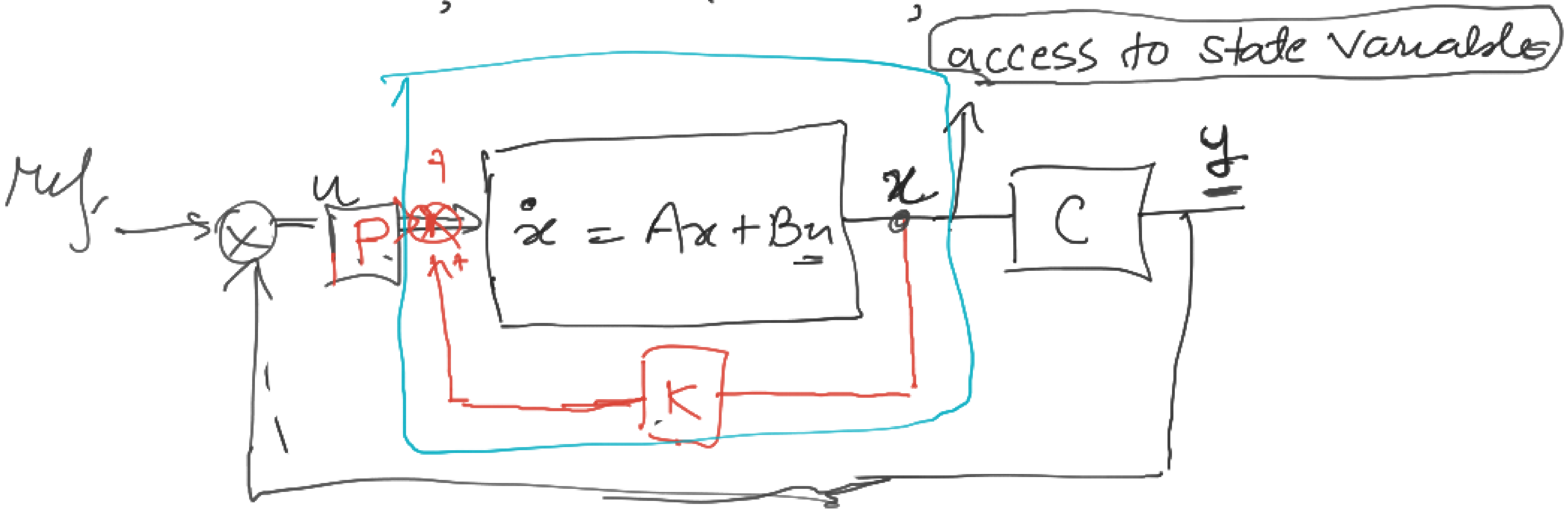
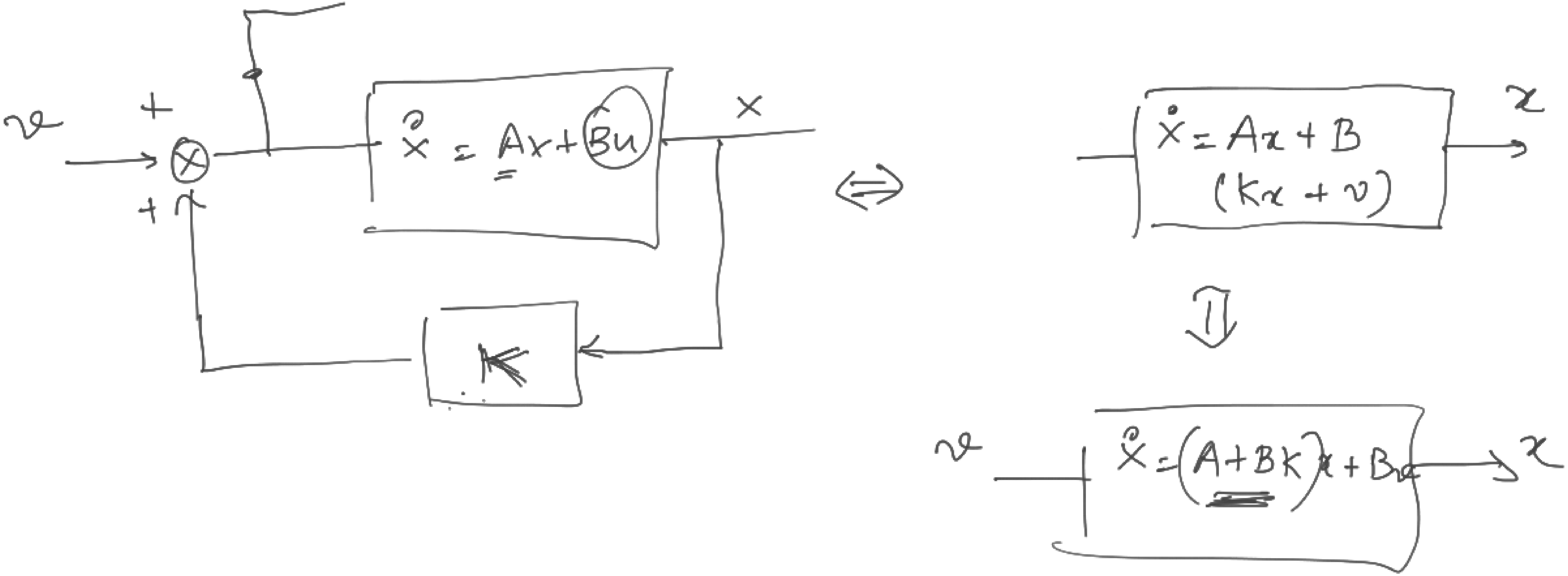
$$\dot{x} = Ax + Bu, \quad y = \underline{Cx}$$

