



Stochastic modelling of multi-layer HAP-LEO systems in 6G for energy saving: An analytical approach

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ABSTRACT

In the current era of communication systems, the next-generation technology known as sixth generation (6G) is anticipated to extensively utilize space-air-terrestrial networks (SAT). High altitude platforms (HAPs) and low earth orbit (LEO) satellites are essential components of SAT network. The presented study proposes an architecture which concentrates on energy saving of HAP in the SAT network. Further, a stochastic model has been proposed which considers three energy saving states of HAP; power saving, standby, and sleep states. It has been assumed that in these states, the energy consumption of HAP is very low or zero. Whenever, there is an arrival of a packet in the system, HAP will start providing service immediately and the whole system will be in active state. The analysis of the stochastic model has been carried out using Markov process (MP) and semi-Markov process (SMP). The system has been analysed in steady-state for both the MP and SMP to obtain the steady-state probabilities. Through the numerical results, the effective contribution of energy saving states in enhancing the energy saving of HAP system has been provided and a comparison for the MP and SMP case has been described in detail. To verify the accuracy of the presented model, a discrete-event simulation approach is employed.

1. Introduction

1.1. Motivation

Sixth generation (6G) technology research is on the horizon as the fifth generation (5G) communication system progressively reaches maturity and expands its commercial applications [1,2]. 6G rely heavily on space-air-terrestrial (SAT) networks, notably in rural areas, marine settings, and emergency situations, where over three billion people still live in countries without regular access to the internet [3,4]. Another objective of 6G is to make ubiquitous access services available to the world's large mobile and Internet of Things (IoT) user populations. This objective can be achieved by using low earth orbit (LEO) satellites and high altitude platforms (HAPs) which cannot only enhance system's performance but also can provide coverage for mobile and IoT users in rural locations, hotspot regions, and emergency situations where ground base stations are absent or inadequately able to provide coverage [5,6].

LEO satellites have a pronounced global coverage advantage over terrestrial cellular systems. They not only has the capacity to supply enough capacity in remote and non-land locations but also has the ability to play a crucial role in emergency communication. They have more advantages in terms of launch costs, shorter launch delays, and

a higher propensity to construct LEO satellite networks by utilizing inter-satellite links (ISLs) [7]. Although these satellites possess certain advantages such as wide coverage area and swift movement, they still exhibit notable drawbacks. Due to the fact that most LEO satellite beams have coverage radii of more than 100 km, LEO cannot meet the demands of extremely dense regions, which require high capacity. Additionally, they move at a very fast speed, making it very challenging to manage resources in a normal radio fashion to ensure the quality-of-service (QoS) of mobile and IoT customers [4]. Due to the substantial path-loss attenuation, a classic satellite communication system is typically limited by the weak link between the satellite and the ground station. Therefore, HAP-LEO multi-layer architecture was introduced to overcome these shortcomings in the communication system. A HAP is positioned above the ground, where it has strong connections to both the satellite and the ground station [8]. As a consequence, it may enhance system performance in terms of throughput or energy consumption. Fig. 1 provides a good overview of aerial and terrestrial cellular coverage of HAP and terrestrial cellular infrastructure.

HAPs are equipped with communication equipment that float in the stratosphere [9]. They are typically stationary or move slowly along predetermined paths. HAPs act as relay stations between terrestrial cellular infrastructure and end-user devices, extending coverage to remote

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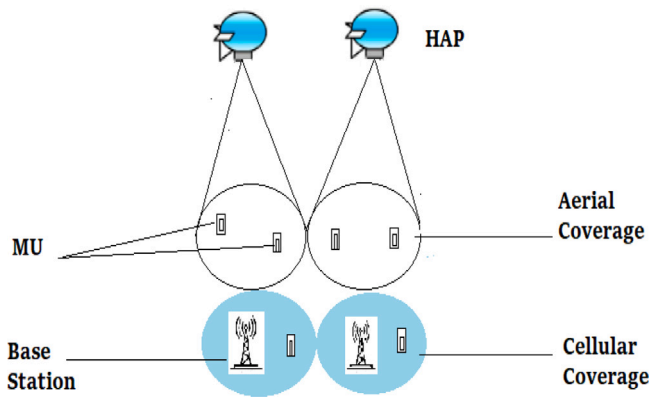


Fig. 1. Aerial and Terrestrial Cellular Coverage.

areas or areas with limited infrastructure. HAP systems utilize various wireless technologies, including LTE (Long-Term Evolution) and 5G/6G, to establish connectivity between the platforms and ground-based infrastructure. They have the potential to bridge the digital divide by offering coverage to under-served regions and providing emergency communication services during disasters. Terrestrial cellular infrastructure comprises a network of base stations, antennas, and associated equipment located on the ground [10]. This infrastructure forms the backbone of cellular communication systems and provides coverage across a wide area. It includes technologies like 2G, 3G, 4G LTE, and 5G, 6G each offering increased data speeds and capacity. HAP systems, offer aerial cellular coverage from the stratosphere, extending connectivity to remote or under-served areas [11]. Terrestrial cellular infrastructure, on the other hand, relies on ground-based base stations and antennas to provide widespread coverage in populated regions.

LEO satellite and HAP research is receiving more attention recently from both industry and academia [12,13]. HAPs are positioned as the stable base stations in the air because to their powerful payload capacity, high altitude, and extensive coverage. The ideal channels to use as backhauls for HAPs are those between HAPs and LEO satellites. Additionally, LEO satellite networks are interconnected globally and can ensure that the data is returned to ground in a timely manner. Fig. 2 represents the architecture of LEO-HAP system. Several studies have been conducted from both modelling perspectives and architecture perspective on HAP/LEO-HAP systems. The HAP-LEO systems have been extensively investigated from different perspectives e.g., architecture, resource management, energy saving, etc. Given that HAPs have curtailed energy resources, these networks can benefit greatly from prudent energy management. In this context, studies centred on reducing the total energy consumption by designing and modelling of these types of system are of great importance.

1.2. Relevant work

An abstract architecture of HAP-LEO system is depicted in Fig. 2. The architecture of a LEO-HAP system involves the integration of two distinct components: the LEO satellite constellation and the HAP platform.

- **LEO Satellite Constellation:** The architecture of the LEO satellite constellation includes the following main elements: LEO satellites, inter-satellite links, and ground station network. LEO satellites are interconnected through inter-satellite links (ISLs), forming a mesh-like network. ISLs allow the satellites to communicate with each other, relay data, and maintain continuous connectivity across the constellation [7]. These links enable data routing and provide redundancy in case of satellite failures.

Ground stations are essential components of the LEO satellite constellation architecture. They serve as gateways between the satellites and the terrestrial network infrastructure. Ground stations communicate with the LEO satellites, receiving and transmitting data, and act as intermediaries for connectivity with the HAP platform and end-user devices [6].

- **HAP:** The architecture of the HAP platform involves the following main elements: HAPs, communication equipment, payload and power systems, and control and navigation systems. HAP are stationed at high altitudes to establish a link between the LEO satellite constellation and ground-based infrastructure [9]. The HAP platform is equipped with communication equipment capable of receiving and transmitting signals to and from the LEO satellites. This equipment ensures seamless connectivity and data transfer between the LEO satellite constellation and the HAP platform. The HAP platform carries various payloads, including communication systems, power systems (such as solar panels or fuel cells), and other necessary equipment. These payloads enable the platform to maintain its position, provide stable communication links, and power its operations. Furthermore, the HAP platform incorporates control and navigation systems to ensure its stability, altitude control, and precise positioning. These systems allow the HAP platform to adjust its location and maintain optimal communication links with the LEO satellite constellation [11].

Researchers have begun to pay particular attention to modelling of networks that include satellites, HAPs and mobile users [8,14–16]. For the emergency situations in [17], a hybrid satellite-aerial-terrestrial network design was presented. They contrasted the benefits and drawbacks of satellites, high- and low-altitude platforms, and terrestrial networks. A few potential network technologies were also highlighted. The resource management of satellite systems has become the focus of another area of ongoing research in recent years [18–21]. The authors of [18] concentrated on the issue of satellite networks and terrestrial networks sharing spectrum. The HAP-based massive access and LEO satellite-based backhaul to support mobile and IoT users in rural locations without ground base stations and HAP-oriented ground stations is investigated in [19]. A hierarchical resource allocation approach to circumvent the complex management among different layers of HAP and LEO was provided by [20]. The article [21] examined the asymmetry in the optical wireless communication channel from and to HAPs. The authors examined how HAPs and satellites can operate together to create a space-air ground IoT network in [6]. A constellation design technique was presented in Ref. [22] for the large network of HAPs that ensures QoS and user demand. In order to cover under-served areas, the millimetre wave employed for HAPs was examined in [23].

Numerous studies have described and illustrated the characteristics of the system, including its reliability and performance, and energy management characteristics through stochastic modelling. A few studies have developed stochastic models for the HAP system in the literature (refer, [24–28]). The authors of paper [24] demonstrated that a relay architecture with unmanned aerial vehicles (UAV) between the ground and satellites can facilitate satellite access with a low overhead. A novel Cooperative Directional inter-cell Handover Scheme (CDHS) for HAP communications systems was proposed in [25]. The author of [29] studied the performance of HAP-LEO networks in a more generic form. They used non-homogeneous Poisson point process to model the uneven distribution of satellites. In [26], the HAP system was modelled considering various arrival and service processes. In the papers [27,28], the authors studied the reliability of the functioning of a flight module of a tethered high-altitude telecommunication platform, utilizing the k -out-of- n : F model. In [30], a HAP-assisted satellite relaying system for up-link data transmission was modelled using Markov process and the obtained results were compared with the simulated ones. Stochastic fractional integrodifferential models are also widely employed to model the communication networks these days (see, [31–34]).

