



Performance Analysis for Tethered HAP Systems: An Analytical Approach

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Abstract. An analytical model for tethered high altitude platform (HAP) systems, that handle voice, video, and data traffic, is presented in this work. The Markovian arrival process (*MAP*) is used to depict the arrival of traffic due to the burstiness and self-similar properties of the incoming traffic. Guard channel policy has been employed for the prioritization of the higher priority traffic. If the incoming traffic is blocked due to the lack of idle channel, it will retry for the service after some random amount of time. Phase-type (*PH*) distributions with distinct parameters have been employed for the representation of the service and retrial processes. The suggested model's behaviour is represented by the level-dependent quasi-birth-death (*LDQBD*) process. Expressions for some significant performance measures are derived. Additionally, queueing examples are offered, which can be acquired as the special cases of the suggested analytical model.

Keywords: Blocking probabilities · High altitude platforms · Level dependent quasi-birth-death process · Markovian arrival process · Phase type distributions

1 Introduction

The confluence of customer demand for personal communication systems has contributed to the global adoption related to wireless communications. The deployment of wireless services also come with the emerging challenges of proper resource allocation, efficient use of spectrum, quality of service for users, etc. The challenge of wireless communication may overcome by aerial platforms carrying communications relay payloads and moving in a largely stationary posture. In contrast to the majority of satellites, a payload might be as simple as a transparent transponder or as complex as a full-fledged base station [14]. A single aerial platform can replace a large number of terrestrial masts, as well as the costs, environmental effect, and backhaul limits that come with them. These aerial

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platforms can be manned or unmanned, with autonomous operation combined with ground control. High-altitude platforms (HAPs) are unmanned aerial base stations which are designed to operate in the stratosphere at a height of between 17 and 22 km. The HAP system is further classified as tethered and untethered. The HAP systems are intended to carry voice, multimedia, file transfer, and web access [11]. To provide QoS for a diverse variety of traffic, service providers must make optimum use of available radio resources. Call Admission Control (CAC) schemes manage radio resources by admitting an arriving traffic based on a pre-defined criterion. References [1, 13] present a comprehensive survey for a thorough examination of CAC issues in wireless communication.

The statistical characteristics of self-similarity and burstiness that characterise contemporary voice, video, and data traffic lead to a correlation in the times between arriving packet. As a result, the exponentially distributed call holding times (CHT) and inter-arrival times assumptions are rendered inaccurate. Furthermore, with new applications, the length of time a mobile user remains or the time spent in a cell, known as cell residence time (CRT), is determined by the user's mobility and the geographic location. To estimate realistic performance measures, traffic arrivals can be modelled using the Markovian arrival process (*MAP*), which has the advantage of smoothing total traffic through statistical aggregations [7]. The CHT and CRT should be represented by phase-type (*PH*) distribution, which is the convolution of different exponential distributions [7]. When the incoming traffic finds no idle channel available in the system, it will join a virtual space termed as orbit to retry for the services after some random amount of time. The phenomenon is referred as a retrial phenomenon and is very ubiquitous in wireless communication scenarios. Though, the retrial phenomenon has been explored in a number of queueing models with the consideration of exponential distribution, the literature considering a general distributed or *PH* distributed retrial times is still moderate (see, [4]).

In this study, we have constructed an analytical retrial queueing model which represents the HAP systems with four types of traffic. Also, to consider a more realistic scenario, the inter-arrival times is represented by *MAP*, and the service and retrial times are denoted by *PH* distributions. The proposed model's behaviour is represented by level dependent quasi-birth-death (*LDQBD*) process. Further, some special cases of the proposed model has been described and the explicit expressions of the generator matrix have been obtained.

The remainder of this article is structured as follows. The suggested multi-server retrial queueing mechanism is briefly described and mathematically developed in Sect. 2. In Sect. 3, explicit expressions of the infinitesimal generator matrix and significant performance measures required to examine the network efficiency are provided by modelling the stochastic behaviour into a *LDQBD* process. The crucial performance measures of the proposed model is covered in Sect. 4. Section 5 presents in-depth presentations of specific cases of the proposed model. Section 6 provides the conclusions and suggestions for further research.

2 Model Description

The HAP will respond to any incoming traffic, including video (vo), voice (ve), and data queries (d). The quality of service (QoS) requirements for video traffic is considered as both soft and hard, depending on the particular application. Video traffic has soft QoS for a few specific applications (like, video on demand), whereas it can have hard QoS for some other ones (like, live video). The video traffic is therefore divided into video 1 (vo1) and video 2 (vo2) traffic. Each inbound traffic is classified as new (n_i , $i = ve, vo1, vo2, d$) and handoff (h_i , $i = ve, vo1, vo2, d$). A continuous time *MAP* is used to represent each traffic's arrival since it enables correlation between the traffic's individual arrival times. Numerous prioritization policies have been put out in the literature for traffic prioritization. One of the most used techniques, fixed guard channel policy (GC), is a decent balance between performance and complexity. In this study, the GC policy is used to maintain connectivity and provide various types of traffic preference. According to this policy, a specific number of channels, let's say g , (g changes for each type of traffic), are set aside for each type of traffic. Some other detailed assumptions are as follows:

