



Stochastic Modelling for Energy Efficiency in LTE-A and LTE-5G Networks

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Abstract. Maximizing energy efficiency in User Equipment (UE) is a critical concern due to the limited power sources available in these devices. This holds true for both LTE-A and LTE-5G networks. To assess energy efficiency in these networks, we've developed three stochastic models: the Markov model, semi-Markov model, Markov regenerative process (MRGP) model. The Markov and semi-Markov model focuses on the discontinuous reception (DRX) mechanism, and the MRGP model pertains to the Modified DRX mechanism. We've derived explicit expressions for steady-state system size probabilities across all these models. Additionally, we've computed energy savings in UE based on the DRX mechanism states and performed sensitivity analyses for both the Markov and semi-Markov models. In the case of the MRGP model, we've determined the optimal system's energy savings and throughput by optimizing parameters like maximum short sleep and short sleep duration. To evaluate energy efficiency in LTE-5G networks, we've explored the trade-off between energy savings, average energy consumption, and throughput. Our analysis of the DRX mechanism provides valuable insights into its performance and lays the groundwork for potential enhancements to existing DRX mechanisms. We believe that these models have the potential to be extended for studying energy savings in hardware and other system components, contributing to overall energy efficiency improvements.

Keywords: Markov model · Semi-Markov process model · Markov regenerative process model · Energy · DRX mechanism

1 Introduction

Long-Term Evolution (LTE) stands as a cornerstone in wireless communication technology (WCT), while LTE Advanced (LTE-A) represents the next-generation standard, boasting advancements beyond LTE networks. LTE-A brings forth a novel cellular network paradigm characterized by high peak data rates, exceptional spectral efficiency, rapid round-trip times, reduced latency, prolonged battery life, and enhanced bandwidth frequency flexibility. Nonetheless, a prevalent challenge in wireless technology remains the heightened power

consumption associated with achieving faster data rates in LTE-A networks. In response to this challenge, we turn our attention to the Discontinuous Reception mechanism (DRX), an energy-saving mechanism designed for User Equipment (UE) within LTE-A networks.

Efficient reduction of power consumption in UE is accomplished by implementing a sleep mode functionality. To bolster energy efficiency, the LTE-A standard has introduced the DRX mechanism, permitting UEs to deactivate components when not actively transmitting or receiving data traffic. Numerous approaches have been proposed to optimize the DRX scheme, as seen in references such as [3, 16], which prioritize Quality of Service (QoS) and energy efficiency considerations. Moreover, the work in [17] presents an uncomplicated yet highly effective application-aware DRX mechanism, tailored to enhance LTE-A network performance. Additionally, [19] introduces a semi-Markov model to assess DRX performance metrics under specific conditions involving an exponential packet arrival rate. Notably, Arunsundar et al. [20] conducted an extensive analysis of the DRX mechanism, considering the impact of long and short cycles on buffering, average latency, DRX-sleep, and energy consumption.

Despite the variety of models explored in existing literature by different researchers, a stochastic model with a particular focus on UE energy savings at the packet level has been notably absent. This paper introduces two distinct stochastic models aimed at evaluating UE performance in LTE-A networks. Our primary objective is to demonstrate that energy savings in UE can be maximized through Markov modeling of the DRX mechanism, as opposed to semi-Markov modeling.

In a related study [21], Philip and Malarkodi introduced a Semi-Markov (SM) Model to assess HD-DRX performance in 5G communication scenarios involving 4G base stations. They employed beam search techniques to enable dual connectivity for User Equipment (UE), connecting to both LTE eNB (4G base station) and NR NodeB (5G New RAN base station). Sallam et al. [22] utilized a Semi-Markov (SM) Process to determine UE state probabilities within the HD-DRX system, leveraging these probabilities to calculate power-saving factors and average delays. Additionally, in the context of mmWave-enabled 5G communication and directional air interfaces, a novel D-DRX mechanism was explored in [1]. This mechanism emphasized the significance of beam searching states following each sleep cycle to enhance power-saving. Maheshwari et al. [1] further proposed three distinct D-DRX variants—Integrated D-DRX (ID-DRX), Standalone D-DRX (SD-DRX), and Cooperative D-DRX (CD-DRX)—and obtained probabilistic results for UE delay and power-saving across these mechanisms. Furthermore, within the realm of wireless technology, artificial intelligence (AI) plays a crucial role, as discussed by Memon et al. [2]. They introduced an AI-based DRX mechanism (AI-DRX) aimed at enhancing energy efficiency in LTE-5G connected devices, with a particular focus on dynamic short and long sleep cycles in DRX through the application of an AI-DRX algorithm for multiple beam communications.