$$x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m, \quad \underline{y} \in \mathbb{R}^p$$

$$A \in \mathbb{R}^{n \times n}, \quad B \in \mathbb{R}^{n \times m}, \quad C \in \mathbb{R}^{p \times n}.$$





$$(A + BK)$$

(A, B) is controllable $\iff$ Can choose K s.t $\Lambda(A + BK)$ are in open left half of the complex plane.

Problems in previous State-feedback Control Scheme.

(i) Needed access to State Variables

(ii) all States cannot be measured

$\Downarrow$

Can we avoid sensing all State Variables?

Notion of observable (C, A) pair?

C, A

Given $\dot{\underline{x}} = \underline{\underline{A}} x$, $\underline{\underline{y}} = \underline{\underline{C}} x$, $x(0) = x_0$

Can I reconstruct the initial Condition?

Is it possible to uniquely determine $x(t)$ from $y(t)$ given (C, A) pair?

$$
\begin{array}{cccc}
y(t), & y^{(1)}(t), & y^{(2)}(t), & \cdots \\
\| & \| & \| & \\
Cx & C\dot{x} & CA\dot{x} & \\
 & \| & \| & \\
 & CAx & CA^2 x &
\end{array}
$$

$$Cx = y$$
$$CAx = y^{(1)}$$
$$\vdots$$
$$CA^{(n-1)}x = y^{(2)}$$

$$\underbrace{\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{(n-1)} \end{bmatrix}}_{\underline{\underline{Q}}} \underline{x} = \begin{bmatrix} y \\ y^{(1)} \\ \vdots \\ y^{(n)} \end{bmatrix}$$

$$\underline{\underline{Q}}\, x = Y$$

If and only if ① is full column rank, we are able to determine x uniquely given measurement y.

