# DFT / DTFS.

(treating only some samples which are repeated)

$x[n]$ is periodic. if $x[n] = x[n+N]$ for some positive integer $N$.

Smallest such integer is fundamental period. If $N$ is fundamental period then

let $\omega_0 = \frac{2\pi}{N}$ be the fundamental frequency

<u>Question</u> : Express $x[n]$ as linear combination of periodic signals. (exponential)

let $\phi_1[n] = e^{j\frac{2\pi}{N}n} \rightarrow \left\{ \begin{array}{l} \text{a signal with is the fundamental period } N. \\ \omega_0 = \frac{2\pi}{N} \end{array} \right.$

$\phi_2[n] = e^{j2\left(\frac{2\pi}{N}\right)n} \longrightarrow \left\{ \begin{array}{l} N/2 \text{ is fund. period.} \\ 2\omega_0 \text{ is fund. freq.} \end{array} \right.$

$\vdots$

$\phi_K[n] = e^{jK\left(\frac{2\pi}{N}\right)n} \rightarrow \left\{ \begin{array}{l} \frac{N}{K} \text{ is fund. period} \\ K\omega_0 - \text{fund. fr.} \end{array} \right.$

if $K = N$ then $\phi_K[n] = e^{jK\frac{2\pi}{N}n} = 1 = \phi_0[n]$

if $K \in N\mathbb{Z}$

if $r \in N\mathbb{Z}$ then $\phi_{K+rN}[n] = e^{j(K+rN)\frac{2\pi}{N}n}$

$= e^{jK\frac{2\pi}{N}n} e^{j2\pi r n} = e^{jK\frac{2\pi}{N}n}$

$= \phi_K[n]$

Thus.

$$\left.\begin{aligned}
\phi_0[n] &= \phi_{\pm rN}[n] \\
\phi_1[n] &= \phi_{1\pm rN}[n] \\
&\vdots \\
\phi_k[n] &= \phi_{k\pm rN}[n]. \\
&\vdots \\
\phi_{N-1}[n] &= \phi_{N-1\pm rN}[n].
\end{aligned}\right\} \text{for } r \in \mathbb{Z} \qquad \text{—— (1)}$$

are the only possible distinct elements that are independent.

We want to express

$$x[n] = \sum_{k} a_k e^{jk\frac{2\pi}{N}n} = \sum_{k} a_k \phi_k[n].$$

but since (1) holds true.

$$x[n] = \sum_{k=\langle N\rangle} a_k e^{j\frac{2\pi}{N}n} = \sum_{k=\langle N\rangle} a_k \phi_k[n]$$

$$= a_0 \phi_0[n] + a_1 \phi_1[n] + \cdots + a_{N-1} \phi_{N-1}[n].$$

$$= a_0 \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} + a_1 \begin{bmatrix} 1 \\ e^{j\frac{2\pi}{N}1} \\ \vdots \\ e^{j\frac{2\pi}{N}(N-1)} \end{bmatrix} + \cdots + a_{N-1} \begin{bmatrix} 1 \\ e^{j\frac{2\pi}{N}(N-1)} \\ \vdots \\ e^{j\frac{2\pi}{N}(N-1)^2} \end{bmatrix}$$

$$\begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{j1\left(\frac{2\pi}{N}\right)} & \cdots & e^{j\left(\frac{2\pi}{N}\right)(N-1)} \\ \vdots & \vdots & & \vdots \\ 1 & e^{j(N-1)\left(\frac{2\pi}{N}\right)} & \cdots & e^{j(N-1)^2\left(\frac{2\pi}{N}\right)} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{N-1} \end{bmatrix}}$$

$$x = W a$$

$W \rightarrow$ DFT Matrix

$\left(\frac{1}{\sqrt{N}} W\right)$ is orthogonal matrix.

Let us check.

Note that $e^{j\frac{2\pi}{N}k}$ is an $N^{th}$ root of unity.

Then if $e^{j\frac{2\pi}{N}k} \neq 1$

then. $$\sum_{n=0}^{N-1} e^{j\left(\frac{2\pi}{N}k\right)n} = 0$$

Since. [let $\Omega_k = e^{j\frac{2\pi}{N}k}$ then $\Omega_k^N - 1 = 0$

then. $(\Omega_k - 1)(\Omega_k^{N-1} + \cdots + \Omega_k + 1) = 0$

$\Downarrow$

$$\sum_{n=0}^{N-1} \Omega_k^n = 0 \Rightarrow \sum_{n=0}^{N-1} e^{j\left(\frac{2\pi k}{N}\right)n} = 0$$

$$\sum_{n=0}^{N-1} \Omega_k^n = \begin{cases} N & \text{if } k \in N\mathbb{Z} \\ 0 & \text{if } k \notin N\mathbb{Z} \end{cases}$$

