



Performance analysis of DRX mechanism using batch arrival vacation queueing system with N -policy in LTE-A networks

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Received: 22 June 2017 / Accepted: 26 June 2018 / Published online: 9 July 2018
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Abstract

Power saving and Quality of Service (QoS) are the two significant aspects of Long Term Evolution-Advanced (LTE-A) networks. DRX (“Discontinuous Reception”) is a mechanism, commonly exercised to enhance the power saving competency of a User Equipment (UE) in LTE-A networks. In this paper, based on the kind of traffic running at the UE, a new appliance is proposed to switch the DRX mechanism from the power active state to the power saving state and vice versa. We mathematically investigate this switching technique in DRX mechanism using the $M^{[X]}/G/1$ vacation queue system with N -policy. Various performance and energy metrics are obtained and examined numerically. Further, the optimal value of N as well as the maximum number of DRX cycles, are computed to obtain the minimal amount of power consumption. This study concludes the selection guidelines for choosing the optimal values of N and the maximum number of DRX cycles.

Keywords LTE -A networks · DRX mechanism · $M^{[X]}/G/1$ vacation queueing system · Power saving · Delay

1 Introduction

Long Term Evolution-Advanced (LTE-A) is the latest mobile broadband technology, which has been initiated by the Third Generation Partnership Project (3GPP). It is designed to keep up with the today’s promptly amplifying data traffic; however, it lacks in maintaining the energy efficiency of the UEs. In addition to energy efficiency, Quality of Service (QoS) is another emerging issue, which should also be managed simultaneously as the multiple types of data and services are handled at one point in time in UEs [1, 2]. Most of the techniques, such as DRX mechanism, sleep mode, Bluetooth Low Energy (BLE, previously known

as Wibree), low-power Wi-Fi, are used to save energy consumption; however, they are also responsible for the poor QoS [3].

Several analytical studies regarding the performance of DRX mechanism have been conducted in the literature, which investigate the performance of Universal Mobile Telecommunications System (UMTS) DRX, a former technology in 3GPP LTE-A networks with a variant $M/G/1$ vacation model [4–6]. The abovementioned literature assumes that the arriving data traffic follows a Poisson distribution, which is an unrealistic assumption as it does not capture the bursty nature of the data traffic. In [7, 8] and [9, 10], authors have analyzed the $M/G/1$ queue with repeated inhomogeneous vacations and homogeneous vacations, respectively. Further, these models are used to study the problems of power saving in UEs. In [9, 10], DRX mechanism is modeled and analyzed, whereas in [7, 8], IEEE802.16e sleep mode and a general power saving scheme are considered for the performance analysis. In [11], two power saving modes for IEEE802.16m and 3GPP LTE-A are analyzed for their performance, and a comparison is presented. This paper concludes that the power saving obtained by enabling the DRX mechanism is significantly substantial in comparison to the power saving obtained by enabling the sleep mode in a UE. In [12], optimal policy in case of a Poisson arrival process is derived for an on-off model and a selection of the best plan among a class

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is performed. In [13], a new sleep mode scheme called the power saving mechanism with binary traffic indications is proposed for the IEEE802.16e standard. It observed that TRF-IND interval increases binary-exponentially. Impact of 3GPP-defined power saving mechanism, on the performance of the users during the continuous connectivity is investigated [14]. Further, the $M/G/1$ queueing model with multiple classes and inhomogeneous vacations is used for traffic modeling. Yeh et al. [15] compares the energy saving techniques in 3GPP and 3GPP2 systems. Qualitative, as well as quantitative comparisons, are conducted for the same. These comparisons may be useful in determining the appropriate parameter for the power saving schemes. In [16], authors have proposed a traffic based DRX cycle adjustment scheme. In this model, partially observable decision process is employed to represent the traffic pattern.

To the best of the authors' knowledge, there is no literature found, which analyze the DRX mechanism with a batch arrival vacation queue modeling technique having the concept of N -policy and at most a fixed number of vacations to reduce the power consumption. Analysis of a queueing model with N -policy and at most J vacations was first initiated theoretically by [17]. However, neither the performance analysis was performed, nor the optimal N was obtained. It motivated us in utilizing this technique to capture the stochastic behavior of DRX mechanism in the LTE-A networks and improve the energy efficiency of the UEs.

The rest of this paper is organized as follows. The DRX mechanism and its analytical model for the performance analysis in LTE-A networks are explained in Section 2. Various performance measures, such as expected packet delay, handover ratio, etc., are obtained in Section 3. Energy efficiency under the power saving is discussed in Section 4. The numerical illustration and the comparison with the existing work are performed in Sections 4.3 and 4.4, respectively. In Section 5, the impact of the arrival and the service rate are discussed on various performance measures obtained in the previous sections. The concluding remarks and some future directions are presented in Section 6.

2 DRX Mechanism and its performance analysis in LTE-A networks

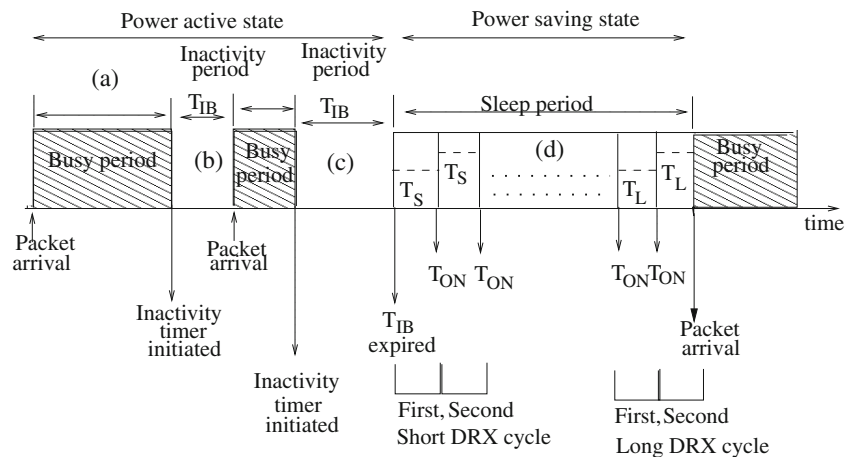
2.1 DRX mechanism in LTE-A networks

DRX operations of a UE are regulated in the eNode B (eNB) by the Radio Resource Control (RRC) component through the Physical Downlink Control Channel (PDCCH) [18]. LTE/LTE-A network supports two types of the RRC modes, i.e., RRC_CONNECTED and RRC_IDLE. These modes are categorized based on the handling of the RRC connection. The fundamental difference between LTE/LTE-A

DRX mechanism and other existing DRX mechanisms is that the LTE/LTE-A DRX mechanism allows the UE to enter a sleep period, while the UE is in RRC_CONNECTED mode, i.e., still registered with the eNB. It leads to more power saving. Throughout this work, we have used the word system exhaustively. DRX mechanism for the UE is termed as the system. Further, DRX mechanism and its parameters are discussed briefly as the following [19–21]:

- *Inactivity timer* (T_{IN}): It is the time period during which the UE waits before starting the DRX. It works as a timer which re-initiates itself after successful reception of the data packets on the PDCCH. On the expiration of the inactivity timer, the UE enters into a short DRX cycle.
- *On duration timer* (T_{ON}): It is the length of a tiny time interval inside a DRX cycle (short or long) at its end. During the T_{ON} period, the UE monitors the PDCCH for the arrival of a new packet. A packet arrival indication at the PDCCH during the T_{ON} period, wakes up the UE and it begins to serve the data packets. The time duration of the T_{ON} period remains same for the short as well as the long DRX cycles.
- *Short DRX cycle*: It is the length of the first DRX cycle after the initiation of DRX mechanism in UEs. This short DRX cycle is repeated for a predetermined number of times if no data packet arrives and ends otherwise. During this period, the UE turns off most of its components. It is divided into two parts, the short DRX sleep period (T_S) and the T_{ON} period.
- *Short DRX cycle timer* (T_D): It is a timer which regulates the number of short DRX cycles before the start of the first long DRX cycle.
- *Long DRX cycle*: It is the length of the first long DRX cycle, which starts on the expiration of T_D short DRX cycles if no data packet arrives ($T_L \geq T_S$). Further, this cycle repeats for a predetermined number of times. It is divided into two parts, the long DRX sleep period (T_L) and the T_{ON} period.
- *Busy period* (T_B): It is the length of time during, which the UE provides service to the data packets after waking up from DRX.

The transition from the inactivity timer period to the short DRX cycle is controlled by T_{IB} , while the transitions among the short DRX cycles are controlled by T_D . Therefore, the DRX mechanism is comprised of the power active and the power saving states. Further, as depicted in Fig. 1, the power active state includes *Busy period*, *Inactivity timer period*, and *ON period*, whereas power saving state includes the length of sleep periods (i.e., the time duration of short and long sleep periods). Also for a detailed functioning of DRX mechanism readers can refer to [20]. The possible values of the DRX parameters are given in Table 1 for LTE-A

Fig. 1 An illustration of the DRX mechanism in LTE-A networks

networks. Thus, the DRX mechanism basically consists of the busy period, the inactivity timer period, and the short and the long DRX cycles.