While Arunsundar et al. [20] previously introduced a Semi-Markov Model (SMM) to derive probabilistic outcomes related to UE power-saving, energy consumption, and delay within the context of the DRX mechanism for LTE-A and LTE-5G networks, the existing literature lacks research that specifically addresses the optimization of energy-saving and energy consumption while considering system throughput and employs a stochastic model utilizing the Markov regenerative process. Therefore, this study aims to bridge this gap by developing a stochastic model for the MD-DRX mechanism in LTE-5G networks using the Markov regenerative process.

The paper is arranged as follows: In Sect. 2, we delve into the various stochastic models, including the Markov model, semi-Markov model, and Markov regenerative process model. Section 3 is dedicated to the analysis of performance measures across all the models. In Sect. 4, we present the numerical results pertaining to the proposed models. Finally, in Sect. 5, we provide concluding remarks.

2 Stochastic Modelling

2.1 Markov Model

In this section, we will begin by providing a detailed explanation of the UE's Discontinuous Reception (DRX) mechanism, which serves as a foundation for comprehending the proposed stochastic models. The DRX mechanism is composed of three distinct periods: working, idle, and sleep periods, as documented in [3, 4]. Within the sleep period, two categories of sleep modes exist, specifically referred to as short sleep and long sleep. To illustrate the structure of the DRX mechanism, please refer to Fig. 1. It's important to note that energy consumption levels vary across these different DRX periods, and these energy values have been sourced from existing literature [5, 6]. Detailed energy consumption values for each period are presented in Table 1.

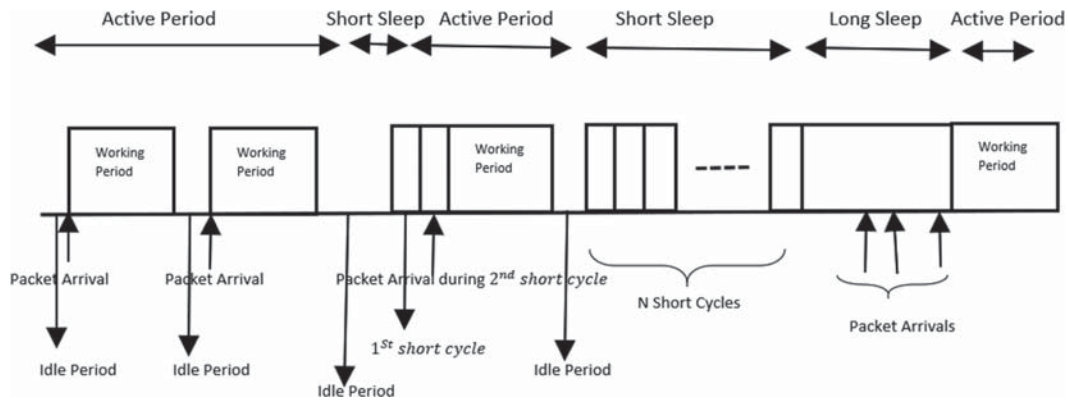


Fig. 1. Structure of DRX Mechanism

Table 1. Energy consumption values of UE in different periods of the DRX mechanism

Period	Energy Consumption (mW/ms)
Idle period	255.5
Working period	500
Short sleep	11
Long sleep	0

In this paper, we initially explore two models: Markov model and the semi-Markov model, which are founded on the sojourn time distributions within sleep periods. In the Markov model, the sojourn time within each state follows an exponential distribution. In contrast, the semi-Markov model diverges from this by having non-exponential sojourn times within sleep periods.

Description of Proposed Model. We introduce a stochastic model that accounts for the various states experienced by the UE in the DRX Mechanism. This stochastic model characterizes each state's duration using independent exponential distributions, ensuring that the overall stochastic process adheres to a continuous-time Markov chain. In the following section, we provide a comprehensive explanation of our proposed Markov model, outlining its four distinct states:

- Idle period: This state represents the condition in which the UE is active and awaiting incoming packets to deliver its services.
- Working period: This state signifies that the UE is actively providing services to the packets.
- Short cycle: This state involves the UE powering down most of its components. If no data packet arrives, this short cycle repeats a set number of times (e.g., N), and it terminates if a data packet does arrive.
- Long sleep: This state is entered when no data packet arrives upon the expiration of the short cycle.

