# Department of Mathematics
# MA6L001 Mathematical Methods (Probability and Statistics)
# Tutorial Sheet No. 1
## Answer for selected Problems

1. $|\Omega| = 12$
   $\Omega = \{DD, NDD, NDND, NNDD, NNDN, NNNN, NNND, NDNN, DNNN, DNDN, DNND, DNDD\}$

2. $\frac{1}{2}$

3. (a) True (b) False (c) True (d) False (e) True

4. $\frac{1}{4}$

5. (a) $\frac{1}{2}$ (b) $\frac{1}{7}$

6. (a) $R^4 +^4 C_3 R^3(1-R) +^4 C_2 R^2(1-R)^2$
   (b) $R^4 +^4 C_3 R^3(1-R) +^2 C_1 R(1-R) *^2 C_1 R(1-R)$

7. $\frac{^{(N-D)}C_n}{^N C_n}$

8. (a) 0.002 (b) 0.7255

9. $\alpha = 1$ - $p, 0 < p < 1$

10. (a) $\alpha = \frac{1}{10}, \beta = \frac{3}{64}$ (b) $\frac{1}{2}$

11. a) 0.75 b) 0.5

12. $(1 - 0.001)^{1200}$

13. 0.99997

14. $P[X \geq 2] = [1 - [(1-p)^n + {}^nC_1 p^n (1-p)^{(n-1)}]] \geq 0.95$ where p = 0.001
    n= 4742 approximately

15. $[1- (0.95)^{52}$ - ${}^{52}C_1(0.05)(0.95)^{51}]$

16. $e^{-0.4}$

17. (a)$D_2$ (b)$D_2$

18. Y is uniformly distributed random variable on the interval (a, b)

19. $f_Y(y) = \sqrt{\frac{2}{\pi}}|y|e^{\frac{-1}{2}y^4}$, $-\infty < y < \infty$

20. $f_Y(y) = \begin{cases} \frac{1}{\pi\sqrt{1-y^2}}, & -1 < y < 1 \\ 0, & otherwise \end{cases}$

21. $P[X = x] = \begin{cases} \frac{\binom{n}{x} p^x q^{n-x}}{\sum_{i=0}^{r-1} \binom{n}{i} p^i q^{n-i}}, & x = 0, 1, 2, 3, .., r-1 \\ 0, & otherwise \end{cases}$

22. $f_X(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma(\phi(\frac{\beta-\mu}{\sigma}) - \phi(\frac{\alpha-\mu}{\sigma}))} e^{(\frac{-1}{2}(\frac{x-\mu}{\sigma})^2)}, & \alpha < x < \beta \\ 0, & otherwise \end{cases}$

23. X = No. of games played, P(X = k)= $p_k(>0)$, k= 4,5,6,7 E(X) = $\frac{93}{16}$

24. $f_Y(y) = \begin{cases} \frac{1}{2\sqrt{3}}, & -\sqrt{3} < y < \sqrt{3} \\ 0, & otherwise \end{cases}$ ; $P(-2 < Y < 2) = 1$

25. 50

26. (a)0
(b) P($\frac{-1}{2} < X \leq 3$)= 1 and P(X=0) = 0.

27. a) $X \sim P(\mu)$ (b) $\sum_{k=1}^{7} \frac{e^{-\mu}\mu^k}{k!}$ ; $\mu = 4$

28. $E[X^2] = \lambda + \lambda^2$ ,$Var(X) = \lambda$, $E[X^3] = \lambda^3 + 3\lambda^2 + \lambda$

29. $e^{\frac{\sigma^2 t^2}{2}}$
$$E[X^n] = \begin{cases} 0, & n - odd \\ \frac{n!}{(\frac{n}{2})! 2^{\frac{n}{2}}}\sigma^n, & n - even \end{cases}$$

30. $\frac{2}{3}$

31. $(a)$ $p_x(1) = 0.2$, $p_x(3) = 0.5$ $p_x(4) = 0.3$ $p_y(1) = 0.4$, $p_y(2) = 0.6$ $(b)$ 0.5

32. $k = 12$, $(1 - e^{-8})(1 - e^{-3})$

33. Let $Y$: r.v. denoting no. of 1's transmitted. $P(Y = n) = \frac{e^{-\lambda(1-p)}(\lambda(1-p))^n}{n!}$, $n = 0, 1, \ldots$

34. $f_{R,\theta}(r, \theta) = \begin{cases} r, & 0 < \theta < \pi/4, 0 < r < \sec\theta \text{ or } \pi/4 < \theta < \pi/2, 0 < r < cosec\theta \\ 0, & \text{otherwise} \end{cases}$
$$f_\theta(\theta) = \begin{cases} \frac{1}{2}\sec^2\theta, & 0 < \theta < \pi/4 \\ \frac{1}{2}cosec^2\theta, & \pi/4 < \theta < \pi/2 \end{cases}$$
$$f_R(r) = \begin{cases} \frac{\pi}{2}r, & 0 < r < 1 \\ r(cosec^{-1}(r) - \sec^{-1}(r)), & 1 < r < \sqrt{2} \end{cases}$$

36. (a) $\frac{3}{8}$ (b) $\frac{3}{8}e^{-\frac{1}{3}}$ (c) $\frac{3}{8}e^{-\frac{1}{3}} + \frac{5}{8}e^{-\frac{1}{5}}$

37. 31

38. 0.1814

39. (a) 0 approx (b) 0.02 is the exact value greater than 0.