



Analysis of Power Management in a Tethered High Altitude Platform Using *MAP/PH*[3]/1 Retrial Queueing Model

Vidyottama Jain^{1(✉)}, V. M. Vishnevsky², Dharmaraja Selvamuthu³,
and Raina Raj³

¹ Central University of Rajasthan, Ajmer, India

vidyottama.jain@curaj.ac.in

² Institute of Control Sciences of Russian Academy of Sciences, Moscow, Russia

vishn@inbox.ru

³ Indian Institute of Technology Delhi, New Delhi, India

dharmar@maths.iitd.ac.in, rainaraj.curaj@gmail.com

Abstract. Due to the increasing demand of the tethered high altitude platform (HAP) systems, it is imperative to assess their power consumption along with their deployment. This study considers the power management of lithium-ion batteries based tethered HAP systems for wireless communications service provisioning. This article discusses a novel model based on a multi-dimensional Markov process applied for the evaluation of the power consumption characteristics of a tethered HAP system. The proposed model takes into account the increment in the load on the functioning of system after the consumption of power in batteries. The underlying study introduces functioning of the system in three modes along with the retrial phenomenon. The arrival of packets follows Markovian arrival process (*MAP*) and the service time is phase-type (*PH*) distributed with distinct parameters in three different modes. The stationary distributions and stability of the system have been derived using the matrix-geometric method. Further, by deriving key system performance measures, numerical examples have been illustrated.

Keywords: High altitude platform systems · Degraded service rate · Markovian arrival process · Phase type distribution · Power management

1 Introduction

For more than two decades, high expectations have been associated with the deployment of satellites or high altitude platform (HAP) for ubiquitous, global

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broadband communications availability [14]. These HAP systems can be classified as tethered HAP or untethered HAP systems [11]. The untethered HAP systems are established at the height of 20 to 50 km whereas the tethered HAP systems are positioned 200–400 metres above land. One of the key obstacles of using HAP to enhance the existing wireless networks is the restricted on-board energy and flight time [2, 7, 12]. According to the Alam et al. [1], batteries in an untethered HAP can last only for less than an hour. The hovering period is further lowered when the payload, communication, and signal processing use energy. As a result, a tethered HAP system is often used, with power supplied by lithium-ion batteries, solar panels, ground-based energy sources with energy transmission via a thin cable-rope, etc. Over the time the energy density of a new light weight lithium-ion battery is predicted to rise by 3% every year [1].

There exist a number of studies describing and illustrating the reliability of the system through queueing system but the literature over the power and energy management considering the queueing system appears to be rather moderate. In the literature, a few studies have constructed queueing models for the tethered HAP system (refer, [6, 9, 13]). However, the applicability of these models has been diminished in the present scenario, due to their consideration of Poisson process for arrival and exponential distribution for service times. In the cutting edge wireless technologies, the input flow of calls possesses burstiness and correlation properties rather than the memory-less property of stationary Poisson flow [5]. The flow of incoming traffic can be correlated, homogeneous, heterogeneous, bursty, etc. In the case of homogeneous traffic (when traffic is of the same type), an apt mathematical model for the arrival process is known as Markovian arrival process (*MAP*) which is a generalization of the stationary Poisson process.

Phase-type (*PH*) distribution forms a versatile family of probability distributions where a process is completed in multiple phases. The exponential, Erlang and hyper-exponential distributions belong to this family. In the HAP system, the service of an arriving packet can be blocked due to the unavailability of the server or insufficient space in the waiting queue. Hence, the blocked packet waits for some time before receiving the service and retry for its service. This phenomenon is referred as retrial phenomenon and is ubiquitous in many real life applications such as traffic centers, communication systems, optical networks, inventory systems, and so on. Therefore, the consideration of retrial phenomenon is essential while evaluating the performance of the system.