Consider the continuous-time Markov chain denoted by $\{(N(t), S(t)), t \geq 0\}$. Here, $\{N(t), t \geq 0\}$ signifies the count of packets in the system at any given time t , while $\{S(t), t \geq 0\}$ indicates the system's state at time t , with its state space defined as $\Omega = \{(i, j); i = 0, 1, \dots; j = 0, 1, \dots, N, N + 1\}$.

The system's states are defined as follows:

- State $(0, 0)$ represents an idle period.
- States $(i, 0)$, where $i = 1, 2, \dots$, represent working periods where the system serves the i^{th} packet.
- States $(0, j)$, where $j = 1, 2, \dots, N$, correspond to short cycles.
- State $(i, N + 1)$ denotes a long sleep state with i packets. During both idle and working periods, the UE actively serves packets.

The idle period before entering the first short cycle follows an exponential distribution with parameter β . Packet arrivals in the system adhere to a Poisson process with parameter λ , and packet service times are exponentially distributed with parameter μ . Short cycle durations and long sleep periods are exponentially distributed with parameters α and ν , respectively.

Initially, the system is in an idle period, waiting for data packets to arrive for service. If a packet arrives during an idle period, the system transitions to state $(1,0)$ and starts serving packets. If another packet arrives before the current one is serviced, the system moves to state $(i+1,0)$, where i packets are already in the system. After each service, the system transitions from state $(i,0)$ to state $(i-1,0)$. In the absence of packet arrivals during an idle period, the system transitions to state $(0,1)$ and continues moving to states $(0,j)$ if no packet arrivals occur in state $(0,j-1)$, where $j = 1, 2, \dots, N$. If a packet arrives in any state $(0,j)$, where $j = 1, 2, \dots, N$, the system transitions back to state $(1,0)$.

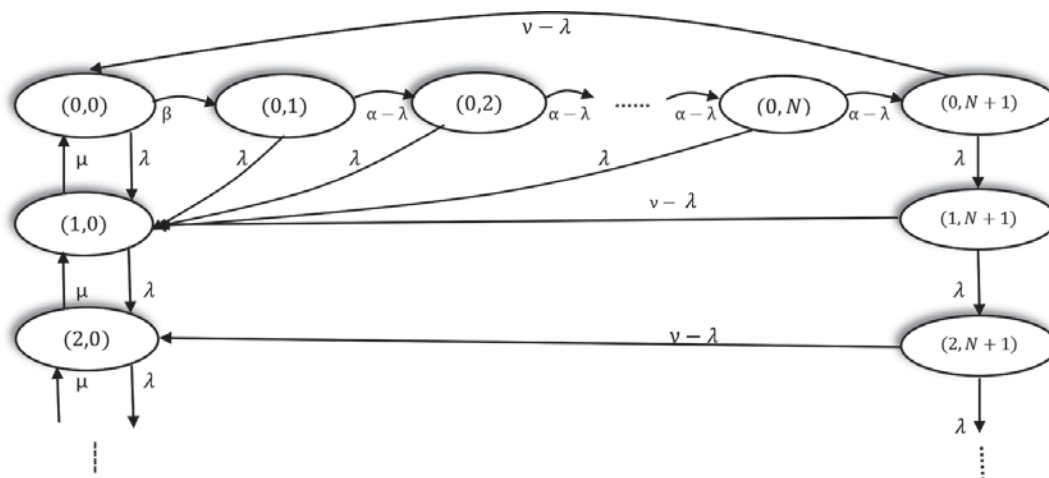


Fig. 2. State transition diagram for the proposed Markov model

If no packet arrives in state $(0,N)$, the system transitions to state $(0,N+1)$. In the event of a packet arrival during this long sleep state, the system moves to state $(i+1,N+1)$, with i packets already in the system. Upon completing the long sleep, if i packets arrive, the system transitions to state $(i,0)$. The state transition diagram for the proposed Markov model is illustrated in Fig. 2. The steady state probabilities of the system are obtained from forward Kolmogorov equation. It is known that, $\lim_{t \rightarrow \infty} P_{i,j}(t) = \pi_{i,j}$ and $\lim_{t \rightarrow \infty} P'_{i,j}(t) = 0$. Hence, Eqs. (1)–(7) becomes

