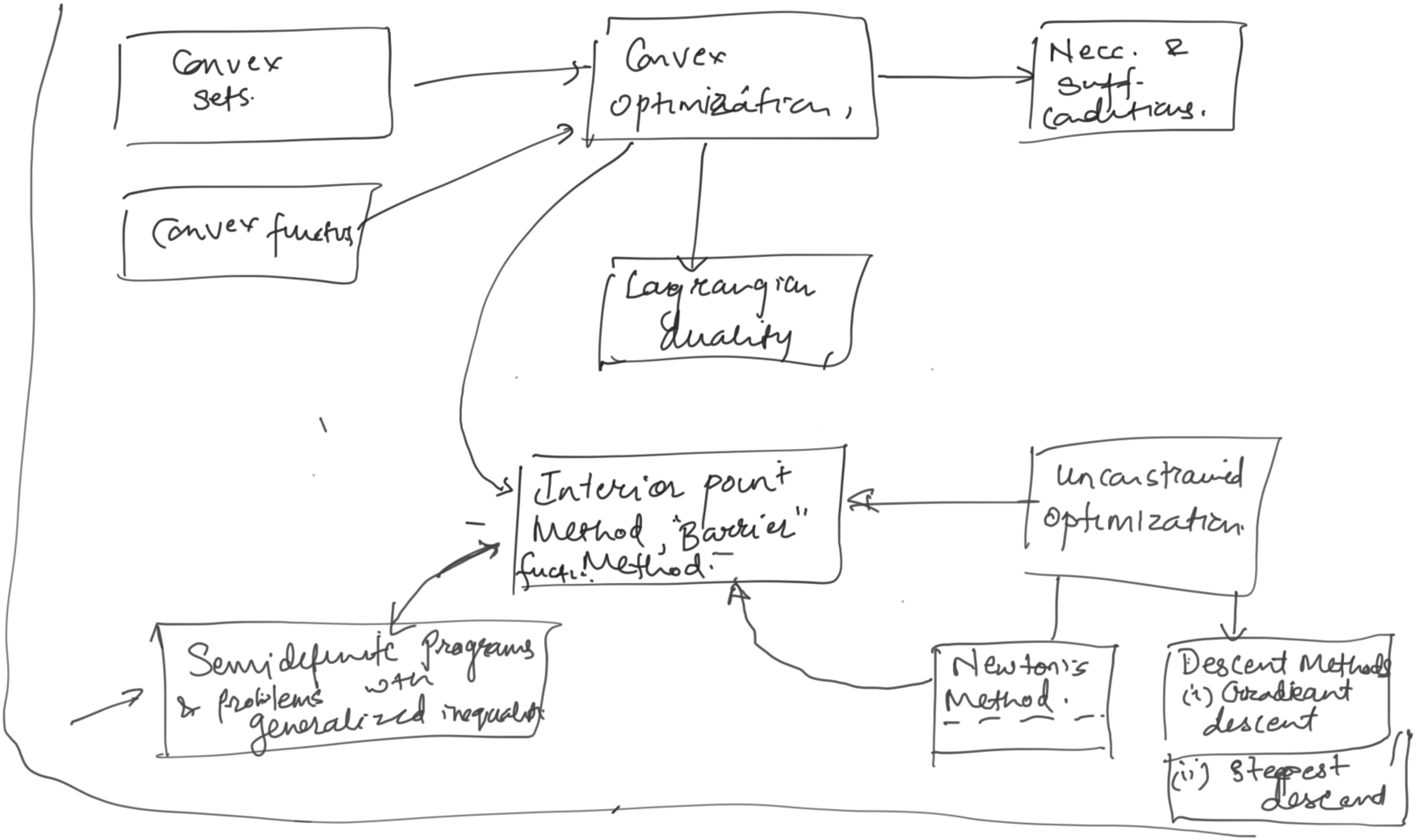
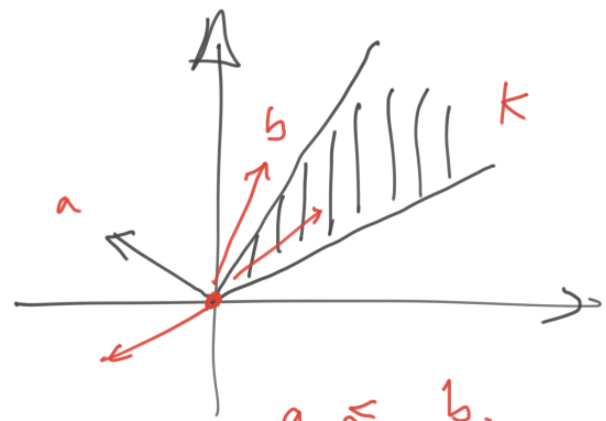

Convex sets.
Convex functns
Convex optimization,
Necc. & suff. conditions.
Lagrangian duality
Interior point Method, "Barrier" fuctn. Method.
Unconstrained optimization.
Semidefinite Programs & problems with generalized inequalities
Newton's Method.
Descent Methods (i) Gradient descent
(ii) Steepest descent

Generalized inequalities.