The proposed study considers a retrial queueing model for the power management of the tethered HAP systems. It has been assumed that the proposed system works in three different modes as per the availability of the power status of batteries: normal mode, power saving mode, and ultra power saving mode. To this end, two threshold parameters have been defined, say K_1 and K_2 . There can be three possible cases:

- If the power level of the batteries is above K_1 , the system will provide service in the normal mode.
- If the power level of the batteries is between K_1 and K_2 , the system will provide service in the power saving mode with a degraded rate.

- If the power level of batteries is below K_2 , the system will provide service in the ultra power saving mode with a degraded rate.

The main purpose behind considering these different modes is that the system will keep on providing service to the incoming packets at the same time the power will be saved as the system will provide service with a degraded rate. This factor can be extremely useful from both customer's point of view as well as from the service provider's point of view. In this study, the concept of retrial phenomenon has also been added for those packets which find the queue full. These arriving packets will join the orbit of infinite capacity to retry for their service after some random amount of time or it might exit the system without obtaining the service.

This paper has the following structure. In Sect. 2, the queueing model for the power management in a tethered HAP system is thoroughly described. Section 3 demonstrates the construction of an infinitesimal generator matrix for the underlying process and provides the steady-state analysis of the system. Expressions for a few essential performance measures to assess the system's efficiency are formulated in Sect. 4. Section 5 presents numerical examples to highlight the qualitative behaviour of the proposed queueing model. Finally, a few concluding remarks and insights for further research are presented in Sect. 6.

2 Model Description

The proposed study investigates a queueing model for a tethered HAP system. The HAP will provide service to all the arriving requests which will be in the packet form. In this work, the HAP will be addressed as a server in the context of providing the services to the arriving packets. The proposed system works in three different modes on the basis of power level of batteries. To this end, two threshold levels have been introduced. If the power status of the batteries in the system is above a pre-defined threshold value, say K_1 , the system will work in the normal mode. When the power status is in between K_1 and K_2 ($K_1 > K_2$), the system goes in power saving mode. If the power status is below the threshold value K_2 , it is considered crucial scenario, therefore, the system operates in ultra power saving mode. In the normal mode, power saving mode, and ultra power saving mode, the server will provide the service with rates, say μ_1 , μ_2 , and μ_3 , respectively. Here, it has been considered that as the system changes its mode of providing the service, the service rate also degrades, i.e., $\mu_1 > \mu_2 > \mu_3$.

The arriving packets, which find the queue full, will join a virtual space named orbit of infinite capacity. From the orbit, the packet will retry for its service following the exponential distribution with rate θ and probability p or it might exit the system also with complementary probability $1 - p$. The batteries will be discharged with a rate, say D_b per sec, following the exponential distribution and the discharged batteries will be recharged with a rate R_b per sec, following the exponential distribution. Figure 1 represents the pictorial representation of the proposed system.

– Arrival Process:

Arrival of packets follows a continuous time *MAP* with dimension M . Let D be the irreducible infinitesimal generator of this *MAP* where $D = D_0 + D_1$. Let π be the unique solution of $\pi D = O$ and $\pi e = 1$, where O is a zero row vector of appropriate size and e is a unit column vector. The arrival intensity of the *MAP* is defined as $\lambda = \pi D_1 e$.

– Service Process:

The service times in these three different modes of the system follow *PH* distribution with different parameters. In the normal mode, the service time follows *PH* distribution with parameters (δ_1, L_1) and dimension M_1 , i.e., $L_1 e + L_1^0 = 0$. In the power saving mode, the service time follows *PH* distribution with parameters (δ_2, L_2) and dimension M_2 , i.e., $L_2 e + L_2^0 = 0$. In the ultra power saving mode, the service time follows *PH* distribution with parameters (δ_3, L_3) and dimension M_3 , i.e., $L_3 e + L_3^0 = 0$. Note that notations $\oplus$ and $\otimes$ are used for the Kronecker sum and the Kronecker product of two matrices, respectively. For more description over Kronecker sum and Kronecker product, authors suggest readers to refer [3].

