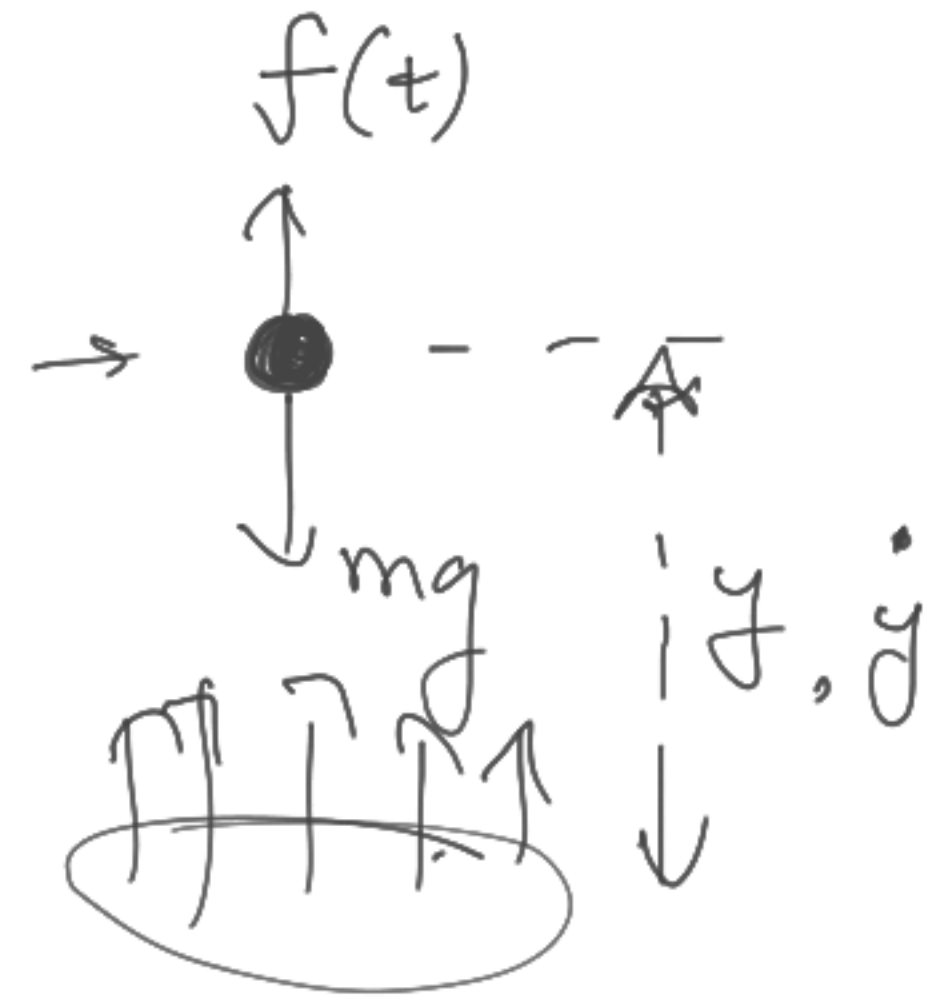


Modelling

- Force - Balance Equations. (Newtonian methods.)
- Lagrangian Method.
- Hamiltonian Based formulation



$$\underline{m\ddot{y} = f(t) - mg.}$$

$$\underline{\mathcal{L} = \frac{1}{2} m \dot{y}^2 - mgy}$$

$$\int_0^t \mathcal{L}(y, \dot{y}) dt$$

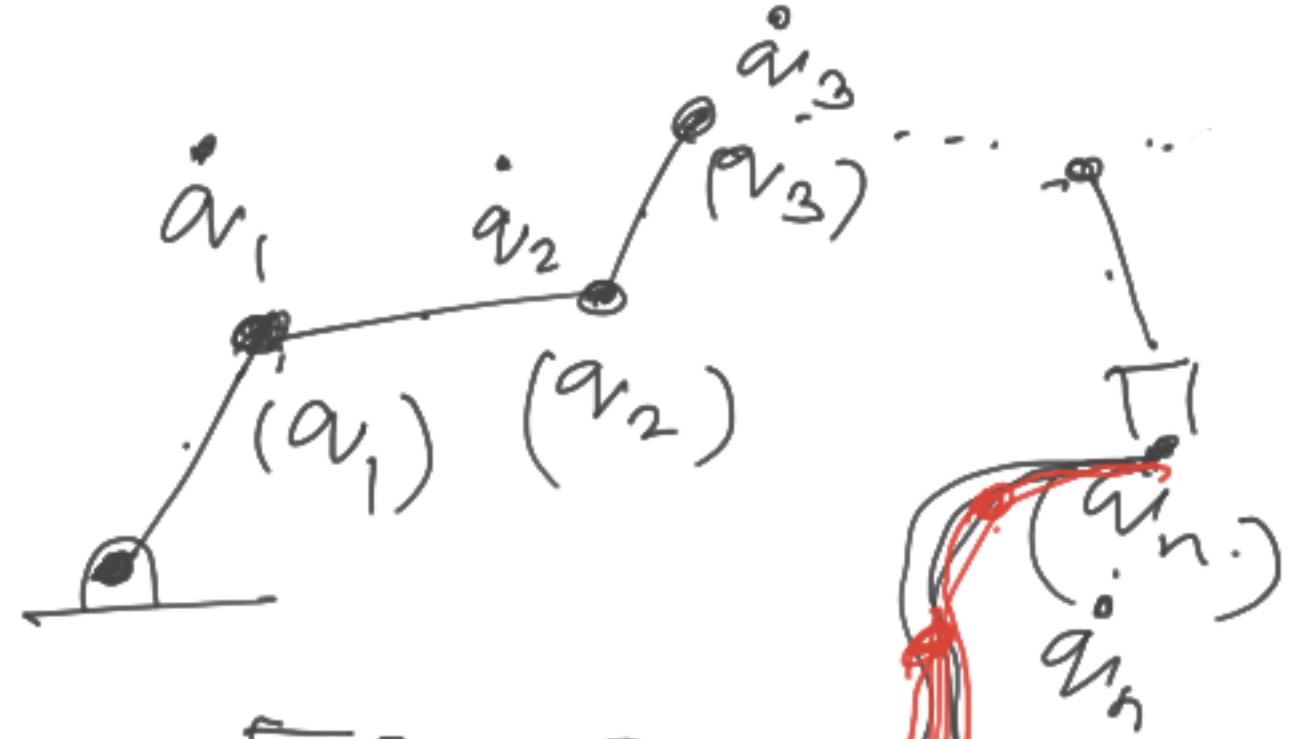
$$\rightarrow \underline{\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{y}} - \frac{\partial \mathcal{L}}{\partial y} = f(t) \Rightarrow \underline{\underline{m\ddot{y} + mg = f(t)}}$$