From Fig. 1, stepwise transitions states (a) to (b), (b) to (c), etc., can be observed as follows: Initially, the system begins either in the busy period or in the inactivity timer in RRC.CONNECTED mode of LTE-A networks. Without loss of generality, we assume that the system starts in the busy period.

- The system remains in busy period if there are packets to be served in the buffer. Otherwise, system moves to the inactivity timer state.
- If, before the expiration of the inactivity timer, a packet arrives then the system moves to the busy period again.
- Further, after completion of the service of the packets in the buffer, it again moves to the inactivity timer. If no packet arrives then the inactivity timer expires, and the system runs on a short DRX cycle in succession.
- These short DRX cycles keep going on until a fixed number (T_D), and then a long DRX cycle starts to take place.
- Further, these cycles keep going on for a predetermined fixed number until a packet arrives and the service begins.

2.2 Selection of the DRX mechanism parameters

The UE employees DRX mechanism under the following assumptions are as follows:

- Consideration of the Internet traffic: Popular data applications, such as Facebook, Twitter, and online messengers have characteristic quite different from traditional Internet applications. They require small but frequent data exchange between the UE and the corresponding network. These recurrent exchange of a little amount of data creates critical challenges as the UE keeps on changing its state between the

RRC.CONNECTED and the RRC.IDLE mode, which leads to draining a significant amount of the battery life. These traffic types are considered as *Background traffic*. In multimedia applications, like YouTube and Skype, the complexity of traffic is a natural consequence of integrating, over the single communication channel. A diverse range of traffic sources, such as video, voice, and data packets, disagree notably in their traffic patterns as well as requirements for the performance. But they show a certain correlation between arrivals and long-range dependence in time [21]. These traffic types are considered as *Self-similar traffic*.

- Adaptative mode of the UE on the basis of the traffic arrival: In case of the background type traffic arrival, short DRX cycle goes on consecutively until a fixed number, say K_1 is reached. Additionally, it requires to switch on for a tiny period to receive notifications (e.g., Whatsapp, Facebook notifications). During the ON period of each short DRX cycle, these notification data packets are received. In case of the self-similar type traffic arrival, packets come in the bulk quantity. Further, they need to be served as early as possible to avoid them from getting lost. Hence, the UE starts to serve and remains in the busy period until all the packets are served.
- Employing N -policy in DRX mechanism: We engaged a policy called as N -policy in the DRX mechanism. According to this policy, packets will be served only if a number of packets in the buffer added up to N during

Table 1 Parameters in DRX mechanism

Parameter	Values
cDRX ON duration timer (ms)	4
DRX inactivity timer (ms)	1–500
Short DRX cycle timer (K_1)(ms)	1–16
Short DRX cycle(average length) (ms)	10–80
Long DRX cycle (average length) (ms)	10–320

the power saving state of the UE. Otherwise, short DRX cycles keep on occurring successively until T_D expires. Subsequently, during the occurrence of long DRX cycles, if the N number of packets are indicated by the PDCCH at the buffer, then service starts quickly in the next ON duration. Consequently, it can be observed that every packet indication is not served immediately [21]. Therefore, employing N -policy leads to more power saving for a UE.

2.3 The analytical model for the performance analysis

In this section, each component of DRX mechanism is mapped to the parts of a vacation queue system. Further, it is followed by presenting an outline for a considered vacation queue model.

In this study, we assume that a UE does not enter a DRX cycle until the entire packet transmission is completed. If there is no indication of the N packets, inactivity timer always starts at the end of the fixed number of DRX cycles, say K_2 . Length of the inactivity timer is considered as equals to the length of the ON duration. Also, this time duration is very small in length, in which energy consumption is not significant in comparison to energy consumption in the busy period. Hence, when the packet arrival rate is low and N is large, then the UE will wait only for a time duration equals to the T_{ON} period. After that, it will move to the DRX mode to save power consumption.

Consider an $M^{[X]}/G/1$ queue model with N -policy and at most K_2 vacations. Now, in reference to the mapping between the DRX mechanism and the aforesaid vacation queue model, when a batch of arriving packets find the server (UE) on vacation (either a short DRX cycle or a long DRX cycle) and the packets in the buffer of the UE are indicated as less than N in number, then the UE goes for the next vacation consecutively. Further, this process continues until, either the number of packets in the buffer reaches N or K_2 vacations expire. Also, packets arriving which join the system form a single waiting line based on the order of their arrivals (i.e., first come first serve (FCFS) discipline).

In a vacation queue model, an *idle period* consists of the vacation and the dormant periods. An idle period together with a busy period in the system is known as a *busy cycle*. A vacation period is defined as the time spent by the customer in a vacation. When a server who returns after completing a fixed number of vacations and does not find any customer in the system then, it waits for the arrival of the first customer in the system. This waiting time of the server is called *dormant period*. A reader can go through references [21–23] to understand the vacation queueing system in detail.

Further, for the clarity of the readers, in simple words, one can say that in a vacation queue model sum of the

vacation and the dormant periods are considered as the idle period. Also, a busy cycle is the sum of the idle period and the busy periods.

Now, to model the DRX mechanism as a vacation queue model, we map its components with a vacation queue model. In this consideration, the short and the long DRX sleep periods are mapped with two different vacation periods. Also, the inactivity timer period of DRX mechanism is mapped with the dormant period of the vacation queue model. The mapping between the DRX mechanism and a vacation queue model, in particular, the $M^{[X]}/G/1$ vacation queue model with N -policy and at most K_2 vacations, is depicted in Fig. 2.

Furthermore, to capture the self-similar nature of massive multimedia packets arrival in LTE-A networks, we assume the input process to be batch arrival. Thus, the batch size is supposed to follow Pareto distribution due to its heavy tail property, which is responsible for aggregating more and more identically and independently distributed (i.i.d.) copies of the renewal reward processes [6].

Now, we briefly describe the system model. In an RRC.CONNECTED state of the UE, the length of the short and long DRX cycles are abstracted as vacation periods and are denoted by V_1 and V_2 , respectively. V_1 's are i.i.d. random variables and duration of long DRX cycle are larger and multiple of the length of a short DRX cycle. Also, Laplace Stieltjes Transform (LST) of V_i is denoted as $\hat{V}_i(\theta) = E[e^{-\theta V_i}]$, $i = 1, 2$. The buffer capacity of the eNB is supposed to be infinite. Also, the proposed model takes into account the downlink packet transmission only. Further, we analyze the system using the bivariate Markov processes, Kolmogorov forward equation, and the supplementary variable technique.

2.4 Description of the system model

As discussed above, we have used an $M^{[x]}/G/1$ queue model with N -policy and at most K_2 vacations to study the DRX mechanism in LTE-A networks. The state transition diagram for the same is depicted in Fig. 3. In this figure, at each state (i, j) , the variables $i \in \{0, 1, 2, \dots\}$ and $j \in \{0, 1, 2\}$ denote the number of packets in the system at time t and state of the UE at time t , respectively. Moreover, for $j \in \{0, 1, 2\}$, 0, 1, and 2 stand for the vacation, the dormant and the busy periods, respectively.

Assume that the packet arrival follows, the compound Poisson process with an arrival rate λ and are served one at a time on an FCFS basis. These successively arriving batches are i.i.d. random variables. Let X be the number of packets in a batch arrival. Then, the probability mass function and Probability Generating Function (PGF) of X are given by $P\{X = k\} = p_k$, $k = 1, 2, 3, \dots$, and $P_X(z) = \sum_{k=1}^{\infty} p_k z^k$, ($|z| \leq 1$), respectively.