(i) any cone (closed, convex, pointed) induces an inequality



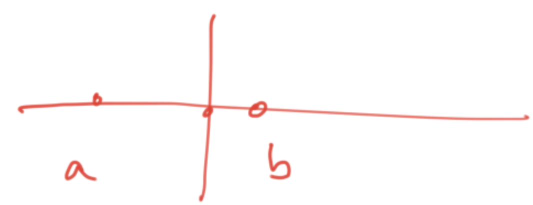
$a \preceq_K b.$

if $b - a \in K.$

$-x \in K, \; x \in K \quad x = 0.$

$a, b \qquad a \preceq_K b$ if $b - a \in K.$

↑ generalized inequality



$a < b$

~~a < b~~ $b - a \in \mathbb{R}_+$

Semi-definite program

$$\min \quad x^T P_0 x + q_0^T x + r_0$$
$$\text{s.t.} \quad x^T P_i x + q_i^T x + r_i \leq 0 \quad i = 1, \dots, m.$$

Convex $\quad P_0, P_i \succeq 0.$

Otherwise $\quad$ non-convex.

$$x^T P_0 x = \text{trace}(P_0 x x^T) = \text{trace}(P_0 X)$$

$$\boxed{X = x x^T} \longrightarrow X \succeq x x^T$$

$$\min \quad \operatorname{trace}(P_0 X) + q_0^T x + r_0$$

$$\text{st} \quad \underbrace{\operatorname{trace}(P_i X) + q_i^T x + r_i \leq 0.}$$

$$\boxed{X = x x^T} \qquad \boxed{X \succeq x x^T}$$

$x x^T$ is p.s.d. with rank one

$$X - x x^T \succeq 0.$$

$$\Updownarrow$$

$$\boxed{\begin{bmatrix} X & x \\ x^T & 1 \end{bmatrix} \succeq 0}$$

Linear matrix inequality

$f(x) \preceq_{K_i} 0.$

$f(x) \leq 0.$

$$\boxed{-\log(-f(x))}$$

Generalized inequality Constrained Program.

$K_i \subseteq \mathbb{R}^{n_i}$

$\min \; f(x)$

K_i are proper cones.

$f_i : \mathbb{R} \to \mathbb{R}^{n_i}$ s.t. $f_i(x) \preceq_{K_i} 0. \quad i = 1, \dots, m.$

e.g. positive Semi-definite Cone

$$\sup \inf_{x \in X} \mathcal{L}(x, \lambda) = f(x) + \sum_{i=1} \lambda_i^T f_i(x)$$

$\lambda_i \succeq_{K_i} 0, \iff \lambda_i \in K_i$

e.g. $\left[\begin{array}{ll} \min & c^T x \\ \text{s.t.} & \boxed{F(x) \preceq 0} \end{array}\right]$ → Linear matrix inequality

$$\boxed{F(x) = x_1 F_1 + x_2 F_2 + \cdots + x_n F_n + G}$$

$$F_i \in \mathbb{S}^p$$

→ Semidefinite program.

A to have eigenvalues in Left half of Complex plane.

$$\Updownarrow$$

$$\left[\begin{array}{l} -P \preceq 0. \\ \boxed{A^T P + PA} \preceq 0 \end{array}\right] \text{ is feasible}$$

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \qquad -P \preceq 0.$$

$$-\underline{F}(p) = -\left(p_{11}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + p_{12}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + p_{22}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) \preceq 0.$$

Barrier function for generalized inequalities

generalized Logarithm for a proper cone. $K \subseteq \mathbb{R}^q$

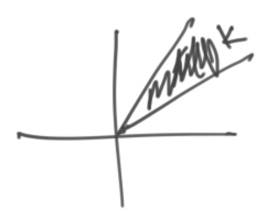
$\psi : \mathbb{R}^q \rightarrow \mathbb{R}$ is concave, closed, twice continuous differentiable

$\mathrm{dom}\,\psi = \mathrm{int}\, K.$

and. $\psi(sy) = \psi(y) + \theta \log s. \quad \forall\, y \in K,\ s > 0.$

$\hookrightarrow \log(ab) = \log a + \log b.$

(i) $\nabla \psi(y) \succ_{K^*} 0 \quad \forall\, y \in K.$



$K^* = \{ y \mid \langle x, y \rangle \geq 0, \forall x \in K \}$

(ii) $y^T \nabla \psi(y) = \theta.$

For positive semidefinite cone

$$\psi(X) = \log\det X. \qquad \nabla\psi(X) = X^{-1} \succeq 0.$$

$$\mathrm{trace}\big(X\,\nabla\psi(X)\big) = \mathrm{tr}\big(X X^{-1}\big) = \mathrm{tr}\, I = n.$$

$$\langle X, \nabla\psi(X)\rangle = \theta.$$

$$\psi(sX) = \log \underline{\det(sX)} = \log\det X + n\log s.$$

generalized logarithm

$$\min \quad f(x)$$

$$\text{s.t.} \quad f_i(x) \preceq_{K_i} 0.$$

$$\boxed{t f(x) - \sum_{i=1}^{m} \psi_i(-f_i(x))}$$

$$\boxed{\min\ t f(x) - \sum_{i=1}^{m} \log(-f_i(x))}$$

$$f(x) - p^* \leq \frac{m}{t}$$

$$\min \quad c^T x.$$

$$\text{s.t.} \quad \underbrace{\boxed{x_1 F_1 + x_2 F_2 + \cdots + x_n F_n + G}}_{= F(x)} \preceq 0.$$

$$F(x) \preceq 0.$$

$$\min \underbrace{t\, c^T x - \log\det(-F(x))}$$

$t^{(0)}$, μ, $x^{(0)}$, ϵ

Repeat.

$\min t\, c^T x - \log\det(-F(x))$

$x \leftarrow \text{Arg}\min$

$\frac{m}{t} < \epsilon$ Stop , otherwise $t \leftarrow \mu t$.

$\cancel{z > 0}$

$$\boxed{\begin{array}{l} x > 0. \\ z^2 - xy > 0. \end{array}} \iff \boxed{\begin{bmatrix} x & z \\ z & x \end{bmatrix} \succeq 0.}$$