- **Arrival Process:** The arrival of each traffic is modeled by using *MAP* and represented by MAP_1 (ve), MAP_2 (vo1), MAP_3 (vo2) and MAP_4 (d). The arrival process for ve traffic is stated briefly. Let C^{ve} be the infinitesimal generator matrix with dimension K . The arrival rates for new and handoff ve traffic are given as $\lambda_{nve} = \pi C_n^{ve} e$ and $\lambda_{hve} = \pi C_h^{ve} e$, respectively. Here, π is the stationary probability vector of the generator C^{ve} and e is column vector with ones entries. On the similar track, the arrival processes for vo1, vo2, and d traffic can be defined.
- **Cell Residence Time:** The CRT (R_s) is *PH* distributed with representation (α, T) and dimension r .
- **Call Holding Time:** The CHT (H_i , $i = ve, vo, d$) of type i traffic has *PH* distribution with representation (β_i, S_i) and dimension h_i .
- **Service Process:** According to Neuts [12], the residual CRT and residual CHT are also *PH* distributed. Let $S^{i,N}$ be the service times of new traffic of type i , defined as $S^{i,N} = \min(H_i, \bar{R}_s)$. Since, the minimum of two *PH* distribution is also a *PH* distribution. Therefore, $S^{i,N}$ is also *PH* distributed with representation (δ_{N_i}, L_{N_i}) and dimension rh_i . Similarly, $S^{i,H}$ is *PH* distributed with representation (δ_{H_i}, L_{H_i}) and dimension rh_i .
- **Retrial Process:** The retrial process is *PH* distributed with representation (δ, L) and dimension n . The inter-retrial times or departure times denoted by $R = \min(R_t, \bar{R}_s)$, which ever happens first. The retrial process has two absorbing states either due to successful retrial or departure from the system
- **Call Admission Control Scheme:** As per the proposed CAC scheme, the incoming traffic is classified in the decreasing order of priority as $h_{ve}, n_{ve}, h_{vo1}, n_{vo1}, h_{vo2}, n_{vo2}, h_d, n_d$. It has been assumed that ve and d traffic are assigned only one channel whereas vo are assigned four channels. Therefore, h_1, h_2, h_3 and h_4 are reserved for $h_{ve}, h_{vo1}, h_{vo2}, h_d$, whereas n_1, n_2 and

n_3 channels are reserved for n_{ve} , n_{vo1} , n_{vo2} , respectively. Since, n_d is of least priority and can tolerate the delay, no channel has been reserved for it. Let M be the total number of available channels. We define the following notations in order to carry out the analysis:

$$\begin{aligned} M_1 &= M - \sum_{i=1}^3 (h_i + n_i) - h_4; & M_2 &= M - \sum_{i=1}^3 (h_i + n_i) \\ M_3 &= M - \sum_{i=1}^2 (h_i + n_i) - h_3; & M_4 &= M - \sum_{i=1}^2 (h_i + n_i) \\ M_5 &= M - h_1 - n_1 - h_2; & M_6 &= M - h_1 - n_1; & M_7 &= M - h_1. \end{aligned}$$

3 Analytical Model

Consider a HAP system with M channels and J orbit size. Let $\{X(t), t \geq 0\}$ be a stochastic process for a given system with the state space

$$\begin{aligned} \Omega = \{i, j, k, l, u_{ve}, u_{vo1}, u_{vo2}, u_d, \mathbf{S}_{i-j-k}^{ve}, \mathbf{S}_j^{vo}, \mathbf{S}_k^d, \mathbf{r}_l; & 0 \leq i \leq M, 0 \leq j \leq j_m, \\ & 0 \leq k \leq k_m, 0 \leq l \leq J, 1 \leq u_m \leq K_m, 1 \leq u_d \leq 3\}, \end{aligned}$$

where,

- i is the number of traffic receiving service,
- j is the number of vo traffic,
- k is the number of d traffic,
- l is the total number of new ve, handoff vo2, new vo2, handoff d and new d traffic in the orbit,
- $u_{ve}, u_{vo1}, u_{vo2}, u_d$ are the phases of MAPs for ve, vo1, vo2 and d traffic arrival process, respectively,
- $\mathbf{S}_{i-j-k}^{ve}, \mathbf{S}_j^{vo}, \mathbf{S}_k^d$ are the set of phase of service for ve, vo (1 and 2) and d traffic, respectively,
- $\mathbf{r}_l$ is the set of retrial phase of l traffic in the orbit,

and, $m = ve, vo1, vo2, j_m = \min\{i, \lceil \frac{M_6}{4} \rceil\}$ and $k_m = \min\{i - j, M_2\}$. Here, $[x]$ means the greatest integer function of x . The infinitesimal generator matrix Q for this LDQBD process and the blocks of the matrix Q are provided as follows.

$$Q = \begin{bmatrix} Q_{01} & Q_{00} & & & \\ \ddots & \ddots & \ddots & & \\ & Q_{i2} & Q_{i1} & Q_{i0} & \\ & & \ddots & \ddots & \ddots \\ & & & Q_{M2} & Q_{M1} \end{bmatrix}$$

$$Q_{i0} = \begin{bmatrix} A_{01}^i & A_{00}^i & & & \\ & \ddots & \ddots & \ddots & \\ & & 0 & A_{j_1}^i & A_{j_0}^i \\ & & & \ddots & \ddots & \ddots \\ & & & & 0 & A_{j_m}^i \end{bmatrix} \quad Q_{i2} = \begin{bmatrix} C_{01}^i & 0 & & & \\ & \ddots & \ddots & \ddots & \\ & & C_{j_2}^i & C_{j_1}^i & 0 \\ & & & \ddots & \ddots & \ddots \\ & & & & C_{j_m}^i & C_{j_{m-1}}^i \end{bmatrix}$$

and $Q_{i1} = \text{diag}(B_0^i, \dots, B_j^i, \dots, B_{j_m}^i)$.