$$\cancel{\overline{W(0,k)}^{T}} \; W(0,k)$$

then

$$\overline{W(0,\ell)}^{T} x = N a_{\ell}.$$

$$W(0,\ell) = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 \\ e^{j\left(\frac{2\pi}{N}\ell\right)} \\ \vdots \\ e^{j\left(\frac{2\pi}{N}\ell\right)(N-1)} \end{bmatrix}$$

$$\overline{W(0,\ell)} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 \\ e^{-j\left(\frac{2\pi}{N}\right)\ell} \\ \vdots \\ e^{-j\left(\frac{2\pi}{N}\ell\right)(N-1)} \end{bmatrix}$$

$$\overline{W(0,\ell)}^{T} W(0,m) = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & e^{-j\left(\frac{2\pi}{N}\right)\ell} & \cdots & e^{-j\left(\frac{2\pi}{N}\ell\right)(N-1)} \end{bmatrix} \times \frac{1}{\sqrt{N}} \begin{bmatrix} 1 \\ e^{j\frac{2\pi m}{N}} \\ \vdots \\ e^{j\left(\frac{2\pi}{N}m\right)(N-1)} \end{bmatrix}$$

$$= \frac{1}{N}\left(1 + \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}(m-\ell)n}\right) = \left\{\frac{1}{N}\left(1 + \sum_{n=1}^{N-1} \left(\Omega_{m-\ell}\right)^{n}\right)\right\}.$$

$$= \left( \sum_{n=0}^{N-1} \left(\Omega_{m-l}\right)^n \right)$$

$$= \begin{cases} N & \text{if } m = l \\ 0 & \text{if } m \neq l \end{cases}$$

Which shows that:

$$\overline{W(0,l)}^T W(0,m) = \begin{cases} N & \text{if } m = l \\ 0 & \text{if } m \neq l \end{cases}$$

Then

$$\overline{W(0,l)}^T x = \overline{W(0,l)}^T W a$$

$$= \overline{W(0,l)}^T \begin{bmatrix} | & | & & | & & | \\ W(0,0), & W(0,1) & \cdots & W(0,l) & \cdots & W(0,N-1) \\ | & | & & | & & | \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_l \\ \vdots \\ a_{N-1} \end{bmatrix}$$

$$= \Big[ \underbrace{\overline{W(0,l)}^T W(0,0)}_{0}, \; \underbrace{\overline{W(0,l)}^T W(0,1)}_{0} \cdots \underbrace{\overline{W(0,l)}^T W(0,l)}_{N} \cdots \underbrace{\overline{W(0,l)}^T W(0,N-1)}_{0} \Big] \begin{bmatrix} a_0 \\ \vdots \\ a_l \end{bmatrix} = N a_l.$$

$$N a_\ell = \overline{W(0;\ell)}^T x$$

$$= \begin{bmatrix} 1 & e^{-j\frac{2\pi}{N}\ell} & \cdots & e^{-j\left(\frac{2\pi}{N}\ell\right)N-1} \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}$$

$$= \sum_{n=0}^{N-1} x[n]\, e^{-j\left(\frac{2\pi}{N}\ell\right)n}.$$

$$a_\ell = \frac{1}{N} \sum_{n=0}^{N-1} x[n]\, e^{-j(\omega_0 \ell)n}.$$

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$$\boxed{\begin{aligned} x[n] &= \sum_{k=0}^{N-1} a_k\, e^{j\omega_0 k n}. \\ a_k &= \frac{1}{N} \sum_{n=0}^{N-1} x[n]\, e^{-jk\omega_0 n}. \end{aligned}}$$

DFT.

$$x[n] = a_0 \phi_0 + a_1\phi_1 + \cdots + a_{N-1} \phi_{N-1}$$

$$x[n] = a_1 \phi_1 + \cdots + a_N \phi_N.$$

but $\phi_N = \phi_0$.

We get

$$a_N = a_0$$

Similarly

$$a_{K+N} = a_K.$$

Thus DFT is also periodic sequence of complex numbers.

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We may as well restrict our attention to indices $0, 1, 2, \ldots, N-1$

Then.

$$x = Wa$$

$$N(Ia) = \overline{W} x$$

$$a = \underbrace{\frac{1}{N} \overline{W}}\, x.$$

$$\frac{1}{\sqrt{N}} \overline{W} \frac{1}{\sqrt{N}} W = I$$

$$\Rightarrow \overline{W} W = NI$$

DFT matrix.
W is inverse DFT matrix.

