ELL 706: "Optimization" for EE

- Assignments → 30% [Programming based]
- Minor → 34% Submission moodle based
- Major → 36%

Lectures → Asynchronous → Teams, Impartus } [Lecture recordings]

→ Tuesday, Friday

Wednesday

Reference Books

1. Convex Optimization by Stephen Boyd, Lieven Vanderberghe
Cambridge University Press.

2. Convex Analysis & optimization by Dmitri Bertesekas, A. Nedic, A. Ozdagar
Athena Scientific.

3. Non-linear Programming, M. Bazaara, H. Sherali, C. Shetty, Wiley.

4. Research Papers ... Later.

- Typical Problem.

Optimization problem.

[[minimizing performance Criteria [cost, effort]

s.t. Constraints Problem specific.]

Minimize. [maximing performance criterion.]

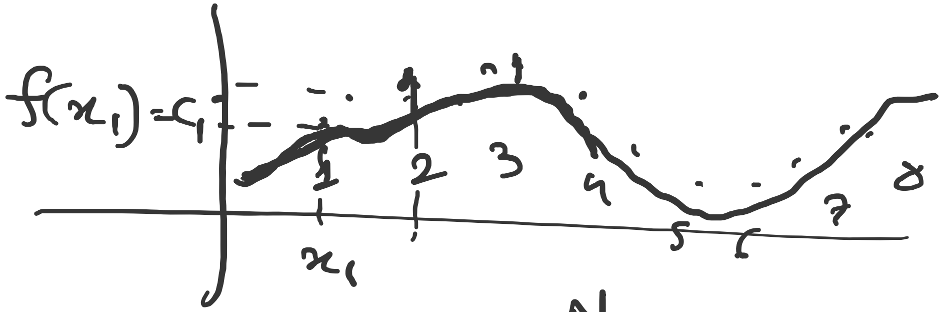
s.t Constraints Problems.

} game theoretic

min max fn

- Mathematical in nature.

$$f(x) = a + bx + cx^2$$



$$c_i = f(x_i) = a + b x_i + c x_i^2$$

$$\min_{(a,b,c)} \sum_{i=1}^{N} \left(f(x_i) - c_i \right)^2 \quad \phi$$

- [Machine learning
- System Identification.] →

Problem

$$\min \ f(x) \qquad x \in \mathbb{R}^n$$

$$\text{s.t.} \quad \begin{bmatrix} g_i(x) \le 0 & i = 1, \dots, m \\ h_i(x) = 0 & i = 1, \dots, p. \end{bmatrix}$$

$f : \mathbb{R}^n \to \mathbb{R}$, $x \mapsto f(x)$ } objective funct.

$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$

$g_i : \mathbb{R}^n \to \mathbb{R}$
$h_i : \mathbb{R}^n \to \mathbb{R}$ } constraints

$x \in \mathbb{R}^n$ satisfying constraints feasible point

Set of all $x \in \mathbb{R}^n$ satisfying is feasible region.

