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MACRO MECHANICAL ANALYSIS OF BAMBOO LAMINA

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ABSTRACT

Dry bamboo culms of *Dendrocalamus* strictus were processed into thin laminas selected from different regions. Tensile properties of bamboo laminae increases from inner to outer region for any cross section and the same is experienced from bottom to top. The trends of the variation of elastic properties and longitudinal tensile strength with direction of a fiber direction of bamboo lamina have been evaluated.

Keywords: Lamina/ply; Laminates; Strength

INTRODUCTION

Aminuddin and Latif (1991) has reported that the bamboo has phenomenal growth rate potential, with some species growing at a rate of 15 to 18 cm daily, with maximum height in just four to six months. Wang et al (1987) stated that there are about 60 to 70 genera and over 1,200 – 1,500 species of bamboo in the world. Dendrocalamus strictus species is commonly recognized as culcutta bamboo, male bamboo and solid bamboo and widely used bamboo in India (Ferrelly,1984). Lee et al. (1994) determined the physical and mechanical properties of Phyllostachys bambusoides bamboo grown in South Carolina, USA. This study concluded that moisture content, height location in the culm, presence of nodes and orientation of the outer bark affect the mechanical and physical properties. Some of the mechanical properties of bamboo pole is enough for structural purposes where specific tensile strength of bamboo is four times better than that of steel (Lakkad and Patel, 1980). Chung (Chung et al., 2002) and Naik (2004) has reported that the tensile , compressive and bending strength of natural round hallow bamboo culms for different species were in the range of 111-219 MPa, 53-100MPa, and 86-229MPa respectively. Tensile strength are proportional to volume fraction of fibers of bamboo where volume fraction of fibers is dense in the outer region (60~65%), sparse (15~20%) in the inner region and increases linearly with height by about 20~ 40% . Therefore the bamboo belongs to the class of material known as functionally graded material (FGM) (Amada et al., 1997 and Janssen, 2000). Jones(1975) and Datto (1991) has reported about stiffness and strength analysis for fibrous composites at macroscopic scale. Furthermore, the majority of the publication reported the mechanical properties of natural hollow bamboo either as a single specimen or as a fiber bundle or as a composite with adhesive while very few reported about some of the variation of elastic properties and strength of bamboo laminae.

MATERIALS AND METHODS

Four year old green bamboo (*Dendrocalamus strictus*) culms were obtained from TERI Gram (Tata Energy and Resource Institute), District Gurgaon (Haryana), India because it grows all over India and it is strong material compare to other species. Moisture content of green bamboo collected were 39% at the time of felling (Digital moisture meter model MD-4G). Moisture content of green bamboo was reduced to 10-12% by sundry. A full length bamboo was labeled at nodes and internodes as shown in Fig. 1. Bamboo was cut length-wise into six slats using radial hydraulic splitting machine. Each slat was sliced using sliver cutting machine and crosscut for suitable dimension using hacksaw respectively for outer, middle and inner as shown in Fig.1. Laminae were prepared from slivers as per ASTM standard D3039. Laminae were in the form of rectangular cross-section which were 200mm overall length, 100mm gage length and 15mm wide with a thickness of 1.5mm as shown in Fig. 2. Three samples were prepared from each location for obtaining variation and average of test results.