Note that the notations $I_w, W_r(i), V_r(i, 1), I(w, s)$ and $\widehat{V}_r(i)$ as defined in [3] have been used to carry out the analysis. A detailed explanation for the blocks of the generator matrix is given as follows.

- **Q_{i0} block:** The blocks $A_{j_1}^i$ and $A_{j_0}^i$ represents the case when due to successful retrial or arrival of fresh traffic, the number of ve or d traffic and vo traffic goes up by one. The main diagonal elements $A_{k_1}^{i,j,1}$ represents the case, when the number of ve traffic goes up by one. Similarly, the upper diagonal elements $A_{k_0}^{i,j,1}$ shows increment in the d traffic. Each of $A_{k_1}^{i,j,1}$ and $A_{k_0}^{i,j,1}$ is a lower diagonal block matrices with main diagonal elements $A_{l,1}^{i,j,1,k,1}$ and $A_{l,1}^{i,j,1,k,0}$, respectively and lower diagonal elements as $A_{l,2}^{i,j,1,k,1}$ and $A_{l,2}^{i,j,1,k,0}$.

- $A_{l,1}^{i,j,1,k,1}$ represents the increment due to the arrival of fresh ve traffic. Similarly, $A_{l,1}^{i,j,1,k,0}$ denotes the increment in the fresh d traffic.
- $A_{l,2}^{i,j,1,k,1}$ and $A_{l,2}^{i,j,1,k,0}$ represents the increment due to the successful retrial of new ve and d traffic from the orbit.

The upper diagonal block $A_{j_0}^i$ is a diagonal matrix with $A_{k_1}^{i,j,0}$, $0 \leq k \leq k_m$ as diagonal entries. Each of $A_{k_1}^{i,j,0}$ is a lower diagonal matrix with main diagonal elements $A_{l,1}^{i,j,0,k,1}$ and lower diagonal elements $A_{l,2}^{i,j,0,k,1}$.

- $A_{l,1}^{i,j,0,k,1}$ represents the increase in the number of vo traffic due to the fresh arrivals of vo (1 and 2) traffic.
- $A_{l,2}^{i,j,0,k,1}$ shows the case when increase in the vo traffic is due to successful retrials by the vo traffic from the orbit.

The expressions of the block entries are given as follows for $0 \leq i \leq M-1$, $0 \leq j \leq j_m$, $0 \leq k \leq k_m$ and $0 \leq l \leq J$:

$$\begin{aligned} A_{l,1}^{i,j,1,k,1} &= \lambda_{VE}^{i,j} \oplus C_0^{vo1} \oplus C_0^{vo2} \oplus C_0^d \oplus W_{ve}(i-j-k) \oplus W_{vo}(j) \oplus W_d(k) \oplus W_r(l), \\ A_{l,1}^{i,j,1,k,0} &= C_0^{ve} \oplus C_0^{vo1} \oplus C_0^{vo2} \oplus \lambda_D^{i,j} \oplus W_{ve}(i-j-k) \oplus W_{vo}(j) \oplus W_d(k) \oplus W_r(l), \\ A_{l,1}^{i,j,0,k,1} &= C_0^{ve} \oplus \lambda_{VO}^{i,j} \oplus C_0^d \oplus W_{ve}(i-j-k) \oplus W_{vo}(j) \oplus W_d(k) \oplus W_r(l), \\ A_{l,2}^{i,j,1,k,1} &= C_0^{ve} \oplus C_0^{vo1} \oplus C_0^{vo2} \oplus C_0^d \oplus W_{ve}(i-j-k) \oplus W_{vo}(j) \oplus W_d(k) \oplus g^{i,j} \widehat{V}_r(l), \\ A_{l,2}^{i,j,1,k,0} &= C_0^{ve} \oplus C_0^{vo1} \oplus C_0^{vo2} \oplus C_0^d \oplus W_{ve}(i-j-k) \oplus W_{vo}(j) \oplus W_d(k) \oplus h^{i,j} \widehat{V}_r(l), \\ A_{l,2}^{i,j,0,k,1} &= C_0^{ve} \oplus C_0^{vo1} \oplus C_0^{vo2} \oplus C_0^d \oplus W_{ve}(i-j-k) \oplus W_{vo}(j) \oplus W_d(k) \oplus m^{i,j} \widehat{V}_r(l), \end{aligned}$$

where

$$\lambda_{VE}^{i,j} = \begin{cases} C_{hv}^{ve} + C_{nv}^{ve}, & 0 \leq i + 3j < M_7 \\ C_{hv}^{ve}, & M_7 \leq i + 3j < M \\ 0, & \text{otherwise} \end{cases}; \quad g^{i,j} = \begin{cases} p_{nve}, & 0 \leq i + 3j < M_7 \\ 0, & \text{otherwise} \end{cases};$$

$$\lambda_D^{i,j} = \begin{cases} C_{hd}^d + C_{nd}^d, & 0 \leq i + 3j < M_1 \\ C_{hd}^d, & M_1 \leq i + 3j < M_2 \\ 0, & \text{otherwise} \end{cases}; \quad h^{i,j} = \begin{cases} p_{hd} + p_{nd}, & 0 \leq i + 3j < M_1 \\ p_{hd}, & M_1 \leq i + 3j < M_2 \\ 0, & \text{otherwise} \end{cases};$$