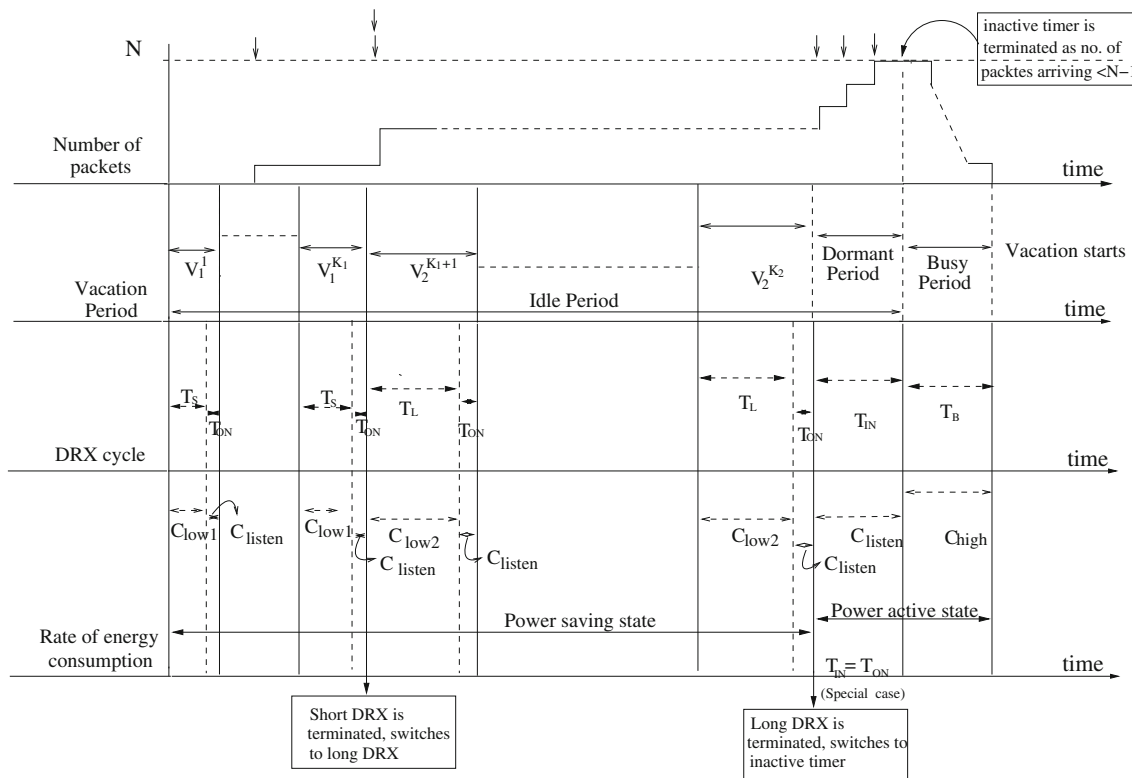


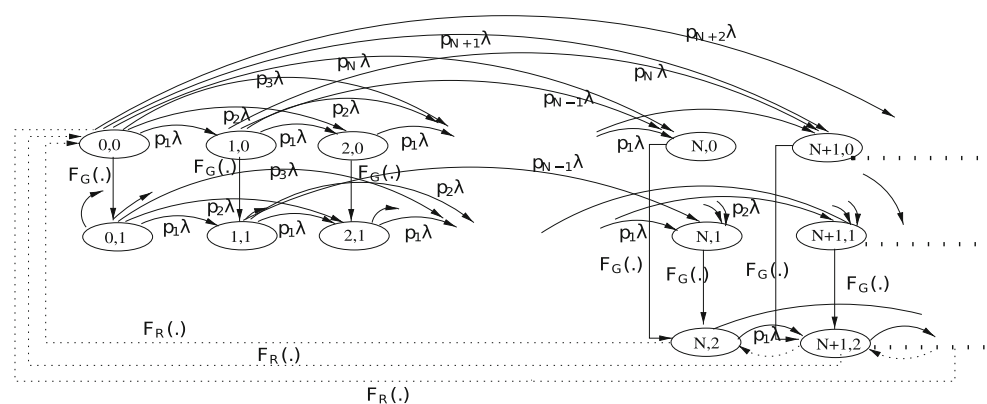
Fig. 2 Mapping between DRX mechanism and vacation queue model considered

Hence, $p_k \lambda$ denotes the rate of arrival of k packets in a batch. Before proceeding to the system model, some notations and random variables adopted in this manuscript are given as follows:

- N : threshold level of packets to begin service ($N \geq 1$),
- $E(\cdot)$: expectation of the random variable ‘ $\cdot$ ’,
- X : a random variable denoting the arriving batch size,
- $P_X(z)$: PGF of X ,
- K_1 : maximum number of short DRX cycles,
- K_2 : the total number of DRX cycles,
- R : service time random variable,

- $F_R(x)$: Cumulative Distribution Function (CDF) of R ,
- $P_R(z)$: PGF of R ,
- $\hat{\beta}(\theta)$: LST of R and is given by $E[e^{-\theta R}]$,
- β_k : the k th finite moment of R ,
- V_i : i th vacation time random variable, $i = 1, 2$,
- $\hat{V}_i(\theta)$: LST of V_i , $i = 1, 2$,
- $F_{V_i}(x)$: CDF of V_i ,
- ρ : traffic intensity,
- $L(t)$: number of packets in the system at time t ,
- L : number of packets in the system in the steady-state,
- $P_L(z)$: PGF of L ,
- $R^c(t)$: consumed service time at time t ,

Fig. 3 State transition diagram for the $M^{[X]}/G/1$ queue model with N -policy and at most K_2 vacations



$d_i^l(t)$	: elapsed time of l th DRX cycle of type i at time t where for $i = 1, l = 1, 2, \dots, K_1$, and for $i = 2, l = K_1 + 1, K_1 + 2, \dots, K_2$,	$W_{l,n}(x, t)$	: the probability that there are exactly n packets in the buffer at time t and elapsed time of l th DRX cycle of type i is lying in between x and $x + dx$, where for $i = 1, l = 1, 2, \dots, K_1$, and for $i = 2, l = K_1 + 1, K_1 + 2, \dots, K_2$,
$G(t)$	: the state of the UE at time t ,	$\mu(x)$	: first-order differential function of random variable R ,
$S(t)$	: the elapsed time in a state of the UE at time t ,	$h_i(x)$	: first-order differential functions of random variable $V_i, i = 1, 2$,
$U_n(x, t)$	: the probability that there are precisely n packets in buffer at time t with consumed service time of the tagged packet, which is being served is lying in between x and $x + dx$,	$W_{l,0}$	: the steady-state probability that no packet appears, while the UE is on l th DRX cycle, $l = 1, 2, \dots, K_2$.
$Y_n(t)$	: the probability that there are n packets in the buffer when the UE is in inactive state at time $t, (0 \leq n < N)$,		

$$G(t) := \begin{cases} 0, & \text{if the UE is in the inactivity timer period in the system at time } t, \\ 1, & \text{if the UE is in the busy period in the system at time } t, \\ 2, & \text{if the UE is in the 1st short DRX cycle in the system at time } t, \\ \vdots & \\ K_1, & \text{if the UE is in the } (K_1 - 1)\text{th short DRX cycle in the system at time } t, \\ K_1 + 1, & \text{if the UE is in the } K_1\text{th short DRX cycle in the system at time } t, \\ K_1 + 2, & \text{if the UE is in the 1st long DRX cycle in the system at time } t, \\ \vdots & \\ K_2 + 1 & \text{if the UE is in the } K_2\text{th } ((K_2 - K_1 - 1)\text{th long) DRX cycle in the system at time } t, \end{cases}$$

where K_1 is the short DRX cycle timer and $K_1 < K_2$. Further, when $G(t) \in \{2, 3, \dots, K_1 + 1\}$, the UE is in the

short DRX cycle, whereas when $G(t) \in \{K_1 + 2, \dots, K_2 + 1\}$, the UE is in long DRX cycle.

$$S(t) := \begin{cases} 0, & \text{if } G(t) = 0, \\ R^c(t), & \text{if } G(t) = 1, \\ d_i^l(t), & \text{if } G(t) = l + 1 \text{ for } i = 1, l = 1, 2, \dots, K_1, \text{ and for } i = 2, l = K_1 + 1, K_1 + 2, \dots, K_2. \end{cases}$$

The service times are i.i.d. random variables, and further, we assume them to have a finite mean. Then, condition for the system to be in the steady-state regime [24] is given by $\lambda E[X]E[R] < 1$, and it is supposed to be true throughout this manuscript.

Now, we obtain the performance measures by developing the steady-state difference-differential equations for the DRX mechanisms by treating the elapsed service time, elapsed time in short, and long DRX cycles as supplementary variables. In the steady-state, for CDFs $F_R(x)$, and $F_{V_i}(x)$, $i = 1, 2$, it is assumed that $F_R(0) = 0, F_R(\infty) = 1, F_{V_i}(0) = 0, F_{V_i}(\infty) = 1$. Also, $F_R(x)$, and $F_{V_i}(x)$ are continuous at $x = 0$, so that

$$\mu(x) = \frac{dF_R(x)}{(1 - F_R(x))}, \text{ and } h_i(x) = \frac{dF_{V_i}(x)}{(1 - F_{V_i}(x))}, i = 1, 2,$$

are the first-order differential (hazard rate) functions of R and $V_i, i \in \{1, 2\}$, respectively [22].

