Department of Mathematics
MAL 140 (Probability and Statistics)
Tutorial Sheet No. 4

1. Suppose that the number of customers who visit SBI, IIT Delhi on a Saturday is a random variable with $\mu = 75$ and $\sigma = 5$. Find the lower bound for the probability that there will be more than 50 but fewer than 100 customers in the bank?

2. Does the random variable X exist for which

$$P[\mu - 2\sigma \leq X \leq \mu + 2\sigma] = 0.6 \ .$$

3. Suppose that the life length of an item is exponentially distributed with parameter 0.5. Assume that 10 such items are installed successively so that the *ith* item is installed immediately after the $(i-1)$th item has failed. Let T_i be the time to failure of the ith item $i = 1, 2, \ldots, 10$ and is always measured from the time of installation. Let S denote the total time of functioning of the 10 items. Assuming that $T_i's$ are independent, evaluate $P(S \geq 15.5)$.

4. A certain industrial process yields a large number of steel cylinder whose lengths are distributed normally with mean 3.25 inches and standard deviation 0.05 inches. If two such cylinders are chosen at random and placed end to end what is the probability that their combined length is less than 6.60 inches?

5. A complex system is made of 100 components functioning independently. The probability that any one component will fail during the period of operation equal 0.10. In order for the entire system to function at least 85 of the components must be working. Compute approximate probability of this.

6. Suppose that $X_i, i = 1, 2, \ldots, 450$ are independent random variables, each having a distribution $N(0, 1)$. Evaluate $P(X_1^2 + X_2^2 + \ldots + X_{450}^2 > 495)$ approximately.
($\Phi(2) = 0.9772$, $\Phi(1.5) = 0.9452$)

7. A computer is adding number, rounds each number off to the nearest integer. Suppose that all rounding errors are independent and uniformly distributed over (-0.5,0.5).
(a) If 1500 numbers are added, what is the probability that the magnitude of the total error exceeds 15?
(b) How many numbers may be added together in order that the magnitude of the total error is less than 10 with probability 0.90?

8. Let $X \sim b(n, p)$. Use CLT to find n such that $P[X > n/2] \geq 1 - \alpha$. Calculate the value of n, when $\alpha = 0.90$ and $p = 0.45$. (Use $P[Z \leq 1.28] = 0.90$)

9. A box contains a collection of IBM cards corresponding to the workers from some branch of industry. Of the workers 20% are minors and 30% adults. We select a IBM card in a random way and mark the age given on this card. Before choosing the next card, we return the first one to the box. We observe n cards in this manner. What value should n have so that the probability that the frequency of cards corresponding to minors lies between 0.10 and 0.22 is 0.95?.

10. Items are produced in such a manner that the probability of item being defective is p (assume unknown). A large number of items say n are classified as defective or non-defective. How large should n be so that we may be 99% sure that the relative frequency of defective differs from p by less than 0.05?

11. A person puts some rupee coins into a piggy-bank each day. The number of coins added on any given day is equally likely to be 1, 2, 3, 4, 5 or 6, and is independent from day to day. Find an approximate probability that it takes at least 80 days to collect 300 rupees? Final answer can be in terms of $\Phi(z)$ where $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-t^2/2} dt$.

12. Assuming population to be $\mathcal{N}\ (\mu, \sigma^2)$, show that sample variance is a consistent estimator for population variance σ^2.

13. Let $X_1, X_2, \dots, X_n$ be a random sample from uniform distribution on an interval $(0, \theta)$. Show that $(\prod_{i=1}^{n} X_i)^{1/n}$ is consistent estimator of θe^{-1}.

14. Let $X_1, X_2, \dots, X_n$ be a random sample from Poisson distribution $P(\lambda)$. Show that $\alpha\overline{X}+(1-\alpha)s^2, 0 \leq \alpha \leq 1$, is a class of unbiased estimators for λ. Also find an unbiased estimator for $e^{-\lambda}$.

15. Let $X_1, X_2, \dots, X_n$ be a random sample from binomial distribution $B(1, p)$. Find an unbiased estimators for p^2 if it exists.

16. Suppose that 200 independent observations $X_1, X_2, \dots, X_{200}$ are obtained from random variable X. We are told that $\sum_{i=1}^{200} X_i = 300$ and that $\sum_{i=1}^{200} X_i^2 = 3754$. Using these values obtain unbiased estimates for $E(X)$ and $V(X)$.

17. Consider the number of students attended MAL 140 lecture classes for 42 lectures. Let $X_1, X_2, \dots, X_{30}$ denote the number students attended in randomly chosen 30 lecture classes. Suppose the observed data is as follows
$100, 90, 85, 95, 88, 82, 92, 84, 88, 87, 82, 88, 82, 91, 92, 91, 82, 90, 82, 87, 92, 70, 84, 79, 88, 81, 82, 78, 81, 82$

    (a) Using method of moments, find the estimators for the population mean and population variance.

    (b) Assume that, X_i's are i.i.d random variables each having discrete uniform distribution with interval 70 and 90, find ML estimators for the population mean and population variance.

18. Using method of moments, find the estimators of the parameters for the following population distributions (a) $\mathcal{N}(\mu, \sigma^2)$ (b) $B(n, p)$.

19. A random variable X has pdf $f(x) = (\beta + 1)x^\beta,\ 0 < x < 1$.

    (a) Obtain the ML estimate of β based on a sample $X_1, X_2, \dots, X_n$.

    (b) Evaluate the estimate if the sample values are 0.3,0.8,0.27,0.35,0.62 and 0.55.

20. Suppose that X has an gamma distribution, that is, the pdf is given by

$$f_X(x) = \frac{\lambda(\lambda x)^{r-1}e^{-\lambda x}}{\Gamma(r)}, \quad x > 0 .$$

    Suppose that r is known. Let $X_1, X_2, \dots, X_n$ be a random sample on X. Obtain the ML estimate of λ based on this sample.

21. A random variable X has pdf

$$f(x) = \frac{1}{2}e^{-|x-\theta|}, \quad -\infty < x < \infty .$$

    Obtain the ML estimates of θ based on a random sample $X_1, X_2, \dots, X_n$.

22. If $X_1, X_2$and X_3 are three independent random variables having the Poisson distribution with the parameter λ show that

$$\hat{\lambda_1} = \frac{X_1 + 2X_2 + 3X_3}{6}$$

    is an unbiased estimator of λ. Also compare the efficiency of $\hat{\lambda_1}$ with that of the alternate estimator.

$$\hat{\lambda_2} = \frac{X_1 + X_2 + X_3}{3} .$$