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$$\lambda_{VO}^{i,j} = \begin{cases} (C_{hvo1}^{vo1} + C_{nvo1}^{vo1}) \oplus (C_{hvo2}^{vo2} + C_{nvo2}^{vo2}), & 0 \leq i + 3j + 4 < M_3 \\ (C_{hvo1}^{vo1} + C_{nvo1}^{vo1}) \oplus C_{hvo2}^{vo2}, & M_3 \leq i + 3j + 4 < M_4 \\ (C_{hvo1}^{vo1} + C_{nvo1}^{vo1}) \oplus C_0^{vo2}, & M_4 \leq i + 3j + 4 < M_5 \\ C_{hvo1}^{vo1} \oplus C_0^{vo2}, & M_5 \leq i + 3j + 4 < M_6 \\ 0, & \text{otherwise} \end{cases};$$

$$m^{i,j} = \begin{cases} p_{hvo2} + p_{nvo2}, & 0 \leq i + 3j + 4 < M_3 \\ p_{hvo2}, & M_3 \leq i + 3j + 4 < M_4 \\ 0, & \text{otherwise.} \end{cases}$$

- **Q_{i2} block:** For each $1 \leq i \leq M$, the blocks C_{j1}^i corresponds to the case of ve or d traffic completion whereas C_{j2}^i represents the traffic completion by the vo traffic. C_{j1}^i is a lower diagonal matrix with $C_{k1}^{i,j,1}$ as main diagonal and $C_{k2}^{i,j,1}$ as lower diagonal elements. Each of $C_{k1}^{i,j,1}$ and $C_{k2}^{i,j,1}$ is a diagonal matrix with diagonal elements $C_{l,1}^{i,j,1,k,1}$ and $C_{l,1}^{i,j,1,k,2}$, respectively.

- $C_{l,1}^{i,j,1,k,1}$ shows reduction in the number of ongoing ve traffic.
- $C_{l,1}^{i,j,1,k,2}$ represents the reduction in the d traffic due to service completion.

For $1 \leq i \leq M$, $0 \leq j \leq j_m$, $0 \leq k \leq k_m$, $0 \leq l \leq J$,

$$C_{l,1}^{i,j,1,k,1} = C_0^{ve} \oplus C_0^{vo1} \oplus C_0^{vo2} \oplus C_0^d \oplus V_{ve}(i - j - k) \oplus W_{vo}(j) \oplus W_d(k) \oplus W_r(l)$$

$$C_{l,1}^{i,j,1,k,2} = C_0^{ve} \oplus C_0^{vo1} \oplus C_0^{vo2} \oplus C_0^d \oplus W_{ve}(i - j - k) \oplus W_{vo}(j) \oplus V_d(k) \oplus W_r(l).$$

C_{j2}^i is a diagonal matrix with diagonal entries as $C_{k,1}^{i,j,2}$. Each of $C_{k,1}^{i,j,2}$ is a diagonal matrix with its diagonal elements $C_{l,1}^{i,j,2,k,1}$. $C_{l,1}^{i,j,2,k,1}$ represents the decrement in the number of vo traffic.

For $1 \leq i \leq M$, $1 \leq j \leq j_m$, $0 \leq k \leq k_m$ and $0 \leq l \leq J$,

$$C_{l,1}^{i,j,2,k,1} = C_0^{ve} \oplus C_0^{vo1} \oplus C_0^{vo2} \oplus C_0^d \oplus W_{ve}(i - j - k) \oplus V_{vo}(j) \oplus W_d(k) \oplus W_r(l).$$

- **Q_{i1} block:** B_j^i represents no change in the number of traffic receiving service where, $B_j^i = \text{diag}(B_0^{i,j}, \dots, B_k^{i,j}, \dots, B_{k_m}^{i,j})$. Each of $B_k^{i,j}$ is a tri-diagonal matrix.

- The main diagonal element $B_{l,1}^{i,j,k}$ exhibits no change in the traffic in service as well as no change in the number of traffic in the orbit.
For $0 \leq i \leq M$, $0 \leq j \leq j_m$, $0 \leq k \leq k_m$, and $0 \leq l \leq J$,

$$B_{l,1}^{i,j,k} = C_0^{ve} \oplus C_0^{vo1} \oplus C_0^{vo2} \oplus C_0^d \oplus W_{ve}(i-j-k) \oplus W_{vo}(j) \oplus W_d(k) \oplus W_r(l).$$

- The lower diagonal elements $B_{l,2}^{i,j,k}$ shows departure of a traffic from the orbit without service.
For $0 \leq i \leq M$, $0 \leq j \leq j_m$, $0 \leq k \leq k_m$, and $1 \leq l \leq J$,

$$B_{l,2}^{i,j,k} = C_0^{ve} \oplus C_0^{vo1} \oplus C_0^{vo2} \oplus C_0^d \oplus W_{ve}(i-j-k) \oplus W_{vo}(j) \oplus W_d(k) \oplus V_r(l, 1).$$