A $\bar{x}$ is minimizer if

$$f(\bar{x}) \le f(x) \text{ for any } x \text{ feasible}$$

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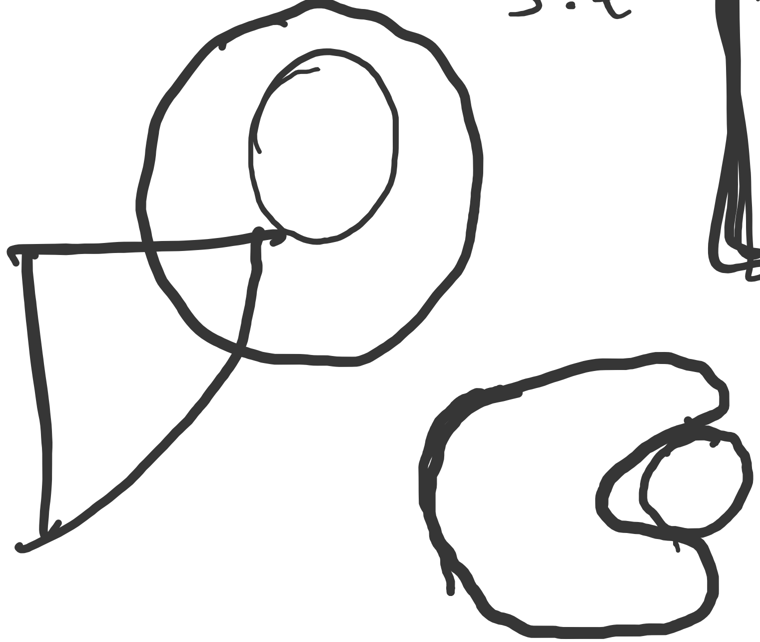
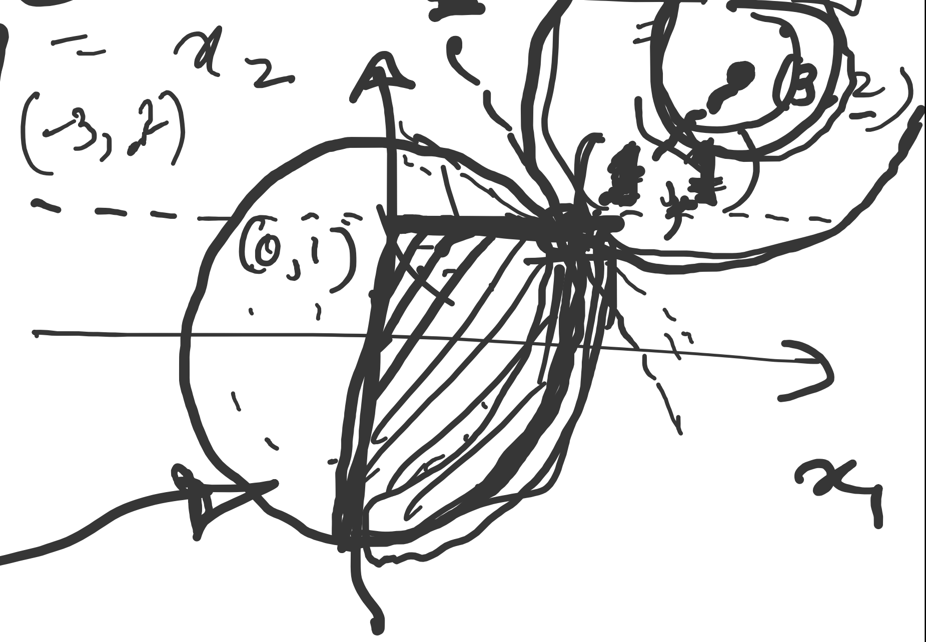
e.g. minimize $f(x) = \left[(x_1 - 3)^2 + (x_2 - 2)^2\right]$

s.t. $\left[\begin{array}{l} x_1^2 + x_2^2 \le 2 \\ x_2 \le 1 \\ x_1 \ge 0 \end{array}\right]$