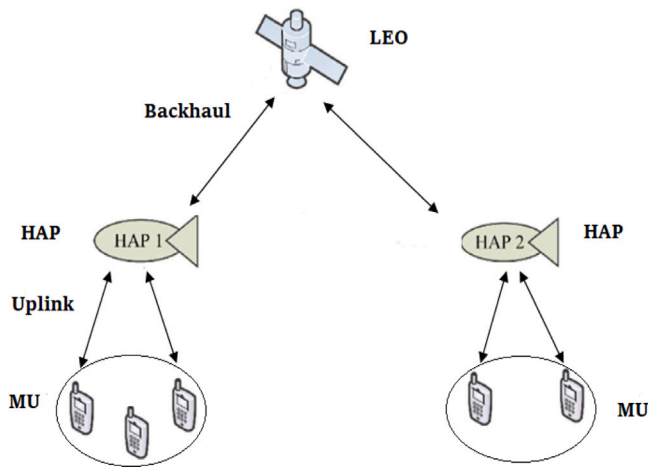


Fig. 2. Architecture of HAP-LEO system.

The restricted on-board energy and flight time are key obstacles in using HAP to enhance existing wireless networks, as noted in several studies [35–37]. According to Alam et al. [38], the batteries in a HAP have a lifespan of less than an hour, which is further reduced when energy is consumed by the payload, communication, and signal processing. Therefore, HAP systems typically rely on power sources such as lithium-ion batteries, solar panels, or ground-based energy sources with energy transmitted via a thin cable-rope. It is predicted that the energy density of lightweight lithium-ion batteries will increase by 3% annually over time [38].

The HAPs network is primarily concerned with finding ways to manage an anticipated 100-fold increase in mobile data traffic while improving energy saving. The benefit could be rather significant if the design allows for the deactivation of unneeded network resources in order to maintain the desired QoS for the specified traffic. If the objective is to increase network energy saving, a low or zero power consumption mode can be crucial in this scenario. To save overall energy consumption, network resources including frequency carriers, power amplifiers, and transceivers (TRXs) can be switched off or into low power mode when not being used. For energy saving in emergency communications, Dong et al. [39] looked into transmission methods in combined HAP-satellite networks. A down-link satellite communication system with HAP assistance was designed in [40] with the goal of reducing the overall energy cost. Islam et al. [41] proposed a Markov decision process based algorithm which switched among the states of transceivers based on the offered traffic. Jain et al. [42] presented an energy saving technique for HAP considering a single server queueing system.

In the wake of a devastating natural disaster in a densely populated coastal city, communication infrastructure has been severely damaged, leaving many areas disconnected and hindering emergency response efforts. The authorities urgently need a reliable and rapid communication system to coordinate rescue operations, assess damages, and deliver essential aid to affected communities. The local government decides to deploy a HAP as a relay node in a SAT network along with LEO satellites and terrestrial base stations. The HAP-LEO-terrestrial network is seen as an ideal solution to reestablish communication links quickly, given its ability to provide wide coverage and high data throughput.

The aim of the proposed model is to apply an analytical aspect of energy management to ensure the efficient functioning of the HAP-LEO-terrestrial network during this disaster response scenario. The proposed model takes into account various factors, such as the energy consumption of HAP's communication equipment, orbital dynamics of LEO satellites, and power constraints of terrestrial base stations. By optimizing the energy usage and coordinating the relay nodes effectively, the network's longevity and sustainability can be significantly

enhanced. The scenario begins with the rapid deployment of the HAP at an optimal altitude, capable of establishing a stable connection with the LEO satellites passing overhead. The HAP acts as a central relay node, collecting data from various terrestrial base stations and forwarding it to the LEO satellites for global dissemination. As the disaster response operations continue, the proposed model monitors and manages the energy consumption of the HAP, ensuring it operates efficiently without depleting its power reserves.

Additionally, the model optimizes the handoff process between LEO satellites and the HAP to maintain continuous communication coverage as the satellites move across the sky. The terrestrial base stations, which may be running on backup power sources, also benefit from the energy management model. It optimizes their transmission schedules and power levels to maximize communication reach while conserving energy. With the HAP-LEO-terrestrial network in place, emergency responders can coordinate their efforts seamlessly, exchange critical information, and receive real-time updates on the disaster's impact. Communication channels are restored in affected areas, allowing residents to reach out for help and enabling aid agencies to deliver necessary supplies more effectively. Beyond the immediate disaster response, the proposed energy management model proves to be valuable in ensuring the network's long-term sustainability during the recovery phase. As fuel and power resources may be limited, the model aids in allocating energy resources optimally, extending the network's operational time and impact.

The proposed model presented in the contribution section stands out from existing stochastic models for HAPs in several significant ways. While the existing literature primarily focuses on modelling networks involving satellites, HAPs, and mobile users, the proposed model specifically addresses the stochastic modelling of a cooperative SAT network that encompasses LEO satellites, HAPs, and mobile users. While previous studies have explored the reliability and performability aspects of HAPs using concepts like k out of n systems, and some have delved into energy management through battery power optimization, none have specifically examined the energy management and throughput aspects of HAPs in the context of a SAT network. The proposed model introduces energy-saving techniques customized for HAPs during periods of low traffic by incorporating standby and sleep states. By strategically switching off transceivers in these states, the model aims to optimize energy consumption in HAPs, which is critical due to the limited on-board energy and flight time. Moreover, the model explicitly considers the dynamic behaviour of LEO satellites and mobile users and addresses it from the perspective of energy-saving measures for HAPs.

1.3. Contribution

The proposed study considers a cooperative SAT network composed of LEO satellites in the space, HAPs in the air and mobile users on the ground as depicted in Fig. 2. Data from users transmitted by HAPs can be assured to return to earth through ISLs between LEO satellites using the store-carry-and-forward technique [7]. In this study, the up-link scenario for the mobile users has been explored.

Energy saving refers to the practice of reducing energy consumption or using energy more efficiently in order to minimize overall energy usage and lower the associated costs and environmental impacts. It involves adopting measures, technologies, and behaviours that aim to conserve energy resources while still achieving desired outcomes or services [41]. As already presented in some pioneering works [39,41,42], the energy saving techniques considers switching off transceivers of HAP during low traffic conditions. To this end, in the proposed stochastic model two energy saving states have been proposed; standby state and sleep state. Another issue in the SAT network is the dynamic behaviour of LEO and mobile users. LEO are orbiting with high speed and their visibility times are limited according to elevation angle and orbital height. The orbits of LEO satellites are always regular and

predictable, making it possible to predict the channel and propagation delay. Normally, LEO satellites take 90 min to 2 h to complete one full cycle which means that satellite is in view for only a few minutes. Therefore, the services must be handed off from satellite to satellite as well as between cells in order to main the continuity of communication. This switching times is observed to be 50 ms to a few seconds depending on the external and internal factors. During the service in a day, these types of dynamic switching scenarios occur often. Therefore, an energy/power saving mode can be activated in HAP during the dynamic relay switching scenarios. HAP can dynamically switch between relaying traffic and sleeping based on the traffic load and availability of LEO satellites. Therefore, a power saving state has been introduced to save energy of HAP in this type of scenario.

The novelty of the presented study is to propose an architecture of SAT network which focuses on the energy saving of the HAP. To check the feasibility of the proposed architecture, a stochastic model has been constructed. The proposed model considers general distribution for the sleep time period of HAP. For the distributions of energy saving states, along with considering the traditional exponential distribution, a non-exponential case has also been considered. The fundamental idea behind considering two types of distributions is to study the energy saving strategies in various environments. For the sake of comprehension, general distribution has been assumed to demonstrate the transitions of energy saving states i.e., power saving state, standby state, and sleep state. Note that, a different distribution can also be considered and replaced with the general one to obtain corresponding results. To this end, the stochastic model has been proposed using Markov process (MP) and semi-Markov process (SMP). In the MP, the arrival, service, and sleep processes follow exponential distributions. Whereas, in the SMP, mean sojourn times of standby and sleep states follow general distribution. A two stage method has been employed to analyse the dynamics of the SMP and to obtain the steady-state probabilities of the system. Through the numerical results, the impact of several factors over energy saving factor have been illustrated. Also, a comparison of results has been provided for the stochastic model using MP and SMP.

The rest of this article is summarized as follows. Section 2 describes the potential queueing system for energy conservation. The system's steady-state analysis has been presented in Section 3. Section 4 covers the key performance indicators of the suggested model. Section 5 presents in-depth presentations of specific cases of the proposed model. Section 6 provides the conclusions and suggestions for further research.

2. Model specification

In this study, the dynamics of a multi-layer LEO-HAP system has been modelled using Markov and semi-Markov models. The proposed model mainly focuses on the energy saving of HAP. The state transition diagram for the proposed model is depicted in Fig. 3. The states of the model are described in detail as follows:

- **Idle State:** This is also termed as the inactive state, denoted as $L_{0,0}$, of the system and the system will be in idle state during this time period. There can be two possible transitions from the idle state. Whenever a packet arrives to the HAP from the users, there will be a transition from idle state to active state $L_{i,0}$, $i = 1, 2, 3, \dots$. If there is no arrival of packets in the system, HAP will move to power saving state.
- **Active State:** When a packet arrive to HAP, it will complete its service and further, it will be relayed to LEO for the service. There will not be any direct arrival to the LEO. Once the packets are provided service by the LEO system, and there is no other packet served in the system, the system will transit to idle state.
- **Power Saving State:** Due to the dynamic motion of LEO satellite or the mobile user, for some time the HAP can switch to power saving state. The HAP can dynamically switch between relaying traffic and sleeping based on the availability of LEO satellites

Table 1

Description of states of the proposed model.

States	Functionalities
Idle state	System remains in the inactive state until the arrival of packets.
Active state	It can be divided into busy states (a) when HAP is providing the service to incoming packets (b) when LEO is providing the service to those packets which have been served by HAP.
Power saving state	- HAP switches to power-saving state due to the dynamic motion of the LEO satellite or the mobile user. - HAP does not provide service and some functions may turn off.
Standby state	- HAP moves to standby state when no packets arrive and the time period of power-saving state is completed. - HAP does not provide service and most of the functions may turn off.
Sleep state	- HAP moves to sleep state when no packets arrive and the time period of standby state is completed. - Sleep cycle in sleep state is considered to be longer than the standby state. - Almost all the components of HAP are turned off except those which sense the arrival of packets.

and mobile user. In this state, HAP will not be providing service and some of the functions of HAP will shut down. From this state, the system can transit to two states. If any packet arrives to the system, HAP will start providing service; otherwise HAP will move to longer sleep state i.e., standby state.