- The upper diagonal elements $B_{l,0}^{i,j,k}$ represents increase in the number of traffic in the orbit.
For $0 \leq i \leq M$, $0 \leq j \leq j_m$, $0 \leq k \leq k_m$, and $0 \leq l \leq J-1$,

$$B_{l,0}^{i,j,k} = C_0^{vo1} \oplus n^{i,j,k} \oplus W_{ve}(i-j-k) \oplus W_{vo}(j) \oplus W_d(k) \oplus W_r(l),$$

where

$$n^{i,j,k} = \begin{cases} C_0^{md} \oplus C_0^{ve} \oplus C_0^{vo2}, & M_1 \leq i+3j < M_2 \\ (C_{hd}^d + C_{nd}^d) \oplus C_0^{ve} \oplus C_0^{vo2}, & M_2 \leq i+3j < M_3 \\ C_{nv}^{vo2} \oplus (C_{hd}^d + C_{nd}^d) \oplus C_0^{ve}, & M_3 \leq i+3j < M_4 \\ (C_{nv}^{vo2} + C_{hv}^{vo2}) \oplus (C_{hd}^d + C_{nd}^d) \oplus C_0^{ve}, & M_4 \leq i+3j < M_7 \\ C_{nv}^{ve} \oplus (C_{nv}^{vo2} + C_{hv}^{vo2}) \oplus (C_{hd}^d + C_{nd}^d) \oplus C_0^{ve}, & M_7 \leq i+3j < M \\ 0, & \text{otherwise.} \end{cases}$$

4 Performance Measures

Let $\mathbf{x}$ be the steady state probability vector of the system. Some important performance measures are given as follows:

1. The probability that there are l traffic in the orbit is given by

$$P_r(l) = \sum_{i=0}^M \sum_{j=0}^{j_m} \sum_{k=0}^{k_m} \mathbf{x}(i, j, k, l) \mathbf{e}, \quad 0 \leq l \leq J.$$

2. The dropping probabilities of handoff ve and vo1, respectively are given as

$$P_{dv} = \sum_i \sum_j \sum_{k=0}^{k_m} \sum_{l=0}^J \mathbf{x}(i, j, k, l) \mathbf{e}, \quad 0 \leq i+3j \leq M, 0 \leq i \leq M, 0 \leq j \leq j_m,$$

$$P_{dvo1} = \sum_i \sum_j \sum_{k=0}^{k_m} \sum_{l=0}^J \mathbf{x}(i, j, k, l) \mathbf{e}, \quad M_6 \leq i+3j \leq M, 0 \leq i \leq M, 0 \leq j \leq j_m.$$

3. The blocking probabilities of new vo1 traffic is given by

$$P_{bvo1} = \sum_i \sum_j \sum_{k=0}^{k_m} \sum_{l=0}^J \mathbf{x}(i, j, k, l) \mathbf{e}, \quad M_6 \leq i+3j \leq M, 0 \leq i \leq M, 0 \leq j \leq j_m.$$

4. The probability that new ve, handoff vo2, handoff d and new d traffic, respectively has to join the orbit is given as follows

$$P_{nv} = (1 - P_r(J)) \sum_i \sum_j \sum_{k=0}^{k_m} \sum_{l=0}^J \mathbf{x}(i, j, k, l) \mathbf{e}, \quad M_7 \leq i + 3j \leq M,$$

$$P_{hvo2} = (1 - P_r(J)) \sum_i \sum_j \sum_{k=0}^{k_m} \sum_{l=0}^J \mathbf{x}(i, j, k, l) \mathbf{e}, \quad M_4 \leq i + 3j \leq M,$$

$$P_{nvo2} = (1 - P_r(J)) \sum_i \sum_j \sum_{k=0}^{k_m} \sum_{l=0}^J \mathbf{x}(i, j, k, l) \mathbf{e}, \quad M_3 \leq i + 3j \leq M,$$

$$P_{hd} = (1 - P_r(J)) \sum_i \sum_j \sum_{k=0}^{k_m} \sum_{l=0}^J \mathbf{x}(i, j, k, l) \mathbf{e}, \quad M_2 \leq i + 3j \leq M,$$

$$P_{nd} = (1 - P_r(J)) \sum_i \sum_j \sum_{k=0}^{k_m} \sum_{l=0}^J \mathbf{x}(i, j, k, l) \mathbf{e}, \quad M_1 \leq i + 3j \leq M.$$

Note that $(1 - P_r(J))$ is the probability that the orbit is not full.

5. The average value of waiting time of traffic in the retrial queue is given by

$$E(T) = L / [\lambda_{nv}(1 - P_{nv}) + \lambda_{hvo2}(1 - P_{hvo2}) + \lambda_{nvo2}(1 - P_{nvo2}) \\ + \lambda_{hd}(1 - P_{hd}) + \lambda_{nd}(1 - P_{nd})].$$

5 Particular Cases

CASE I: Poisson Arrivals, Exponential Service and Retrial Times, Four Type of Traffic [6]

Let the state space for the stochastic process $\{X(t), t \geq 0\}$ is defined as : $\Omega = \{(i, j, k, l); 0 \leq i \leq M; 0 \leq j \leq j_m; 0 \leq k \leq k_m; 0 \leq l \leq J\}$. The expression for the block matrices of the generator matrix Q for this particular case is given as follows.

- First, we give the expression of $A_{l,1}^{i,j,1,k,1}$ and $A_{l,2}^{i,j,1,k,1}$. For $0 \leq i \leq M-1$, $0 \leq j \leq j_m$, $0 \leq k \leq k_m$, $A_{l,1}^{i,j,1,k,1} = \lambda_{VE}^{i,j}$, $l = 0, 1, \dots, J$; $A_{l,2}^{i,j,1,k,1} = g^{i,j} * l * \mu_{rc}$, $l = 1, \dots, J$, where

$$\lambda_{VE}^{i,j} = \begin{cases} \lambda_{hv} + \lambda_{nv}, & 0 \leq i + 3j < M_7 \\ \lambda_{hv}, & M_7 \leq i + 3j < M; \\ 0, & \text{otherwise} \end{cases} \quad g^{i,j} = \begin{cases} p_{nv}, & 0 \leq i + 3j < M_7 \\ 0, & \text{otherwise.} \end{cases}$$