$$\begin{aligned}
 0 &= -(\beta + \lambda)\pi_{0,0} + \mu\pi_{1,0} + (\nu - \lambda)\pi_{0,N+1}, \\
 0 &= -(\mu + \lambda)\pi_{1,0} + \lambda\pi_{0,0} + \mu\pi_{2,0} + \lambda(\pi_{0,1} + \pi_{0,2} + \dots + \pi_{0,N}) + (\nu - \lambda)\pi_{1,N+1}, \\
 0 &= -(\mu + \lambda)\pi_{i,0} + \lambda\pi_{i-1,0} + \mu\pi_{i+1,0} + (\nu - \lambda)\pi_{i,N+1}, \quad i = 2, 3, \dots \\
 0 &= -\alpha\pi_{0,1} + \beta\pi_{0,0},
 \end{aligned}$$

$$\begin{aligned}
0 &= -\alpha\pi_{0,j} + (\alpha - \lambda)\pi_{0,j-1}, j = 2, 3, \dots, N \\
0 &= -\nu\pi_{0,N+1} + (\alpha - \lambda)\pi_{0,N}, \\
0 &= -\nu\pi_{i,N+1} + \lambda\pi_{i-1,N+1}, i = 1, 2, \dots
\end{aligned}$$

The stability conditions for the existence of these steady state probabilities are $\lambda \leq \alpha$, $\lambda < \mu$ and $\lambda < \nu$.

Theorem 1. *Steady-state probabilities of UE being in different states, i.e., $(0, 0)$, $(0, j)$, $(i, N+1)$ and $(n, 0)$ for $n = 1, 2, \dots$; $1 \leq j \leq N$; $i \geq 0$ are given by*

$$\begin{aligned}
\pi_{0,j} &= \frac{\beta(\alpha - \lambda)^{j-1}}{\alpha^j} \pi_{0,0}, j = 1, 2, \dots, N. \\
\pi_{i,N+1} &= \frac{\beta\lambda^i(\alpha - \lambda)^N}{\nu^{i+1}\alpha^N} \pi_{0,0}, i = 0, 1, \dots \\
\pi_{n,0} &= A_n \pi_{0,0}, n \geq 1
\end{aligned}$$

where

$$\begin{aligned}
A_n &= \left[\frac{\mu b^{n+1}}{(\mu - \lambda)} + \frac{\beta b^{n-1}((\alpha - \lambda)^N - \alpha^N)}{\alpha^N(\mu - \lambda)} \right] [b^{n+1} - b^{n-1}] \\
&\quad + \frac{\beta b^n(\alpha - \lambda)^N}{(\mu - \lambda)\alpha^N} \left[\frac{(\nu - \lambda)b^{n+2}(1 - (\frac{\lambda}{\nu b^2})^{n+1})}{(\nu b^2 - \lambda)} \right. \\
&\quad \left. - b^n \left(1 - \left(\frac{\lambda}{\nu} \right)^{n+1} \right) + \left(\frac{\lambda}{\nu} \right)^{n+1} (b^{-n} - b^n) \right]. \\
\pi_{0,0} &= \frac{1}{1 + \sum_{i=1}^{\infty} A_i + \sum_{j=1}^N \frac{\beta(\alpha - \lambda)^{j-1}}{\alpha^j} + \sum_{i=0}^{\infty} \frac{\beta\lambda^i(\alpha - \lambda)^N}{\nu^{i+1}\alpha^N}}.
\end{aligned}$$

For the proof of above theorem readers can refer to [24]. Using the probability $\pi_{0,0}$, we can compute all the probabilities $\pi_{0,j}$, $\pi_{i,N+1}$; $1 \leq j \leq N$; $i \geq 0$ and $\pi_{n,0}$ for $n = 1, 2, \dots$

2.2 Semi-Markov Process

The state space in this proposed semi-Markov model aligns with that of the proposed Markov model. However, the key difference lies in the sojourn time within states related to the sleep period, which follows a non-exponential distribution.

Let I_0 represent a state where the system lies in an idle period with no packets, signifying that the system is awaiting packets. W_k denotes a state where the UE is in a working period with k packets, corresponding to the $(k, 0)^{th}$ state. S_j signifies a state where the UE is in the j^{th} short cycle, corresponding to the $(0, j)^{th}$ state. L_i represents a state where the UE is in a long sleep mode with i packets to serve, indicating the $(i, N+1)^{th}$ state. Here, $k = 1, 2, \dots$, $i = 0, 1, \dots$, and $j = 1, 2, \dots, N$.