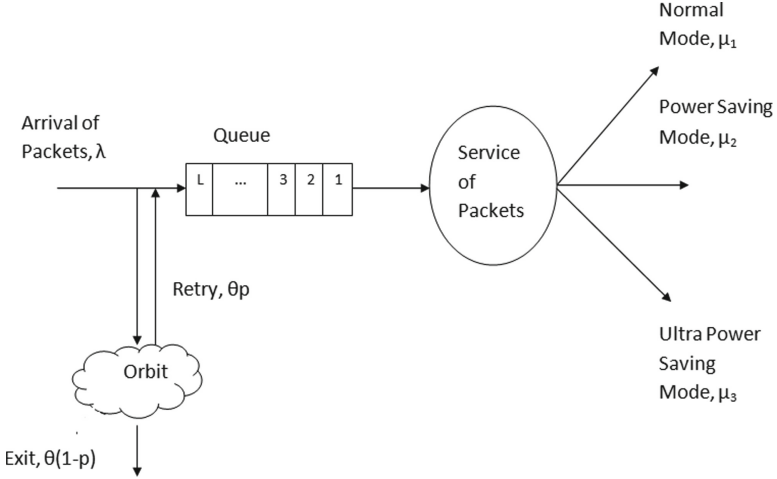


Fig. 1. A schematic diagram of the proposed model.

3 Stochastic Modeling

The state space of the underlying process $\{X(t), t \geq 0\}$ is defined as follows.

$$\Omega = \{(i, j, k, s_1, s_2, s_3, v); i \geq 0, 0 \leq j \leq L, k = 0, 1, 2, 1 \leq s_1 \leq M_1, \\ 1 \leq s_2 \leq M_2, 1 \leq s_3 \leq M_3, 1 \leq v \leq M\},$$

where,

- i denotes number of packets in the orbit,
- j denotes number of packets in the queue,
- k denotes the mode of the system,
 - $k = 0$ denotes the normal mode of the system,
 - $k = 1$ denotes the power saving mode of the system,
 - $k = 2$ denotes the ultra power saving mode of the system,
- s_1 is the number of phases in normal mode for PH distribution,
- s_2 is the number of phases in power saving mode for PH distribution,
- s_3 is the number of phases in ultra power saving mode for PH distribution,
- v is the number of phases for MAP .

The generator matrix of the stochastic process $\{X(t), t \geq 0\}$ is given as follows:

$$Q = \begin{pmatrix} Q_{0,0} & Q_{0,1} & 0 & 0 & 0 & \cdots \\ Q_{1,0} & Q_{1,1} & Q_{1,2} & 0 & 0 & \cdots \\ 0 & Q_{2,1} & Q_{2,2} & Q_{2,3} & 0 & \cdots \\ 0 & 0 & Q_{3,2} & Q_{3,3} & Q_{3,4} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots \end{pmatrix}.$$

We define the following notations in order to carry out the mathematical analysis.

- I_u is an identity matrix of dimension u .
- $\mathbf{0}$ is a zero matrix of appropriate dimension.

Upper Diagonal:

$$Q_{i,i+1} = \begin{pmatrix} C_0 & 0 & 0 & \cdots & 0 \\ 0 & C_1 & 0 & \cdots & 0 \\ 0 & 0 & C_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & C_L \end{pmatrix}; i \geq 0,$$

$$C_j = \begin{cases} \mathbf{0}; 0 \leq j \leq L-1, \\ \begin{pmatrix} D_1 \otimes \delta_1 & 0 & 0 \\ 0 & D_1 \otimes \delta_2 & 0 \\ 0 & 0 & D_1 \otimes \delta_3 \end{pmatrix}; j = L. \end{cases}$$