If I call $\Omega$ as $N^{th}$ root of unity.

then IDFT matrix

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & \cdots & 1 \\ 1 & \Omega & \Omega^2 & \cdots & \Omega^k & \cdots & \Omega^{N-1} \\ 1 & \Omega^2 & \Omega^4 & \cdots & \Omega^{2k} & \cdots & \Omega^{2(N-1)} \\ \vdots & \vdots & \vdots & & & & \vdots \\ 1 & \Omega^{N-1} & \Omega^{2(N-1)} & \cdots & \Omega^{(N-1)k} & \cdots & \Omega^{(N-1)(N-1)} \end{bmatrix}$$

One can create IDFT matrix.

by taking seq.

$$\begin{bmatrix} 1 \\ \Omega \\ \Omega^2 \\ \vdots \\ \Omega^{N-1} \end{bmatrix}$$ then take element powers.

$$\begin{bmatrix} 1 \\ \Omega \\ \Omega^2 \\ \vdots \\ \Omega^{N-1} \end{bmatrix} .\wedge k.$$

DFT matrix is just $\frac{1}{N}\overline{W}$.

Since

$$x = W a$$

and $$\boxed{a = \frac{1}{N} \overline{W} x}$$ matrix form of DFT.

Linearity is evident.

Remaining properties can be obtaind from Summation form.

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Parseval's ~~Relation~~ Thm. is evident from matrix for.

$$\sum_{n=0}^{N-1} |x[n]|^2 = \overline{x}^T x = \overline{a}^T \overline{W}^T W a$$

$$= \overline{a}^T N(I) a = N \overline{a}^T a$$

$$= N \sum_{k=0}^{N-1} |a_k|^2$$

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Example.

$$x[n] = \begin{cases} 1 & -N_1 \leq n \leq N_1 \\ 0 & -N \leq n \leq -N_1 \text{ and } N_1 < n \leq N \end{cases}$$

$$a_k = \frac{1}{N} \sum_{n=-N_1}^{N_1} e^{-jk\left(\frac{2\pi}{N}\right)n}.$$

$$= \frac{1}{N} \sum_{m=0}^{2N_1} e^{-jk\left(\frac{2\pi}{N}\right)(m-N_1)}$$

$$= \frac{e^{jk\frac{2\pi}{N}N_1}}{N} \sum_{m=0}^{2N_1} e^{-kj\frac{2\pi}{N}m}$$

$$= \frac{e^{jk\frac{2\pi}{N}N_1}}{N} \; \frac{1 - e^{-jk\frac{2\pi}{N}(2N_1+1)}}{1 - e^{-jk\frac{2\pi}{N}}}$$

$$= \frac{e^{jk\frac{2\pi}{N}N_1} - e^{+jk\frac{2\pi}{N}(N_1 - 2N_1 - 1)}}{1 - e^{-jk\frac{2\pi}{N}}}$$

$$= \frac{e^{jk\frac{2\pi}{N}N_1} - e^{-jk\frac{2\pi}{N}(N_1+1)}}{1 - e^{-jk2\pi/N}}$$

$$= \frac{\cancel{e^{-jk\frac{2\pi}{2N}}}\left[e^{jk\frac{2\pi}{N}(N_1+1/2)} - e^{-jk\frac{2\pi}{N}(N_1+\frac{1}{2})}\right]}{\cancel{e^{-jk\frac{2\pi}{2N}}}\left[e^{jk\frac{2\pi}{2N}} - e^{-jk\frac{2\pi}{2N}}\right]}$$

