Department of Mathematics
MTL 106 (Introduction to Probability Theory and Stochastic Processes)
Tutorial Sheet No. 9
Answer for selected Problems

1. e^{-5t}

2. $\mathbf{Q} = \begin{pmatrix} -\lambda & \lambda & 0 & 0 & 0 \\ \mu & -(\lambda+\mu) & \lambda & 0 & 0 \\ 0 & 2\mu & -(\lambda+2\mu) & \lambda & 0 \\ 0 & 0 & 3\mu & -(\lambda+3\mu) & \lambda \\ 0 & 0 & 0 & 4\mu & -4\mu \end{pmatrix}$

   and

   $$\frac{dP_0(t)}{dt} = -\lambda P_0(t) + \mu P_1(t),$$
   $$\frac{dP_1(t)}{dt} = \lambda P_0(t) - (\lambda+\mu)P_1(t) + 2\mu P_2(t),$$
   $$\frac{dP_2(t)}{dt} = \lambda P_1(t) - (\lambda+2\mu)P_2(t) + 3\mu P_3(t),$$
   $$\frac{dP_3(t)}{dt} = \lambda P_2(t) - (\lambda+3\mu)P_3(t) + 4\mu P_4(t),$$
   $$\frac{dP_4(t)}{dt} = \lambda P_3(t) - 4\mu P_4(t).$$

13. $\frac{1}{\lambda}$

15. $P\{X(t)=0 \mid X(0)=0\} = e^{-\lambda t}$, $P\{X(t)=1 \mid X(0)=0\} = 1-e^{-\lambda t}$, $P\{X(t)=2 \mid X(0)=0\} = 0$.

17. $9e^{-8}$

21. $\frac{\lambda T^2}{2}$

25. (a) $T_j \sim \text{Exp}(\lambda)$, i.e., $P(T_j \le t) = 1-e^{-\lambda t}$, $t \ge 0$.

    (b) T_{2023}, T_{2024}, T_{2025} are i.i.d. Exp(λ)
    Joint PDF is given by $f_{T_{2023},T_{2024},T_{2025}}(t_1,t_2,t_3) = \lambda^3 e^{-\lambda(t_1+t_2+t_3)}$, $t_1,t_2,t_3 \ge 0$.

28. (a) $e^{-0.25}$

    (b) $5e^{-2}$

32. $E[N(T_1)T_1] = \frac{2}{\lambda}$ and $Var[N(T_1)T_1] = \frac{26}{\lambda^2}$.