Now, we denote $\{(L(t), S(t)) : t \geq 0\}$ as a bivariate Markov process, where $\{S(t), t \geq 0\}$ is an underlying process and $\{L(t), t \geq 0\}$ is an observable process with state space $\{0, 1, \dots\}$. Next, we consider the following probabilities:

$$\begin{aligned} Y_n(t) &:= P\{L(t) = n, S(t) = 0\}, \\ &\quad n = 0, 1, 2, \dots, N - 1, \\ U_n(x, t)dx &:= P\{L(t) = n, S(t) = R^c(t); \\ &\quad x < R^c(t) \leq x + dx\}, \\ &\quad x > 0, n = 1, 2, \dots, \\ W_{l,n}(x, t)dx &:= P\{L(t) = n, S(t) = d_i^l(t); x < d_i^l(t) \\ &\quad \leq x + dx\}, x > 0, \\ &\quad \text{where for } i = 1, l = 1, 2, \dots, K_1, \\ &\quad \text{and for } i = 2, l = K_1 + 1, K_1 + 2, \dots, \\ &\quad K_2, n = 0, 1, 2, \dots \end{aligned}$$

For the steady-state analysis, consider

$$Y_n := \lim_{t \rightarrow \infty} Y_n(t) \text{ for } n = 0, 1, \dots, N-1, \\ \text{and limiting densities for } X > 0 \text{ as}$$

$$U_n(x) := \lim_{t \rightarrow \infty} U_n(x, t), n = 1, 2, \dots,$$

$$W_{l,n}(x) := \lim_{t \rightarrow \infty} W_{l,n}(x, t), n = 0, 1, \dots, l = 1, \dots, K_2.$$

Let $\{P_Y(z)\}$, $\{P_U(x, z)\}$, and $\{P_{W_l}(x, z)\}$ be the PGFs of $\{Y_n\}$, $\{U_n(x)\}$, and $\{W_{l,n}(x)\}$, respectively, and are given by

$$P_Y := \sum_{n=0}^{N-1} z^n Y_n, P_U(x, z) := \sum_{n=1}^{\infty} z^n U_n(x), \text{ and } P_{W_l}(x, z) \\ := \sum_{n=0}^{\infty} z^n W_{l,n}(x), (|z| \leq 1), l = 1, \dots, K_2.$$

Further, consider $P_U(z) := \int_0^{\infty} P_U(x, z) dx$ and $P_W(z) := \sum_{l=1}^{K_2} P_{W_l}(z)$, where $P_{W_l}(z) := \int_0^{\infty} P_{W_l}(x, z) dx$.

2.5 The steady-state equations

Since, $\{L(t), S(t) : t \geq 0\}$ is Markovian in continuous time, then the equation for this process can be observed by using the usual Erlangian for transitions occurring in δt time [24]. Following are the arguments of Kolmogorov forward equations, which govern the system under steady-state conditions [25]:

$$\lambda Y_n = \int_0^{\infty} W_{K_2,n}(x) h_i(x) dx + \lambda (1 - \delta_{n,0}) \sum_{k=1}^n p_k Y_{n-k}, \\ \text{for } x > 0, i = 1, 2, n = 0, 1, 2, \dots, N-1, \quad (1)$$

$$\frac{d}{dx} U_n(x) + [\lambda + \mu(x)] U_n(x) = \lambda \sum_{k=1}^n p_k U_{n-k}(x), \\ \text{for } x > 0, n = 1, 2, \dots, \quad (2)$$

$$\frac{d}{dx} W_{l,0}(x) + [\lambda + h_i(x)] W_{l,0}(x) = 0, \text{ where for } i = 1, \\ l = 1, 2, \dots, K_1, \text{ and for } i = 2, l = K_1 + 1, \dots, K_2, \quad (3)$$

Also, K_1 denotes the last (i.e., K_1^{th}) short DRX cycle and $\delta_{n,0}$ denotes the Kronecker's delta function. Furthermore, we have

$$\frac{d}{dx} W_{l,n}(x) + [\lambda + h_i(x)] W_{l,n}(x) \\ = \lambda \sum_{k=1}^n p_k W_{l,n-k}(x), x > 0, n = 1, 2, \dots, \\ \text{where for } i = 1, l = 1, \dots, K_1, \\ \text{and for } i = 2, l = K_1 + 1, \dots, K_2. \quad (4)$$

Consider, the boundary conditions at $x = 0$ and normalizing condition as follows:

$$U_n(0) := \int_0^{\infty} U_{n+1}(x) \mu(x) dx, n = 1, 2, \dots, N-1, \quad (5)$$

$$U_n(0) := \int_0^{\infty} W_{1,n}(x) h_1(x) dx + \dots \\ + \int_0^{\infty} W_{K_1,n}(x) h_1(x) dx \\ + \int_0^{\infty} W_{K_1+1,n}(x) h_2(x) dx + \dots \\ + \int_0^{\infty} W_{K_2,n}(x) h_2(x) dx, \\ + \int_0^{\infty} U_{n+1}(x) \mu(x) dx \\ + \lambda \sum_{k=0}^{N-1} Y_k p_{n-k}, n = N, N+1, \dots, \quad (6)$$

$$W_{l,n}(0) := \begin{cases} \int_0^{\infty} U_1(x) \mu(x) dx, n = 0 \\ 0, n = 1, 2, \dots, \end{cases} \quad (7)$$

$$W_{l,n}(0) := \begin{cases} \int_0^{\infty} W_{l-1,n}(x) h_1(x) dx, n = 0, 1, \dots, N-1, l = 2, 3, \dots, K_1 \\ \int_0^{\infty} W_{l-1,n}(x) h_2(x) dx, n = 0, 1, \dots, N-1, l = K_1 + 1, K_1 + 2, \dots, K_2 \\ 0, n = N, N+1, \dots, l = 2, 3, \dots, K_2. \end{cases} \quad (8)$$

Thus, the normalizing condition is given by

$$\sum_{n=0}^{N-1} Y_n + \sum_{n=1}^{\infty} \int_0^{\infty} U_n(x) dx \\ + \sum_{l=1}^{K_2} \left[\sum_{n=0}^{\infty} \int_0^{\infty} W_{l,n}(x) dx \right] = 1. \quad (9)$$

2.6 The steady-state probability that no packet appears, while the UE is in power saving state

From Eq. 4, we have

$$W_{l,0}(x) = W_{l,0}(0) [1 - F_{V_i}(x)] e^{-\lambda x}, \text{ where for } i = 1, \\ l = 1, 2, \dots, K_1, \text{ and for } i = 2, l = K_1 + 1, \\ K_1 + 2, \dots, K_2. \quad (10)$$

Now, multiplying the above equation by $h_1(x)$ and $h_2(x)$ for $i = 1, 2$, respectively, and then integrating with respect to x from 0 to ∞ on both the sides, we get

$$\frac{\lambda Y_0}{\zeta_0} = W_{K_1,0}(0), \quad \frac{\lambda Y_0}{\zeta'_0} = W_{K_2-K_1,0}(0),$$

where $\zeta_0 = \widehat{V}_1(\lambda)$ and $\zeta'_0 = \widehat{V}_2(\lambda)$. (11)

Solving Eq. 11 recursively from $l = K_1 - 1, \dots, 1$ and $l = K_2 - 1, K_2 - 2, \dots, K_2 - K_1$, we get

$$W_{l,0}(0) = \begin{cases} \frac{\lambda Y_0}{\zeta_0^{K_1-l+1}}, & l = 1, 2, \dots, K_1 \\ \frac{\lambda Y_0}{\zeta'_0 K_2 - K_1 - l + 1}, & l = K_1 + 1, K_2 + 1, \dots, K_2. \end{cases} \quad (12)$$

Thus, on integrating Eq. 10 with respect to x from 0 to ∞ , we have

$$W_{l,0} = \begin{cases} \frac{Y_0(1-\zeta_0)}{\zeta_0^{K_1-l+1}}, & l = 1, 2, \dots, K_1 \\ \frac{Y_0(1-\zeta_0)}{\zeta'_0 K_2 - K_1 - l + 1}, & l = K_1 + 1, K_2 + 1, \dots, K_2. \end{cases}$$

Therefore,

$$W_0 = \sum W_{l,0} = \frac{Y_0(1-\zeta_0^{K_1})}{\zeta_0^{K_1}} + \frac{Y_0(1-\zeta'_0 K_2 - K_1)}{\zeta'_0 K_2 - K_1}. \quad (13)$$

2.7 The model solution

Now, we find the various PGFs, which are used later to obtain the performance measures for the system.