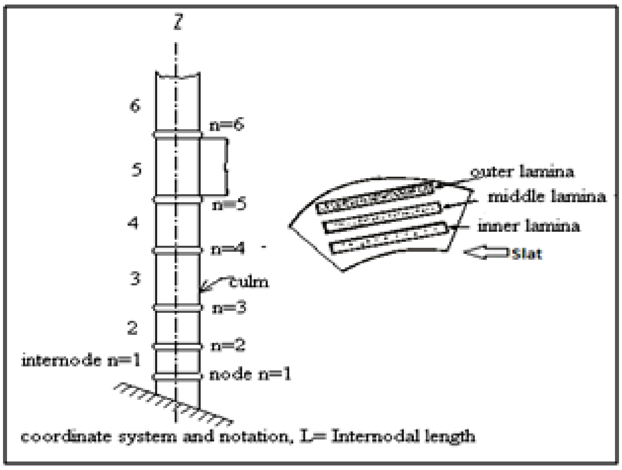

Fig. 1 : Bamboo culms and location of laminas on slats

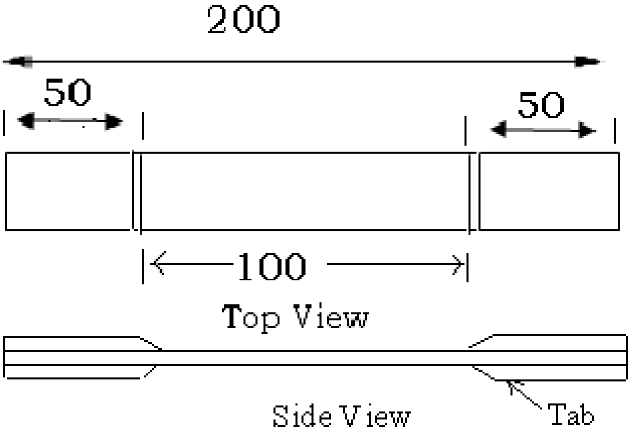

Fig. 2 : Sketch of lamina specimens with tab (all dimension in mm)

TENSILE TESTING

The experiments were performed on universal testing machine (Instron) under axial loading. Averages of three measurements were taken of each lamina specimens. The laminae were carefully positioned at the center of the cross-head with its end faces exactly perpendicular to the longitudinal axis to get accurate results. The experiments were conducted at a constant crosshead speed 2mm/min. Tensile failure strength and its young`s modulus i.e. stiffness were recorded from machine for all laminae along the length of bamboo selected from outer, middle and inner region of cross section of culms as shown in Table 1 and Table 2 respectively. Strength (i.e. maximum tensile failure stress) and stiffness (i.e. young modulus) of a lamina from top middle region is 193MPa and 18GPa respectively. Other properties of lamina for same species of bamboo were taken from literature. The summarized elastic constants of lamina are given in Table 3.

Table 1 Tensile Failure Stress (MPa) (max value assigned as specimen 1 and lowest as 3)

Regions	**Internodal Number**					
	Specimen No.	**1**	**4**	**8**	**11**	**14**
Outer region	1	250.3	290.6	329.4	322.2	324.1
	2	235.0	256.7	260.3	280.2	290.7
	3	228.5	230.3	245.1	248.4	287.9
	Ave.	237.93	259.2	278.2	283..6	300.9
Middle region	1	225.4	197.2	232.1	222.1	268.6
	2	162.6	178.3	210.7	199.9	230.2
	3	130.8	135.5	165.8	158.0	186.7
	Ave.	172.93	170.33	202.86	**193.0**	228.5
Inner Region	1	118.1	110.1	203.9	204.3	228.2
	2	92.6	106.5	150.3	162.5	227.1
	3	83	94.7	141.2	152.6	203.2
	Ave.	97.9	103.76	165.13	173.13	219.5

Table 2 : Tensile Young Modulus GPa (max value assigned as specimen 1 and lowest as 3)

Regions	**Internodal Number**					
	Specimen No.	**1**	**4**	**8**	**11**	**14**
Outer region	1	15.6	16.2	16.4	17.6	16.6
	2	14.4	15.7	16.5	15.3	16.0
	3	13.8	13.8	16.6	12.8	16.21
	Ave.	14.6	15.2	16.50	15.23	16.27
Middle region	1	16.8	17	18.8	16.5	15.66
	2	14.1	16.2	16.4	17.2	15.6
	3	13	15.4	16.7	20.3	16.8
	Ave.	14.63	16.2	17.3	**18.0**	16.02
Inner Region	1	8.1	8.7	14.1	9.4	14.3
	2	8.0	9.1	12.2	11.29	14.0
	3	7.8	8.2	11.8	8.2	12.8
	Ave.	7.96	8.66	12.7	9.63	13.7

Table 3 : Elastic properties

Properties of bamboo lamina	**Values**
Young Modulus along fiber direction	E_1=18GPa (test result)
Young Modulus across fiber direction	E_2= 2GPa (available data)
Shear modulus	G_{12}= 0.672 GPa(available data)
Poisson ratio	υ_{12}= 0.32 (available data)
Maximum failure tensile stress	X_t = 193 MPa (test result)
Maximum failure compressive stress	Y_t =8 MPa (test result)
Maximum interlaminar shear stress	S = 9.2 MPa (available data)

RESULTS AND DISCUSSIONS

Experimental investigation indicate that tensile failure strength and young modulus both increases with height and also increases from inner region to outer region due to increase in volume fraction of fibers as given in Table 1 and Table 2 respectively. Maximum tensile strength and stiffness (young modulus) at the top middle region lamina were 193MPa and 18GPa respectively. These test results are used at random for stiffness and strength analysis.