Let $F_{m,n}(t)$ denote the Cumulative Distribution Function (CDF) associated with the transition from state m to state n , where $m, n \in \Omega$, with $\Omega = \{I_0, S_j, W_k, L_i, i = 0, 1, \dots; k = 1, 2, \dots; j = 1, 2, \dots, N\}$. Several papers

have discussed the sojourn time distribution for various states within the DRX mechanism [5, 14]. Therefore, the sojourn time distribution for various states in the semi-Markov model is presented in Table 2. The transitions between states are subject to various influencing factors, resulting in a stochastic behavior. Within this dynamic process, the distribution of transitions between states is directly impacted by the real-time conditions of DRX mechanisms. Consequently, the sojourn time in certain states adheres to deterministic distribution patterns, with time epochs exhibiting the Markov property. As a result, this stochastic process can be conceptualized as a Semi-Markov Process (SMP) [13]. The state transition diagram for the proposed semi-Markov model is depicted in Fig. 3. The state space of this model is given as $\Omega = \{I_0, W_k, S_j, L_i, k = 1, 2, \dots; i = 0, 1, \dots; j = 1, 2, \dots, N\}$. The steady-state probabilities $\pi_m \forall m \in S$ exist under the stability conditions, $\lambda < \mu$, $\lambda \leq \alpha$ and $\lambda < \nu$.

Table 2. Distributions of sojourn times in semi-Markov model

CDF	Distribution	Parameter
$F_{I_0, S_1}(t)$	Deterministic	$\frac{1}{\beta}$
$F_{S_j, S_{j+1}}(t)$	Deterministic	$\frac{1}{\alpha}$
$F_{S_N, L_0}(t)$	Deterministic	$\frac{1}{\alpha}$
$F_{W_k, W_{k+1}}(t)$	exponential	λ
$F_{W_1, I_0}(t)$	exponential	μ
$F_{W_k, W_{k-1}}(t)$	exponential	μ
$F_{S_j, W_1}(t)$	exponential	λ
$F_{L_i, L_{i+1}}(t)$	exponential	λ
$F_{L_0, I_0}(t)$	Deterministic	$\frac{1}{\nu}$
$F_{L_i, W_i}(t)$	Deterministic	$\frac{1}{\nu}$

Theorem 2. *The steady-state probabilities for the semi-Markov model are given as:*

$$\pi_{I_0} = \frac{1 - e^{-\frac{\lambda}{\beta}}}{\lambda M},$$

$$\pi_{W_1} = \frac{1 - e^{-\left(\frac{\lambda}{\beta} + \frac{\lambda N}{\alpha} + \frac{\lambda}{\nu}\right)}}{\mu M},$$

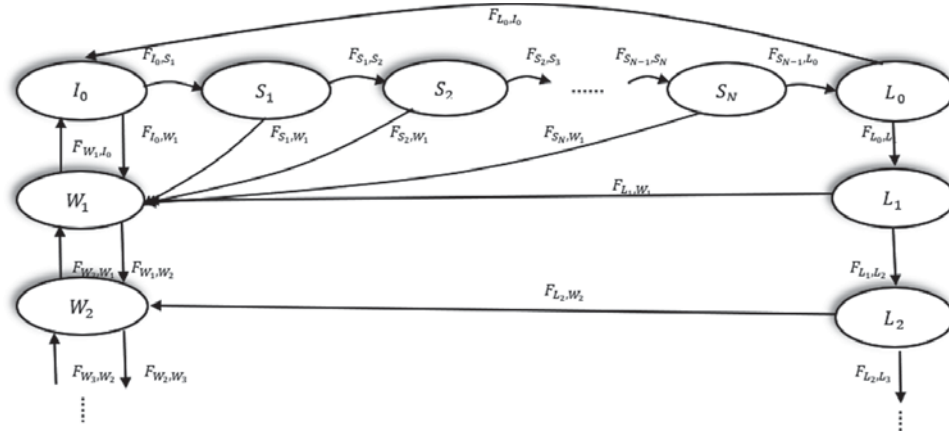


Fig. 3. State transition diagram for the proposed semi-Markov model

$$\pi_{W_k} = \frac{1}{\mu M} \left[\left(\frac{\lambda}{\mu} \right)^{k-1} \left(1 - e^{-\left(\frac{\lambda}{\beta} + \frac{\lambda N}{\alpha} + \frac{\lambda}{\nu} \right)} \right) + \right. \\ \left. + \frac{\left(\frac{\lambda}{\mu} \right) \left(1 - e^{-\frac{\lambda}{\nu}} \right)^2}{\left(\frac{\lambda}{\mu} - 1 + e^{-\frac{\lambda}{\nu}} \right)} e^{-\left(\frac{\lambda}{\beta} + \frac{\lambda N}{\alpha} \right)} \left[\left(\frac{\lambda}{\mu} \right)^{k-2} - \left(1 - e^{-\frac{\lambda}{\nu}} \right)^{k-2} \right] \right], \quad k = 2, 3, \dots$$