n-link manipulator

$$K = \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

$$P = \sum_{i=1}^3 m_i g^T r_{ci}$$

$$q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}$$



$$L = K - P, \quad \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = \tau_k$$

$$\boxed{M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + g(q) = \tau}$$

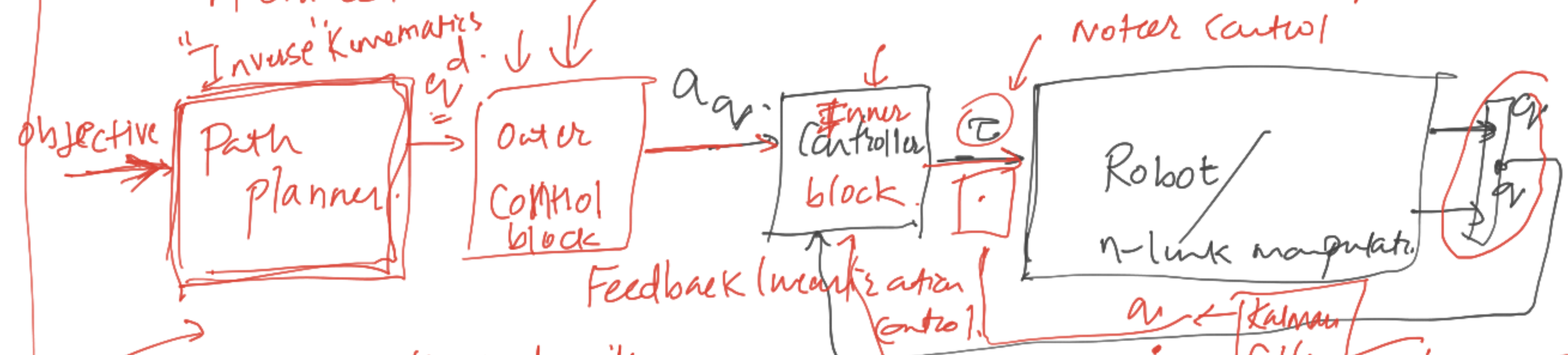
Non-linear
Differential
Equation.

Torque control method

$$\underline{M}(q) \ddot{q} + \underline{C}(q, \dot{q}) \dot{q} + \underline{g}(q) = \underline{\tau}$$

Adaptive Control *

Architecture for control n-link manipulator.



Uncertainty

$$\underline{\tau} = \underline{M}(q) \underline{a}_q + \underline{C}(q, \dot{q}) \dot{q} + \underline{g}(q)$$

$M(q)$ is always "invertible"? $\Rightarrow \underline{\ddot{q}} = \underline{a}_q$

- Linear Syste
- * Discrete time Kalman filter
- * Extended Kalman filter
- * Unscented Kalman filter
- * Particle filter

q^d , \dot{q}^d known. \ddot{q}_K^d , v_K^d

$$\ddot{x} = Ax + Bu \quad \begin{matrix} K \\ (A+BK) \end{matrix}$$

given desired trajectory q^d calculate

- v , \dot{v} , $\ddot{v} = \underline{\underline{v}} - \underline{\underline{v}}^d$, $\ddot{\underline{\underline{v}}} = \ddot{\underline{\underline{q}}} - \ddot{\underline{\underline{q}}^d}$

$$\underline{\underline{v}} = \underline{\underline{v}}_q = \underline{\underline{v}}^d - K_1 \ddot{\underline{\underline{v}}} - K_2 \dot{\underline{\underline{v}}}$$

$$\ddot{\underline{\underline{v}}} + K_1 \dot{\underline{\underline{v}}} + K_2 \underline{\underline{v}} = 0$$

$$\ddot{\underline{\underline{v}}} + 2\zeta\omega_n \dot{\underline{\underline{v}}} + \omega_n^2 \underline{\underline{v}} = 0$$

$$K_1 = \begin{bmatrix} \omega_n^2 \\ \dots \end{bmatrix}, \quad K_2 = \begin{bmatrix} 2\zeta\omega_n \\ \dots \end{bmatrix}$$