Multiplying Eq. 4 by z^n and summing it over $n, n = 1, 2, \dots$, and then using Eq. 11 to solve it, we have

$$P_{W_l}(0, z) = \begin{cases} \frac{\lambda Y_0}{\zeta_0^{K_1-l+2}} \sum_{n=0}^{N-1} z^n \eta_n, & l = 1, 2, \dots, K_1, \\ \frac{\lambda Y_0}{\zeta'_0 K_2 - K_1 - l + 2} \sum_{n=0}^{N-1} z^n \eta'_n, & l = K_1 + 1, K_2 + 1, \dots, K_2, \end{cases} \quad (14)$$

$$\Rightarrow \sum_{l=1}^{K_2} P_{W_l}(0, z) = \frac{\lambda Y_0}{\zeta_0^{K_1}} \left(1 + \frac{1 - \zeta_0^{K_1-1}}{1 - \zeta_0} \sum_{n=0}^{N-1} z^n \eta_n \right) + \frac{\lambda Y_0}{\zeta'_0 K_2 - K_1} \left(1 + \frac{1 - \zeta'^{K_2-K_1-1}}{1 - \zeta'_0} \sum_{n=0}^{N-1} z^n \eta'_n \right), \quad (15)$$

where $\zeta(z) = \widehat{V}_1[\lambda(1 - P_X(z))]$, $\zeta'(z) = \widehat{V}_2'[\lambda(1 - P_X(z))]$, and when $z = 0$,

$$\begin{aligned} \eta_n &= \sum_{a_1+2a_2+\dots+na_n=n} (-\lambda)^{(\sum_{k=1}^n a_k)} \zeta_0^{(\sum_{k=1}^n a_k)} \prod_{k=1}^n \frac{p_k^{a_k}}{a_k!}, \eta'_n \\ &= \sum_{a_1+2a_2+\dots+na_n=n} (-\lambda)^{(\sum_{k=1}^n a_k)} \zeta'_0^{(\sum_{k=1}^n a_k)} \prod_{k=1}^n \frac{p_k^{a_k}}{a_k!}, \\ \text{then } \eta_0 &= \zeta_0 = \widehat{V}_1(\lambda), \zeta_{0,k} = \frac{d^k \widehat{V}_1(\theta)}{d\theta^k} \\ &\times \left|_{\theta=\lambda} \text{ and } \eta'_0 = \zeta'_0 = \widehat{V}_2'(\lambda), \zeta'_{0,k} = \frac{d^k \widehat{V}_2'(\theta)}{d\theta^k} \right|_{\theta=\lambda}. \end{aligned}$$

Also, multiplying Eqs. 3 and 4 by z^n and summing it over $n, n = 1, 2, \dots$, and then, using Eq. 12 to solve it, we get

$$\begin{aligned} P_{W_l}(x, z) &= P_{W_l}(0, z)[1 - F_{V_1}(x)]e^{-\lambda[1-P_X(z)]x}, \\ & \quad l = 1, 2, \dots, K_1, \\ P_{W_l}(x, z) &= P_{W_l}(0, z)[1 - F_{V_2}(x)]e^{-\lambda[1-P_X(z)]x}, \\ & \quad l = K_1 + 1, K_1 + 2, \dots, K_2, \end{aligned}$$

and further using Eq. 13, for $l = 1, 2, \dots, K_2$, we get

$$\begin{aligned} P_{W_l}(z) &= \int_0^\infty P_{W_l}(x, z) dx \\ &= \frac{\lambda Y_0}{\zeta_0^{K_1-l+2}} \sum_{n=0}^{N-1} z^n \eta_n \frac{\zeta(z) - 1}{\lambda(P_X(z) - 1)} \\ &\quad + \frac{\lambda Y_0}{\zeta'_0 K_2 - K_1 - l + 2} \sum_{n=0}^{N-1} z^n \eta'_n \frac{\zeta'(z) - 1}{\lambda(P_X(z) - 1)}. \end{aligned}$$

Further, by taking the sum over $l = 1, 2, \dots, K_2$,

$$\begin{aligned} P_W(z) &= \left(\frac{\lambda Y_0}{\zeta_0^{K_1}} \left[1 + \frac{1 - \zeta_0^{K_1-1}}{1 - \zeta_0} \sum_{n=0}^{N-1} z^n \eta_n \right] \right) \\ &\quad \times \frac{\zeta(z) - 1}{\lambda(P_X(z) - 1)} \\ &\quad + \left(\frac{\lambda Y_0}{\zeta'_0 K_2 - K_1} \left[1 + \frac{1 - \zeta'^{K_2-K_1-1}}{1 - \zeta'_0} \sum_{n=0}^{N-1} z^n \eta'_n \right] \right) \\ &\quad \times \frac{\zeta'(z) - 1}{\lambda(P_X(z) - 1)}. \end{aligned} \quad (16)$$

Now, to obtain PGF of Y , Eqs. 1 and 11 are solved and for $n = 0, 1, \dots, N-1$, we obtain

$$\begin{aligned}\lambda P_Y(z) &= \frac{\lambda Y_0}{\zeta_0} \sum_{n=0}^{N-1} z^n \psi_n + \frac{\lambda Y_0}{\zeta'_0} \sum_{n=0}^{N-1} z^n \psi'_n, \text{ where } \psi_n \\ &= \sum_{i=0}^n \pi_i \eta_{n-i}, \psi'_n = \sum_{i=0}^n \pi_i \eta'_{n-i}, \pi_n \\ &= \sum_{i=1}^n p_i \pi_{n-i},\end{aligned}\quad (17)$$

and π_n is the steady-state probability that the system has n packets during an inactivity timer period.

Now, multiplying Eq. 2 by z^n and taking the sum over $n, n = 1, 2, \dots$, then on solving the obtained differential equation, we get

$$P_U(x, z) = P_U(0, z)[1 - F_R(x)]e^{-\lambda[1-P_X(z)]x}. \quad (18)$$

Furthermore, by taking natural log on both the sides of Eq. 18 and differentiating with respect to x , and then using Eqs. 16 and 17, we get

$$\begin{aligned}P_U(z) &= \frac{zY_0}{z - P_R(z)} \left[\left\{ \frac{\zeta(z) - 1}{\zeta_0^{K_1}} \left(1 + \frac{1 - \zeta_0^{K_1-1}}{1 - \zeta_0} \sum_{n=0}^{N-1} z^n \eta_n \right) + \frac{P_X(z) - 1}{\zeta_0} \sum_{n=0}^{N-1} z^n \psi_n \right\} \frac{P_R(z) - 1}{P_X(z) - 1} \right. \\ &\quad \left. + \left\{ \frac{\zeta'(z) - 1}{\zeta_0'^{K_2-K_1}} \left(1 + \frac{1 - \zeta_0'^{K_2-K_1-1}}{1 - \zeta_0'} \sum_{n=0}^{N-1} z^n \eta'_n \right) + \frac{P_X(z) - 1}{\zeta_0'} \sum_{n=0}^{N-1} z^n \psi'_n \right\} \frac{P_R(z) - 1}{P_X(z) - 1} \right],\end{aligned}\quad (19)$$

where $P_R(z)$ is the PGF of the random variable R . Thus, the unknown constant Y_0 is determined by using the normalization condition obtained in Eq. 9. First, multiplying Eq. 9 by z^n and summing over n , for $n = 0$ to ∞

and then substituting $z = 1$, we have $P_Y(1) + P_U(1) + P_W(1) = 1$.

Therefore, $P_W(1) + P_Y(1) = 1 - \rho$ with a stability condition $\rho < 1$, where $\rho = \lambda E[X]E[R]$. (20)

Also, using Eqs. 14 and 16, we obtain Y_0 as follows:

$$\frac{(1 - \rho)}{\frac{\lambda E[V_1]}{\zeta_0^{K_1}} \left(1 + \frac{1 - \zeta_0^{K_1-1}}{1 - \zeta_0} \sum_{n=0}^{N-1} z^n \eta_n + \frac{1}{\zeta_0} \sum_{n=0}^{N-1} \psi_n \right) + \frac{\lambda E[V_2]}{\zeta_0'^{K_2-K_1}} \left(1 + \frac{1 - \zeta_0'^{K_2-K_1-1}}{1 - \zeta_0'} \sum_{n=0}^{N-1} z^n \eta'_n + \frac{1}{\zeta_0'} \sum_{n=0}^{N-1} \psi'_n \right)}.\quad (21)$$

Further, $P_L(z) = P_Y(z) + P_U(z) + \sum_{l=1}^{K_2} W_l(z)$ is the PGF of the system size distribution at stationary point of time and is given by the following:

$$P_L(z) = \frac{(z - 1)P_R(z)}{z - P_R(z)} \left[\frac{\zeta(z) - 1}{\lambda(P_X(z) - 1)} \sum_{l=1}^{K_2} W_l(0, z) + P_Y(z) \right]. \quad (22)$$

3 Performance measures

3.1 Expected number of packets in the system

$$\begin{aligned}E(L) &:= \lim_{z \rightarrow 1} \frac{d}{dz} P_L(z) = \rho + \frac{\lambda E[X(X-1)]E[R] + (\lambda(E[X])^2)E[R]^2}{2(1 - \rho)} + \frac{\lambda^2 E[X]E[V_1^2]}{2} \\ &\quad \times \frac{1 + \frac{1 - \zeta_0^{K_1-1}}{1 - \zeta_0} \sum_{n=0}^{N-1} \eta_n + \lambda E[V_1] \left(\frac{1 - \zeta_0^{K_1-1}}{1 - \zeta_0} \sum_{n=0}^{N-1} n \eta_n \right) + \zeta_0^{K_1-1} \sum_{n=0}^{N-1} n \psi_n}{\lambda E[V_1] \left(\frac{1 - \zeta_0^{K_1-1}}{1 - \zeta_0} \sum_{n=0}^{N-1} \eta_n \right) + \zeta_0^{K_1-1} \sum_{n=0}^{N-1} \psi_n} + \frac{\lambda^2 E[X]E[V_2^2]}{2} \\ &\quad \times \frac{1 + \frac{1 - \zeta_0'^{K_2-K_1-1}}{1 - \zeta_0'} \sum_{n=0}^{N-1} \eta'_n + \lambda E[V_2] \left(\frac{1 - \zeta_0'^{K_2-K_1-1}}{1 - \zeta_0'} \sum_{n=0}^{N-1} n \eta'_n \right) + \zeta_0'^{K_2-K_1-1} \sum_{n=0}^{N-1} n \psi'_n}{\lambda E[V_2] \left(\frac{1 - \zeta_0'^{K_2-K_1-1}}{1 - \zeta_0'} \sum_{n=0}^{N-1} \eta'_n \right) + \zeta_0'^{K_2-K_1-1} \sum_{n=0}^{N-1} \psi'_n}.\end{aligned}\quad (23)$$

3.2 Expected length of the sleep period of the UE

Let W be the number of packets in the system during the short or long DRX cycle in the steady-state.