- **Standby State:** Whenever there is no arrival of packets in the system, and the time period of power saving state is completed, HAP will move to stand-by state. HAP will stay in this state until the occurrence of one of the two scenarios. During the time period of this state, if any packets arrive to the system, HAP will start providing service to the arriving packet. If the time to remain in the standby state expires, HAP will move to sleep state.
- **Sleep State:** The sleep cycle of sleep state is considered to be longer than the standby state. Also, the energy consumption in this state is considered to be very low as compared to standby state. HAP will stay in this state until the arrival of any packet. If the sleep cycle is completed and there is no arrival of packets, the system will move to idle state.

Table 1 provides concrete information about the states of the system. In the absence of a packet, the HAP remains in an idle state, denoted as $L_{0,0}$, for a specific time period. The service for an arriving packet takes place in two stages. In the first stage, the HAP provides the initial service, transitioning the system to state $L_{1,0}$. Only one packet can receive service at a time, causing incoming packets to queue up and wait for their turn. Once the HAP completes its service, the packet is transmitted to the LEO, resulting in a transition from state $L_{1,0}$ to $L_{0,1}$. Generally, the system transitions from state $L_{i,j}$ to $L_{i-1,j+1}$ as the stages progress. In the second stage, the packet receives service from the LEO and exits the system, leading to a transition from state $L_{i,j}$ to $L_{i,j-1}$.

When no packets arrive in the system, the HAP enters a power-saving state after the idle time period. In this state, the HAP remains active but, due to the rapid movement of the LEO, the packet service cannot be completed. As a result, the HAP switches off certain components and transitions to the state denoted as P . After the specified time period in state P , if no packets have arrived, the HAP moves to a standby state, further conserving energy by turning off additional components. This transition is shown from state P to SB . Following the specified time period in the standby state, the system transitions to a longer sleep state, denoted as S , in the absence of packet arrivals. After the sleep period elapses, the system returns to the idle state, denoted as $L_{0,0}$.

Table 2 provides the notations for all the processes and their respective description. The distributions for the arrival, service and sleep

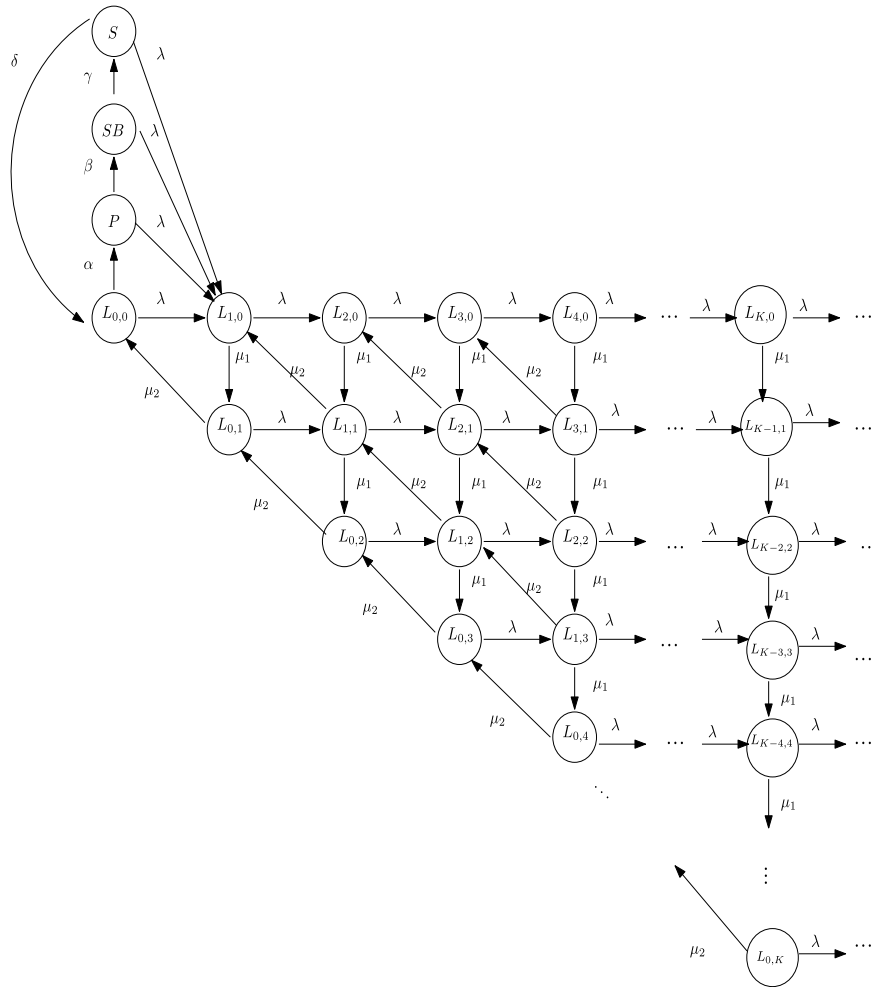


Fig. 3. State transition diagram for the proposed model.

Table 2
Notations and their description.

Notation	Description
λ	Arrival of packets
μ_1	Service of packets from HAP
μ_2	Service of packets from LEO
$1/\alpha$	Mean sojourn time of idle state
$1/\beta$	Mean sojourn time of power saving state
$1/\gamma$	Mean sojourn time of standby state
$1/\delta$	Mean sojourn time of sleep state

processes are described in Tables 3 and 4 for the SMM. In case of MM, the arrival, service and sleep processes will follow exponential distribution.

3. Steady-state analysis

3.1. Semi-Markov model

In this section, the steady-state analysis of the proposed model will be presented. Consider the underlying system state process $\{X(t), t \in T\}$ where $X(t)$ is the state of the system at time t . The sequence of system states $X = \{L_{i,j}, P, SB, S; i \geq 0, j \geq 0\}$ (including idle state, active state, power saving state, standby state, sleep state) corresponds to the time

instances $T = \{t_0, t_1, t_2, \dots\}$ (e.g., at time instance t_0 the system is in idle state, at time instance t_0 the system is in active state, at time instance t_0 the system is in sleep state, and so on). This sequence of states with the time instances is the embedded discrete time Markov chain. A Markov chain is a mathematical model that describes a sequence of events or states in a system where the probability of transitioning to the next state depends only on the current state and is independent of the previous states. For a discrete-time Markov chain, if $X(t_n)$ represents the state of the system at time t_n , the transition probabilities can be represented as $P\{X(t_n + 1) = j | X(t_n) = i\}$, where i and j are the states of the system.

Due to the general distribution for mean sojourn time of some states, and Markov attributes satisfied time epochs T , the underlying stochastic process can be appropriately represented by a semi-Markov process (SMP) [43]. The state space of the underlying stochastic process is defined as

$$\Omega = \{L_{i,j}, P, SB, S, i = 0, 1, 2, \dots; j = 0, 1, 2, \dots\}.$$

Under the stability conditions $\lambda < \mu$, $\lambda < \gamma$, $\lambda < \beta$, $\lambda < \delta$, the steady-state probabilities of the SMP exist and are unique. To obtain these probabilities, two stage method is employed. For this method, the elements of kernel matrix $K(t)$ defined as $K_{m,n}(t) = P\{Y_1 = n, t_1 \leq t | Y_0 = m\}$, where $m, n \in \Omega$ and $\{(Y_n, t_n), n \geq 0\}$ is the underlying Markov renewal sequence of random variables. Alternatively, $K_{m,n}(t)$ can be understood as the probability that if the SMP has just entered state m ; the next transition occurs within time t and the next state will be n . Let $F_{m,n}$ be the cumulative distribution function (CDF) for the transition from state m to n of the state space. The non-zero elements

Table 3
Distributions of mean sojourn times in semi-Markov model for general distributions.

CDF	General distribution	Mean	General distribution	CDF
$F_{L_{0,0},P}(t)$	General	$\frac{1}{\alpha}$	Deterministic	$F(x) = \begin{cases} 1 & x \leq 1/k \\ 0 & x > 1/k \end{cases}$
$F_{P,SB}(t)$	General	$\frac{1}{\beta}$		
$F_{SB,S}(t)$	General	$\frac{1}{\gamma}$	Hypo-exponential	$F(x) = 1 - \frac{v_2}{v_2 - v_1} e^{-v_1 x} + \frac{v_1}{v_2 - v_1} e^{-v_2 x}$
$F_{S,L_{0,0}}(t)$	General	$\frac{1}{\delta}$		

Table 4
Distributions of mean sojourn times in semi-Markov model for exponential distribution.

CDF	Distribution	Parameter
$F_{L_{0,0},L_{1,0}}(t)$	Exponential	λ
$F_{L_{1,0},L_{1,0}}(t)$	Exponential	λ
$F_{S,L_{1,0}}(t)$	Exponential	λ
$F_{SB,L_{1,0}}(t)$	Exponential	λ
$F_{L_{i,j},L_{i-1,j+1}}(t)$	Exponential	μ_1
$F_{L_{i,j},L_{i,j-1}}(t)$	Exponential	μ_2
$F_{L_{0,j},L_{0,j-1}}(t)$	Exponential	μ_2

of kernel matrix are obtained by using the CDF defined in Table 3 and 4.