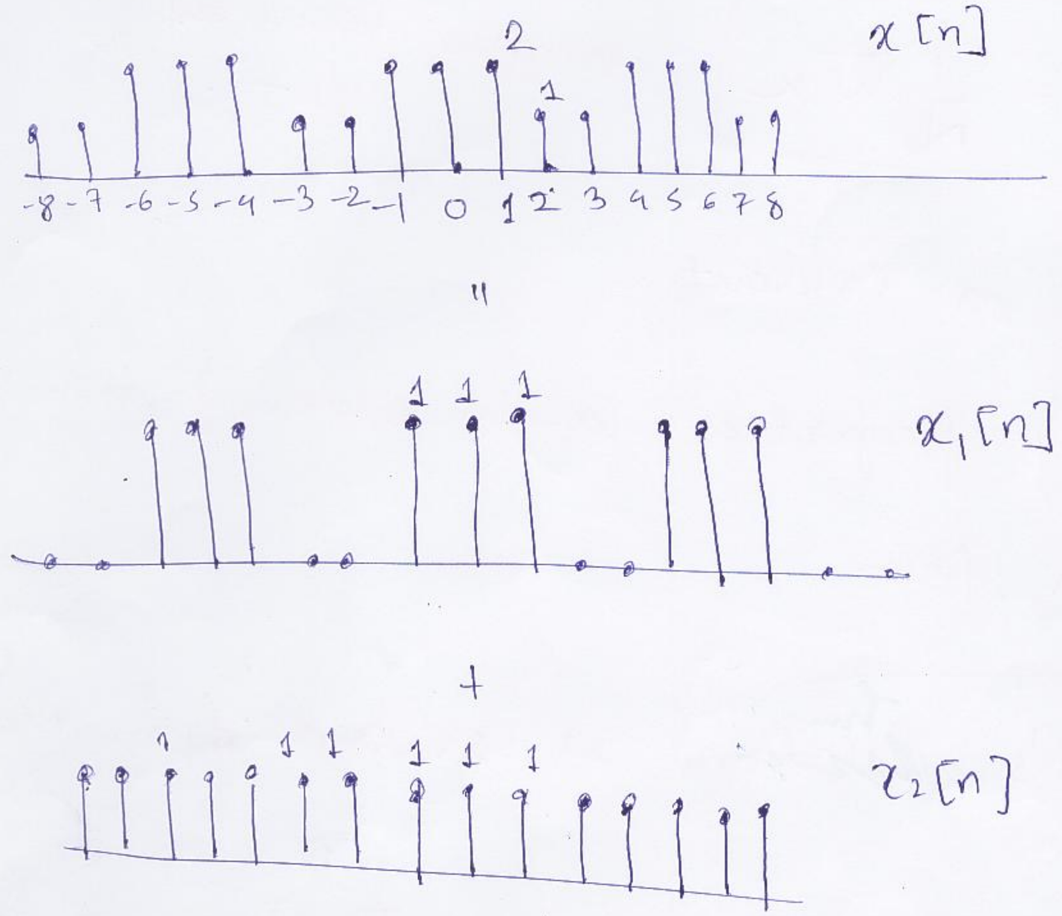
$$= \frac{1}{N} \frac{\sin\left[\frac{2\pi k (N_1 + 1/2)}{N}\right]}{\sin\left(\frac{\pi k}{N}\right)}, \quad k \neq 0, \pm N, \pm 2N \ldots$$

$$a_k = \frac{2N_1 + 1}{N}, \quad k = 0, \pm N, \pm 2N \ldots$$

Properties of DFT / DTFS.

| | | |
|---|---|---|
| Linearity | $Ax[n] + By[n]$ | $Aa_k + Bb_k$ |
| Time shifting. | $x[n - n_0]$ | $a_k e^{-j\left(\frac{2\pi}{N}\right) n_0}$ |
| Conjugation. | $x^*[n]$ | $a^*_{-k}$ |
| Time Reversal | $x[-n]$ | $a_{-k}$ |
| Time scaling | $x_{(m)}[n] = \begin{cases} x\left[\frac{n}{m}\right], & \text{if } n/m \\ 0 & \text{otherwise} \end{cases}$ | $\frac{1}{m} a_k$ |
| Periodic Convolution. | $\sum_{r=\langle N \rangle} x[r] y[n-r]$ | $N a_k b_k$ |
| | $x[n] - x[n-1]$ | $\left(1 - e^{-jk\left(\frac{2\pi}{N}\right)}\right) a_k$ |

Example.




$$x[n] = x_1[n] + x_2[n]$$

$$x_1 \longleftrightarrow b_k = \begin{cases} \dfrac{1}{5} \cdot \dfrac{\sin\left(\frac{3\pi k}{5}\right)}{\sin\left(\frac{\pi k}{5}\right)} & \text{for } k \neq 0, \pm 5, \pm 10 \ldots \\[2ex] \dfrac{3}{5}, & k = 0, \pm 5, \pm 10 \ldots \end{cases}$$

lly. $x_2 \longleftrightarrow c_k$, but $c_k = \begin{cases} 0 & k \neq 0, \pm 5, \ldots \pm 10 \\ 1 & \text{for } k = 0, \ldots \end{cases}$

## Fourier series & LTI systems

$$x(t) = e^{st} \longrightarrow \boxed{h(t)} \longrightarrow y(t) = e^{st}\, H(s)$$

$$H(s) = \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{+j\omega_0 k t} \longrightarrow \boxed{h(t)} \longrightarrow$$

$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(j\omega_0 k)\, e^{j\omega_0 k t}$$

- One can design $H$ s.t. it picks only certain frequencies / attenuates other ones.

for example if we want that there be no higher harmonics then $H(j\omega_0 k)$ must decrease as $k$ increases.

In general if I apply for any freq. $\omega$.

$$x(t) = e^{j\omega t} \longrightarrow \boxed{h(t)} \longrightarrow y(t) = e^{j\omega t} H(j\omega)$$

Then $H(j\omega)$ determines what would happen to $e^{j\omega t}$ in the output.

