Department of Mathematics
MTL 106 (Introduction to Probability Theory and Stochastic Processes)
Tutorial Sheet No. 10

1. Consider a system with two servers and unlimited space for the waiting room. The arrival rate is λ and the service rate is μ. Write down the Kolmogorov backward equation for the above process.

2. Consider the Queueing model where customers arrives as a Poisson process with a rate of 5 per minute and service time follows an exponential distribution with rate μ service is provided to one customer at a time. Given that 90% of time the system contains less than or equal to 4 customers, then find the value of μ.

3. In an airport, there is a separate runway for arrivals only. An arriving aircraft joins a single queue for the runway. Here, the arrival follow Poisson process with the rate 20 arrivals per hour and service time is exponential distribution with a rate 27 arrivals per hour. Find the mean waiting time in the queue.

4. The time spent on a job has an exponential distribution, with a mean of 30 minutes, according to a television repairman. If he fixes the sets in the order they arrive and if the arrivals are roughly Poisson, with a mean rate of 10 every eight-hour day. Find the expected number of units in the queue?

5. In an $M/M/1$ queueing system with service rate μ and customers arrival follows Poisson process with rate λ, where $\lambda < \mu$. Suppose one extra identical server can be added, then which of the following scenarios is better in terms of mean waiting time.

   (a) Separate the two operators. Therefore, there will be two $M/M/1/\infty$ queues. In this case, it is assumed that an arriving customer can wither join the first queue or the second with equal probability.

   (b) Combine the two operators together. Therefore, it will be considered as $M/M/2/\infty$ queue.

6. A woman is the only proprietor of a unisex hair salon. She operates the salon on a first-come, first-served basis and does not take appointments. She discovers that Friday mornings are her busiest time, therefore she is thinking about hiring a part-time helper and perhaps even moving to a bigger space. She chooses to conduct a thorough analysis of the circumstance before making a decision because she has a master's degree in operations research (OR) before beginning her profession. As a result, she carefully records each Friday morning and discovers that there are 5 clients on average every hour, consistent with a Poisson process. Customers were always patient because of her stellar reputation. Consider the system as $M/M/1$ queue and calculate the following measures:

   (a) What is the mean number of customers in the shop?

   (b) What is the total number of customers waiting for a haircut?

   (c) What is the percentage of time an arrival can walk right in without having to wait at all?

   (d) If the waiting room has only four seats at present, what is the probability that a customer, upon arrival, will not be able to find a seat and have to stand?

7. Consider a single-server hair salon, where the owner serves customers alone. Customers arrive according to a Poisson process with a rate of 4 per hour, and the service time follows an exponential distribution. While customers are willing to wait for service, the waiting area can hold a maximum of 5 customers. Determine the mean time a customer spends in the system.

8. Suppose M machines are subject to failures and repairs. The failure times are exponential distributed with parameter μ, and repair times are exponentially distributed with parameter λ. Let $X(t)$ denote the number of machines that are failed at time t. If there is only one repairman, then under appropriate reasonable assumptions, let $\{X(t), t \geq 0\}$ be a birth-death process on $S = \{0, 1, \ldots, M\}$ with birth rates $\lambda_n = (M-n)\lambda, \quad n = 0, 1, \ldots, M-1$ and death rate $\mu_n = \mu, \quad n = 1, 2, \ldots, M$. Find the stationary distribution for this process.

9. A Poisson process with a rate of two per hour is formed when jobs arrive at a server with single service at a time. Such jobs service times have an exponential distribution with a mean of 20 minutes. In the system, only maximum four jobs can be active at any time. Find the likelihood that a work will be rejected due to storage space, assuming that the percentage of processing power used by smaller jobs is insignificant. Find out how many mean number of jobs in the system at steady-state as well.

10. Every Wednesday night, free vision screenings are provided by the eye clinic at City Hospital. Three ophthalmologists are on call. The real-time is found to be spread with potential distribution around the average test time of 20 minutes. Patients are accepted on a first-come, first-served basis and clients arrive according to a Poisson process with a mean of 6/hr. Consider the system as $M/M/c$ queueing system and compute the following:

    (a) What is the mean number of people waiting?

    (b) What is the total amount of mean time a patient spends at the clinic?

    (c) What is the average percentage idle time of each of the doctors?

11. Consider a $M/M/c/c$ system with two types of customers, preemptive priority policy and retrial phenomenon. When a type 2 customer arrives to the system and finds all the servers occupied, type 2 customer joins orbit of infinite capacity to avail the liberty of retrial. If the system is occupied with c type 2 customer and a type 1 customer arrives, it is given preemptive priority over a type 2 customer. As per this policy, the type 1 customer will preempt the service of a type 2 customer and starts service in its place. The preempted type 2 customer will be dropped from the system. Compute the probability when any type of customer is busy in the system and mean number of customers in the system.

12. Consider a vehicle emission inspection facility with three examination booths that can fit a single vehicle each. It makes sense to suppose that vehicles queue up such that the first vehicle to move up to an open stall will do so. At most four automobiles can wait at once in the station (there are seven total). During the busiest times, there is typically one automobile arriving every minute according to a Poisson distribution. The service time is exponential with mean 6 minutes. The chief inspector is interested in knowing the typical number using the system at busy times, the typical wait time (including service), and the anticipated number of people per hour who are unable to enter the station due to capacity.

13. In a parking lot, the vehicles arrive with the arrival rate $\lambda = 5$ and the service time is exponentially distributed with rate $\mu = 7$. Obtain the optimal value of c such that $\pi_c < 0.05$.