$$K_{L_{0,0},L_{1,0}}(t) = \int_0^t (1 - F_{L_{0,0},P}(x)) dF_{L_{0,0},L_{1,0}}(x), \quad (1)$$

$$K_{L_{0,0},P}(t) = \int_0^t (1 - F_{L_{0,0},L_{1,0}}(x)) dF_{L_{0,0},P}(x), \quad (2)$$

$$K_{P,SB}(t) = \int_0^t (1 - F_{P,L_{1,0}}(x)) dF_{P,SB}(x), \quad (3)$$

$$K_{P,L_{1,0}}(t) = \int_0^t (1 - F_{P,SB}(x)) dF_{P,L_{1,0}}(x), \quad (4)$$

$$K_{SB,L_{1,0}}(t) = \int_0^t (1 - F_{SB,S}(x)) dF_{SB,L_{1,0}}(x), \quad (5)$$

$$K_{SB,S}(t) = \int_0^t (1 - F_{SB,L_{1,0}}(x)) dF_{SB,S}(x), \quad (6)$$

$$K_{S,L_{0,0}}(t) = \int_0^t (1 - F_{S,L_{1,0}}(x)) dF_{S,L_{0,0}}(x), \quad (7)$$

$$K_{S,L_{1,0}}(t) = \int_0^t (1 - F_{S,L_{0,0}}(x)) dF_{S,L_{1,0}}(x), \quad (8)$$

$$K_{L_{1,0},L_{i+1,0}}(t) = \int_0^t (1 - F_{L_{1,0},L_{i-1,1}}(x)) dF_{L_{1,0},L_{i+1,0}}(x), i = 1, 2, 3, \dots, \quad (9)$$

$$K_{L_{i,j},L_{i-1,j+1}}(t) = \int_0^t (1 - F_{L_{i,j},L_{i,j-1}}(x)) dF_{L_{i,j},L_{i-1,j+1}}(x), i = 1, 2, 3, \dots, j = 1, 2, 3, \dots, \quad (10)$$

$$K_{L_{1,0},L_{i-1,1}}(t) = \int_0^t (1 - F_{L_{1,0},L_{i+1,0}}(x)) dF_{L_{1,0},L_{i-1,1}}(x), i = 1, 2, 3, \dots, \quad (11)$$

$$K_{L_{0,j},L_{0,j-1}}(t) = \int_0^t dF_{L_{0,j},L_{0,j-1}}(x), j = 1, 2, 3, \dots, \quad (12)$$

$$K_{L_{i,j},L_{i,j-1}}(t) = \int_0^t (1 - F_{L_{i,j},L_{i-1,j+1}}(x)) dF_{L_{i,j},L_{i,j-1}}(x), i = 1, 2, 3, \dots, j = 1, 2, 3, \dots \quad (13)$$

Define $K_{m,n}(\infty) = \lim_{t \rightarrow \infty} K_{m,n}(t)$, where $m, n \in \Omega$. Hence, matrix $K(\infty)$ with entries $K_{m,n}(\infty)$ gives the one step transition probabilities for the model and row sum of matrix $K(t)$ becomes 1 as $t \rightarrow \infty$. The steady-state probabilities of SMP are defined by v_i , where $i \in \Omega$. This constitutes elements of vector v , and is obtained by solving

$$v = vK(\infty), \sum_{i \in \Omega} v_i = 1. \quad (14)$$

The steady-state probability of state i , for the SMP model is given by

$$\pi_i = \frac{v_i h_i}{\sum_{j \in \Omega} v_j h_j}, i \in \Omega, \quad (15)$$

where h_i is the expected sojourn time that the process spends in each state i . These mean sojourn times are computed as follows.

$$h_P = \int_0^\infty (1 - F_{P,SB}(t))(1 - F_{P,L_{1,0}}(t)) dt, \quad (16)$$

$$h_S = \int_0^\infty (1 - F_{S,L_{0,0}}(t))(1 - F_{S,L_{1,0}}(t)) dt, \quad (17)$$

$$h_{SB} = \int_0^\infty (1 - F_{SB,S}(t))(1 - F_{SB,L_{1,0}}(t)) dt, \quad (18)$$

$$h_{L_{0,0}} = \int_0^\infty (1 - F_{L_{0,0},SB}(t))(1 - F_{L_{0,0},L_{1,0}}(t)) dt, \quad (19)$$

$$h_{L_{i,0}} = \int_0^\infty (1 - F_{L_{i,0},L_{i+1,0}}(t))(1 - F_{L_{i,0},L_{i-1,1}}(t)) dt, \quad (20)$$

$$h_{L_{0,i}} = \int_0^\infty (1 - F_{L_{0,i},L_{0,i+1}}(t)) dt, i = 1, 2, 3, \dots, \quad (21)$$

$$h_{L_{i,j}} = \int_0^\infty (1 - F_{L_{i,j},L_{i-1,j+1}}(t))(1 - F_{L_{i,j},L_{i-1,j}}(t)) dt, \quad (22)$$

$$i = 1, 2, 3, \dots, j = 1, 2, 3, \dots$$

Using Eq. (15) and the values of mean sojourn times from Eqs. (16)–(22), the steady-state probabilities of the system are computed. The detailed description of the computation process is given in Appendix.

3.2. Markov model

Let $\{N_1(t), t \geq 0\}$ denotes the number of packets at HAP at any time t , $\{N_2(t), t \geq 0\}$ denotes the number of packets at LEO at any time t and $\{S(t), t \geq 0\}$ denotes the state of the system at any time t . Let $\{(N_1(t), N_2(t), S(t)), t \geq 0\}$ represents the three dimensional continuous time Markov chain of the underlying system. The state transition diagram for the proposed model is same as depicted in Fig. 3. The state space of the underlying process is given by

$$\Omega' = \{(i, j, k); i = 0, 1, 2, \dots; j = 0, 1, 2, \dots; k = 0, 1, 2, 3\}.$$

Here,

- $k = 0$ denotes the idle and busy state of the system,
- $k = 1$ denotes the power saving state of HAP,
- $k = 2$ denotes the standby state of HAP,
- $k = 3$ denotes the sleep state of HAP.

Let $P_{i,j,k}(t) = P\{N_1(t) = i, N_2(t) = j, S(t) = k\}$, $i \geq 0, j \geq 0, k = 0, 1, 2, 3$ be the probability that there are i packets at HAP, there are j packets at LEO and the system is in k state at any time t . The steady-state probabilities of the system are defined as

$$\lim_{t \rightarrow \infty} P_{i,j,k}(t) = \pi_{i,j,k}; i, j, k \in \Omega.$$

$\pi_{i,j,k}$ is called the steady-state probability of the system. Under the stability conditions $\lambda < \mu$, $\lambda < \gamma$, $\lambda < \beta$, $\lambda < \delta$, the steady-state probabilities of the Markov model exist. In order to obtain the steady-state probabilities of the system, the following steady-state equations are written.

$$\pi_{0,0,0}(\lambda + \alpha) = \mu_2 \pi_{0,1,0} + \delta \pi_{0,0,3},$$

$$\pi_{0,0,1}(\lambda + \beta) = \alpha \pi_{0,0,0},$$

$$\pi_{0,0,2}(\lambda + \gamma) = \beta \pi_{0,0,1},$$

$$\pi_{0,0,3}(\lambda + \delta) = \gamma \pi_{0,0,2},$$

$$\pi_{i,0,0}(\lambda + \mu_1) = \lambda(\pi_{0,0,1} + \pi_{0,0,2} + \pi_{0,0,3} + \pi_{0,0,4}),$$

$$\pi_{i,0,0}(\lambda + \mu_1) = \lambda \pi_{i-1,0,0}, i = 1, 2, \dots,$$

$$\pi_{i,j,0}(\lambda + \mu_2 + \mu_1) = \mu_2 \pi_{i,j-1,0} + \mu_1 \pi_{i-1,j+1,0} + \lambda \pi_{i-1,j,0},$$

$$i = 1, 2, \dots, j = 1, 2, \dots$$

The above mentioned equations can be solved through backward substitution. The steady-state probabilities of the system can be obtained by solving the steady-state equations along with the normalization condition i.e.,

$$\sum_{i,j,k \in \Omega'} \pi_{i,j,k} = 1.$$

4. Performance measures

This section represents the following performance measures for long run probabilities of the proposed model.

1. **Steady-State Probabilities:** Let the steady-state probabilities of the system being in idle state, active state, power saving state, standby state and sleep state are given by $P_I, P_A, P_P, P_{SB}, P_S$, respectively.

- **SMM:**

$$\begin{aligned} P_I &= \pi_{L_{0,0}}, \\ P_A &= \sum_{i=0}^{\infty} \sum_{j=0, i \neq j \neq 0}^{\infty} \pi_{L_{i,j}}, \\ P_P &= \pi_P, \\ P_{SB} &= \pi_{SB}, \\ P_S &= \pi_S. \end{aligned}$$

- **MM:**

$$\begin{aligned} P_I &= \pi_{0,0,0}, \\ P_A &= \sum_{i=0}^{\infty} \sum_{j=0, i \neq j \neq 0}^{\infty} \sum_{k=0}^3 \pi_{i,j,k}, \\ P_P &= \pi_{0,0,1}, \\ P_{SB} &= \pi_{0,0,2}, \\ P_S &= \pi_{0,0,3}. \end{aligned}$$

2. Expected number of packets at the HAP system ($E[H]$) and LEO system ($E[L]$) are provided by following expressions.

- **SMM:**

$$\begin{aligned} E[H] &= \sum_{i=0}^{\infty} \sum_{j=0, i \neq j \neq 0}^{\infty} i \pi_{L_{i,j}}, \\ E[L] &= \sum_{i=0}^{\infty} \sum_{j=0, i \neq j \neq 0}^{\infty} j \pi_{L_{i,j}}. \end{aligned}$$

- **MM:**

$$\begin{aligned} E[H] &= \sum_{i=0}^{\infty} \sum_{j=0, i \neq j \neq 0}^{\infty} \sum_{k=0}^3 i \pi_{i,j,k}, \\ E[L] &= \sum_{i=0}^{\infty} \sum_{j=0, i \neq j \neq 0}^{\infty} \sum_{k=0}^3 j \pi_{i,j,k}. \end{aligned}$$

3. **Energy Saving Factor:** The energy saving state of the HAP system is defined as the proportion of time spent in sleep state and standby state by the HAP system. Hence, it is obtained as:

- **SMM:**

$$P_E = \pi_P + \pi_S + \pi_{SB}.$$

- **MM:**

$$P_E = \pi_{0,0,1} + \pi_{0,0,2} + \pi_{0,0,3}.$$

4. **Average Energy Consumption:** Let $E_I, E_A, E_P, E_{SB}, E_S$ be the energy consumed by the HAP in idle state, active state, power

saving state, standby state, sleep state, respectively. Therefore, the average energy consumption can be computed as follows

$$E_{cons} = P_I E_I + P_A E_A + P_P E_P + P_{SB} E_{SB} + P_S E_S.$$

5. **Throughput of the System:** The throughput of the system is calculated by multiplying the probability of system being in available state to service rate i.e.,

$$\text{Throughput } (TP) = P_A \times \mu.$$

5. Numerical illustration

In this section, numerical investigations are carried out to examine the viability of the suggested model in order to attain steady-state outcomes. The absence of precise model parameterization is addressed by examining the sensitivity of proposed model's characteristics to modest variations in specific parameter values considering parameter estimation is outside the purview of this paper. Table 5 comprises the values of the parameters required for producing the results. It is crucial to recall that these numbers are selected merely for the sake of numerical illustration.

