MA6L001 (Probability and Statistics)
Tutorial Sheet No. 2
Answer for selected Problems

Sample No.	Sample value	X
1	4,5,5	$\frac{14}{3}$
2	4,5,7	$\frac{16}{3}$
3	4,5,7	$\frac{16}{3}$
4	5,5,7	$\frac{17}{3}$

Table 1: Sample mean X for each sample

X	frequency(f)	f(X)	Xf(X)	X^2 f(X)
$\frac{14}{3}$	1	$\frac{1}{4}$	$\frac{14}{12}$	$\frac{196}{36}$
$\frac{16}{3}$	2	$\frac{2}{4}$	$\frac{32}{12}$	$\frac{512}{36}$
$\frac{17}{3}$	1	$\frac{1}{4}$	$\frac{17}{12}$	$\frac{289}{36}$
Total	4	1	$\frac{63}{12}$	$\frac{997}{36}$

Table 2: Sampling distribution of X

I Semester 2019-20

1. mean of Sampling distribution ,$\mu_X = \sum Xf(X) = 5.25$
   Variance of sampling distribution, ${\sigma_X}^2 = 0.3632$
   Population mean ,μ= 5.25
   Population variance, $\sigma^2 = 1.0897$
   ${\sigma_X}^2 < \sigma^2$

2. $P(Z < -1.7678)$ where Z follows standard normal distribution.

3. $\bar{X}_1$ follows $N(\mu_1, \frac{\sigma_1^2}{n_1})$
   $\bar{X}_2$ follws $N(\mu_2, \frac{\sigma_2^2}{n_2})$
   $\bar{X}_1 - \bar{X}_2$ follows $N(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2})$

6. $\bar{X} - S^2$

7. 1.5, 16.6

8. Median.

9. (a) Unbiased estimator of $\mu = m_1$ and unbiased estimator of $\sigma = m_2 - m_1^2$
   (b) Unbiased estimator of $p = 1 + m_1 - \frac{m_2}{m_1}$ and unbiased estimator of $n = m_1/unbiasedestimatorofp$
   where $m_k = \frac{\sum_{i=1}^n X_i^k}{n}$.

10. Median.

11. $H_0 : \mu < 180$
    Reject H_0 if $\overline{X} > \mu_0 + \frac{\sigma}{\sqrt{n}} z_\alpha$
    where $\mu_0 = 180, n = 10, \sigma = 5$ and z_α is taken from standard normal table at 1) $\alpha = 0.05$ and 2) $\alpha = 0.1$.

12. $H_0 : \mu \geq 40$
    Reject H_0 if $\overline{X} \leq \mu_0 + \frac{s}{\sqrt{n}} t_{n-1,1-\alpha}$
    where $\mu_0 = 40, n = 15, s^2$ is sample variance and $t_{n-1,1-\alpha}$ is taken from t-distribution table at $\alpha = 0.05$

13. $H_0 : \sigma > \sigma_0$
Reject H_0 if $s^2 < \frac{\sigma_0^2}{n-1}\chi^2_{n-1,1-\alpha}$
where $\sigma_0 = 1.2, n = 12, s = 1.3$ and $\chi^2_{n-1,1-\alpha}$ is taken from χ^2 distribution table at $\alpha = 0.1$

14. $H_0 : \mu_1 - \mu_2 = 0$
Reject H_0 if $|\overline{X} - \overline{Y}| \geq t_{n_1+n_2-2,\alpha/2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
where $s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$

15. $H_0 : \sigma_1^2 < \sigma_2^2$
Reject H_0 if $\frac{s_1^2}{s_2^2} > F_{n-1,n-1,\alpha}$
where n = 10 and $F_{n-1,n-1,\alpha}$ is taken from F- distribution table at $\alpha = 0.05$

16. Let X = no. of accidents in a week.
$H_0 : X follows P(\lambda)$
where $\lambda = \overline{X}$, O_i be observed values of x_i and
E_i be expected values of $x_i = np_i$ where $p_i = \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$
$D_0 = \sum \frac{(O_i-E_i)^2}{E_i}$
Reject H_0 if $D_0 > \chi^2_{3,0.05}$

17. H_0 : Earthquake is equally likely to occur on any of 7 days.
O_i be observed values of x_i and
E_i be expected values of $x_i = np_i$ where $p_i = \frac{1}{7}$
$D_0 = \sum \frac{(O_i-E_i)^2}{E_i}$
Reject H_0 if $D_0 > \chi^2_{6,0.05}$

18. H_0 : having cellular phone and being involved in accident are independent.
O_{ij} be observed values of x_{ij} and
E_{ij} be expected values of x_{ij}
$D_0 = \sum_i \sum_j \frac{(O_{ij}-E_{ij})^2}{E_{ij}}$
Reject H_0 if $D_0 > \chi^2_{1,0.05}$

19. H_0 : smoking and lung cancer are independent.
O_{ij} be observed values of x_{ij} and
E_{ij} be expected values of x_{ij}
$D_0 = \sum_i \sum_j \frac{(O_{ij}-E_{ij})^2}{E_{ij}}$
Reject H_0 if $D_0 > \chi^2_{1,0.01}$

20. X follows B(1,p) bernoulli
$H_0 : p \geq p_0$
Reject H_0 if z_0 = -0.4082 $\leq -z_\alpha$ where $p_0 = \frac{3}{5}$

21. $H_0 : \mu_1 = \mu_2 = \mu_3$
Grand mean $\overline{y} = 82.44$
Between sum of squares, BSS = $\sum_{i=1}^{3} 15(\overline{y}_i - \overline{y})^2$
Within sum of squares, WSS = $(14)(s_1^2 + s_2^2 + s_3^2)$

Reject H_0 if $\frac{BSS/2}{WSS/42} \geq F_{2,42,0.05}$

I Semester 2019-20