ELL702 Non-linear Systems

MTh 5-6:30pm IV 323, https://web.iitd.ac.in/~deepakpatil/eel702.html

Evaluation

- Minor exam 30%
- Major exam 40%
- Assignments/Research Papers → 30%

Reference books

(i) W. M. Haddad & V. Chellaboina, Non-linear Dynamical systems: A Lyapunov based approach. Princeton University Press, 2008

(ii) Hassan Khalil, Non-linear Systems 3rd ed. Prentice Hall, 1993.

(iii) M. Vidyasagar, Non-linear Systems Analysis
Prentice Hall. 1993

(iv) M. Hirsch & S. Smale Differential eqnations,
Dynamical Systems & Linear
Algebra. Academic Press.
1974.

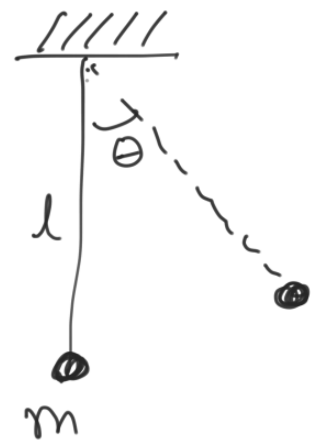
$$u \longrightarrow \boxed{f} \longrightarrow f(u)$$

$\alpha u \qquad\qquad\qquad \alpha f(u)$

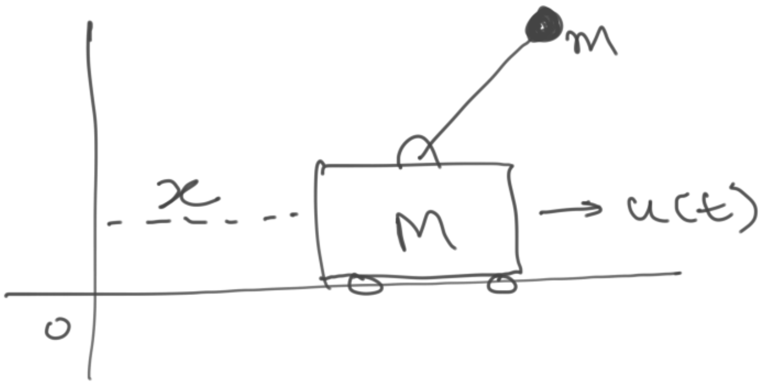
$$\begin{array}{ccc} u_1 & \qquad\qquad & f(u_1) \\ + & & + \\ u_2 & & f(u_2) \end{array}$$

Examples :

(1) Pendulum.



$$mg\sin\theta + ml\ddot{\theta} + Kl\dot{\theta} = 0.$$



$$m l^2 \ddot{\theta} - m g l \sin\theta + m \ddot{x} l \cos\theta = 0$$

$$\boxed{m \frac{d^2}{dt^2}\left(x + l\sin\theta\right) + M\ddot{x} - u(t) = 0}$$

$x_1 = \theta$, $x_2 = \dot{\theta}$

$x_3 = x$, $x_4 = \dot{x}$

$$\underbrace{\begin{bmatrix} ml\cos\theta & M+m \\ ml & m\cos\theta \end{bmatrix}} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} ml\dot{\theta}^2 \sin\theta \\ mg\sin\theta \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

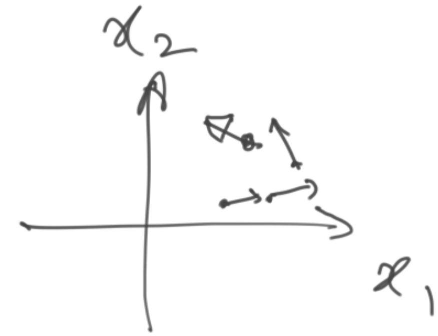
$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} x_2 \\ \cdots\cdots \\ x_4 \\ \cdots\cdots \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u(t)}$$

$$y = \cancel{\theta}\, x_1$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, x_3, x_4, u, t) \\ f_2(x_1, x_2, x_3, x_4, u, t) \\ f_3(x_1, x_2, x_3, x_4, u, t) \\ f_4(x_1, x_2, x_3, x_4, u, t) \end{bmatrix}$$

$$y = h(x_1, x_2, x_3, x_4, u, t)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2)\\ f_2(x_1, x_2)\end{bmatrix}$$



In general, $\dot{x} = f(x, u, t)$

$$y = h(x, u, t)$$

$$\begin{aligned} x &: \mathbb{R}_+ \longrightarrow \mathbb{R}^n \\ &\quad t \longmapsto x(t) \\ u &: \mathbb{R}_+ \longrightarrow \mathbb{R}^m \\ &\quad t \longmapsto u(t) \\ y &: \mathbb{R}_+ \longrightarrow \mathbb{R}^p \\ &\quad t \longmapsto \mathbb{R}^p \end{aligned}$$

$$\begin{aligned} f &: \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}_+ \longrightarrow \mathbb{R}^n \\ &\quad (x, u, t) \longmapsto f(x, u, t) \\ h &: \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}_+ \longrightarrow \mathbb{R}^p \\ &\quad (x, u, t) \longmapsto h(x, u, t) \end{aligned}$$