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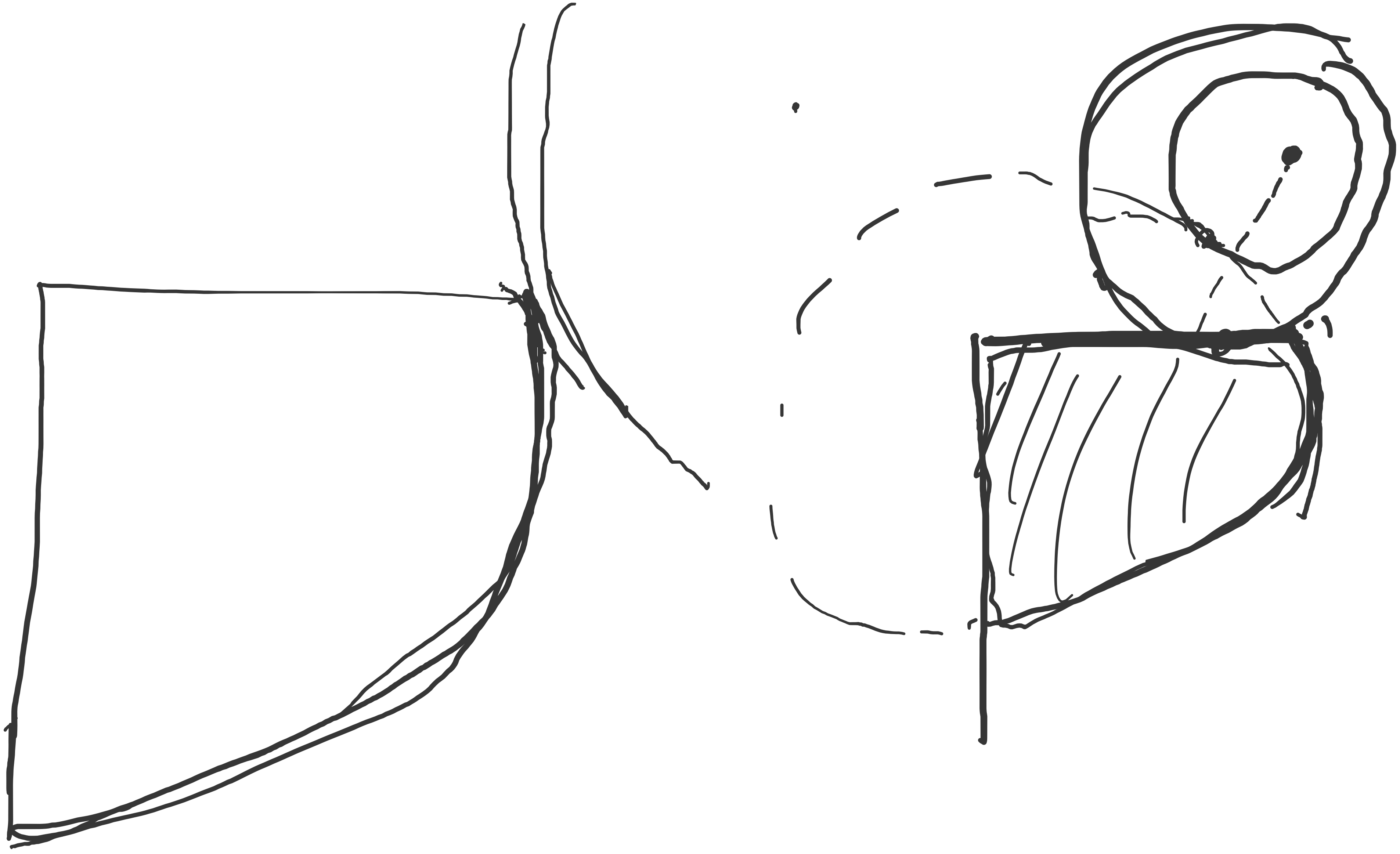
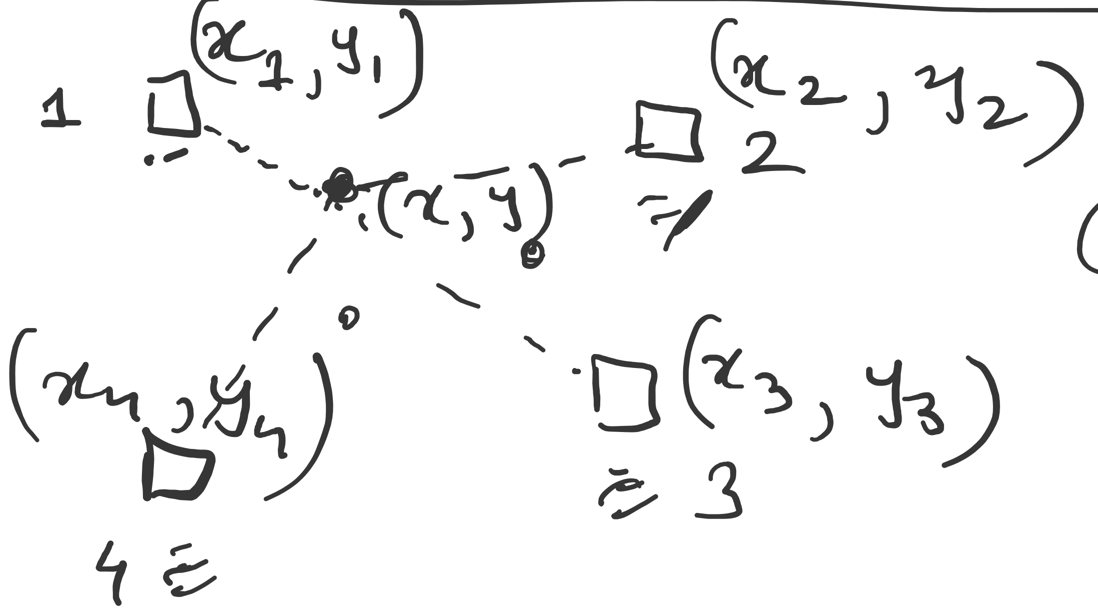