5.1. Sensitivity analysis

Figs. 4–5 exhibit the trade-off between throughput (TP), energy consumption (E_{cons}), and energy saving (P_E). The main motivation of this sensitivity analysis is to demonstrate the behaviour of proposed stochastic model with respect to several parameters of the system. This sensitivity analysis has been done for the deterministic sleep distribution. Further, a comparison of results has been provided for the exponential and hypo-exponential case as well.

The increment in λ increases the TP due to the increasing number of packets in the system. Further, it can also be observed that TP increases with respect to μ because more packets will be served and consequently, TP increases. The similar type of behaviour of TP can be seen in Figs. 4(b)–4(d). Similarly, the average energy consumption of the system also increases with respect to λ . Since, the increasing number of packets in the system will increase the overall energy consumption as well. Figs. 4(b)–4(d) exhibit the trade-off between TP and E_{cons} for the sleeping time of different energy saving states i.e., $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$. A decrement can be observed in the values of TP with respect to $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$. This finding can be interpreted as follows. In this scenario, some of the components of HAP will be switched off and there will be no service of packets. Therefore, TP will decrease with the longer sleeping time of energy saving states. Similarly, E_{cons} also decreases with respect to the increasing sleeping times of the energy saving states. Since, some components of HAP will be turned off in this state which subsequently reduces the energy consumption of HAP.

Fig. 5 exhibits the trade-off between TP and P_E . The behaviour of TP is similar as explained in Fig. 4 with respect to the parameters of the model. Fig. 5(a) shows the trade-off between TP and P_E for λ and $1/\alpha$. A decrement can be observed in the values of P_E as the values of λ increases. Since, HAP will be in active state providing service to the incoming packets. Therefore, less energy will be saved in this scenario. Whereas, if the values of $1/\alpha$ are increased, an increment can be observed in the values of P_E . The similar type of trade-off can be observed in Figs. 5(b)–5(d).

The trade-offs demonstrated in this sensitivity analysis highlight the interdependencies between throughput, energy consumption, and energy saving in the proposed stochastic model. Both throughput and energy consumption are observed to increase as the number of packets in the system increases. When packets are served at a faster rate, the throughput of the system increases. However, this also leads to an increase in energy consumption. It is important to consider that the energy of the HAP is a limited resource that needs to be strategically utilized to ensure the system's throughput is not compromised.

Table 5
Default values of parameters.

Parameters	λ [42]	μ_1 [42]	μ_2 [42]	
Possible values	0.5–2/sec	0.5–3/sec	0.5–3/sec	
Parameters for deterministic distribution	$1/\alpha$	$1/\beta$	$1/\gamma$	$1/\delta$
Possible values	1–5 ms	1–10 ms	1–15 ms	1–30 ms
Parameters for exponential distribution	α	β	γ	δ
Possible values	0.1–3.5 s	0.1–3/sec	0.1–2.5/sec	0.1–2/sec
Parameters for hypo-exponential distribution	$\alpha_1 = \alpha_2 = \alpha/2$	$\beta_1 = \beta_2 = \beta/2$	$\gamma_1 = \gamma_2 = \gamma/2$	$\delta_1 = \delta_2 = \delta/2$
Possible values	0.1–3.5 s	0.1–3/sec	0.1–2.5/sec	0.1–2/sec

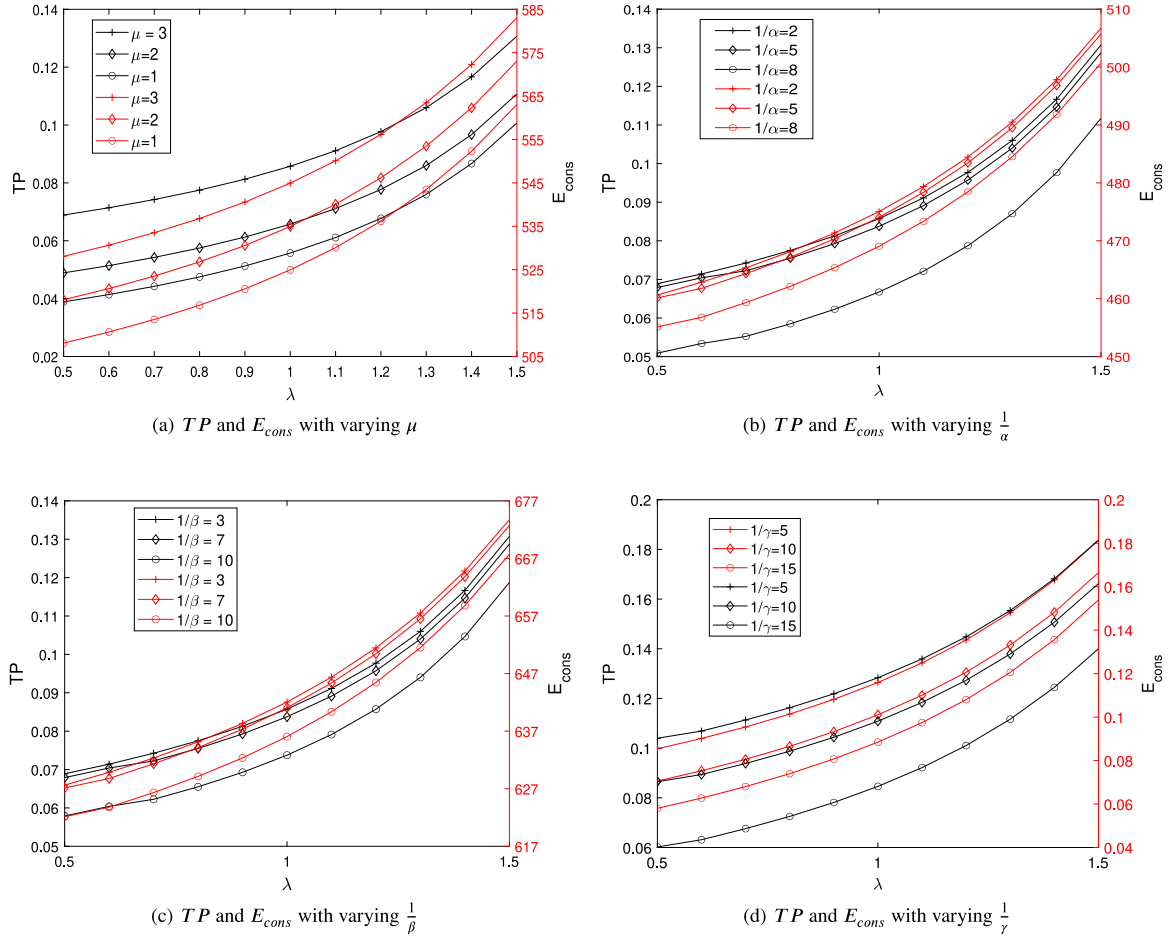


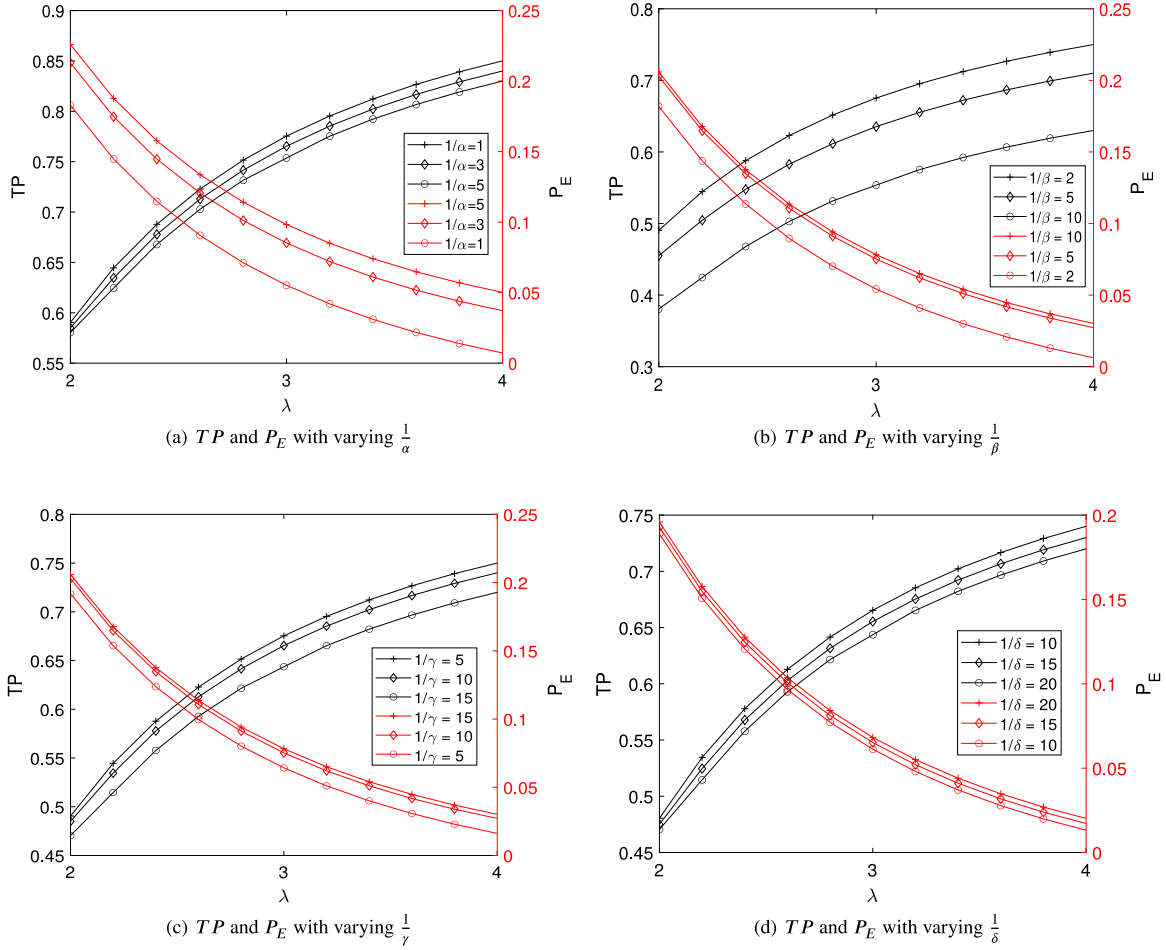
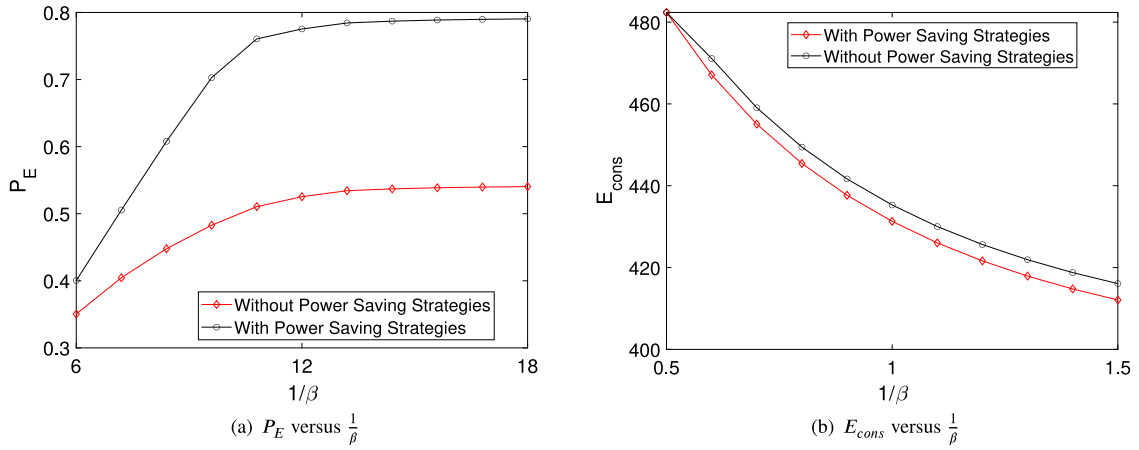
Fig. 4. Trade-off between TP (bit/ms) and E_{cons} (mW).