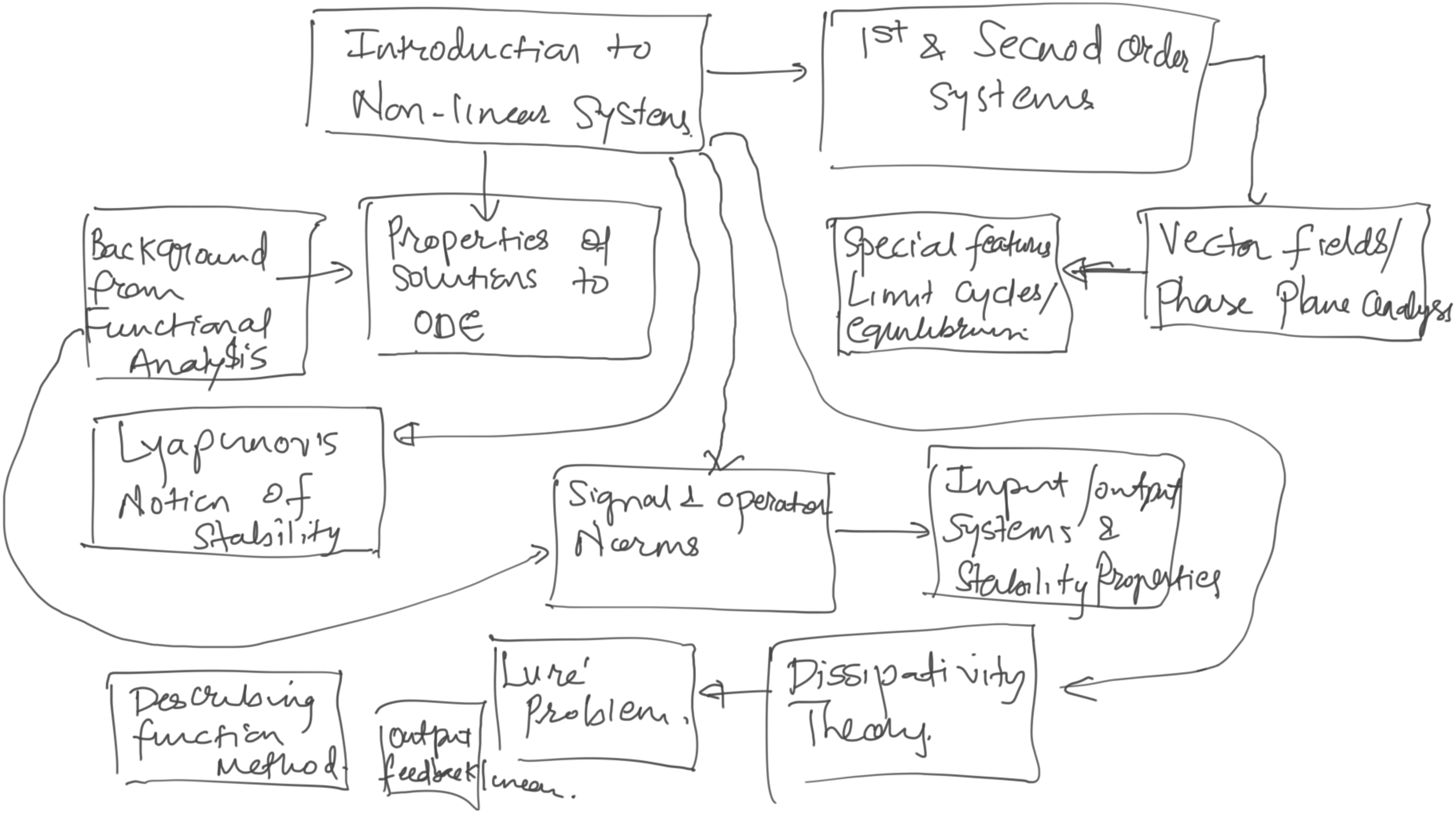
Predator-Prey Model.

$x \rightarrow$ no of Preys.

$y \rightarrow$ no of Predators.

$$\dot{x} = K_1 x - K_2 y x$$

$$\dot{y} = K_3 x y - K_4 y$$



Introduction to Non-linear Systems
1st & Secnod Order systems
Background from Functional Analysis
Properties of solutions to ODE
Special features Limit cycles/ equilibrium
Vector fields/ Phase Plane Analysis
Lyapunov's Notion of Stability
Signal & operator Norms
Input/output Systems & Stability Properties
Describing function Method
output feedback linear.
Lure' Problem.
Dissipativity Theory.