Lower Diagonal:

$$Q_{i+1,i} = \begin{pmatrix} A_0 \hat{A}_0 & 0 & \cdots & 0 & 0 \\ 0 & A_1 \hat{A}_1 & \cdots & 0 & 0 \\ 0 & 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & A_{L-1} \hat{A}_{L-1} \\ 0 & 0 & 0 & \cdots & 0 & A_L \end{pmatrix}; i \geq 0,$$

$$A_j = \begin{pmatrix} (i+1)\theta(1-p)I & 0 & 0 \\ 0 & (i+1)\theta(1-p)I & 0 \\ 0 & 0 & (i+1)\theta(1-p)I \end{pmatrix}; 0 \leq j \leq L,$$

$$\hat{A}_j = \begin{pmatrix} (i+1)\theta pI & 0 & 0 \\ 0 & (i+1)\theta pI & 0 \\ 0 & 0 & (i+1)\theta pI \end{pmatrix}; 0 \leq j \leq L.$$

Main Diagonal:

$$Q_{i,i} = \begin{pmatrix} B_0 & \hat{B}_0 & 0 & \cdots & 0 & 0 \\ \bar{B}_1 & B_1 & \hat{B}_1 & \cdots & 0 & 0 \\ 0 & \bar{B}_2 & B_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & B_{L-1} & \hat{B}_{L-1} \\ 0 & 0 & 0 & \cdots & \bar{B}_L & B_L \end{pmatrix}; i \geq 0,$$

$$\bar{B}_j = \begin{pmatrix} I_M \otimes L_1^0 & 0 & 0 \\ 0 & I_M \otimes L_2^0 & 0 \\ 0 & 0 & I_M \otimes L_3^0 \end{pmatrix}; 1 \leq j \leq L,$$

$$\hat{B}_j = \begin{pmatrix} D_1 \otimes \delta_1 & 0 & 0 \\ 0 & D_1 \otimes \delta_2 & 0 \\ 0 & 0 & D_1 \otimes \delta_3 \end{pmatrix}; 0 \leq j \leq L-1,$$

$$B_j = \begin{cases} \begin{pmatrix} D_0 - (i\theta + D_b)I & D_b I & 0 \\ R_b I & D_0 - (i\theta + D_b + R_b)I & D_b I \\ 0 & R_b I & D_0 - (i\theta + R_b)I \end{pmatrix}; j = 0, \\ \begin{pmatrix} D_0 - (i\theta + D_b)I \oplus L_1 & D_b I & 0 \\ R_b I & D_0 - (i\theta + D_b + R_b) \oplus L_2 & D_b I \\ 0 & R_b I & D_0 - (i\theta + R_b)I \oplus L_3 \end{pmatrix}; \\ 1 \leq j \leq L. \end{cases}$$

3.1 Steady-State Analysis

Let $z_s = \{z_s(0), z_s(1), z_s(2), \dots\}$ be the steady-state probability vector of generator matrix Q satisfying

$$z_s Q = 0; z_s e = 1,$$

where,

$$\begin{aligned} z_s(0) &= (z_s(0,0), z_s(0,1), z_s(0,2), z_s(0,3), \dots, z_s(0,L)), \\ z_s(1) &= (z_s(1,0), z_s(1,1), z_s(1,2), z_s(1,3), \dots, z_s(1,L)), \\ &\vdots \\ z_s(i) &= (z_s(i,0), z_s(i,1), z_s(i,2), z_s(i,3), \dots, z_s(i,L)), \\ z_s(i,j) &= (z_s(i,j,0), z_s(i,j,1), z_s(i,j,2)), i \geq 0, 0 \leq j \leq L. \end{aligned}$$