Let $h(t) = e^{-t} u(t)$.

$$H(j\omega) = \int_{-\infty}^{\infty} e^{-\tau} u(\tau) e^{-j\omega\tau} d\tau$$

$$= \int_{0}^{\infty} e^{-\tau} e^{-j\omega\tau} d\tau = \frac{1}{1+j\omega}$$

If I apply.

$$x(t) = \sum_{k=\cancel{0}-3}^{3} b_k e^{jk2\pi t}$$

then
$$y(t) = \sum_{k=-3}^{3} b_k H(jk2\pi) e^{jk2\pi t}$$

$$H(jk2\pi) = \frac{1}{1+j2\pi k} = \frac{1-j2\pi k}{1+4\pi^2k^2}$$

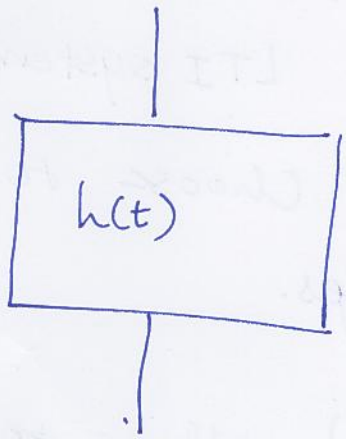
$$|H| = \frac{1}{1+4\pi^2k^2} + \frac{4\pi^2k^2}{1+4\pi^2k^2} = \frac{\cancel{1+4\pi^2k^2}\ 1}{(1+4\pi^2k^2)}$$

as $k\uparrow$ $H\downarrow$ acts a a low pass filter.

Fourier Series and LTI systems.

Consider applying a periodic signal as an input to an LTI system.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t}$$




$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(j\omega_0 k) e^{j\omega_0 k t}.$$

(obtained from impulse response)

$H(j\omega)$ is a crucial function that determines how much gain a particular frequency experiences

It can be viewed as a ~~[illegible]~~ a frequency response of the LTI system

Recall.

$$e^{st} \longrightarrow \boxed{h(t)} \longrightarrow H(s)e^{st}$$

$$y(t) = \int_{-\infty}^{\infty} e^{s(t-\tau)} h(\tau)\, d\tau$$

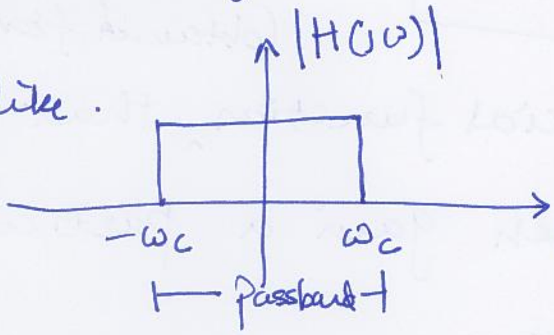
$$= e^{st} \underbrace{\int_{-\infty}^{\infty} e^{-s\tau} h(\tau)\, d\tau}_{H(s)}$$

## Filters:

From the function $H(j\omega) = \int_{-\infty}^{\infty} e^{-j\omega\tau} h(\tau)\, d\tau$

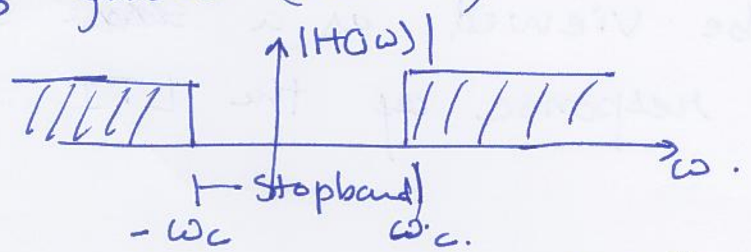
We can see that if we are able to shape this function as per our wish we can decide which frequencies to allow to pass through LTI system. In the output you can choose to see selected frequencies.

(i) Lowpass filter (Ideal) allows ~~to~~ only frequencies upto certain threshold ($\omega_c$ - cutoff freq.) to pass through. Ideally the graph of $|H(j\omega)|$ v/s $\omega$ would look like.

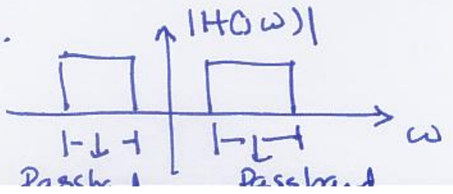



Remember $H(j\omega)$ is gain seen by $e^{j\omega t}$. So if gain is zero at for some $\omega$. it won't appear in output.

