

Department of Mathematics
MTL 106 (Introduction to Probability Theory and Stochastic Processes)
Tutorial Sheet No. 8
Answer for selected Problems

1.

(a) $P(X_2 = 3) = 0.279$, (b) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2) = 0.0048$, (c) $P(X_{30} = 3 \mid X_{27} = 1) = 0.312$.

3. The TPM $\mathbf{P} = [P_{ij}]$ is given by

$$P_{ij} = \begin{cases} a_j, & i = 0, \\ a_{j-i+1}, & i \geq 1, j \geq i-1, \\ 0, & i \geq 1, j < i-1. \end{cases}$$

6. $\{1, 2, 3\}$

7. (a) State 0 is absorbing. $\{1, 2\}$ is a closed recurrent class. $\{3, 4\}$ are transient states.

(b) $(\pi_1, \pi_2) = (\frac{2}{5}, \frac{3}{5})$

(c) $[\mu_{ij}] = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$ where $i, j \in \{3, 4\}$

(d) $\frac{9}{16}$

12. $\frac{4}{27}$

15. The chain is transient.

21. $\mathbf{P} = \begin{pmatrix} 1-v_0 & 0 & v_0 & 0 & 0 & 0 & \cdots \\ 1-v_1 & 0 & 0 & v_1 & 0 & 0 & \cdots \\ 1-v_2 & 0 & 0 & 0 & v_2 & 0 & \cdots \\ 1-v_3 & 0 & 0 & 0 & 0 & v_3 & \cdots \\ 1-v_4 & 0 & 0 & 0 & 0 & 0 & v_4 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

23. $\pi_j = \frac{1}{K}$ for $j = 0, 1, \dots, K-1$

28. (a) $f_{j0}^{(n)} = \begin{cases} \binom{n-1}{j-1} p^{n-j} q^j, & n \geq j, \\ 0, & n < j. \end{cases}$

(b) $\frac{j}{q}$

29. $\frac{1}{4}$