Stiffness Analysis

The process of forming the stress-strain relationship is termed the stiffness analysis where [Stresses σ] = [Stiffness E] [Strains ε]. Let combined stress-system in x-y and 1-2 axes is shown in Fig 3.

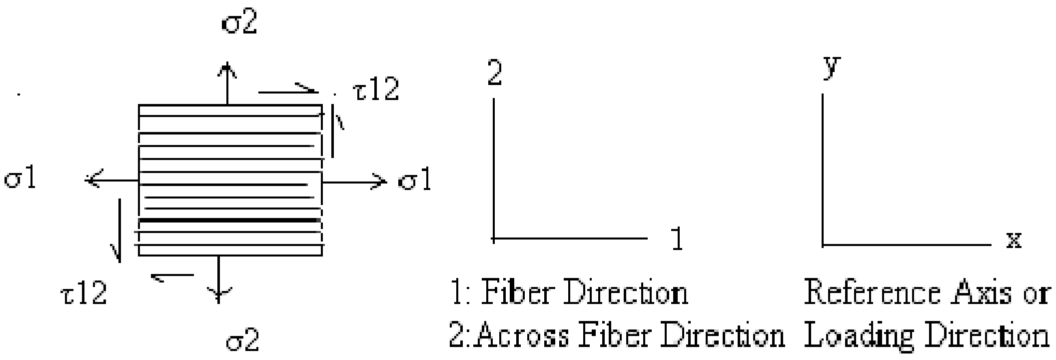

Fig. 3 : Positive combined stress-system in x-y and 1-2 axes

Then full stress-strain relationship in terms of reduced stiffness terms denoted by C_{ij} or simply C is given below in matrix form

	ε_1	ε_2	γ_{12}
σ_1	C_{11}	C_{12}	0
σ_2	C_{21}	C_{22}	0
τ_{12}	0	0	C_{33}

(4.1)

Where, σ and ε indicate direct stress and strain respectively. Similarly τ and γ indicate shear stress and strain respectively. For composite laminae/plies analysis, E_1=18GPa, E_2= 2GPa, G_{12}= 0.672GPa and υ_{12}= 0.32 are known. However there is need of minor poisons ratio υ_{21} also. It can be determined as (By assuming homogenous orthotropic).

$$\frac{\upsilon_{21}}{\upsilon_{12}} = \frac{E_2}{E_1}$$

$$\upsilon_{21} = \frac{2}{18} x\ 0.32 = 0.035$$

therefore $1-\upsilon_{12}\upsilon_{21}=1-0.32\mathrm{x}0.035=1-0.0112$

$=0.9888$

$$C_{11} = \frac{E_1}{1-\upsilon_{12}\upsilon_{21}} = \frac{18}{0.9888} = 18.2038\ Gpa$$

$$C_{12} = \upsilon_{21} C_{11} = 0.6371\ Mpa$$

$$C_{22} = \frac{E_2}{1-\upsilon_{12}\upsilon_{21}} = \frac{2}{0.9888} = 2.0222\ Gpa$$

$$C_{33} = G_{12} = 0.672\ Gpa$$

Then reduced stiffness terms are

$$C = \begin{bmatrix} 18.2038 & 0.6371 & 0 \\ 0.6371 & 2.0222 & 0 \\ 0 & 0 & 0.672 \end{bmatrix} Gpa \tag{4.2}$$

The first step in calculating the lamina stiffness is to determine reduced stiffness terms of the laminates. Reduced stiffness values of the bamboo lamina are given in equation (4.2). Next, the transformed reduced stiffness terms for a lamina angle of 0^0 (i.e. loading direction is coincident with fiber direction) is obtained from equation (4.3), where m=cosθ =1, n =sinθ =0.