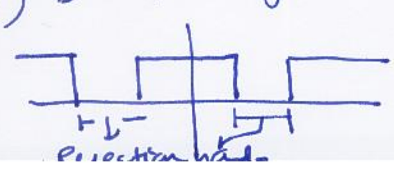
(ii) highpass filter (Ideal)




(iii) Band pass.



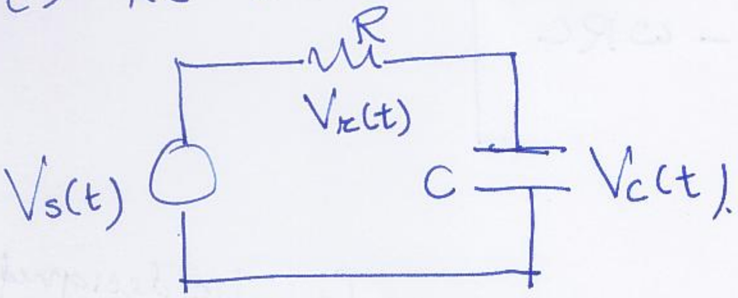

(iv) Band - reject



We will see later that ideal filters of this sort are not practically possible to realise using passive components. However digitally you may ~~s~~ realise them (upto some degree of accuracy).

Passive filters.

(i) RC series circuit can act like a lowpass filter or highpass filter (depending upon what is output)




Choose $V_s(t)$ as input
$V_c(t)$ as output.

then. if

$$V_s(t) = e^{j\omega t} \quad \text{and } V_c(t) = e^{j\omega t} H(j\omega)$$

We must determine $H(j\omega)$.

$$\frac{V_s - V_c}{R} = C\frac{dV_c}{dt}$$ (~~using~~ current through the circuit)

$$\frac{\cancel{e^{j\omega t}}\,1 - \cancel{e^{j\omega t}} H(j\omega)}{RC} = H(j\omega)\, j\omega\, \cancel{e^{j\omega t}}$$

$$H(j\omega) = \frac{1}{1 + RCj\omega}$$ for each value $\omega$ $H(j\omega)$ is a complex number.

It is a complex function of $\omega$.

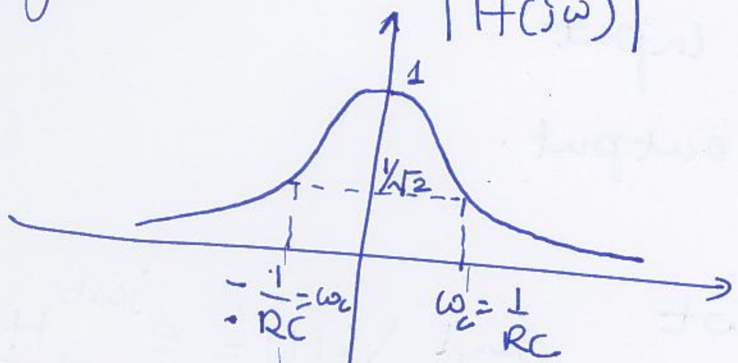
Now one can talk about the magnitude and phase of $H(j\omega)$

$$|H(j\omega)| = \left| \frac{1 - j\omega RC}{1+\omega^2 R^2 C^2} \right|$$

$$= \frac{1}{1+\omega^2 R^2 C^2}$$

$$\angle H(j\omega) = \tan^{-1}\left[ -\omega RC \right]$$

Magnitude plot.



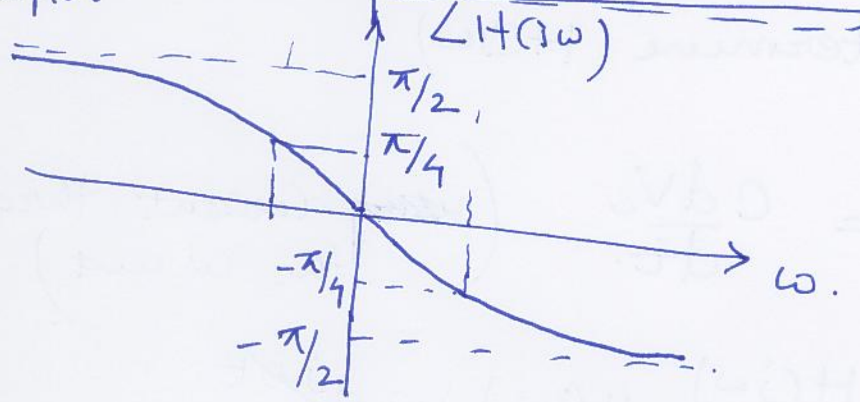

Can be designed by appropriately choosing R and C values.

~~after~~ for $\omega > \omega_c$

$$|H(j\omega)| < \frac{1}{\sqrt{2}}$$

for $\omega \ggg \omega_c$ we get enough attenuation

Phase plot




Excercise: Highpass filter:

Choose $V_R(t)$ as output voltage and repeat the analysis to conclude ~~that~~ Highpass filt.