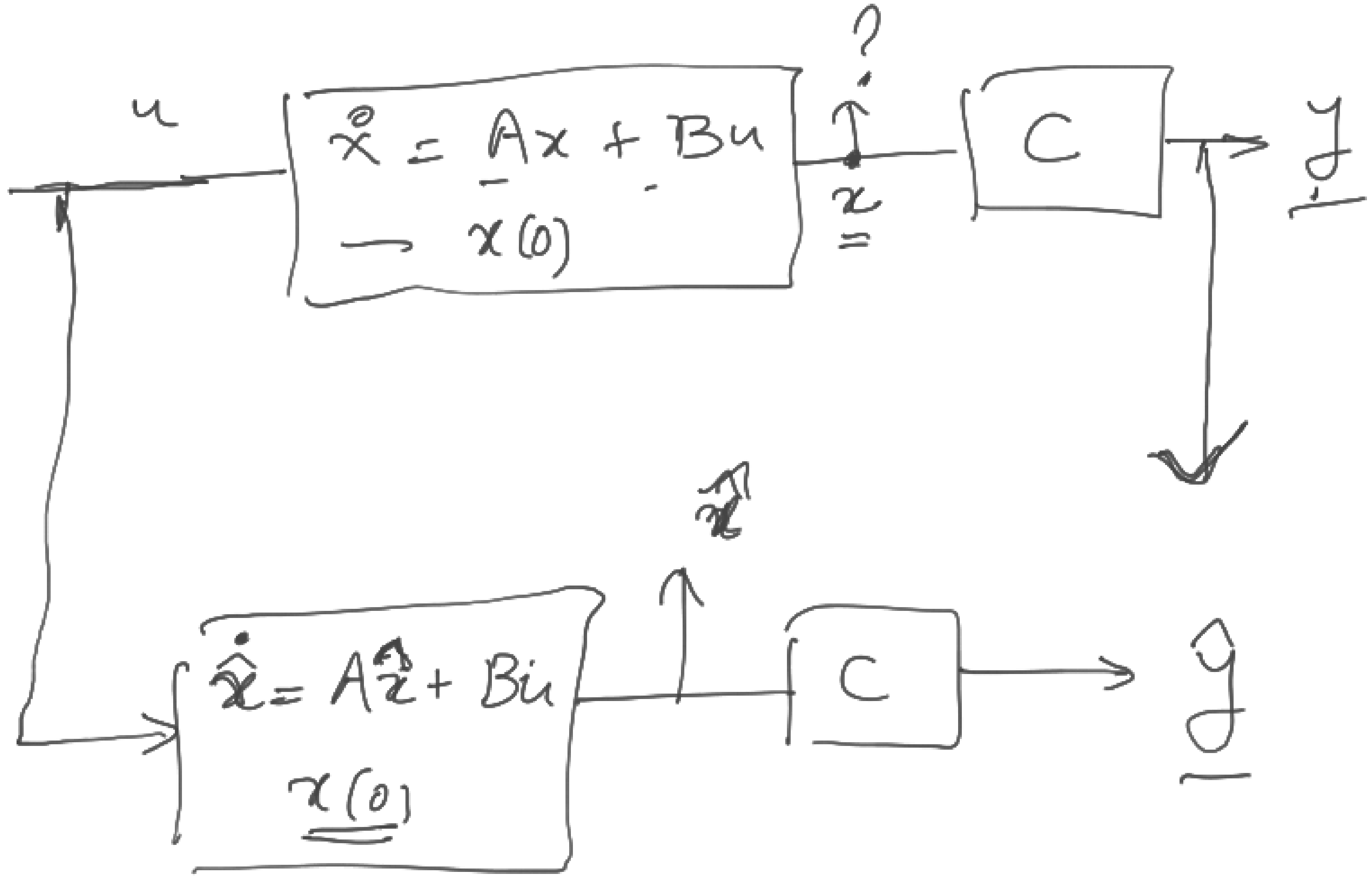
y , $y^{(1)}$, $y^{(2)}$, ...

$$\text{Rank} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} = n$$ we say

(C, A) pair is observable.

$\text{Rank}\,[B, AB, \ldots, A^{n-1}B] = n$, (A, B) is controllable.

$x(0)$



$y \neq \hat{y}$?