"Minimum time" Rendezvous



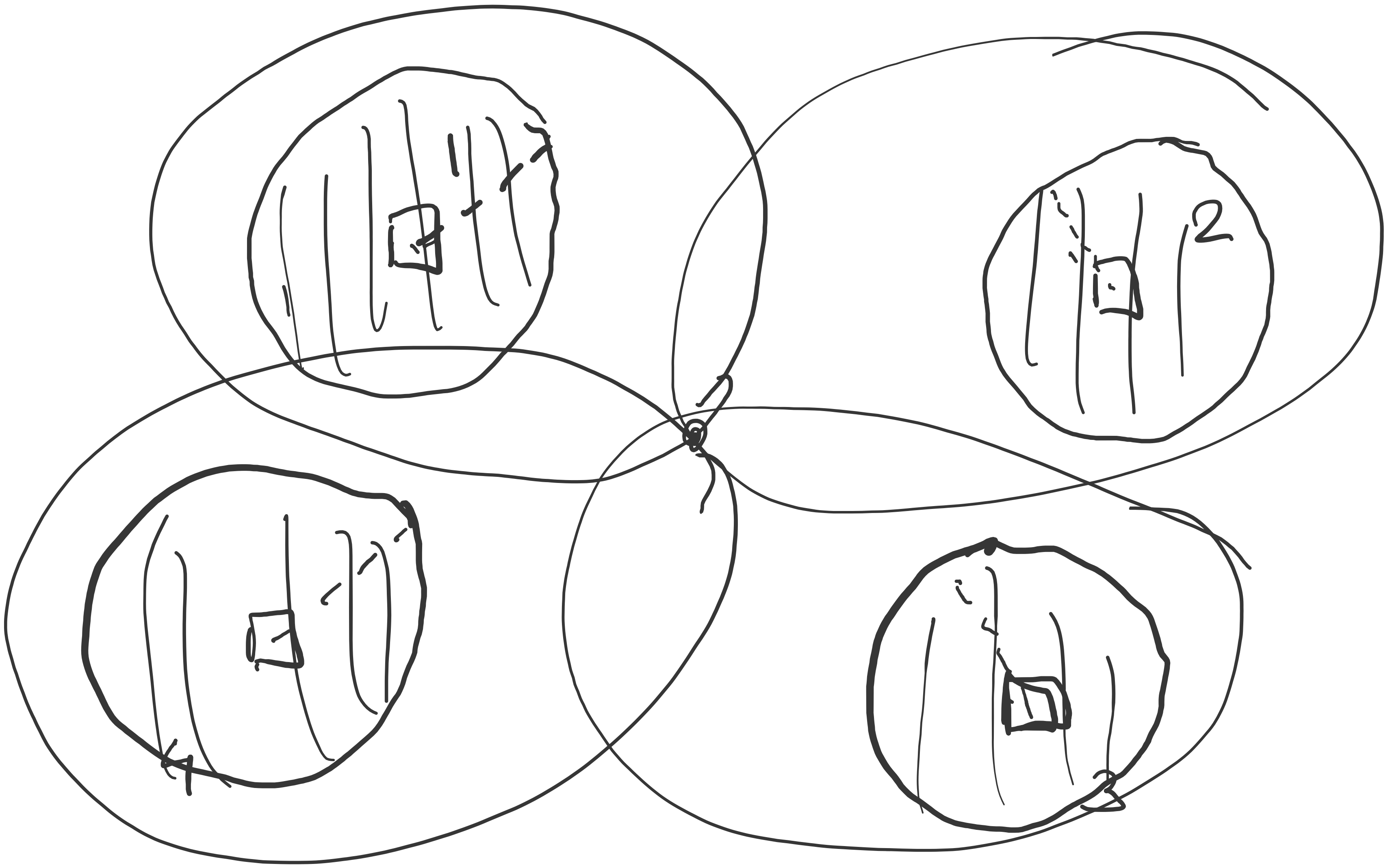
$$\min_{(x,y)} \left\{ \max_i \sqrt{(x - x_i)^2 + (y - y_i)^2} \right\}$$