	C_{11}	C_{22}	C_{12}	C_{33}
$\overline{C_{11}}$	m^4	n^4	$2m^2n^2$	$4m^2n^2$
$\overline{C_{22}}$	n^4	m^4	$2m^2n^2$	$4m^2n^2$
$\overline{C_{33}}$	m^2n^2	m^2n^2	$-2\ m^2n^2$	$(m^2 -n^2)^2$
$\overline{C_{12}}$	n^2n^2	m^2n^2	m^4+n^4	$-4m^2n^2$
$\overline{C_{13}}$	m^3n	$-mn^3$	$mn^3 – m^3n$	$2\ (mn^3–m^3n)$
$\overline{C_{23}}$	mn^3	$-\ m^3n$	m^3n-mn^3	$2\ (m^3n -mn^3)$

(4.3)

Then, the transformed reduced stiffness terms $(\overline{C_{ij}})_0$ will be given in equation (4.4).

$$\bar{C} = \begin{bmatrix} 18.2038 & 0.6371 & 0 \\ 0.6371 & 2.0222 & 0 \\ 0 & 0 & 0.672 \end{bmatrix} Gpa \tag{4.4}$$

From equation (4.2) and (4.4), it is clear that $\overline{C_{ij}}$ =C_{ij} if loading direction is similar with lamina fiber direction i.e. at 0^0.

Transformation of elastic constants

When θ ranging from 0^0 to 90^0 with increment of 10^0. Then the variation of elastic properties with direction of a ply angle in unidirectional lamina / ply is given here under.

1. Variation of the young's modulus in the reference x-direction, E_x is given by following equation.

$$\frac{1}{E_x}=\frac{m^4}{E_1}+\frac{n^4}{E_2}+m^2n^2\left(\frac{1}{G_{12}}-\frac{2v_{12}}{E_1}\right)$$

2. Variation of the young's modulus in the reference y-direction is given by

$$\frac{1}{E_y}=\frac{n^4}{E_1}+\frac{m^4}{E_2}+m^2n^2\left(\frac{1}{G_{12}}-\frac{2v_{12}}{E_1}\right)$$

3. Variation of in plane shear modulus with direction is given by following equation

$$\frac{1}{G_{xy}}=m^2n^2\left(\frac{4}{E_1}+\frac{4}{E_2}+\frac{8v_{12}}{E_1}\right)(m^2-n^2)^2\frac{1}{G_{12}}$$

4. The variation of major poissons ratio with respect to the reference x-y axis is given by following equation.

$$v_{xy}=E_x[(m^4+n^4)\frac{v_{12}}{E_1}-m^2n^2(\frac{1}{E_1}+\frac{1}{E_2}-\frac{1}{G_{12}})$$

5. The minor poisson's ratio in the reference axes is obtained from the following relationships,

$$v_{yx}=\frac{E_y}{E_x}.v_{xy}$$

6. The variation of shear coupling coefficient $\eta_{xy,x}$ with respect so the reference x-y axis is given by following equation.

$$\eta_{xy,x}=E_x\left[m^3n\left(\frac{1}{G_{12}}-\frac{2v_{12}}{E_1}-\frac{2}{E_1}\right)-mn^3\left(\frac{1}{G_{12}}-\frac{2v_{12}}{E_1}-\frac{2}{E_2}\right)\right]$$

7. The variation of the shear coupling coefficient $\eta_{xy,y}$,with direction is obtained from following equation.

$$\eta_{xy,y}=E_y\left[mn^3\left(\frac{1}{G_{12}}-\frac{2v_{12}}{E_1}-\frac{2}{E_1}\right)-m^3n\left(\frac{1}{G_{12}}-\frac{2v_{12}}{E_1}-\frac{2}{E_2}\right)\right]$$

The trends of the variation of elastic properties with direction of a unidirectional ply shown in Figs. 4 to 7 are typical one, although the actual magnitude of the elastic constants will depend on the material used.