Due to the consideration of PH distributions, it is tedious task to obtain the closed form expression of z_s . Therefore, the methodology provided by Neuts-Rao [10] has been adopted to solve the system. Using the theory of Neuts-Rao truncation method, obtain N , a positive integer such that $Q_{i,i+1} = Q_2$, $Q_{i,i} = Q_1$, and $Q_{i+1,i} = Q_0$. Therefore, the modified generator matrix Q^* will be as follows

$$Q^* = \begin{pmatrix} Q_{0,0} & Q_{0,1} & 0 & 0 & 0 & 0 \\ Q_{1,0} & Q_{1,1} & Q_{1,2} & 0 & 0 & 0 \\ 0 & Q_{2,1} & Q_{2,2} & Q_{2,3} & 0 & 0 \\ 0 & 0 & Q_{3,2} & Q_{3,3} & Q_{3,4} & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots \\ \vdots & \vdots & \vdots & & \ddots & \ddots & \ddots \\ \vdots & \vdots & \vdots & & Q_0 & Q_1 & Q_2 \\ \vdots & \vdots & \vdots & & Q_0 & Q_1 & Q_2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \end{pmatrix}.$$

Let z'_s be the steady-state vector of $H = Q_0 + Q_1 + Q_2$. Clearly, the generator matrix H with finite states is irreducible, aperiodic and positive recurrent. In this case, z'_s is the unique solution to the following system of linear equations $z'_s H = 0$; $z'_s e = 1$, where $z'_s = (z'_s(0), z'_s(1), \dots, z'_s(L))$ and e is a column vector of ones with the appropriate dimension. The following result provides a necessary and sufficient condition under which the system is stable.

Theorem 1. *The underlying queueing system is stable if and only if*

$$\begin{aligned} & z'_s(L, 0)(D_1 \otimes \delta_1)e + z'_s(L, 1)(D_1 \otimes \delta_2)e + z'_s(L, 2)(D_1 \otimes \delta_3)e \\ & < [N\theta(1-p) \sum_{j=0}^{L-1} z'_s(j)e + N\theta p \sum_{j=0}^L z'_s(j)e]. \end{aligned}$$

From the [10], the proof of the theorem follows immediately on noting

$$z'_s Q_0 e < z'_s Q_2 e.$$

When the system is considered without retrial phenomenon, the proposed system maps to a classical queueing system and the necessary and sufficient stability condition becomes λ/μ . To compute the steady-state probability vector, the methodology of matrix-geometric method has been applied. For the detailed study over matrix-geometric method, readers are suggested to refer [8, 10].

4 Performance Measures

In this section, the important system performance measures along with their expressions for the proposed model are listed as follows.

1. The expected number of packets in queue:

$$E[Q] = \sum_{i=0}^{\infty} \sum_{j=0}^L \sum_{k=0}^2 j z_s(i, j, k) e.$$

2. The expected number of packets in orbit:

$$E[L] = \sum_{i=0}^{\infty} \sum_{j=0}^L \sum_{k=0}^2 i z_s(i, j, k) e.$$

3. The probability that the server is idle:

$$P_{idle} = \sum_{k=0}^2 z_s(0, 0, k)e.$$

4. The probability for the server being in normal mode:

$$P_{normal} = \sum_{i=0}^{\infty} \sum_{j=0}^L z_s(i, j, 0).$$

5. The probability for the server being in power saving mode:

$$P_{power} = \sum_{i=0}^{\infty} \sum_{j=0}^L z_s(i, j, 1).$$

6. The probability for the server being in ultra power saving mode:

$$P_{ultra} = \sum_{i=0}^{\infty} \sum_{j=0}^L z_s(i, j, 2).$$

7. The overall rate of retrial:

$$\theta^* = \sum_{i=0}^{\infty} \sum_{j=0}^L \sum_{k=0}^2 \theta i z_s(i, j, k).$$

5 Numerical Illustration

In this section, illustration of the key performance measures has been discussed that bring out the qualitative aspects of the model under study. For the numerical computation, the representative matrices for the *MAP* are referred from [4] as follows,

$$D_0 = \begin{pmatrix} -0.81 & 0 \\ 0 & -0.02 \end{pmatrix}, \quad D_1 = \begin{pmatrix} 0.8054 & 0.0053 \\ 0.01145 & 0.0116 \end{pmatrix}.$$

