Department of Mathematics
MTL 390 (Non-parametric tests & Time Series)
Tutorial Sheet No. 7

1. Consider the data arranged in ascending order given below

   -0.9772,-0.8027,-0.3275,-0.2356,-0.2016,-0.1601,0.1514,
   0.2906,0.3705,0.3952,0.4634,0.6314,1.1002,1.4677,1.9352.

   Using one-sample Kolmogorov-Smirnov test, test whether the data comes from standard normal distribution or not at the significance level 0.01.

2. The following data were obtained from a table of random numbers

   0.464 0.137 2.455 -0.323 -0.068
   0.906 -0.513 -0.525 0.595 0.881
   -0.482 1.678 -0.057 -1.229 -0.486
   -1.787 -0.261 1.237 1.046 -0.508

   Using one-sample Kolmogorov-Smirnov test, test whether the data comes from standard normal distribution or not at the significance level 0.01.

3. Using two-sample Kolmogorov-Smirnov test, determine whether the two samples in Table 1 come from the same distribution or not at 5% level of significance.

4. A bank manager claims that the median number of customer per day is no more than 750. A teller doubts the accuracy of this claim. The number of bank customers per day for 16 randomly selected days are listed below.

   775 765 801 742 754 753 739 751
   745 750 777 769 756 760 782 789

   (a) Suggest what non-parametric test can be applied to test the claim?
   (b) At 0.05 significance level, can the teller reject the bank manager claim?

5. The following data represent lifetimes (hours) of batteries for two different brands:

   Brand A: 40 30 40 45 55 30
   Brand B: 50 50 45 55 60 40

   (a) Using Median test, check whether the two samples come from the same distribution.
   (b) Using two sample K-S test, check whether the two samples come from the same distribution.

6. Fifteen 3-year-old boys and fifteen 3-year-old girls were observed during two sessions of recess in a nursery school. Each child's play was scored for incidence and degree of aggression in Table 2:

   Is there evidence to suggest that there are gender differences in the incidence and amount of aggression? Use run test.

Table 1: Data for two sample K-S test

Age	21-22	23-24	25-26	27-28	29-30	31-32	33-34	35-36	37-38	39-40
Men	4	11	5	7	0	5	9	13	20	6
Women	7	4	1	11	12	4	2	4	8	9

Table 2: Data for Boys and Girls

Boys	96	65	74	78	82	121	68	79	111	48	53	92	81	31	40
Girls	12	47	32	59	83	14	32	15	17	82	21	34	9	15	51

7. To determine if a particular development program improves students marks or not, following data was collected. Using two sample run test, examine if there is any change in marks or not.

   before: 35.5 27.6 21.3 24.8 36.7 30.0
   after: 31.8 32.8 39.2 36 30 34.5 37.4

8. Tommy's climbing store sold climbing ropes during the period 2000-2004 according to the table below:

Year	2000	2001	2002	2003	2004
Sale	12342	13429	13243	14231	14378

   Make a forecast for Tommy's selling 2005 using

   (a) Exponential smoothing method using $\alpha = 0.5$.

   (b) Single moving average smoothing using $k = 4$

   (c) 4 and 5 years Weighted average smoothing.

9. Consider the following table for monthly demand of a good in a store:

Month	1	2	3	4	5	6
Demand	650	700	810	800	900	700

   estimate the demand in the month 7 using the methods: exponential smoothing with $\alpha = 0.1$, 3 months weighted average smoothing with weights $(0.2, 0.3, 0.5)$ and using simple 3 months moving average smoothing. Find out which method provides the best estimate out of the three by comparing Mean absolute deviations values.

10. Given the stationary AR(2) process:

$$X_t = \frac{5}{6}X_{t-1} - \frac{1}{6}X_{t-2} + e_t$$

   (a) Find ρ_0, ρ_1, and ρ_2.

   (b) Find ϕ_k for $k = 1, 2, \ldots$.

   (c) Find the general form for the autocorrelation function.

11. Show that the moving average process $X_n = e_n + \beta e_{n-1}$ is weakly stationary, where e_n is a white noise process with mean 0 and variance σ^2 .

12. Is the process $X_n = X_{n-1} + 2X_{n-2} + e_n$ stationary?

13. Give a derivation of the equation:

$$\gamma_0 = \alpha_1\gamma_1 + \alpha_2\gamma_2 + \alpha_3\gamma_3 + \sigma^2$$

   for the AR(3) process

$$X_n = \mu + \alpha_1(X_{n-1} - \mu) + \alpha_2(X_{n-2} - \mu) + \alpha_3(X_{n-3} - \mu) + e_n$$

   .

14. Show that the moving average process $X_n = 3 + e_n - e_{n-1} + 0.25e_{n-2}$ is weakly stationary, where e_n is a white noise process with mean 0 and variance 1 .

15. Is the MA(2) process $X_t = 2 + e_t - 5e_{t-1} + 6e_{t-2}$ invertible?

16. $\{X_t\}$ is a stationary ARMA(1,2) time series defined at integer times by the relationship:

$$X_t = \alpha X_{t-1} + e_t + \beta e_{t-2}$$

where $\alpha,\ \beta$ are constants and $\{e_t\}$ is a purely random process with mean 0 and constant variance σ^2 .

(a) Show that for any integer s :

$$Cov(X_s, e_s) = \sigma^2,\ Cov(X_s, e_{s-1}) = \alpha\sigma^2,\ Cov(X_s, e_{s-2}) = (\alpha^2 + \beta)\sigma^2$$

(b) Let γ_k denotes auto covariance at lag k, i.e, $\gamma_k = cov(X_s, X_{s-k})$

i. Write down three equations involving $\gamma_0,\ \gamma_1,$ and γ_2.

ii. Hence find expression for $\gamma_0,\ \gamma_1,$ and γ_2 in terms of $\alpha,\ \beta,$ and σ^2

(c) Let ρ_k denote the autocorrelation at lag k. Find the values of $\rho_0,\ \rho_1,\ \rho_2,$ and ρ_3 in the case where $\alpha = -0.4$ and $\beta = -0.9$.

17. Show that the process $12X_t = 10X_{t-1} - 2X_{t-2} + 12e_t - 11e_{t-1} + 2e_{t-2}$ is both stationary and invertible.

18. Classify the process $2X_t = 7X_{t-1} - 9X_{t-2} + 5X_{t-3} - X_{t-4} + e_t - e_{t-2}$.