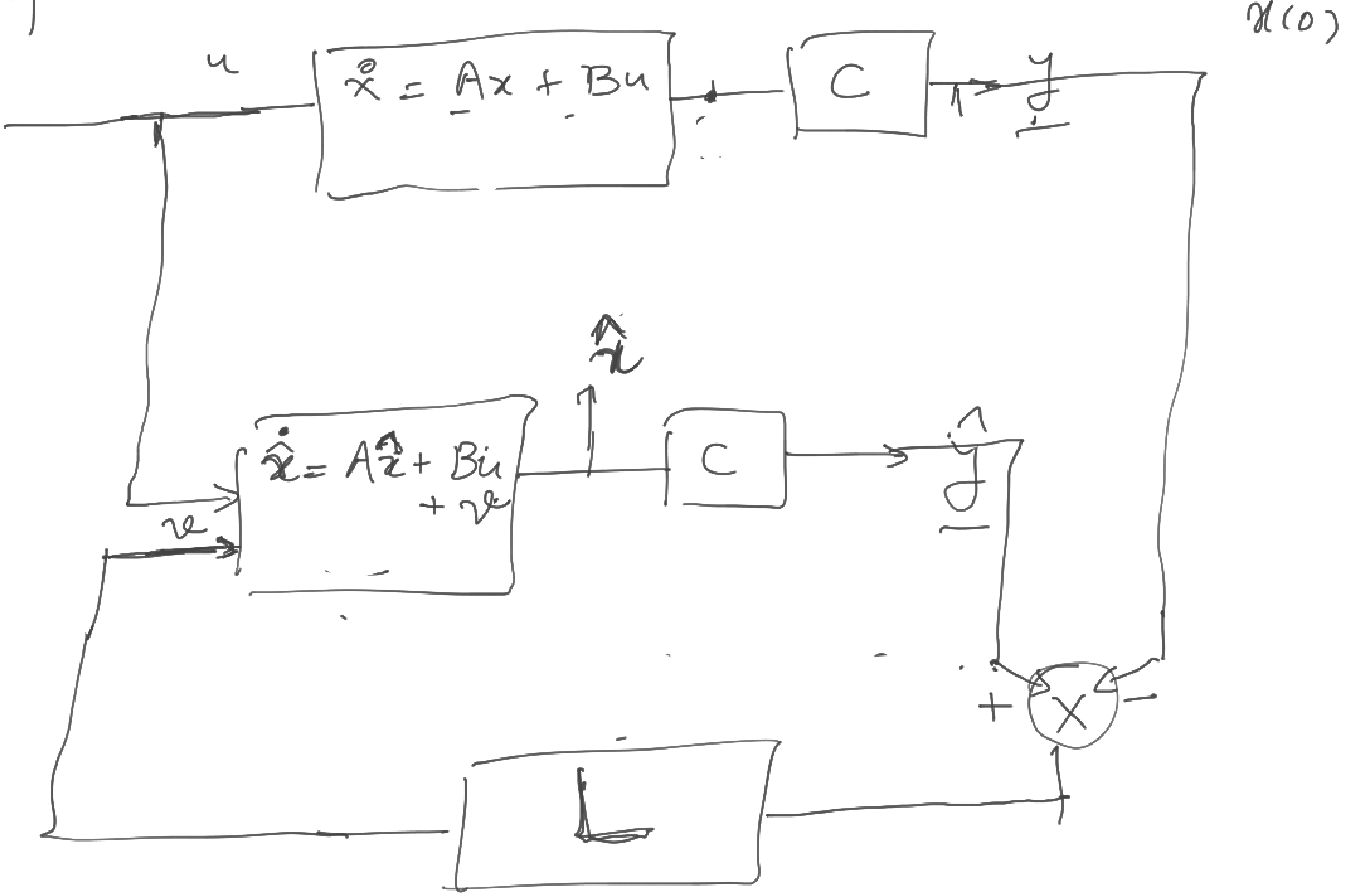
$y - \hat{y}$

$e = x - \hat{x}$, $\quad \dot{e} = Ae$

$e \to 0$ only if $\Lambda(A) \subset \mathbb{C}^-$

$\hat{x} \to x$ very slowly.

Asymptotic Observer.



$$e = x - \hat{x}$$

$\hat{x}(t) \to x(t)$

$$\dot{e} = Ax + \cancel{Bu} - A\hat{x} - \cancel{Bu} - v$$

$$= Ae - L(\hat{y} - y)$$

$$= Ae - LC(\underline{\hat{x} - x})$$

$$= Ae + LCe$$

$$= (\underline{A + LC})e$$

If $\Lambda(\underline{A+LC}) \subset \mathbb{C}^-$ then $e(t) \to 0$ as $t \to \infty$

$$(A + BK)$$

$$A + LC$$

$$A^T + C^T L^T$$ same form.