Similarly, the trade-off between sleeping time periods of the HAP and energy consumption can be explained. Frequent activation of sleep time periods reduces energy consumption and increases energy saving for the HAP. However, this compromise in energy consumption comes at the expense of a compromised system throughput. Although the arrival of packets cannot be controlled by the system, the service rate and sleep time period can be determined by the system.

Thus, it becomes crucial to strike a balance between the service rate, sleep time period, and energy consumption to optimize the system's performance. Careful consideration should be given to managing the limited energy resources of the HAP, ensuring that energy-saving measures do not significantly impact the system's throughput. By intelligently configuring the service rate and sleep time period, it is possible to achieve an optimal trade-off between energy consumption and throughput, thereby maximizing the system's overall efficiency.

5.2. Comparison of results with and without power saving strategies

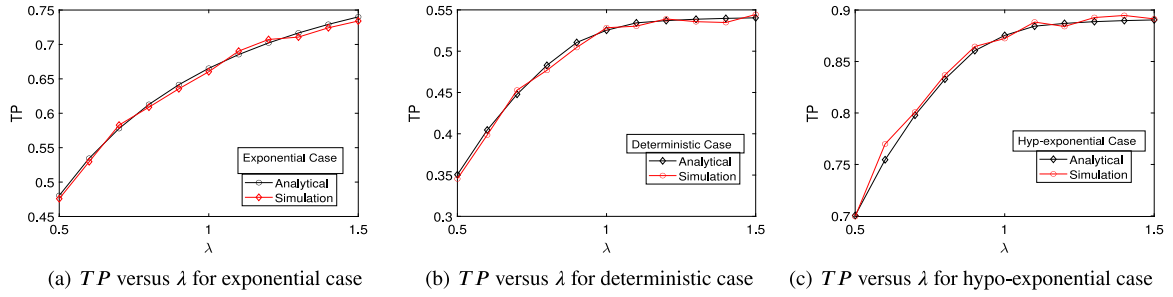
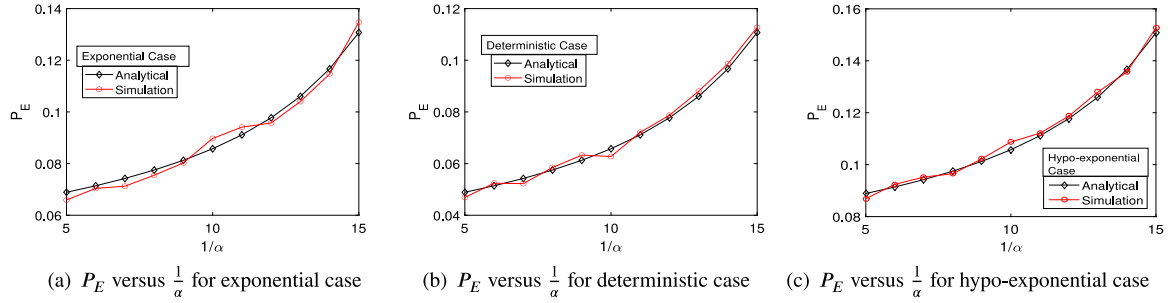
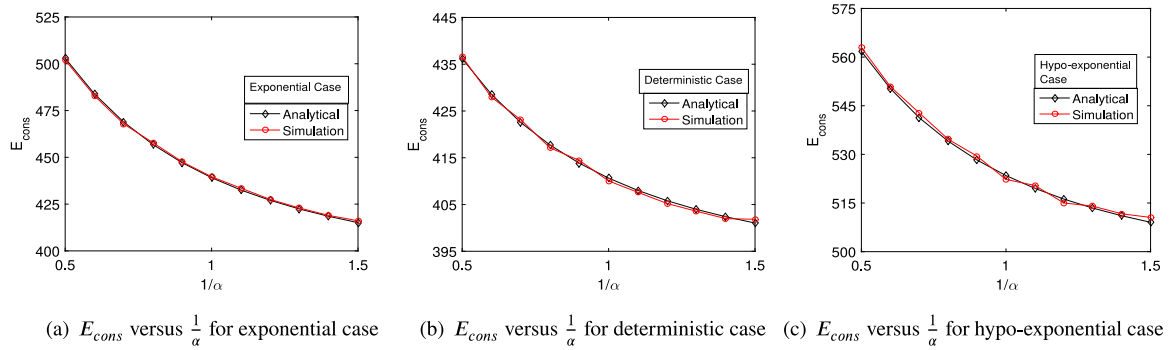
Figs. 6(a) and 6(b) demonstrates the impact of power saving strategies on the P_E and E_{cons} . The main component of the proposed model is the power saving state of HAP. This state will be active whenever there is a handover scenario of LEO satellites or beams are not properly aligned. These duration are relatively small as compared to sleeping states of HAP but these scenarios occur frequently in a particular day. Therefore, cumulatively a significant portion of energy is consumed in these scenarios. Hence, it is necessary to switch off some components of HAP to save energy in this case. Fig. 6(a) exhibit an increasing behaviour with respect to sleeping time of standby state and Fig. 6(b) shows a decreasing behaviour of E_{cons} . A significant difference can be observed in the values of P_E and E_{cons} due to the consideration of power saving state.

Fig. 5. Trade-off between TP (bit/ms) and P_E .Fig. 6. Comparison of values of P_E and E_{cons} with and without power saving strategies.

A huge difference can be observed in the values of P_E using the power saving strategies which indicates that more energy can be saved by utilizing the power saving state. Similarly, a significant difference can be observed in the values of E_{cons} also. The energy consumption reduces after the consideration of power saving strategies. This finding validates that the addition of power saving state while modelling can aid in reducing the energy consumption and consequently can increase the energy saving of HAP.

5.3. Comparison of analytical and simulated results

In this experiment, comparisons are drawn between the theoretical and simulation findings for the P_E , E_{cons} and TP of HAP. The identical values of the parameters that are taken into account to arrive at the analytical values, as specified in Table 5, are used in a MATLAB program to this end. The simulation is run for 30 times for the identical set of parameters and 30 values of the metric are obtained. To examine

Fig. 7. Comparison of analytical and simulated results for TP (bit/ms).Fig. 8. Comparison of analytical and simulated results for P_E .Fig. 9. Comparison of analytical and simulated results for E_{cons} (mW).

the accuracy of the simulation results, the t-distribution test is used. “The t distribution is used for smaller sample sizes to account for the uncertainty in the standard deviation of the sample”. Next, the mean and standard deviation of the 30 values are computed and confidence interval (CI) for 99% accuracy of the results are obtained. The C.I. for the 95% accuracy of the results can be calculated by:

$$C.I. = \bar{X} \pm \frac{s * t_{0.01,k-1}}{\sqrt{k}}$$

where $\bar{X}$ is sample mean, s is the standard deviation, $t_{0.01,k-1}$ is tabulated value for 95% C.I. and $k - 1$ degree of freedom, and k is sample size.

In the graphs, the findings from the analysis are depicted using solid lines, whereas the simulated outcomes are represented as dotted lines. Figs. 7–9 exhibit a comparative extensive study of results of the performance metrics for analytical and simulated cases. Fig. 7 exhibit the analytical and simulated observations of TP for all three distributions with respect to λ . The analytical and simulated results for the exponential and deterministic cases exhibit a close alignment, indicating a high level of agreement. However, in the hypo-exponential case, there are notable differences between the values for a few specific scenarios. Nonetheless, for the majority of cases, the results from both methods converge and demonstrate a significant level of concurrence.

On the similar line, Fig. 8 demonstrate the analytical and simulated results of P_E on $1/\alpha$ for three different scenarios. From these figures, it can be observed that majority of the analytical results matches with the simulated ones. Similar observation can be concluded from Fig. 9. The graphs indicate that findings of the both methods correspond quite well, demonstrating the approximate accuracy of the proposed model and verifying the hypothesized model.