$$\min_{(x,y)} f(x, y)$$

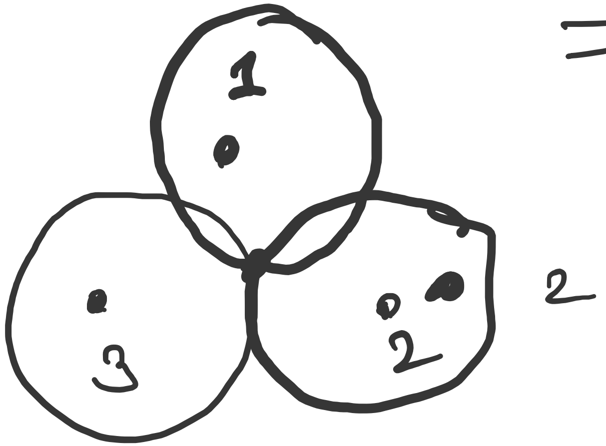
$$f(x, y) = \max_i \sqrt{(x - x_i)^2 + (y - y_i)^2}$$

[Knowing Geometry of problem is often useful!]

Approach 1



Step 1: List $^{N}C_3$ triplets of

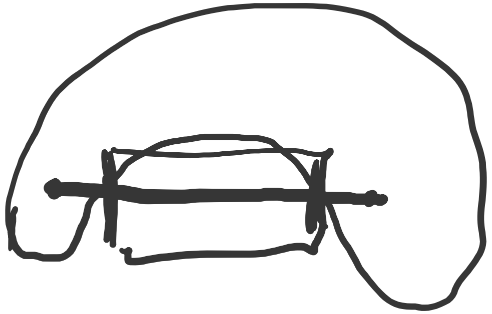


Step 2: Choose whichever triplet has the largest radii. Intersection pt for that triplet is the min time rendezvous pt.