Let B_1 and B_2 be the number of packets in the system during the short and the long DRX sleep periods in the steady-state, respectively. Then,

$$E(W) := \lim_{z \rightarrow 1} \frac{d}{dz} P_W(z) = E(B_1) + E(B_2), E(B_1) \text{ and } E(B_2) \text{ are given by}$$

$$\begin{aligned} E(B_1) &:= \frac{\lambda Y_0}{\zeta_0^{K_1}} \left[\left(\frac{1 - \zeta_0^{K_1-1}}{1 - \zeta_0} \sum_{n=1}^{N-1} n \eta_n \right) \left(\frac{\zeta_1}{\lambda E(X)} \right) \right. \\ &\quad \left. + \left(1 + \frac{1 - \zeta_0^{K_1-1}}{1 - \zeta_0} \sum_{n=0}^{N-1} \eta_n \right) \times \left(\frac{E[X] \lambda^2 \zeta_2 - E[X^2] \lambda \zeta_1}{2\lambda} \right) \right], \\ E(B_2) &:= \frac{\lambda Y_0}{\zeta_0^{K_2-K_1}} \left[\left(\frac{1 - \zeta_0^{K_2-K_1-1}}{1 - \zeta_0'} \sum_{n=1}^{N-1} n \eta'_n \right) \right. \\ &\quad \times \left(\frac{\zeta_1'}{\lambda E(X)} \right) + \left(1 + \frac{1 - \zeta_0^{K_2-K_1-1}}{1 - \zeta_0'} \sum_{n=0}^{N-1} \eta'_n \right) \\ &\quad \times \left(\frac{E[X] \lambda^2 \zeta_2' - E[X^2] \lambda \zeta_1'}{2\lambda} \right) \Big], \text{ where} \\ \zeta_i &= \frac{d^i \hat{V}_1(\lambda(1 - P_X(z)))}{dz^i} \Big|_{z=1}, i = 1, 2. \end{aligned}$$

During the power saving state, let T_W be the time spent in the sleep in the steady-state. Further, let T_{B_1} and T_{B_2} be the time spent in the system during the short and the long DRX cycles, respectively. Thus, we have

$$\begin{aligned} E(T_W) &:= E(T_{B_1}) + E(T_{B_2}), \text{ where } E(T_{B_1}) \\ &:= \frac{E(B_1)}{\lambda E(X)} \text{ and } E(T_{B_2}) := \frac{E(B_2)}{\lambda E(X)}. \end{aligned} \quad (24)$$

3.3 Expected length of the inactivity timer period of the UE

Since Y_n is the number of packets in the system during inactivity timer period in the steady-state, then

$$E(Y) := \lim_{z \rightarrow 1} \frac{d}{dz} P_Y(z) = \frac{Y_0}{\zeta_0} \sum_{n=0}^{N-1} n \psi_n + \frac{Y_0}{\zeta_0'} \sum_{n=0}^{N-1} n \psi'_n.$$

Let T_Y be the length of inactivity timer period of the UE in the steady-state, then we have

$$E(T_Y) := \frac{E(Y)}{\lambda E(X)}. \quad (25)$$

3.4 Expected length of the busy period of the UE

Let T_U be the length of the busy period of the UE in the steady-state. Then,

$$E(T_U) := \frac{\rho_H}{1 - \rho_H} E(T_Y), \quad (26)$$

where $\rho_H = \lambda \beta_1 a_1$, with β_1 and a_1 are the first moment of service time distribution and $E[z^X]$, respectively.

3.5 Expected length of the busy cycle

Let T_B be the length of a busy cycle (termed as a DRX cycle in a DRX mechanism), which is defined as the time interval between two consecutive busy period ending instants and is given by the following:

$$E(T_B) = E(T_W) + E(T_Y) + E(T_U). \quad (27)$$

Also, using the property of linearity of expectation Eq. 16 is procured where $E(T_Y)$, $E(T_W)$, and $E(T_U)$ are obtained in Eqs. 24, 25, and 26, respectively.

3.6 Expected packet delay

Let D denotes the delay of the tagged packet, which is to be transmitted. Thus, using Little's law [26], we have

$$\begin{aligned} E(D) &= \frac{E(L)}{\lambda E(X)} = \frac{1}{\lambda E(X)} \left[\rho + \frac{\lambda E[X(X-1)]E[R] + (\lambda(E[X])^2)E[R^2]}{2(1-\rho)} + \frac{\lambda^2 E[X]E[V_1^2]}{2} \right. \\ &\quad \times \frac{1 + \frac{1 - \zeta_0^{K_1-1}}{1 - \zeta_0} \sum_{n=0}^{N-1} \eta_n + \lambda E[V_1] \left(\frac{1 - \zeta_0^{K_1-1}}{1 - \zeta_0} \sum_{n=0}^{N-1} n \eta_n \right) + \zeta_0^{K_1-1} \sum_{n=0}^{N-1} n \psi_n}{\lambda E[V_1] \left(\frac{1 - \zeta_0^{K_1-1}}{1 - \zeta_0} \sum_{n=0}^{N-1} \eta_n \right) + \zeta_0^{K_1-1} \sum_{n=0}^{N-1} \psi_n} + \frac{\lambda^2 E[X]E[V_2^2]}{2} \\ &\quad \times \frac{1 + \frac{1 - \zeta_0^{K_2-K_1-1}}{1 - \zeta_0'} \sum_{n=0}^{N-1} \eta'_n + \lambda E[V_2] \left(\frac{1 - \zeta_0^{K_2-K_1-1}}{1 - \zeta_0'} \sum_{n=0}^{N-1} n \eta'_n \right) + \zeta_0^{K_2-K_1-1} \sum_{n=0}^{N-1} n \psi'_n}{\lambda E[V_2] \left(\frac{1 - \zeta_0^{K_2-K_1-1}}{1 - \zeta_0'} \sum_{n=0}^{N-1} \eta'_n \right) + \zeta_0^{K_2-K_1-1} \sum_{n=0}^{N-1} \psi'_n} \Big]. \end{aligned} \quad (28)$$

From Eq. 28, we observe that expected packet delay is a function of N . Trade-off between expected packet delay and arrival rate is presented graphically in Section 4.4 on Page 14 for different values of N in Fig. 4a, b, respectively. From these figures, it is noted that with an increase in N , expected packet delay increases, with an increase in the arrival rate of the data packets. The aforesaid scenario is predictable, because, as N increases, the tagged packet has to wait for the accumulation of at least N packets in the buffer.

3.7 Special cases

Case(i) On taking single type vacation, the model described above becomes an $M^{[X]}/G/1$ single type vacation queue model with N -policy and at most K_2 vacations. In this case, expressions obtained for W_0 , $P_W(z)$, $P_Y(z)$, and $P_U(z)$ are found to be consonant with expressions of Ω_0 , $\Omega(z)$, $R_N(z)$, and $P(z)$ as mentioned in [17], respectively.

Case(ii) When $K_2 = 0$, (i.e., no vacation), the proposed model reduces to the $M^{[X]}/G/1$ queue system with N -policy. Then, we have $P_Y(z) = \frac{(1-\rho) \sum_{n=0}^{N-1} \pi_n z^n}{\sum_{n=0}^{N-1} \pi_n}$.

Also, for $z = 1$, we get $P_Y(1) = 1 - \rho$ and $Y_0 = \frac{1-\rho}{\sum_{n=0}^{N-1} \pi_n}$, where $\pi_n = \sum_{i=1}^n p_i \pi_{n-i}$, which agrees with the result obtained in Theorem 3.1 of [22].

Case(iii) Consider the criteria of no batch arrival and no vacation in the system. Also, we take $N = 1$. Thus, the system reduces to the $M/G/1$ queue system.

Let $\text{Var}(R) = \sigma_s^2 < \infty$. We have $P\{X = 1\} = 1$, $E[X] = 1$, $E[X^2] = 1$, $P_X(z) = z$ and $E[V_1] = E[V_2] = 0$, $K_2 = 0$, $E[V_1^2] = E[V_2^2] = 0$ as there is no vacation to consider.