- For $0 \leq i \leq M-1$, $0 \leq j \leq j_m$, $0 \leq k \leq k_m-1$, $A_{l,1}^{i,j,1,k,0} = \lambda_D^{i,j}$, $l = 0, 1, \dots, J$; $A_{l,2}^{i,j,1,k,0} = h^{i,j} * l * \mu_{rc}$, $l = 1, 2, \dots, J$ where,

$$\lambda_D^{i,j} = \begin{cases} \lambda_{hd} + \lambda_{nd}, & 0 \leq i + 3j < M_1 \\ \lambda_{hd}, & M_1 \leq i + 3j < M_2; \\ 0, & \text{otherwise,} \end{cases} \quad h^{i,j} = \begin{cases} p_{hd} + p_{nd}, & 0 \leq i + 3j < M_1 \\ p_{hd}, & M_1 \leq i + 3j < M_2 \\ 0, & \text{otherwise.} \end{cases}$$

- For $0 \leq i \leq M-1$, $0 \leq j \leq j_m-1$, $0 \leq k \leq k_m$, the expressions for $A_{l,1}^{i,j,0,k,1}$ and $A_{l,2}^{i,j,0,k,1}$: $A_{l,1}^{i,j,0,k,1} = \lambda_{VO}^{i,j}$, $l = 0, 1, \dots, J$; $A_{l,2}^{i,j,0,k,1} = m^{i,j} * l * \mu_{rc}$, $l = 1, \dots, J$, where

$$\lambda_{VO}^{i,j} = \begin{cases} \lambda_{hvo1} + \lambda_{nvo1} + \lambda_{hvo2} + \lambda_{nvo2}, & 0 \leq i + 3j + 4 < M_3 \\ \lambda_{hvo1} + \lambda_{nvo1} + \lambda_{hvo2}, & M_3 \leq i + 3j + 4 < M_4 \\ \lambda_{hvo1} + \lambda_{nvo1}, & M_4 \leq i + 3j + 4 < M_5 \\ \lambda_{hvo1}, & M_5 \leq i + 3j + 4 < M_6 \\ 0, & \text{otherwise} \end{cases}$$

$$m^{i,j} = \begin{cases} p_{hvo2} + p_{nvo2}, & 0 \leq i + 3j + 4 < M_3 \\ p_{hvo2}, & M_3 \leq i + 3j + 4 < M_4 \\ 0, & \text{otherwise.} \end{cases}$$

- For $1 \leq i \leq M$, $0 \leq j \leq j_m$, $0 \leq k \leq k_m$ and $0 \leq l \leq J$, $C_{l,1}^{i,j,1,k,1} = (i - j - k) * \mu_{ve}$; $C_{l,1}^{i,j,2,k,1} = j * \mu_{vo}$; $C_{l,1}^{i,j,1,k,2} = k * \mu_d$. Finally, we consider the blocks of Q_{i1} . For $0 \leq i \leq M$, $0 \leq j \leq j_m$, $0 \leq k \leq k_m$ and $0 \leq l \leq J$,

$$B_{l,1}^{i,j,k} = -(\lambda_{VE}^{i,j} + \lambda_{VO}^{i,j} + \lambda_D^{i,j}) - (i - j - k) * \mu_{ve} - j * \mu_{vo} - k * \mu_d - l * \mu_{rq} - l * \mu_{rc}$$

$$B_{l,0}^{i,j,k} = n^{i,j,k}; \quad B_{l,2}^{i,j,k} = l * \mu_{rq},$$

where

$$n^{i,j,k} = \begin{cases} \lambda_{nd}, & M_1 \leq i + 3j < M_2 \\ \lambda_{hd} + \lambda_{nd}, & M_2 \leq i + 3j < M_3 \\ \lambda_{nvo2} + \lambda_{hd} + \lambda_{nd}, & M_3 \leq i + 3j < M_4 \\ \lambda_{hvo2} + \lambda_{nvo2} + \lambda_{hd} + \lambda_{nd}, & M_4 \leq i + 3j < M_7 \\ \lambda_{nv} + \lambda_{hvo2} + \lambda_{nvo2} + \lambda_{hd} + \lambda_{nd}, & M_7 \leq i + 3j < M \\ 0, & \text{otherwise.} \end{cases}$$

CASE II: MAP Arrivals, Exponential Service and Retrial Times, One Type of Traffic [5]

Let the state space of the underlying stochastic process is given as:

$$\{(i, l, u), 0 \leq i \leq M, 0 \leq l \leq J, 1 \leq u \leq K\}.$$

The block matrix entries are given as follows.

- Q_{i0} is a lower diagonal matrix with A_{l1}^i as main diagonal and A_{l2}^i as lower diagonal entries. The matrices A_{l1}^i and A_{l2}^i corresponds to the increment in the number of traffic in service due to fresh arrivals of traffic and due to successful retrials of traffic, respectively. The expressions are given as:

$$A_{l1}^i = C_N + C_H, \quad 0 \leq l \leq J; \quad A_{l2}^i = l\mu_{rc}I, \quad 1 \leq l \leq J.$$

- Consider the block Q_{i2} . It is a diagonal matrix with diagonal elements C_{l1}^i . The expression for C_{l1}^i is given as $C_{l1}^i = i\mu I$.