# Discrete time system Response to Periodic Signals

$$x[n] = z^n \longrightarrow \boxed{h[n]} \longrightarrow y[n] = \sum_{k=-\infty}^{\infty} z^{n-k} h[k]$$

$$= z^n \sum_{k=-\infty}^{\infty} z^{-k} h[k]$$

$$= z^n H(z)$$

If we choose Complex exponential.

$$x[n] = e^{j\omega n} \longrightarrow \boxed{h[n]} \longrightarrow y[n] = e^{j\omega n} \cdot H(e^{j\omega})$$

Thus.

$$x[n] = \sum_{k=0}^{N-1} a_k e^{j\omega_0 k n}$$

then

$$y[n] = \sum_{k=0}^{N-1} a_k e^{j\omega_0 k n} H(e^{j\omega_0 k})$$

We can select which coefficients to choose and get rid off from the output.

$H(e^{j\omega})$ is a sort of frequency response for discrete time systems.

## First order Recursive DT filters

$$y[n] - a\,y[n-1] = x[n]$$

$$H(e^{j\omega})e^{j\omega n} - aH(e^{j\omega})e^{j\omega(n-1)} = e^{j\omega n}$$

$$H(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}} = \frac{1}{1-a\cos\omega + ja\sin\omega}$$

$$= \frac{(1-a\cos\omega) - ja\sin\omega}{(1-a\cos\omega)^2 + a^2\sin^2\omega}$$

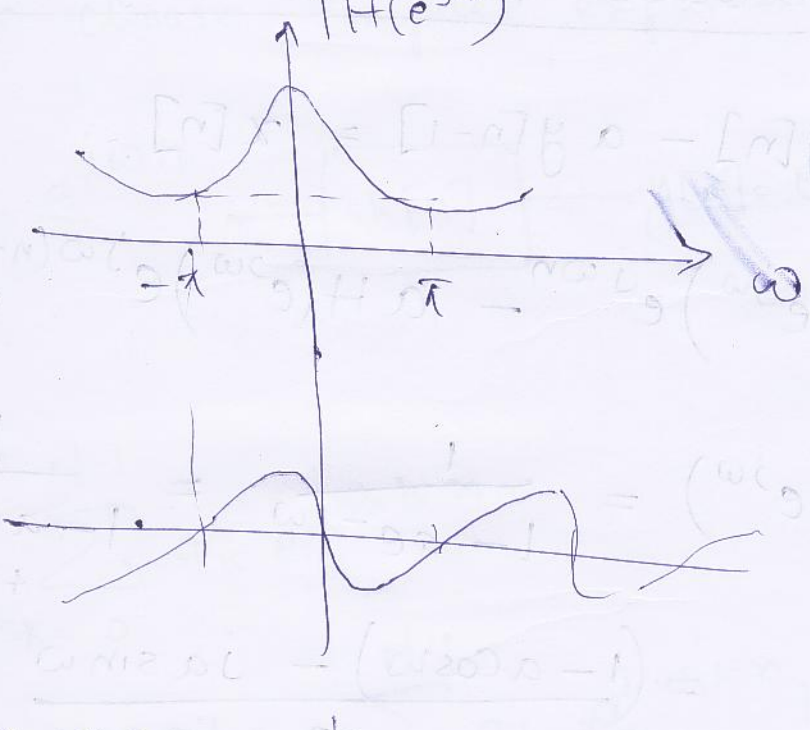
$$|H(e^{j\omega})| = \left(\frac{\cancel{(1-a\cos\omega)^2 + (a\sin\omega)^2}}{[(1-a\cos\omega)^2 + (a\sin\omega)^2]^{\cancel{2}}}\right)^{1/2}$$

$$= \left(\frac{1}{(1-a\cos\omega)^2 + (a\sin\omega)^2}\right)^{1/2}$$

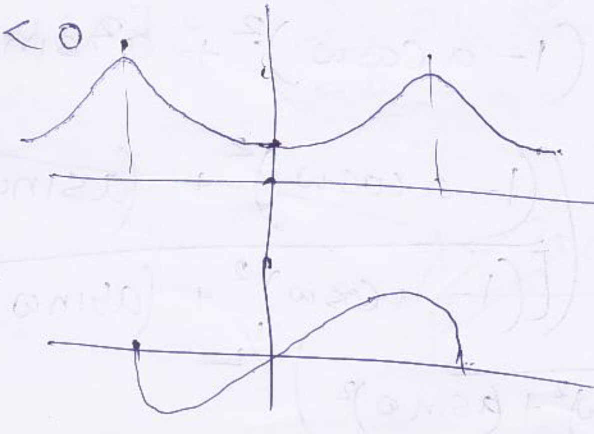
$$= \frac{1}{\sqrt{1 \mp 2a\cos\omega + a^2}}$$

$$\angle H(e^{j\omega}) = \tan^{-1}\left[\frac{-a\sin\omega}{1 - a\cos\omega}\right]$$

if $a > 0$. but $a < 1$.