Hence, $\eta_n = \eta'_n = 0$, $\psi_n = \psi'_n = 0$, and Eq. 23 reduces to

$$\begin{aligned} E(L) &= \rho + \frac{\lambda E[X(X-1)]E[R] + (\lambda(E[X]))^2 E[R]^2}{2(1-\rho)} \\ &= \rho + \frac{(\lambda(E[X]))^2 E[R]^2}{2(1-\rho)}, \\ &= \rho + \frac{(\lambda\sigma_s)^2 + (\lambda E[R])^2}{2(1-\rho)} \\ &= \rho + \frac{\rho^2 + (\lambda\sigma_s)^2}{2(1-\rho)}, \end{aligned}$$

where $\rho = \lambda E[R] < 1$. (29)

The above formula is consistent with the result obtained in Equation (5.11) at page 212, of [26].

4 Energy efficiency under the power saving

In this Section, we develop the total cost function (i.e., average power consumption per second) for the $M^{[X]}/G/1$ queue system with N -policy and at most K_2 vacations, in which N and K_2 are decision variables. Let IN , BU , SD , and LD be the states in which UE remains in the inactivity timer period, busy period, short DRX cycle and long DRX cycle, respectively. Further, consider C_{listen} , C_{low1} , C_{low2} , C_{high} , and C_{switch} as the power consumption (in mW/s) per unit time of the UE's receiver during the inactivity timer period, short DRX sleep period, long DRX sleep period, busy period and switching from the power saving state to the power active state and vice versa, respectively.

Define

$$\begin{aligned} P_i &= \lim_{t \rightarrow \infty} P(i \text{ is the state of the UE at time } t), \\ i &\in \{IN, B, SD, LD\}, \\ &= \frac{E(\text{amount of time spent in the state } i \text{ during } B)}{E(B)}, \end{aligned}$$

where $E(B) = E(T_Y) + E(T_U) + (E(T_{B_1}) - E[l_1]\tau) + (E(T_{B_2}) - E[l_2]\tau)$ is the expected length of busy cycle.

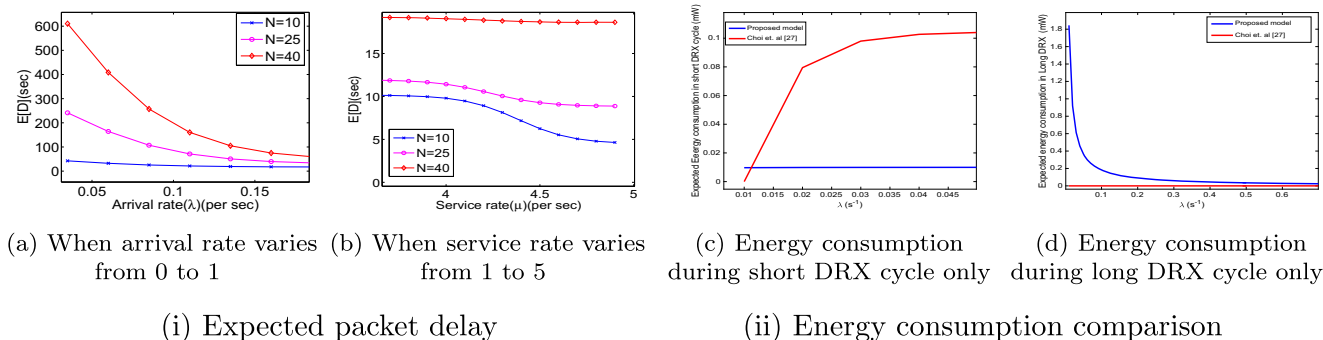


Fig. 4 (i) Expected packet delay. (ii) Energy consumption comparison

$E[l_1]\tau$ and $E[l_2]\tau$ are the small length of time intervals, which are taken into consideration for the accuracy in the numerical analysis. Then, the total cost function for the power consumption denoted as $g(N, K_2)$, is given by

$$g(N, K_2) = P_{IN}C_{listen} + P_{SD}C_{low1} + P_{LD}C_{low2} + P_{BU}C_{high} - C_{switch}\gamma, \text{ where} \quad (30)$$

$$P_{IN} = \frac{E(T_Y)}{E(B)}, P_B = \frac{E(T_U)}{E(B)}, P_{SD} = \frac{E(T_{B1}) - E[l_1]\tau}{E(B)},$$

$$P_{LD} = \frac{E(T_{B2}) - E[l_2]\tau}{E(B)},$$

with $E[l_1]$ and $E[l_2]$ are the average number of short DRX cycles and the average number of long DRX cycles, respectively and are given by

$$E[l_1] = \frac{E[T_{B1}]}{E[V_1]}, E[l_2] = \frac{E[T_{B2}]}{E[V_2]}. \quad (31)$$

The symbol τ is the duration of the ON period in a short or long DRX cycle (i.e., same for both short and long DRX cycles), which is taken as a constant. Further, γ is defined as the number of switches from the power saving state to the power active state per unit time and is called the *handover ratio*. It is a measure which gives the additional energy consumption by virtue of the insertion of the DRX mechanism and is given by the following:

$$\gamma = \frac{E[T_B]}{2(E[l_1] + E[l_2])}, \quad (32)$$

where $E[T_B]$, $E[l_1]$, and $E[l_2]$ are given in Eqs. 27 and 31, respectively.

The complete denominator in Eq. 32 is multiplied by 2 to capture all the possible switches, i.e., from the power saving state to the power active state and vice versa.

Further, to obtain the minimum value of total cost function, $g(N, K_2)$, the optimal values of the control parameters N and K_2 , namely N^* and K_2^* are determined [23, 27]. Based on the concept of dynamic optimization, it is possible to calculate the optimal values (N^* , K_2^*) [28]. Following is the procedure to obtain joint optimal values (N^* , K_2^*) that yields the minimum total cost per unit time. However, one should show the convexity or unimodularity of $g(N, K_2)$ to implement this method.

- Step 1. Set $N = 1$. Find $K_2^*(N) = \min \{K_2 \geq 1 | g(N, K_2 + 1) - g(N, K_2) > 0 \text{ and compute } g(N, K_2^*(N))\}$.
- Step 2. Compute $K_2^*(N + 1)$ and $g(N + 1, K_2^*(N + 1))$.
- Step 3. If $g(N + 1, K_2^*(N + 1)) > g(N, K_2^*(N))$, then STOP. The optimal values are $(N^*, K_2^*) = (N^*, K_2^*(N))$.
Else, GOTO Step 2.

The extensive numerical experiment presented in Section 6 confirms the efficiency of this procedure.

4.1 System utility

The system utility denoted as U_s is defined as the ratio of the transmission time of the packets to the total time spent in power active state in a busy cycle of the DRX mechanism and is given by the following:

$$U_s = \frac{E[U]}{E[T_B]} = \frac{\rho_H}{(1 - \rho_H)} \cdot \frac{E[T_Y]}{E[T_B]}, \quad (33)$$

where $E[T_Y]$ and $E[T_B]$ are given in Eqs. 25 and 27, respectively.

4.2 Energy gain

Define E_{DRX} and $E_{no DRX}$ as the energy consumption when DRX mechanism is enabled and not enabled in a UE, respectively [29]. Then, $E_{DRX} = g(N, K_2)$ and for $E_{no DRX}$, $C_{low1} = C_{low2} = C_{high}$ in the cost function $g(N, K_2)$. Denoting G as the energy gain of the UE, we have the following:

$$G = \frac{E_{no DRX} - E_{DRX}}{E_{no DRX}}, \quad (34)$$

where $g(N, K_2)$ is obtained in Eq. 30.

4.3 Numerical results

The batch arrival process is considered due to the self-similar property of the immense multimedia packets shown in the wireless mobile networks. Let X be the Pareto distributed random variable with a PDF given by the following:

$$f(x) = \frac{\alpha x_m^\alpha}{x^{(\alpha+1)}} \text{ for } x \geq x_m,$$

where α and x_m are the positive real numbers representing shape and location parameters, respectively. Similar to [21], we have considered $m = 2000$ ms, $x_m = 2$ ms, and $\alpha = 2.1$. Symbol α is a factor to determine the degree of self-similarity for the multimedia traffic in 3GPP LTE-A networks.