- Finally, consider the blocks Q_{i1} . It is a tri-diagonal matrix with main diagonal elements B_{l1}^i , as

$$B_{l1}^i = \begin{cases} C_0 - (i\mu + l\mu_{rq} + l\mu_{rc})I, & i = 0, 1, \dots, M-1, \quad l = 0, 1, \dots, J, \\ C_0 + C_H - (i\mu + l\mu_{rq})I, & i = M, \quad l = 0, 1, \dots, J-1, \\ -(i\mu + l\mu_{rq})I, & i = M, \quad l = J. \end{cases}$$

Similarly, B_{l0}^i represents the upper diagonal given as

$$B_{l0}^i = \begin{cases} \mathbf{0} & 0 \leq i \leq M-1, \quad 0 \leq l \leq J-1, \\ C_N, & i = M, \quad 0 \leq l \leq J-1. \end{cases}$$

Lastly, B_{l2}^i is given by : $B_{l2}^i = l\mu_{rq}$; $i = 0, 1, \dots, M, \quad l = 1, 2, \dots, J$.

CASE III: Poisson Arrivals, Phase Type Service and Exponential Retrial Times, One Type of Traffic [2]

Let the state space for the stochastic process $\{X(t), t \geq 0\}$ is given with the state space $\{(i, l, \mathbf{S}_i), 0 \leq i \leq M, 0 \leq l \leq J\}$.

- The expressions of A_{l1}^i and A_{l2}^i , for $0 \leq i \leq M-1$, are $A_{l1}^i = \lambda I, \quad 0 \leq l \leq J, \quad A_{l2}^i = l\mu_{rc}I, \quad 1 \leq l \leq J$.
- C_{l1}^i represents decrement in the ongoing traffic due to service completion by one of the $i, 1 \leq i \leq M$ traffic and is given by $V(i)$, for $l = 0, 1, \dots, J$.
- B_{l1}^i represents no service completion whereas B_{l0}^i represents the increment in the number of traffic in orbit. The decrement in the number of traffic in orbit is possible when the traffic leaves the orbit and is given by B_{l2}^i . The expressions are given below.

$$B_{l1}^i = \begin{cases} -(\lambda + l\mu_{rq} + l\mu_{rc})I, & i = 0, \quad 0 \leq l \leq J \\ W(i) - (\lambda + l\mu_{rq} + l\mu_{rc})I, & 1 \leq i \leq M-1, \quad 0 \leq l \leq J \\ W(i) - (\lambda - l\mu_{rq})I, & i = M, \quad 0 \leq l \leq J-1. \\ W(i) - l\mu_{rq}I, & i = M, \quad l = J \end{cases}$$

$$B_{l0}^i = \begin{cases} \mathbf{0}, & 0 \leq i \leq M-1; \\ \lambda I, & i = M \end{cases}; \quad B_{l2}^i = l\mu_{rq}, \quad 0 \leq i \leq M, \quad 1 \leq l \leq J.$$

CASE IV: MAP Arrivals, Exponential Service, No Retrials, One Type of Traffic [8]

Let the state space for the stochastic process $\{X(t), t \geq 0\}$ is given with the state space $\{(i, u), 0 \leq i \leq M, 1 \leq u \leq K\}$.

- The lower diagonal elements Q_{i2} represent the decrement in the number of ongoing traffic. For $i = 1, 2, \dots, M, \quad Q_{i2} = i\mu I$.
- The upper diagonal element Q_{i0} represent increment in the number of ongoing traffic due to handoff or new ve traffic arrival as

$$Q_{i0} = \begin{cases} C_H + C_N, & 0 \leq i < M-g \\ C_H, & M-g \leq i < M \\ 0, & \text{otherwise.} \end{cases}$$

- The main diagonal elements Q_{i1} corresponds to no change in the number of ongoing traffic given as follows.

$$Q_{i1} = \begin{cases} C_0 - i\mu I, & 0 \leq i < M - g \\ C_0 + C_N - i\mu I, & M - g \leq i < M \\ -i\mu I, & i = M. \end{cases}$$

CASE V: Poisson Arrivals, Exponential Service, No Retrials, One Type of Traffic [9]

Let $\{X(t), t \geq 0\}$ be the stochastic process, where $X(t)$ represents the number of ongoing traffic at time t .

- The lower diagonal elements Q_{i2} represent the decrement in the number of ongoing traffic. For $i = 1, 2, \dots, M$, $Q_{i2} = i\mu$.
- The upper diagonal element Q_{i0} represent increment in the number of ongoing traffic due to handoff or new ve traffic arrival as

$$Q_{i0} = \begin{cases} \lambda_N + \lambda_H, & 0 \leq i < M - g \\ \lambda_H, & M - g \leq i < M \\ 0, & \text{otherwise.} \end{cases}$$

- The main diagonal elements Q_{i1} corresponds to no change in the number of ongoing traffic given as follows.

$$Q_{i1} = \begin{cases} -\lambda_N - \lambda_H - i\mu, & 0 \leq i < M - g \\ -\lambda_H - i\mu, & M - g \leq i < M \\ -i\mu, & i = M. \end{cases}$$

CASE VI: Poisson Arrivals, Exponential Service, Exponential Retrials, One Type of Traffic [10]

Let $\{X(t), t \geq 0\}$ be the stochastic process with a state space $\{(j, k) : 0 \leq j \leq M; 0 \leq k \leq J\}$.