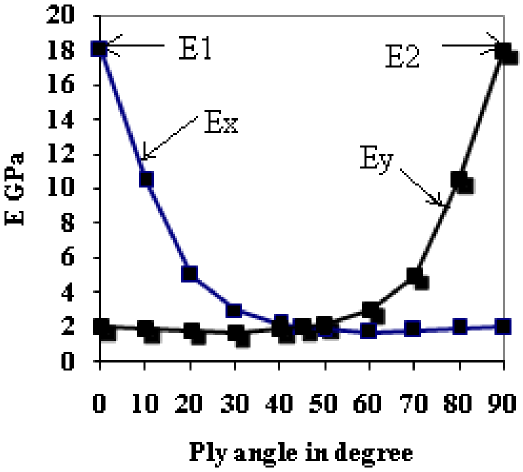

Fig. 4 : Young`s modulus variation with lamina/ ply angle : Unidirectional ply

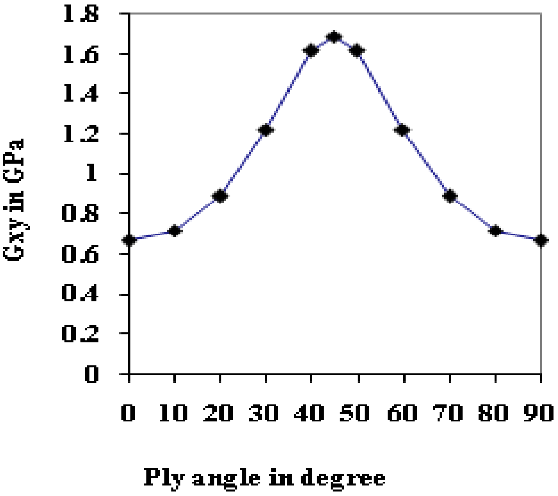

Fig. 5 : Shear modulus variation with ply angle

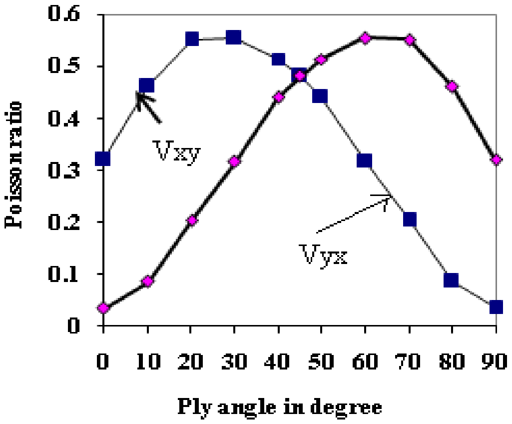

Fig. 6 : Poisson `s ratio variation with ply angle

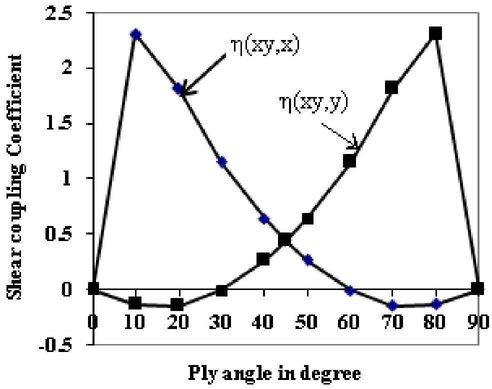

Fig. 7 : Shear coupling coefficient variation variation with ply angle

Strength analysis and mode of failure in lamina under tensile loading

Variation of the longitudinal tensile strength with the ply angle θ of a bamboo lamina before the failure was determined by Tsai-Hill Criteria as given below.

Consider values of $\theta = 0^0, 2^0, 4^0, 6^0, 8^0$ and 10^0 and then in 10^0 increments up to 90^0.

Let the ply be oriented at an arbitrary positive angle θ with reference to the x-axis (Fig.8) and subjected to a tensile stress σ_x in the x-direction.

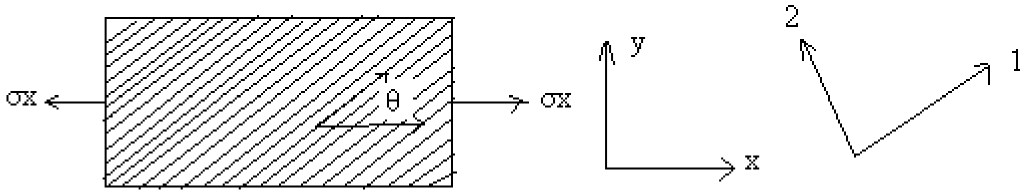

Fig. 8 : Longitudinal tensile stress only on a unidirectional generally orthotropic ply

In this case $\sigma_y = 0$, $\sigma_{xy} = 0$ .Then we can transform the stress into the material axes 1-2 by using following equation.