The correlation coefficients and variation coefficients are $C_r = 0.2$ and $C_v = 12.34$. Let *PH* distributions parameters for the service rates of normal mode, power saving mode and ultra power saving mode are

$$\begin{aligned} \delta_1 &= (0.05, 0.95), \quad L_1 = \begin{pmatrix} -0.03104 & 0 \\ 0 & -2.441 \end{pmatrix}, \\ \delta_2 &= (0.1, 0.9), \quad L_2 = \begin{pmatrix} -0.03359 & 0 \\ 0 & -2.5262 \end{pmatrix}, \\ \delta_3 &= (0.5, 0.5), \quad L_3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}. \end{aligned}$$

To obtain the numerical results, we have set $\theta = 2/s$, $L = 30$, $D_b = 3/s$, $R_b = 3/s$, $p = 0.5$. To demonstrate the feasibility of the developed model, some interesting observations of the proposed system are described through the following numerical experiments. These experiments will present the behaviour of performance measures with respect to arrival, service and retrial rates.

Example 1: Here, the behaviour of the expected number of packets in the system and the idle probability is investigated with respect to arrival rate λ and service rate μ .

In Fig. 2(a), the total number of packets in the system, $E[Q]$ appears to decrease with the increment in service rate μ . It should be noted that $E[Q]$ increases with respect to the increment in the value of arrival rate of packets λ . This finding can be understood as follows. When the arrival rate of packets increases in the system, it will also increase the total number of packets in the system. If the system provides service with an increasing rate, the packets will be served with an increasing rate, and consequently, $E[Q]$ decreases.

Figure 2(b) exhibits the impact of the probability of server being in idle (P_{idle}) situation with respect to λ and μ . From the graph, it can be observed that if the arrival of packets is increased in the system, the possibility of the server being idle decreases. Hence, P_{idle} decreases with respect to the increment in λ . Also, a reverse observation can be seen when the service rate is increased in the system. If the service rate increases in the system for a fixed value of λ , the chances of the server being in idle situation also increases. Therefore, P_{idle} appears to be an increasing function of μ .

Example 2: Here, the behaviour of the expected number of packets in the orbit ($E[L]$) and the overall retrial rate (θ^*) is analyzed with respect to λ , μ , and retrial rate θ .

Figure 3(a) demonstrates that $E[L]$ is an increasing function of λ and decreasing function of μ . The explanation of this finding is very obvious and can be given as follows. If λ increases in the system, there will be more arrival of packets, and these packets which are not able to join the queue will enter to the orbit to retry later on. Therefore, it will increase the number of packets in the orbit. If the service rate is increased, the probability of these packets getting the service also increases, hence, $E[L]$ decreases in the system.

In the Fig. 3(b), the overall rate of retrial θ^* appears to be an increasing function of λ and increasing function of retrial rate θ . If the arrival of packets is increased in the system, more packets will be in the orbit and therefore, the overall retrial rate also increases. Similarly, if the retrial rate of these packets increases, the overall retrial rate of packets will also increase in the system.

Example 3: In this example, the probability of the server being in three different modes have been analyzed with respect to the discharging rate D_b and recharging rate R_b of the batteries.

Figure 4(a) shows the behaviour of the system being in normal mode P_{normal} with respect to discharging rate D_b and recharging rate R_b of batteries. It can be observed that P_{normal} exhibits a decreasing behaviour with respect to D_b and increasing behaviour with respect to R_b . This finding can be illustrated as follows. If D_b is increased in the system, the batteries will be discharged at a rapid speed and soon the charging level of the batteries will be below the threshold level K_1 (pre-defined). Hence, P_{normal} decreases in the system. Whereas, if the batteries are recharged with fast rate, P_{normal} increases as the system will be able to remain above the threshold level for some more time.