5.4. Effect of various distributions on performance measures

This experiment provides a comparison of obtained results for exponential, deterministic and hypo-exponential distributed energy saving states. Fig. 10(a) denotes the behaviour of P_E with respect to λ for all three distributions. It can be observed that the value of P_E is high in hypo-exponential distributed cases in comparison to deterministic and exponential ones. The results shows that the value of P_A is almost 20% more for the hypo-exponential case. The lowest value of P_E is obtained for deterministic case. Fig. 10(b) depicts the behaviour of E_{cons} with respect to λ . The results exhibits that the value of E_{cons} is almost 10% more for the hypo-exponential case in comparison to exponential and deterministic case. On the similar track, Fig. 10(c) describes the behaviour of TP in all three cases. It has been observed that TP is having approximately 8–9% more values for hypo-exponential case as compare

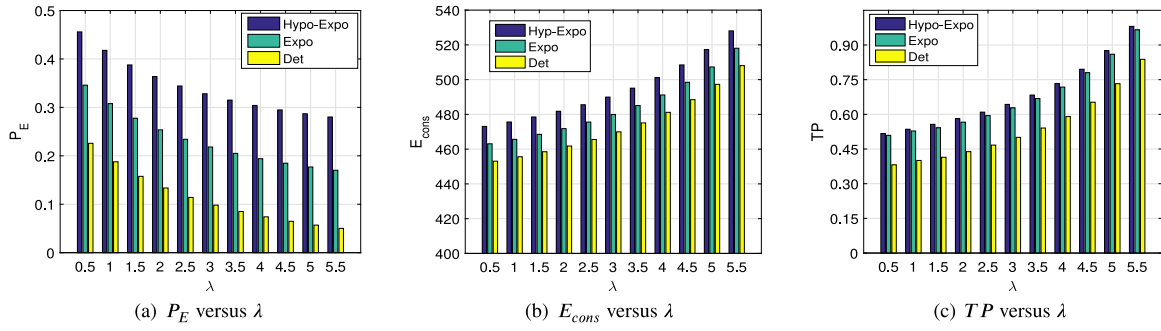


Fig. 10. Comparison of P_E , E_{cons} , and TP for different distributions.

to rest of the two cases. The main purpose of considering the energy saving states in different distributions is to attempt to represent the realistic scenario through modelling. The experiment demonstrates that the choice of energy saving distribution has a substantial impact on the values of performance measures. The reason is the difference between the sleep rates of these distributions. The significant differences observed in the results indicate that each metric analysed is significantly influenced by the type of energy saving distribution employed.

The choice of distribution parameters relies on the specific needs of the system. If the sleep time period of the HAP is deterministically structured, but an exponential distribution is used for modelling, the resulting outcomes would be inflated. Conversely, if the sleep time period adheres to hypo-exponential distributions, but the results are calculated based on an exponential distribution, the performance measures would be underestimated. Consequently, the selection of the most suitable distribution becomes crucial when assessing the system's performance.

6. Conclusions and future works

In this study, a novel energy-saving technique has been presented for the HAP in the SAT network. In order to achieve this, a stochastic model has been developed taking into account the multi-layer SAT network scenario. The proposed architecture has been modelled employing the semi-Markov and Markov processes. The model considers five states, i.e., idle state, active state, power saving state, standby state, and sleep state. To capture the behaviour of the presented system, various mean sojourn times of the states have been considered. This study aims at analysing the energy saving techniques of HAP in the multi-layer architecture. The standby and sleep state were already designed for HAP which helps in saving the energy of HAP in case of no packets for the service. In the proposed technique, a power saving state has been introduced, which is defined to save the energy of HAP in some particular scenarios e.g., during the frequent handover of services in LEO satellites, mismatching of beam alignments in HAP-terrestrial and/or HAP-LEO. During a day, the frequency of occurring these scenarios is relatively high which increases the overall energy consumption of HAP.

The proposed stochastic model considers three different scenarios of mean sojourn times for energy saving states; exponential, deterministic and hypo-exponential distributions. The fundamental objective of evaluating various distributions and taking them into consideration is to attempt to construct a more realistic scenario of energy-saving states. Analytical results have been computed using the two-stage method for the SMP model. The introduced power saving state aids in lowering energy consumption and in saving more energy of HAP, in accordance with rigorous sensitivity and trade-off analyses of performance measures. Also, a comparison has been provided for all the performance measures for different distributions of energy saving states. Additionally, the suggested model is verified via simulation, which produces results that closely align with the analytical results, demonstrating the validity of the underlying model.

The extended work will be concentrated on experimenting the proposed model with modified sleep strategy and a simulation based comparison will be provided. Furthermore, a stochastic game based energy saving analysis will be provided in future work.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Appendix A. Case 1: Deterministic distribution

The CDF of deterministic distribution is given by

$$F(x) = \begin{cases} 1 & x \leq 1/k \\ 0 & x > 1/k. \end{cases}$$

Therefore,

$$F_P(x) = \begin{cases} 1 & x \leq 1/\alpha \\ 0 & x > 1/\alpha \end{cases}, \quad F_{SB}(x) = \begin{cases} 1 & x \leq 1/\beta \\ 0 & x > 1/\beta \end{cases},$$

$$F_S(x) = \begin{cases} 1 & x \leq 1/\gamma \\ 0 & x > 1/\gamma \end{cases}, \quad F_I(x) = \begin{cases} 1 & x \leq 1/\delta \\ 0 & x > 1/\delta \end{cases}.$$

Using the Eqs. (1)–(13), the entries of kernel matrix are computed and displayed as follows.

$$K_{L_{0,0},L_{1,0}} = \begin{cases} 1 - e^{-\lambda/\alpha}, & t \geq \frac{1}{\alpha} \\ 1 - e^{-\lambda t}, & t < \frac{1}{\alpha} \end{cases}, \quad K_{L_{0,0},P} = \begin{cases} e^{-\lambda/\alpha}, & t \geq \frac{1}{\alpha} \\ 0, & t < \frac{1}{\alpha} \end{cases},$$

$$K_{P,SB} = \begin{cases} e^{-\lambda/\beta}, & t \geq \frac{1}{\beta} \\ 0, & t < \frac{1}{\beta} \end{cases}, \quad K_{SB,S} = \begin{cases} e^{-\lambda/\gamma}, & t \geq \frac{1}{\gamma} \\ 0, & t < \frac{1}{\gamma} \end{cases},$$

$$K_{P,L_{1,0}} = \begin{cases} 1 - e^{-\lambda/\beta}, & t \geq \frac{1}{\beta} \\ 1 - e^{-\lambda t}, & t < \frac{1}{\beta} \end{cases}, \quad K_{SB,L_{1,0}} = \begin{cases} 1 - e^{-\lambda/\gamma}, & t \geq \frac{1}{\gamma} \\ 1 - e^{-\lambda t}, & t < \frac{1}{\gamma} \end{cases},$$

$$K_{S,L_{0,0}} = \begin{cases} e^{-\lambda/\delta}, & t \geq \frac{1}{\delta} \\ 0, & t < \frac{1}{\delta} \end{cases}, \quad K_{S,L_{1,0}} = \begin{cases} 1 - e^{-\lambda/\delta}, & t \geq \frac{1}{\delta} \\ 1 - e^{-\lambda t}, & t < \frac{1}{\delta} \end{cases},$$

$$K_{L_{i,0},L_{i+1,0}} = \frac{\lambda}{\lambda + \mu_1} (1 - e^{-(\lambda + \mu_1)t}), \quad K_{L_{i,j},L_{i-1,j+1}} = \frac{\mu_1}{\mu_2 + \mu_1} (1 - e^{-(\mu_2 + \mu_1)t}),$$

$$K_{L_{i,0},L_{i-1,1}} = \frac{\mu_1}{\lambda + \mu_1}(1 - e^{-(\lambda + \mu_1)t}), \quad K_{L_{0,j},L_{0,j-1}} = (1 - e^{-\mu_2 t}),$$

$$K_{L_{i,j},L_{i,j-1}} = \frac{\mu_2}{\mu_2 + \mu_1}(1 - e^{-(\mu_2 + \mu_1)t}).$$

On solving the set of Eqs. (14), the steady-state probabilities can be obtained in the following form.

$$v_P = e^{-\lambda/\alpha} v_{L_{0,0}}, \quad v_{SB} = e^{-\lambda/\alpha - \lambda/\beta} v_{L_{0,0}},$$

$$v_S = e^{-\lambda/\alpha - \lambda/\beta - \lambda/\gamma} v_{L_{0,0}}, \quad v_{0,1} = \frac{1}{\mu_2} (1 - e^{-\lambda/\alpha - \lambda/\beta - \lambda/\gamma - \lambda/\delta}) v_{L_{0,0}},$$

$$v_{L_{0,i}} = \left[e^{-\lambda/\beta - \lambda/\alpha} - e^{-\lambda/\gamma} - \left(1 - e^{-\lambda/\gamma} + \frac{1 - e^{-\lambda/\beta}}{e^{\lambda/\beta}} \right. \right. \\ \left. \left. + \frac{1 - e^{-\lambda/\alpha}}{e^{\lambda/\alpha + \lambda/\beta}} \right) \frac{\mu_1}{\lambda + \mu_1} A_1 \right] v_{L_{0,0}}, i = 1, 2, \dots,$$

where,

$$A_1 = \frac{[1 - \frac{\lambda}{\lambda + \mu_1} \frac{\mu_2}{\mu_2 + \mu_1} - \frac{\lambda \mu_1}{(\lambda + \mu_1)^2} \frac{\mu_2}{\mu_2 + \mu_1} (1 + \frac{\lambda}{\lambda + \mu_1} \frac{\mu_2}{\mu_2 + \mu_1})]^{i-1}}{1 - \frac{\lambda}{\lambda + \mu_1} \frac{\mu_2}{\mu_2 + \mu_1} - \left(\frac{\lambda}{\lambda + \mu_1} \frac{\mu_2}{\mu_2 + \mu_1} \right)^2 \frac{\mu_1}{\lambda + \mu_1} (1 - 2 \frac{\lambda \mu_1}{(\lambda + \mu_1)^2} \frac{\mu_2}{\mu_2 + \mu_1})}.$$

$$v_{L_{1,0}} = \left[1 - e^{-\lambda/\gamma} + \frac{1 - e^{-\lambda/\beta}}{e^{-\lambda/\beta}} + 1 - e^{-\lambda/\alpha} (e^{-\lambda/\delta - \lambda/\beta}) \right] \times A_1 v_{L_{0,0}},$$