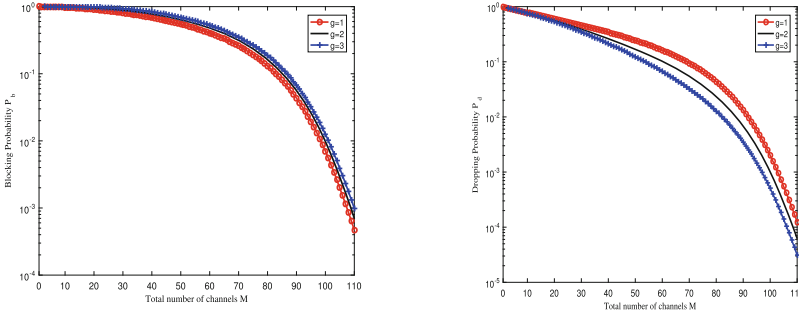
- The elements of the upper diagonal Q_{i0} are represented by

$$A_{i2} = jp\mu_r, 0 \leq j \leq J; A_{i1} = \begin{cases} \lambda = \lambda_H + \lambda_N, & 0 \leq l \leq M - g - 1, \\ \lambda_H, & M - g \leq l \leq M - 1. \end{cases}$$

- The elements of the lower diagonal are given by $Q_{i2} = i\mu$.
- The elements of the main diagonal, denoted as Q_{i1} represented as $B_{i2} = k(1 - p)\mu_r, 0 \leq k \leq J; B_{i1} = -(\lambda + i\mu + k\mu_r), 0 \leq k \leq J;$

$$B_{i0} = \begin{cases} 0, & 0 \leq l \leq M - g - 1, \\ \lambda_N, & M - g \leq l \leq M. \end{cases}$$

After performing the numerical illustration for $g = 1, 2, 3$ and $J = 0$, Fig. 1(a) and Fig. 1(b) plot loss probabilities (the blocking probability P_b and the dropping probability P_d) with respect to M . We observe by these figures that for no retrial, i.e., for $J = 0$, P_b and P_d decreases with M for a fixed value of g , P_b increases with g for a fixed value of M , and P_d decreases with g for a fixed value of M .



(a) Dependence of P_b over M for $J = 0$. (b) Dependence of P_d over M for $J = 0$.

Fig. 1. Dependence of loss probabilities over M for $J = 0$.

6 Conclusions and Future Directions

In this paper, we consider a ve, vo and d traffic supported tethered HAP systems with *MAP* and *PH* distributions for arrival, service and retrial processes. A very well known GC scheme has been implemented for the prioritization of the higher priority traffic. The proposed study has been modeled by a *LDQBD* process. An explicit expression for the generator matrix Q has been obtained. Further, formulae of some important performance measures have been derived. The suggested model is a generalization of various queueing models, a few of these are included here as special cases. For the sake of numerical illustration, a few results have been shown for the special case queueing model.

In the future, a CAC scheme with dynamic resource reservations for ve, vo and d traffic will be proposed and an extensive numerical illustration will be presented with efficient algorithms for the computation. For the reported model, resource allocation and energy optimization problems may be constructed.

References

1. Ahmed, M.H.: Call admission control in wireless networks: a comprehensive survey. *IEEE Commun. Surv. Tutorials* **7**(1–4), 50–69 (2005)
2. Alfa, A.S., Isotupa, K.S.: An M/PH/k retrial queue with finite number of sources. *Comput. Oper. Res.* **31**(9), 1455–1464 (2004)
3. Alfa, A.S., Li, W.: A homogeneous pcs network with Markov call arrival process and phase type cell residence time. *Wireless Netw.* **8**(6), 597–605 (2002). <https://doi.org/10.1023/A:1020329719692>
4. Alfa, A.S., Li, W.: PCS networks with correlated arrival process and retrial phenomenon. *IEEE Trans. Wireless Commun.* **1**(4), 630–637 (2002)
5. Artalejo, J.R., Chakravarthy, S.R.: Computational analysis of the maximal queue length in the MAP/M/c retrial queue. *Appl. Math. Comput.* **183**(2), 1399–1409 (2006)

6. Dharmaraja, S., Jindal, V., Alfa, A.S.: Phase-type models for cellular networks supporting voice, video and data traffic. *Math. Comput. Model.* **47**(11–12), 1167–1180 (2008)
7. Dudin, A.N., Klimenok, V.I., Vishnevsky, V.M.: *The Theory of Queuing Systems with Correlated Flows*. Springer, Cham (2020). <https://doi.org/10.1007/978-3-030-32072-0.pdf>
8. Gaver, D., Jacobs, P., Latouche, G.: Finite birth-and-death models in randomly changing environments. *Adv. Appl. Probab.* **16**(4), 715–731 (1984)
9. Harine, G., Marie, R., Puigjaner, R., Trivedi, K.: Loss formulas and their application to optimization for cellular networks. *IEEE Trans. Veh. Technol.* **50**(3), 664–673 (2001)
10. Jain, V., Raj, R., Dharmaraja, S.: Numerical optimization of loss system with retrial phenomenon in cellular networks. *Int. J. Oper. Res.* (2022). In Press
11. Kurt, G.K., et al.: A vision and framework for the high altitude platform station (haps) networks of the future. *IEEE Commun. Surv. Tutorials* **23**(2), 729–779 (2021)
12. Neuts, M.F.: *Matrix-Geometric Solutions in Stochastic Models: An Algorithmic Approach*. Courier Corporation, North Chelmsford (1994)
13. Slalmi, A., Chaibi, H., Saadane, R., Chehri, A., Jeon, G.: 5G NB-IoT: efficient network call admission control in cellular networks. *Concurr. Comput. Pract. Exp.* **33**(22), e6047 (2021)
14. Tozer, T., Grace, D.: High-altitude platforms for wireless communications. *Electron. Commun. Eng. J.* **13**(3), 127–137 (2001)