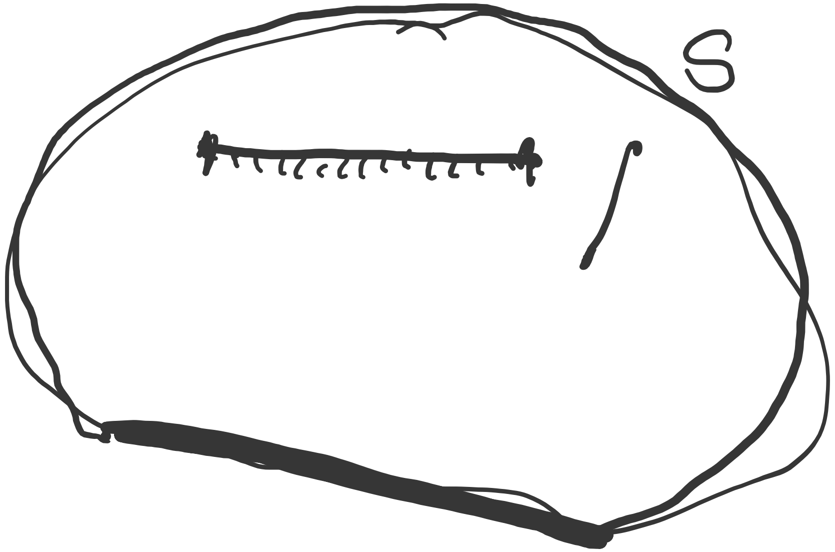
Helly's Thm:

Suppose N convex sets in $\mathbb{R}^d$. bounded.

If all triplets ${}^{N}C_{d+1}$ have a point in common; then N convex sets also have a pt in common.

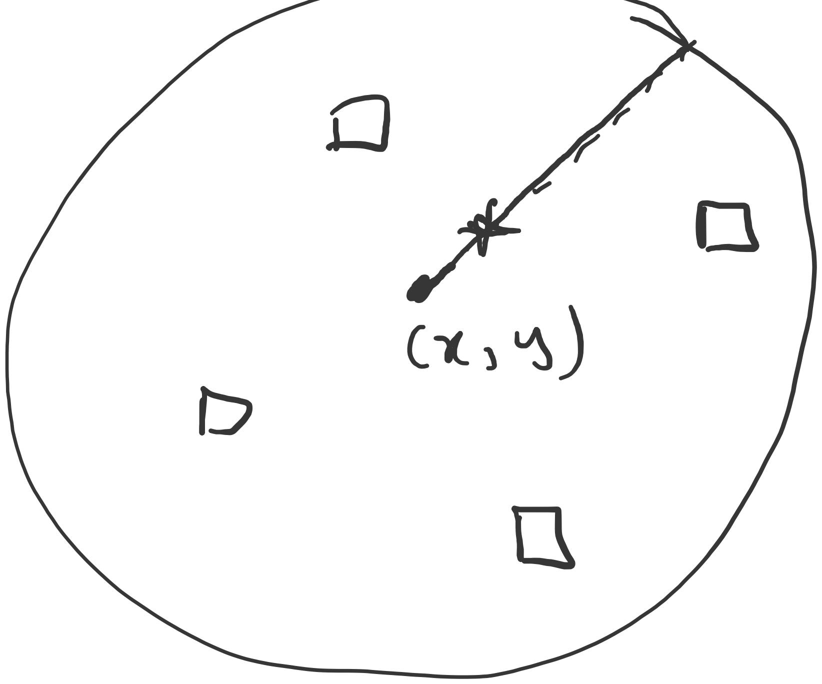


not convex



Convex.