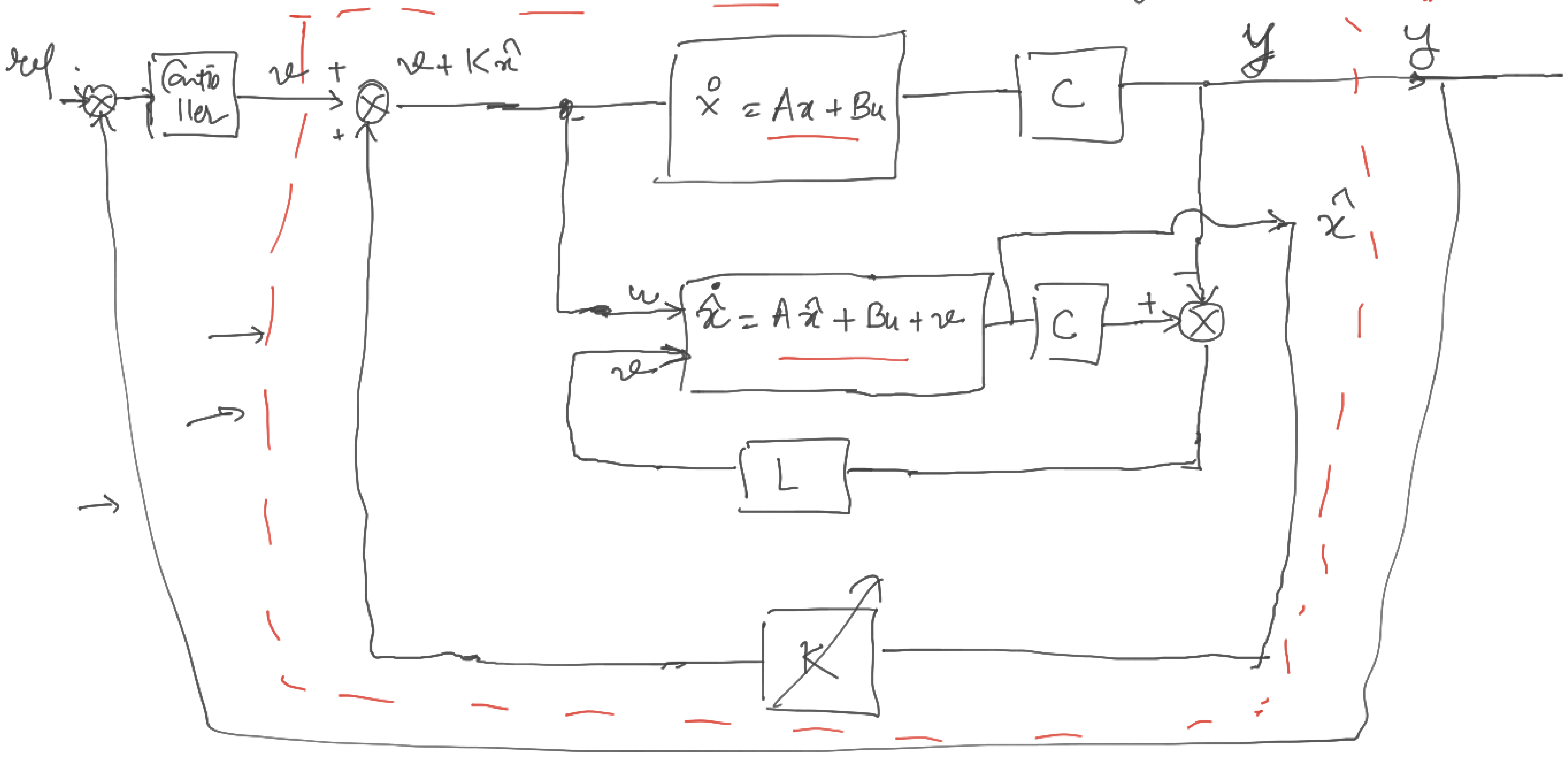
$$\text{place}\left(A,\ B,\ p(\lambda)\right) = K$$

$$\text{place}\left(A^T,\ C^T,\ p(\lambda)\right) = L^T$$

Pole placement is possible iff.

~~If~~ (C, A) Pair is observable

Combined observer-controller design.



$$\dot{x} = Ax + B(K\hat{x} + v)$$

$$= Ax + BK\hat{x} + Bv + BKx - BKx$$

$$\dot{\hat{x}} = A\hat{x} + B(v + K\hat{x}) + LC(\hat{x} - x)$$

$$= A\hat{x} + BK\hat{x} + LC(\hat{x} - x) + Bv$$

$$\dot{\hat{x}} = (A + BK)\hat{x} + LC(\hat{x} - x) + Bv.$$

$$\dot{e} = \dot{x} - \dot{\hat{x}} = (A+BK)x + BK(-e) + \cancel{Bv}$$

$$- (A + BK)\hat{x} - LC(-e) - \cancel{Bv}$$

$$= (A + BK)e - BKe + LCe$$

$$= (A + LC)e.$$

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A+BK & -BK \\ 0 & A+LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} v$$

$$\dot{x} = Ax + BK\hat{x} + Bv.$$

$$e = x - \hat{x}$$

$$\hat{x} = x - e.$$

$$\dot{x} = (A+BK)x - BKe + Bv.$$

$$\rightarrow \begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} \underline{A+BK} & -BK \\ 0 & \underline{A+LC} \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} v$$

(i) Design L s.t $\Lambda(A+LC) \subset \mathbb{C}^-$

$\Rightarrow \quad e(t) \to 0 \quad$ as $t \to \infty$.

$$\dot{x} = (\underline{A+BK})x + Bv.$$

Red block on previous slide.