. Water Resour. Mangmnt.*, 14(1), 1-12, 2000a.

Swamee, P.K., G.C. Mishra and B.R. Chahar, Comprehensive design of minimum cost irrigation canal sections, *J. Irrig. and Drain. Engrg., ASCE*, 126(5), 322-327, 2000b.

Swamee, P.K., G.C. Mishra and B.R. Chahar, Design of minimum earthwork cost sections, *J. Water Resour. Mangmnt.*, 15(1), 17-30, 2001.

Swamee, P.K., G.C. Mishra and B.R. Chahar, Optimal design of transmission canal, *J. Irrig. and Drain. Engrg., ASCE*, 128(4), 234-243, 2002.