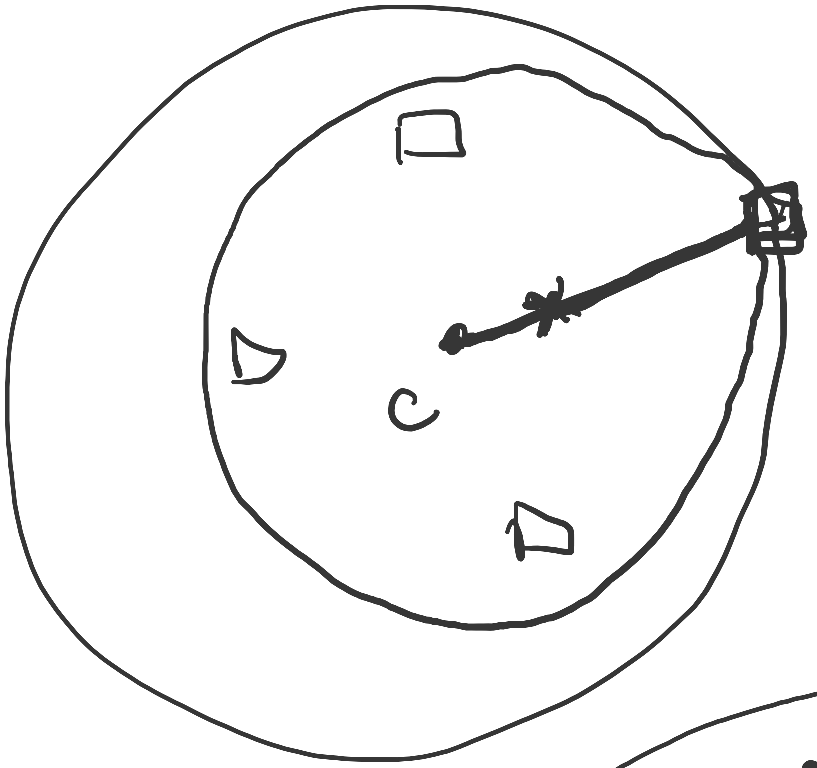
Minimum enclosing circle.

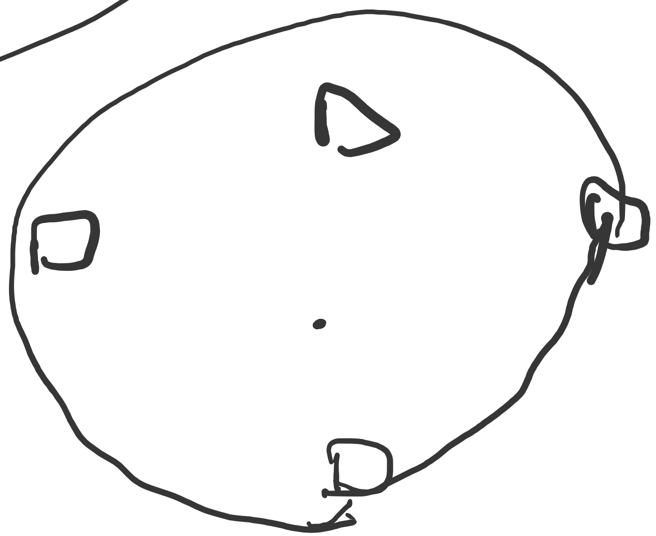


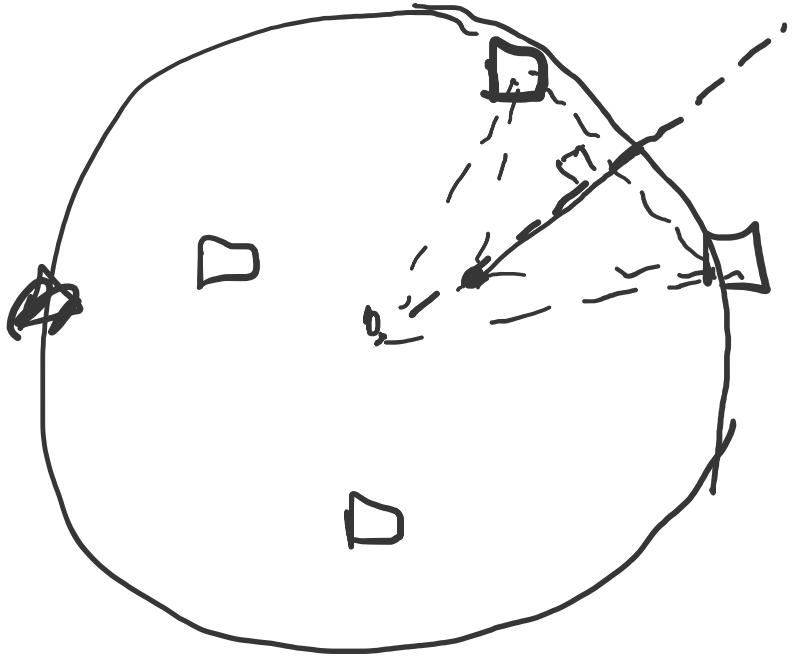
Step 1: Choose a center & radii s.t. all pts inside it

