Tutorial Sheet 3

(1) $\Phi(d_1) = 0.7$

Ans Investor needs to purchase 700 stocks.

(2) Delta of a European put option

$$= \frac{\partial}{\partial S} P^E(0) = 1 - \Phi(d_1)$$

where $$d_1 = \frac{\ln\left(\frac{S(0)}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

(3) $V(0) = \tilde{x}S(0) + y - 50{,}000\, P^E(0)$

$\tilde{x} = -12{,}570$, $C^E(0) = 80.9126$, $P^E(0) = 84.707$

$y = 6523{,}092$

Value of the portfolio after 1 day

$$= -11098.26$$

(4) For that stock prices the value of the port-folio is local maxima. This is giving some type of risk.

(5) Data is insufficient.

$$\mu = \mu_{rf} + \frac{\mu_M - \mu_{rf}}{\sigma_M}\,\sigma$$

(6) $$w = \frac{C^{-1}e}{e^T C^{-1} e}, \quad C = \begin{bmatrix} .0529 & -.00897 & .01365 \\ -.00897 & .0676 & .00966 \\ .01365 & .00966 & .0441 \end{bmatrix}$$

$$\Rightarrow C^{-1} = \begin{bmatrix} 21.5097 & 3.9285 & -7.5183 \\ 3.9285 & 15.9084 & -4.7182 \\ -7.5183 & -4.7182 & 26.0368 \end{bmatrix}$$

$w = \begin{bmatrix} .382 \\ .324 \\ .294 \end{bmatrix}$, Mean = .0944

Variance = 0.0213

Standard deviation = 0.1459

Minimum Variance point = (0.1459, 0.0944).

(7) (a) $E\left[\int_0^t X(s)\, dW(s)\right] = 0$

(b) $Var\left[\int_0^t X(s)\, dW(s)\right] = \int_0^t E(X^2(s))\, ds$

(9) Solution. $X(t) = \sigma W(t) + S(0)e^{\mu t}$

Distribution of $X(t) \sim N(S(0)e^{\mu t}, \sigma^2 t)$

(10) P(call option will be exercised)

$= P(S(4) > K)$

$= P\left(\frac{W(4)}{2} > \ln\left(\frac{12}{40 \times e^{.3648}}\right) \times \frac{1}{2 \times 0.24}\right)$

$= P\left(\frac{W(4)}{2} > -3.2682\right) = \Phi(-3.2682)$

(15) conditional distribution of $w(t)$ given $w(s) = c$ with $s < t$ is $N(c, t-s)$.