Similar behaviour can be observed from the Fig. 4(b) for the system being in power saving mode P_{power} with respect to D_b and R_b . Through the same explanation, this result can be explained. If the charging level of the batteries falls below the threshold value K_1 but remains above K_2 , the system will remain in the power saving mode. If D_b increases, the batteries will be rapidly discharged, and the charging level of the batteries will soon fall below the K_1 . Hence, P_{power} decreases in the system. If the

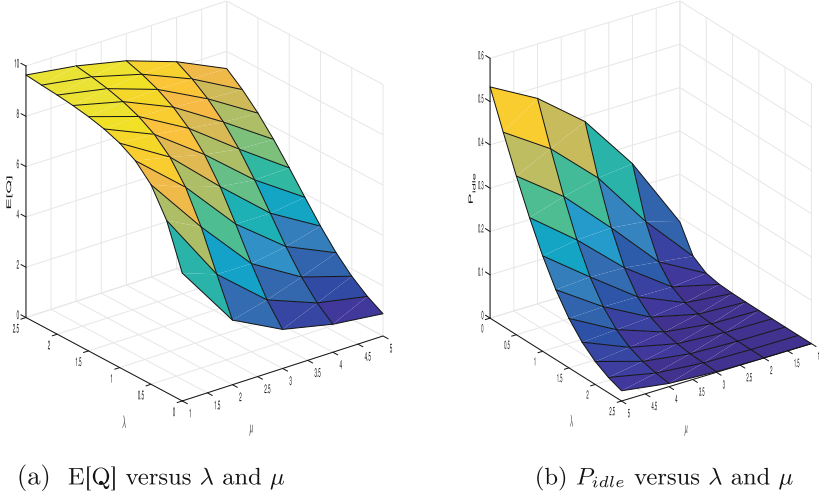


Fig. 2. Behaviour of expected number of packets in the system $E[Q]$ and P_{idle} with respect to the arrival rate λ and service rate μ .

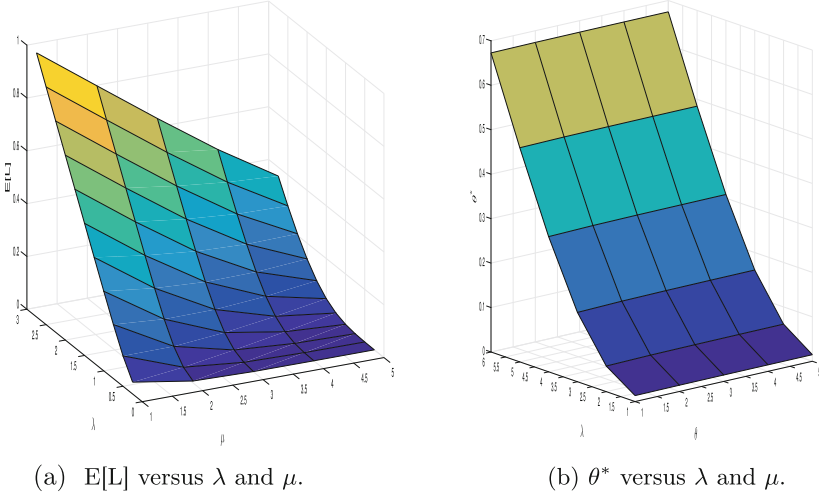


Fig. 3. Behaviour of the expected number of packets in orbit $E[L]$ with respect to the arrival rate λ and service rate μ and overall rate of retrial θ^* with respect to the arrival rate λ and retrial rate θ .

batteries are recharged quickly, P_{power} increases since the system will be able to stay over the threshold level for a longer period of time.