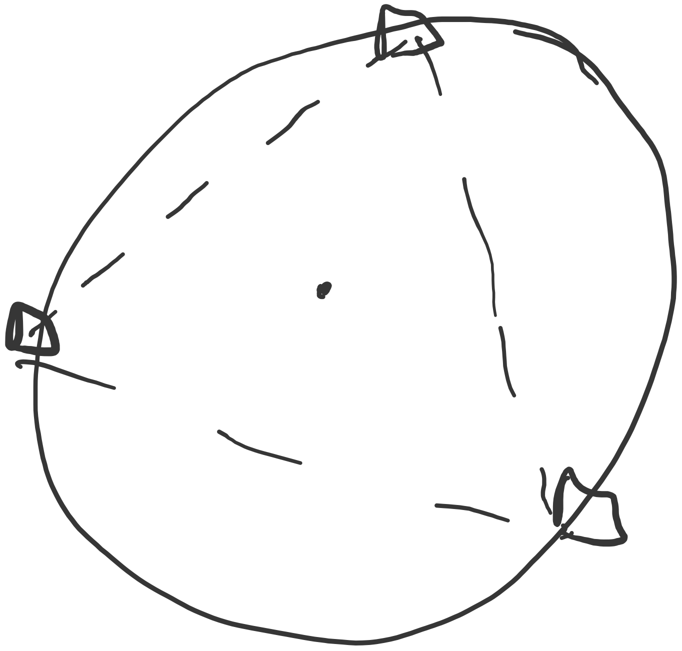
Step 2: reduce the radius s.t. one pt lies on the circle

Step3: Move the center towards the pt on the circle s.t radius reduces



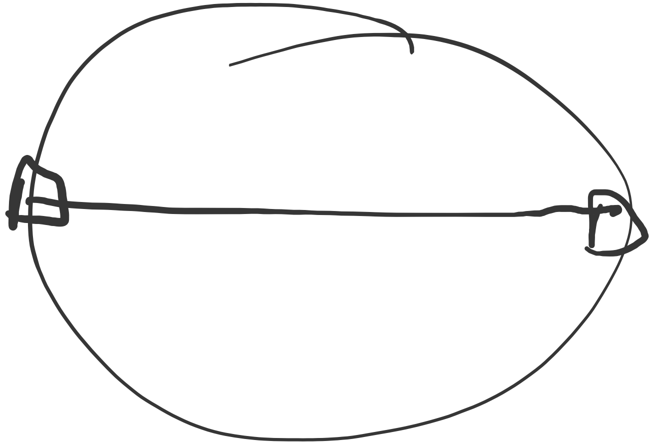






1885

J. J. Sylvester.



Outline of course

1. Basics of Linear algebra]
2. Convex analysis, Sets, descriptions Implicit explicit
3. Existence, Uniqueness of solutions to general optimization Problem
4. Duality
5. Linear Programming, Quadratic Programming, Semi-definite Programming, LMI Problems] solvers
6. Interior pt methods, gradient descent, branch & bound.
7. NLP. → Polynomial cost functions, polynomial ineq. constrs