	σ_x	σ_y	τ_{xy}
σ_1	m^2	n^2	$2mn$
σ_2	n^2	m^2	$-2mn$
τ_{12}	$-mn$	mn	m^2-n^2

Where m= cos θ, n=sin θ

Thus $\sigma_1 = m^2\,\sigma_x = \sigma_x \cos^2\theta$

$\sigma_2 = n^2\,\sigma_x = \sigma_x \sin^2\theta \qquad \tau_{12} = -\sigma_x mn = -\sigma_x \cos\theta \sin\theta$

The stresses in the material axes system, given in terms of the angle θ and the unknown value σ_x, are then substituted in a failure criteria with the failure index set to 1 and $\sigma_x = \sigma_x$ for lamina failure just to occur. The resulting equation is then solved for σ_x for particular values of θ.

Now consider the Tsai-Hill failure criterion of following equation, and set the failure index to 1 for lamina failure just to occur:

$$\text{F.I}= \left(\frac{\sigma_1}{X}\right)^2+\left(\frac{\sigma_2}{Y}\right)^2+\left(\frac{\sigma_{12}}{S}\right)^2-\frac{\sigma_1\sigma_2}{X^2}$$

Where X = Xt and Y= Y_t corresponding to the sign of σ_1 and σ_2 .Substituting the appropriate values, we get

$$\left(\frac{\sigma_x \cos^2\theta}{193}\right)^2+\left(\frac{\sigma_x \sin^2\theta}{8}\right)^2+\left(\frac{-\sigma_x \cos\theta \sin\theta}{9.2}\right)^2-\frac{\cos^2\theta \sin^2\theta \sigma_x^2}{(193)^2}=1$$

$$\Rightarrow \sigma_x^2=\frac{1}{\frac{\cos^4\theta}{(193)^2}+\frac{\sin^4\theta}{(8)^2}+\frac{\cos^2\theta \sin^2\theta}{(9.2)^2}-\frac{\cos^2\theta \sin^2\theta}{(193)^2}}$$

σ_x value is then calculated for particular values of θ which gives graphical presentation as shown in Fig.9 for the variation of the longitudinal tensile strength σ_x with varying ply angles.

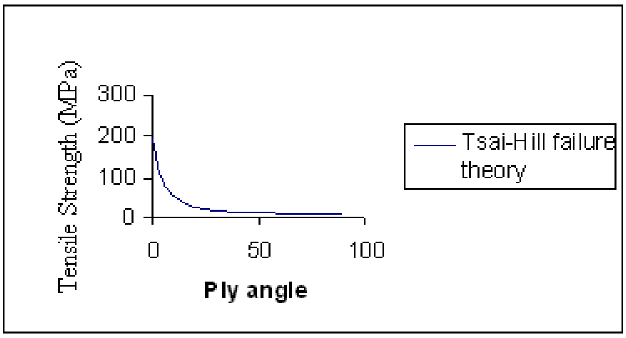

Fig. 9 : **Graphical presentation for the variation of the longitudinal tensile strength σ_x with varying lamina/ply angles**

CONCLUSIONS

(1) Tensile strength and Young's modulus of bamboo increases from inner to outer region across any cross section and from bottom to top of bamboo culms.

(2) When $\theta=0^0$,i.e., a case of a specially orthotropic ply in which the fiber direction of lamina is coincident with the reference x-direction, then the generally orthotropic ply reduces to a specially orthotropic case for $\theta=0^0$.

(3) The in plane shear modulus is largest when $\theta=45^0$ and is symmetric about this point. Thus, 45^0 plies offer the greatest resistance to shear.

(4) The young`s modulus value is greatest in the fiber direction, that is when $\theta=0^0$. This value falls off rapidly with a small change in the fiber orientation. Thus, the maximum membrane resistance is offered when the fiber are aligned in the direction of the applied membrane load. The least membrane resistance is offered in the transverse direction when $\theta=90^0$.

(5) Shear coupling effects are induced when the ply is no longer specially orthotropic, that is for ply angles other than $\theta=0^0$ or $\theta=90^0$ orientations.

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