As discussed by [30], we have $C_{high} = 0.28$ mJ/s, $C_{listen} = 0.12$ mJ/s, $C_{low1} = 0.01$ mJ/s, $C_{low2} = 0.001$ mJ/s, $\tau = 0.02$ s, and $C_{switch} = 0.2$ μ J/s. Now, on taking $V_1 \sim \text{Exp}(2)$ and $V_2 \sim \text{Exp}(0.5)$ for LTE-A networks, we get the minimum value of the cost function $g(N, K_2) = 0.0056$ for $N = 3$, $K_2 = 40$, when $\mu = 5/s$ and λ varies from 0 to 1 per second. Also, minimum value

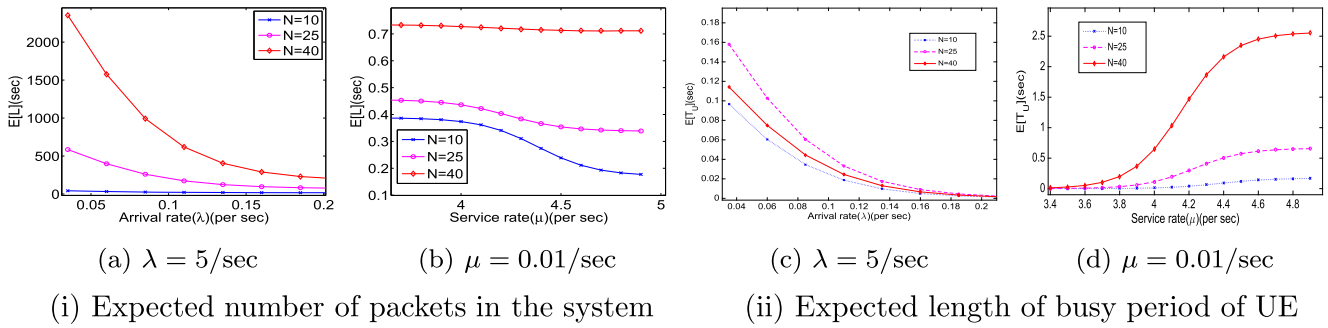


Fig. 5 (i) Expected number of packets in the system. (ii) Expected length of busy period of UE

of the cost function $g(N, K_2) = 0.0056$ for $N = 1$, $K_2 = 5$, when $\lambda = 0.01$ s and μ varies from 1 to 5 per second.

Values of N and K_2 are kept optimal such that if the arrival of packets is slower than the usual, then the accumulation of N packets in the buffer may take longer time but prior to that K_2 (maximum) DRX cycles will expire. Thus, there will be no excess power consumption and packet delay, and power loss will be minimal.

4.4 Comparison with the existing work

In this section, we compare performance results of this manuscript with an existing work. For instance, Choi et al. [30] have used an $M/G/1$ queue modeling approach for the performance analysis of DRX mechanism in LTE-A networks. Figure 4c, d depicts the comparison of energy consumption of the proposed model with that of Choi et al. [30].

5 Impact of the arrival and the service rates on the performance measures

Following are the observations on the impact of the arrival and the service rates on the performance measures which are obtained in this paper.

1. Impact of the arrival rate λ on the performance measures for a fixed service rate, $\mu = 5/s$:

From Fig. 5a, it is inferred that the expected number of packets in the system decreases as the λ increases and becomes constant after a particular value of λ . Moreover, increasing the value of N also leads to an increase in the expected number of packets.

Figure 5c depicts the behavior of the busy period of the UE as λ varies. As λ increases the length of the busy period decreases, whereas the length of the busy period increases with N , which is an intuitive observation.

From Fig. 6a, we observed that as λ increases, the expected length of the inactivity timer period of the UE decreases sharply because when the packets arrive during the inactivity timer, then the timer ends there to start serving the packets. Furthermore, with an increase in N , expected length of the inactivity timer period decreases.

The curve obtained for the expected length of the busy cycle in Fig. 6c decreases sharply as λ increases and decreases with N at that moment.

2. Impact of the service rate μ on performance measures for a fixed arrival rate, $\lambda = 0.01/s$:

From Fig. 5b, we observe that with an increase in μ the expected number of packets in the system decreases slightly and becomes constant after a certain value of μ .

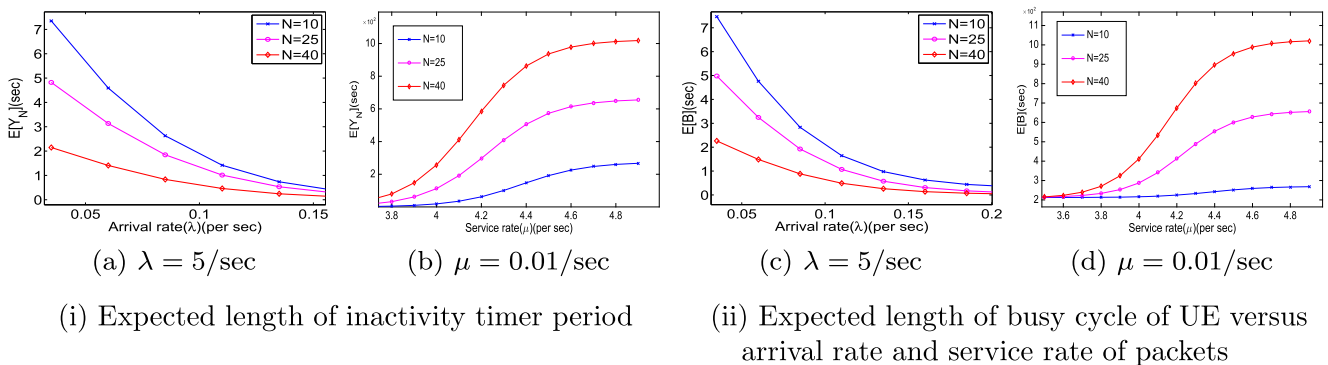


Fig. 6 (i) Expected length of inactivity timer period. (ii) Expected length of busy cycle of UE versus arrival rate and service rate of packets

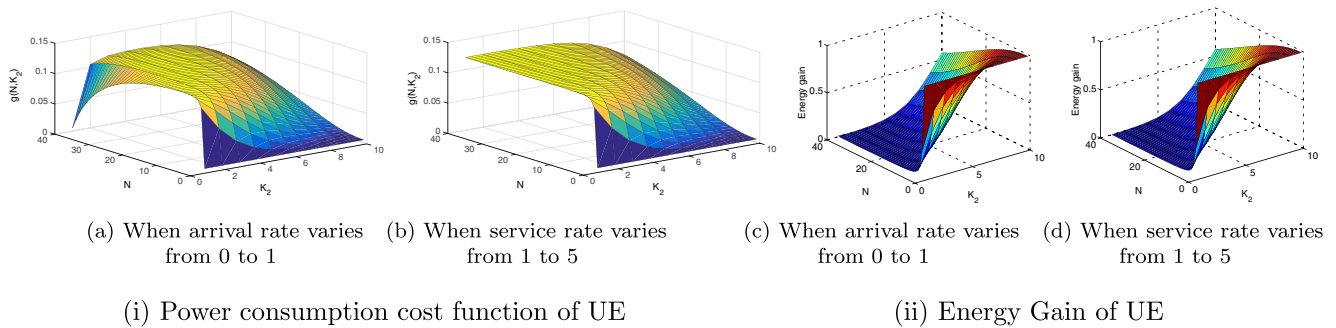


Fig. 7 (i) Power consumption cost function of UE. (ii) Energy gain of UE

Moreover, increasing the value of N also leads to a rise in the expected number of packets.

Figure 5d depicts the behavior of the busy period of the UE with respect to μ . As μ increases, the length of the busy period increases steadily and becomes constant at some point.

We observe from Fig. 6b that as μ increases, the expected length of the inactivity timer period of the UE increases at a slow pace and becomes constant after some point; also in parallel, it increases with an increase in N .

The curve obtained for the expected length of the busy cycle in Fig. 6d increases smoothly and becomes constant after some point, and at the same time increases with an increase in N , which is a conventional observation.

In Fig. 7(i), it is observed that K_2 does not affect the power consumption cost function $g(N, K_2)$. K_2 impacts power consumption substantially, but only at a very low input values as shown in Fig. 7a, b, whereas N affects the power consumption greatly. On the other hand, in Fig. 7(ii), with an increase in K_2 , energy gain increases exponentially and becomes constant after some point, whereas N does not impact the energy gain.

6 Conclusion and future work

In this paper, the performance of DRX mechanism is investigated for a UE in LTE-A networks. To study the DRX mechanism for a UE, batch arrival vacation queue system $M^{[X]}/G/1$ having N -policy with at most a fixed number of vacations is exercised. Several performance measures, such as energy gain, power saving factor, and expected packet delay, etc. are obtained and examined. One of the main contributions of this paper lies in making use of the vacation queue model, to get the optimal N and the maximum number of DRX cycles, which leads to a significant power saving. The ceiling over the number of packets waiting in the system and the maximum number of DRX cycles

are obtained. The aforesaid approach is a novel research direction considered in this work.

Further, the bursty nature of the traffic carried in LTE-A networks requires an understanding of the steady-state as well as transient nature of the system for the different types of traffics, such as the multimedia and the background traffic. In view of this, in future, we are planning to obtain the transient distribution of the system content using the fluid queue approach for improving the QoS.

Acknowledgements Authors are thankful to the editor and two anonymous reviewers for their valuable suggestions and comments, which helped improve the paper to great extent.

Funding information This study received financial support from the Department of Telecommunications (DoT), India. Further, the first author would like to thank the Council of Scientific and Industrial Research (CSIR), India for providing her financial support through Senior Research Fellowship.

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