On the similar track, Fig. 5 demonstrates the behaviour for the system being in ultra power saving mode P_{ultra} with respect to D_b and R_b . Here, it can be observed that P_{ultra} appears to be decreasing function of D_b and increasing function of R_b . The explanation for this finding remains the same as mentioned above for the P_{normal} and

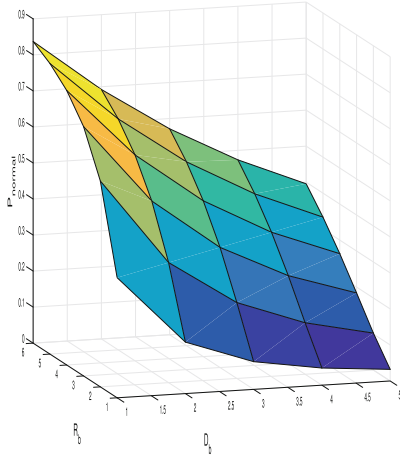
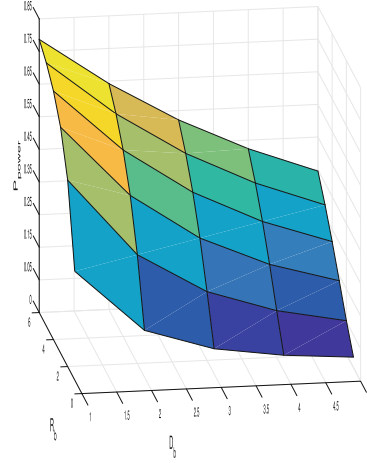
(a) P_{normal} versus D_b and R_b .(b) P_{power} versus D_b and R_b .

Fig. 4. Behaviour of the probability of server being in normal mode P_{normal} and power saving model P_{power} with respect to battery discharging rate D_b and battery recharging rate R_b .

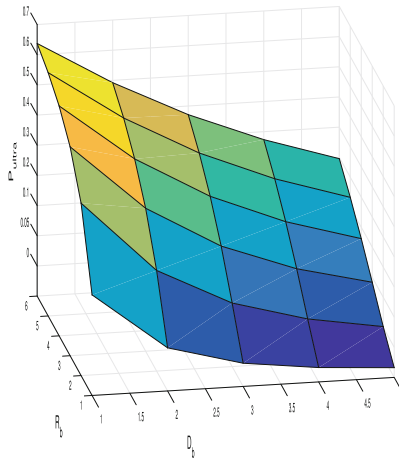


Fig. 5. Behaviour of the probability of server being in ultra power saving mode P_{ultra} with respect to battery discharging rate D_b and battery recharging rate R_b .

P_{power} . The value of P_{ultra} is less for the same value of D_b and R_b in comparison to P_{normal} and P_{power} which is obvious as when the system is in critically low charging level of batteries the probability value of being in the state also becomes less.

6 Conclusion and Future Directions

In the presented study, power management for the tethered HAP system is modeled by using a retrial queueing system. The power management in the tethered HAP system is explored through three different power saving modes, named as, normal mode, power saving mode and ultra power saving mode. The key concept behind introducing different modes is to keep the system active for a longer duration by saving its power while the service is degraded. It is assumed that the service rate is degraded from μ_1 to μ_2 when the system moves from normal mode to power saving mode, and the service rate is degraded from μ_2 to μ_3 when the system moves from power saving mode to ultra power saving mode, i.e., $\mu_1 > \mu_2 > \mu_3$. To handle the burstiness and correlation properties of the incoming traffic, this study considers more generalized *MAP* and *PH* distributions for the arrival and service processes of the system, respectively. The results of computing the stationary state probabilities and stability of the considered system are obtained using the matrix-geometric method. Through the numerical illustration, it has been shown that the probability of the system being in normal, power saving and ultra power saving mode are majorly affected by the discharging and recharging rate of the batteries. Also, the impact of arrival, service and retrial rates has been shown over a few selected performance measures of the system.

In the future, the proposed system can be thoroughly examined by taking into account multi server scenarios and different distributions, e.g., batch Markovian arrival process, general distribution, etc. The presented model can also be explored through various power saving optimization strategies in tethered HAP systems. Future research will focus heavily on a number of topics, particularly pertaining to the tethered HAP system's performance and availability.

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