$a < 0$



- Non-recursive DT filter

$$y[n] = \sum_{k=-N}^{N} b_k \, x[n-k]$$

<u>Moving average filters.</u>

Ex. $y[n] = \frac{1}{3}\left(x[n-1] + x[n] + x[n+1]\right)$

Three pt. mov. avg. filter

$$h[n] = \frac{1}{3}\left(\delta[n+1] + \delta[n] + \delta[n-1]\right)$$

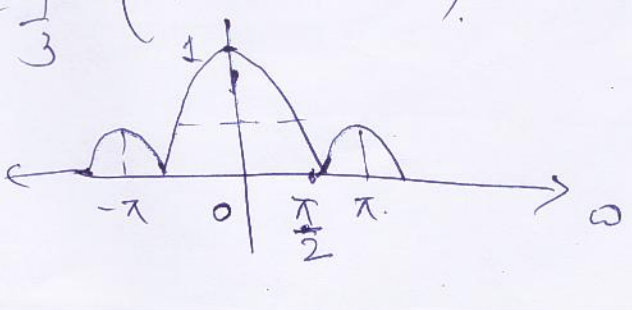
$$y(n) = e^{j\omega n} H(e^{j\omega}) = \frac{1}{3}\left(e^{j\omega(n-1)} + e^{j\omega n} + e^{j\omega(n+1)}\right)$$

$$= \frac{1}{3}\left(e^{j\omega n} e^{-j\omega} + e^{j\omega n} + e^{j\omega n} e^{j\omega}\right)$$

$$= \frac{1}{3}\left(e^{-j\omega} + 1 + e^{j\omega}\right) e^{j\omega n}$$

$$H(e^{j\omega}) = \frac{1}{3}\left(e^{-j\omega} + 1 + e^{j\omega}\right)$$

$$= \frac{1}{3}\left(1 + 2\cos\omega\right)$$



Suppose $N + M + 1$

$$y[n] = \frac{1}{N+M+1} \sum_{k=-N}^{M} x[n-k]$$

$$H(e^{j\omega}) = \frac{1}{N+M+1} \sum_{k=-N}^{M} e^{-j\omega k}$$

$m = N + k.$

$$= \frac{1}{N+M+1} \sum_{k=0}^{M+N} e^{-j\omega(m-N)}$$

$$= \frac{1}{N+M+1} e^{j\omega N} \left( \frac{1 - e^{-j\omega(M+N+1)}}{1 - e^{-j\omega}} \right.$$

$$\cancel{e^{j\omega\left(\frac{N-M}{2}\right)}}$$

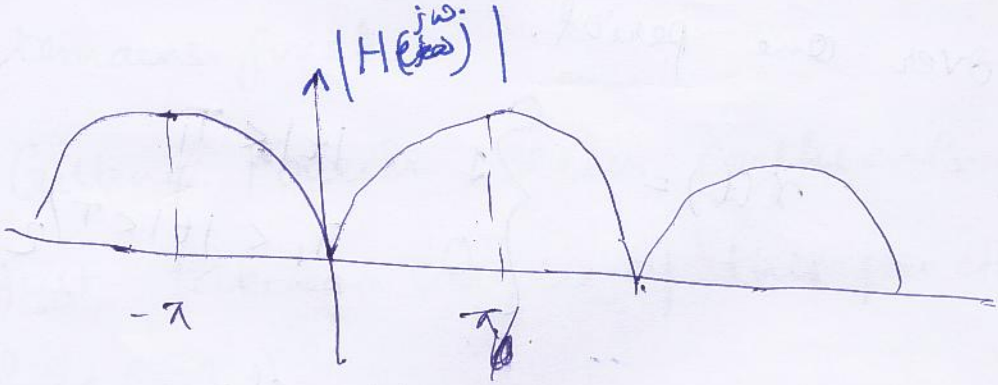
$$= \frac{1}{N+M+1} e^{j\omega\frac{(N-M)}{2}} \frac{\sin\omega(M+N+\frac{1}{2})}{\sin\omega/2}$$

High pass filter.

(1) $$y[n] = \frac{x[n] - x[n-1]}{2}$$

$$H(e^{j\omega}) = \frac{1}{2}\left(1 - e^{-j\omega}\right)$$

$$= j\, e^{j\omega/2} \sin \omega/2$$




Highpass filter