$$v_{L_{i,0}} = \frac{\left(\frac{\lambda}{\lambda + \mu_1} \right)^{i-1} v_{L_{1,0}}}{1 - \frac{\lambda}{\lambda + \mu_1} \frac{\mu_2}{\mu_2 + \mu_1} - \frac{\lambda}{\lambda + \mu_1} \frac{\mu_2}{\mu_2 + \mu_1} \frac{\mu_1}{\lambda + \mu_1} (1 + \frac{\lambda}{\lambda + \mu_1} \frac{\mu_2}{\mu_2 + \mu_1})}, i = 2, 3, \dots,$$

$$v_{L_{i,j}} = \frac{\left(\frac{\lambda}{\lambda + \mu_1} \right)^{j+1} \left(\frac{\mu_1}{\lambda + \mu_1} \right)^{i-1} v_{L_{1,0}}}{1 - \frac{\lambda}{\lambda + \mu_1} \frac{\mu_2}{\mu_2 + \mu_1} - \frac{\lambda}{\lambda + \mu_1} \frac{\mu_2}{\mu_2 + \mu_1} \frac{\mu_1}{\lambda + \mu_1} (1 + \frac{\lambda}{\lambda + \mu_1} \frac{\mu_2}{\mu_2 + \mu_1})},$$

$$i = 1, 2, 3, \dots, j = 1, 2, 3, \dots$$

By using the normalization condition, $\sum_{i \in \Omega} v_i = 1$, the value of $v_{L_{0,0}}$ can be obtained, hence the value of $v_i, i \in \Omega$ are computed. The value of $v_{L_{0,0}}$ is given as follows

$$v_{L_{0,0}} = \left[e^{-\lambda/\beta - \lambda/\delta} + e^{-\lambda/\beta - \lambda/\alpha} + e^{-\lambda/\beta - \lambda/\alpha} - e^{-\lambda/\gamma} \right. \\ \left. + 2 \left(e^{\lambda/\beta + \lambda/\alpha} - e^{-\lambda/\gamma - \lambda/\delta} - A_1 A_2 \right) \right. \\ \left. + (1 - A_7) A_2 \frac{(1 + 2A_3 A_4) + A_5 (1 - A_3)}{1 - A_3 - (A_3 A_7)(1 - 2A_3 A_7)} \right. \\ \left. + \left(1 + \frac{A_8 (1 - A_3) + (A_8)^2 + 2(A_8)^3 + A_6 B}{(1 - A_3 - (A_3 A_7)^2)(1 - 2A_3 A_7)} \right) \right]^{-1}.$$

Here

$$A_2 = \left[1 - e^{-\lambda/\delta} - e^{-\lambda/\gamma} + \frac{1 - e^{-\lambda/\beta}}{e^{-\lambda/\beta}} + 1 - e^{-\lambda/\alpha} (e^{-\lambda/\alpha - \lambda/\beta}) \right],$$

$$A_3 = \frac{\lambda}{\lambda + \mu_1} \frac{\mu_2}{\mu_2 + \mu_1}, A_4 = \left(\frac{\lambda \mu_1}{(\lambda + \mu_1)^2} \right)^2 \frac{\mu_2}{\mu_2 + \mu_1},$$

$$A_5 = \frac{\lambda \mu_1}{(\lambda + \mu_1)^2},$$

$$A_6 = \left(\frac{\lambda}{\lambda + \mu_1} \right)^3 \frac{\mu_1}{\lambda + \mu_1} \left(1 + \frac{\mu_2}{\mu_1 + \mu_2} \right) + \left(\frac{\lambda}{\lambda + \mu_1} \right)^2 \frac{\mu_1}{\lambda + \mu_1},$$

$$A_7 = \frac{\mu_1}{\lambda + \mu_1}, A_8 = \frac{\lambda}{\lambda + \mu_1},$$

$$B = (A_8)^3 (A_7)^2 + (A_8)^2 (A_7)^2 (1 + 2A_3) + (A_8)^3 (A_7)^3.$$

The mean sojourn time of each state $i \in \Omega$ can be computed using Eqs. (16)–(22) as follows.

$$h_P = \frac{1 - e^{-\lambda/\beta}}{\lambda}, h_S = \frac{1 - e^{-\lambda/\delta}}{\lambda}, h_{SB} = \frac{1 - e^{-\lambda/\gamma}}{\lambda},$$

$$h_{L_{0,0}} = \frac{1 - e^{-\lambda/\alpha}}{\lambda}, h_{L_{i,0}} = \frac{1}{\lambda + \mu_1}, h_{L_{0,i}} = \frac{1}{\mu_2}, h_{L_{i,j}} = \frac{1}{\mu_2 + \mu_1}.$$

Appendix B. Case 2: Hypo-exponential distribution

The CDF of hypo-exponential distribution is given as follows

$$F(x) = 1 - \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_1 x} + \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 x}.$$

Therefore,

$$F_P(x) = 1 - \frac{\alpha_2}{\alpha_2 - \alpha_1} e^{-\alpha_1 x} + \frac{\alpha_1}{\alpha_2 - \alpha_1} e^{-\alpha_2 x},$$

$$F_{SB}(x) = 1 - \frac{\beta_2}{\beta_2 - \beta_1} e^{-\beta_1 x} + \frac{\beta_1}{\beta_2 - \beta_1} e^{-\beta_2 x},$$

$$F_S(x) = 1 - \frac{\gamma_2}{\gamma_2 - \gamma_1} e^{-\gamma_1 x} + \frac{\gamma_1}{\gamma_2 - \gamma_1} e^{-\gamma_2 x},$$

$$F_I(x) = 1 - \frac{\delta_2}{\delta_2 - \delta_1} e^{-\delta_1 x} + \frac{\delta_1}{\delta_2 - \delta_1} e^{-\delta_2 x}.$$

Using the Eqs. (1)–(13), the entries of kernel matrix are computed and displayed as follows.

$$K_{L_{0,0},L_{1,0}} = \frac{\lambda}{\alpha_2 - \alpha_1} \left[\frac{\alpha_2 (1 - e^{-(\lambda + \alpha_1)t})}{\lambda + \alpha_1} - \frac{\alpha_1 (1 - e^{-(\lambda + \alpha_2)t})}{\lambda + \alpha_2} \right],$$

$$K_{L_{0,0},P} = \frac{\alpha_2 \alpha_1}{\alpha_2 - \alpha_1} \left(\frac{1 - e^{-(\lambda + \alpha_1)t}}{\lambda + \alpha_2} - \frac{1 - e^{-(\lambda + \alpha_2)t}}{\lambda + \alpha_1} \right),$$

$$K_{P,SB} = \frac{\beta_2 \beta_1}{\beta_2 - \beta_1} \left(\frac{1 - e^{-(\lambda + \beta_1)t}}{\lambda + \beta_2} - \frac{1 - e^{-(\lambda + \beta_2)t}}{\lambda + \beta_1} \right),$$

$$K_{SB,S} = \frac{\gamma_2 \gamma_1}{\gamma_2 - \gamma_1} \left(\frac{1 - e^{-(\lambda + \gamma_1)t}}{\lambda + \gamma_2} - \frac{1 - e^{-(\lambda + \gamma_2)t}}{\lambda + \gamma_1} \right),$$

$$K_{P,L_{1,0}} = \frac{\lambda}{\beta_2 - \beta_1} \left[\frac{\beta_2 (1 - e^{-(\lambda + \beta_1)t})}{\lambda + \beta_1} - \frac{\beta_1 (1 - e^{-(\lambda + \beta_2)t})}{\lambda + \beta_2} \right],$$

$$K_{SB,L_{1,0}} = \frac{\lambda}{\gamma_2 - \gamma_1} \left[\frac{\gamma_2 (1 - e^{-(\lambda + \gamma_1)t})}{\lambda + \gamma_1} - \frac{\gamma_1 (1 - e^{-(\lambda + \gamma_2)t})}{\lambda + \gamma_2} \right],$$

$$K_{S,L_{0,0}} = \frac{\delta_2 \delta_1}{\delta_2 - \delta_1} \left(\frac{1 - e^{-(\lambda + \delta_1)t}}{\lambda + \delta_2} - \frac{1 - e^{-(\lambda + \delta_2)t}}{\lambda + \delta_1} \right),$$

$$K_{S,L_{1,0}} = \frac{\lambda}{\delta_2 - \delta_1} \left[\frac{\alpha_2 (1 - e^{-(\lambda + \delta_1)t})}{\lambda + \delta_1} - \frac{\delta_1 (1 - e^{-(\lambda + \delta_2)t})}{\lambda + \delta_2} \right],$$

$$K_{L_{i,0},L_{i+1,0}} = \frac{\lambda}{\lambda + \mu_1} (1 - e^{-(\lambda + \mu_1)t}),$$

$$K_{L_{i,j},L_{i-1,j+1}} = \frac{\mu_1}{\mu_2 + \mu_1} (1 - e^{-(\mu_2 + \mu_1)t}),$$

$$K_{L_{i,0},L_{i-1,1}} = \frac{\mu_1}{\lambda + \mu_1} (1 - e^{-(\lambda + \mu_1)t}),$$

$$K_{L_{0,j},L_{0,j-1}} = (1 - e^{-\mu_2 t}),$$

$$K_{L_{i,j},L_{i,j-1}} = \frac{\mu_2}{\mu_2 + \mu_1} (1 - e^{-(\mu_2 + \mu_1)t}).$$

The mean sojourn time of each state $i \in \Omega$ can be computed using Eqs. (16)–(22) as follows.

$$h_P = \frac{\beta_2 \beta_1}{(\lambda + \beta_1)(\lambda + \beta_2)}, h_S = \frac{\delta_2 \delta_1}{(\lambda + \delta_1)(\lambda + \delta_2)},$$

$$h_{SB} = \frac{\beta_2 \beta_1}{(\lambda + \beta_1)(\lambda + \beta_2)}, h_{L_{0,0}} = \frac{\alpha_2 \alpha_1}{(\lambda + \alpha_1)(\lambda + \alpha_2)},$$

$$h_{L_{i,0}} = \frac{1}{\lambda + \mu_1}, h_{L_{0,i}} = \frac{1}{\mu_2}, h_{L_{i,j}} = \frac{1}{\mu_2 + \mu_1}.$$

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