Department of Mathematics
MTL 106 (Introduction to Probability Theory and Stochastic Processes)
Tutorial Sheet No. 2
Answer for selected Problems

4. Minimal sigma field $= \{\phi, \{a,b,c\}, \{d\}, \Omega\}$

6. $f(x) = \begin{cases} 1, & x \in E \\ -1, & x \in E^c \end{cases}$ where E is a non-measurable subset of $\mathbf{R}$

8. 11

9. $P(X = i) = \binom{5}{i} \frac{1}{10}^i \frac{9}{10}^{5-i}$, $i = 0$ to 5

10. $P(X = i) = \frac{\binom{7}{i}}{\binom{10}{i}} \frac{4}{7}$, $i = 1$ to 7

11. $\alpha = 1 - p, 0 < p < 1$

12. (i) $\frac{1}{81}$
    (ii) $\frac{16}{81}$, $\frac{56}{81}$, $\frac{15}{81}$
    (iii) $F_X(x) = \frac{(x+1)^2}{81}$ where $x \in \mathbf{Z}^+ \cap \{0\}$
    (iv) 6

13. $c = 1$, $\frac{1}{6}$

14. $0.1, 0.1, 0.01$

15. $P(Y = y) = e^{-\lambda p} \frac{(\lambda p)^y}{y!}$, $y = 0, 1, 2, \ldots$
$$P(|Y-4| = y) = \begin{cases} e^{-\lambda p}\left(\frac{(\lambda p)^{4+y}}{(4+y)!} + \frac{(\lambda p)^{4-y}}{(4-y)!}\right), & y = 1,2,3,4 \\ e^{-\lambda p}\left(\frac{(\lambda p)^{4+y}}{(4+y)!}\right), & o.w. \end{cases}$$

16. $P(X = x) = \begin{cases} \frac{1}{10}, & x = 3,4,8,9 \\ \frac{2}{10}, & x = 5,6,7 \\ 0, & o.w. \end{cases}$

17. $P(X = x) = P(X = 14 - x) = \frac{min\{x, 14-x\} - 1}{36}$, $x = 2$ to 12

20. (i) $\frac{26}{11} e^{-15}$
    (ii) $Y \sim Bin(10, 1 - 51e^{-50})$
    (iii) doesn't exist

21. (a) 0.64
    (b) $1 - (0.36)^3$, $1 - (0.7225)^8$

22. Yes, continuous, $f_X(x) = 2e^{-x}(1 - e^{-x})$

23. $\frac{1}{4}$

24. 1

25. $\frac{24}{25}$

26. $\frac{1}{2}$

27. $F_Y(x) = \begin{cases} 0, & x < 0 \\ F_X(x), & x \geq 0 \end{cases}$

28. $P(X=x) = \begin{cases} 0.4, & x = 3,5 \\ 0.2, & x = 4 \end{cases}$

29. $f_X(x) = 4x + 1, \quad for\ x \in (0, 1/2)$
$P(X > 1/2) = 0, \quad P(1/4 < x < 1/2) = 5/8, \quad P(X < 1/2 | X > 1/4) = 1$

31. $P(X \leq t) = \begin{cases} 0, & t < 0 \\ \frac{1}{2}t^2, & 0 \leq t \leq 1 \\ 1 - \frac{1}{2}(2-t)^2, & 1 \leq t \leq 2 \\ 1, & t > 2 \end{cases}$

32. $P(X = x) = \frac{\binom{n-x}{m-1}}{\binom{n}{m}}, \quad for\ x = 1, 2, 3, \ldots, n-m+1$
$P(X \geq 3/2) = \frac{n-m}{n}$

33. $F_2(x) = \begin{cases} \frac{2x}{r^2}, & 0 < x < r \\ 0, & \text{elsewhere} \end{cases}$

35. $a = 1$, $b = 2$
$f_X(x) = \begin{cases} x, & 0 \leq x < 1 \\ 2 - x, & 1 \leq x < 2 \\ 0, & o.w. \end{cases}$
$f_X(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{2}, & 0 \leq x < 1 \\ 2x - \frac{x^2}{2} - 1, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$

37. $F_1(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{4}, & 0 \leq x < 1 \\ 1, & x \geq 1 \end{cases}$ and
$F_2(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \leq x < 1 \\ 1, & x \geq 1 \end{cases}$
$\alpha = \frac{2}{3}$ and $P(1/2 \leq X \leq 1 | X > 1/4) = \frac{2}{3}$.

39. (i) $\frac{1}{b\sqrt{2\pi}} e^{-\frac{(y-a)^2}{2b^2}}, \quad -\infty < y < \infty.$
(ii) $\frac{1}{\sqrt{2\pi}} e^{-\frac{e^{2y}}{2}}.e^y, \quad -\infty < y < \infty.$
(iii) $\frac{1}{\sqrt{2zpi}} e^{-z/2}, \quad 0 < z < \infty.$