$$\pi_{S_j} = \frac{e^{-\frac{\lambda}{\beta}} \left(e^{-\frac{\lambda}{\alpha}} \right)^{j-1} \left(1 - e^{-\frac{\lambda}{\alpha}} \right)}{\lambda M}, \quad j = 1, 2, \dots, N$$

$$\pi_{L_i} = \frac{e^{-\left(\frac{\lambda}{\beta} + \frac{\lambda N}{\alpha} \right)} \left(1 - e^{-\frac{\lambda}{\nu}} \right)^{i+1}}{\lambda M}, \quad i = 0, 1, \dots$$

For the proof of above theorem readers can refer to [24].

2.3 Markov Generative Process Model

In LTE networks, the management of any DRX mechanism is overseen by the RRC layer. A device or User Equipment (UE) in LTE can operate in either the $RRC_{CONNECTED}$ mode or the RRC_{Idle} mode. In RRC_{Idle} mode, the UE's sole activity is monitoring paging signals during the paging cycle, during which it neither receives nor transmits data packets. Conversely, all data exchange between the eNB and UE occurs within the $RRC_{CONNECTED}$ mode. Consequently, this work primarily focuses on the utilization of the MD-DRX mechanism in the $RRC_{CONNECTED}$ mode, as illustrated in Fig. 4. The DRX mechanism is designed to conserve power by intermittently monitoring the physical downlink control channel (PDCCH). Here, we elucidate the MD-DRX mechanism employed by UEs in LTE-5G networks. The proposed MD-DRX mechanism encompasses the following operational states: inactive, beam searching, active, ON, long sleep, and short sleep. The short sleep state comprises a maximum of M short sleep periods. These states are mentioned as:

- **Active state:** In the active state, the UE receives/transmits data packets, representing a state of availability where it actively provides service to each packet.
- **Inactive state:** Following the active state, the UE transitions into an inactive state for a randomly determined duration. If a packet arrives before this inactive duration expires, the UE will wait until it ends, then move into the ON period. Conversely, if no packets are available in the buffer when the inactive duration concludes, the UE switches to the short sleep state.
- **Short sleep state:** During the short sleep state, the UE powers down most of its energy hungry components and can go through a maximum of M short sleep cycles. After each short sleep if the system finds empty, the UE can have up to M short sleep cycles in this state. However, if no data packets arrive until the M^{th} short sleep, the UE switches to the beam searching state. Conversely, if data packets do arrive during the short sleep, the UE will wait until the duration concludes before transitioning to the ON period state.
- **Beam searching state:** After completing a maximum of M short sleep and long sleep cycles, the UE enters this state. During the beam searching state, the UE conducts measurements on available beams to identify the optimal beam pair for communication. While engaged in this beam alignment process, if the UE receives a data packet, it will remain in this state until the duration expires before transitioning to the ON period state. However, if no suitable beam pair or data packets are detected during the beam searching state, the UE switches to the long sleep state.
- **Long sleep state:** In the long sleep state, the UE can receive data packets but cannot serve them. When the long sleep duration ends, if the system's buffer is empty, the UE transitions to the beam searching state. However, if there is at least one data packet in the buffer when the long sleep duration is completed, the UE enters the ON period.
- **ON period state:** When the inactive state, short sleep state, beam searching state, or long sleep state ends, and the UE's buffer contains at least one data packet, the UE transitions to the ON period. In this state, the UE briefly prepares to start serving packets before entering the active state.

We assume that the active state's packet service time follows a deterministic distribution characterized by the parameter $\frac{1}{\mu}$. Additionally, we consider the inter-arrival times of packets to be independent and following an exponential distribution with the parameter λ .

Description of the Proposed Model. The system state bivariate stochastic process for the MD-DRX mechanism is represented as $\{(N(t), S(t)); t \geq 0\}$, where $\{N(t); t \geq 0\}$ indicates the number of packets in the system at time t , and $\{S(t); t \geq 0\}$ denotes the state of the system at time t with state space Ω . The state space Ω encompasses various combinations of the number of packets and system states. Consider $\Omega \equiv \varsigma_{(i,j)} \in \{0, 1, \dots\} \times \{0, 1, 2, 3, 4, 5, 6, \dots, M + 4\}$. Here, $i \in \{0, 1, 2, \dots\}$ symbolizes the number of

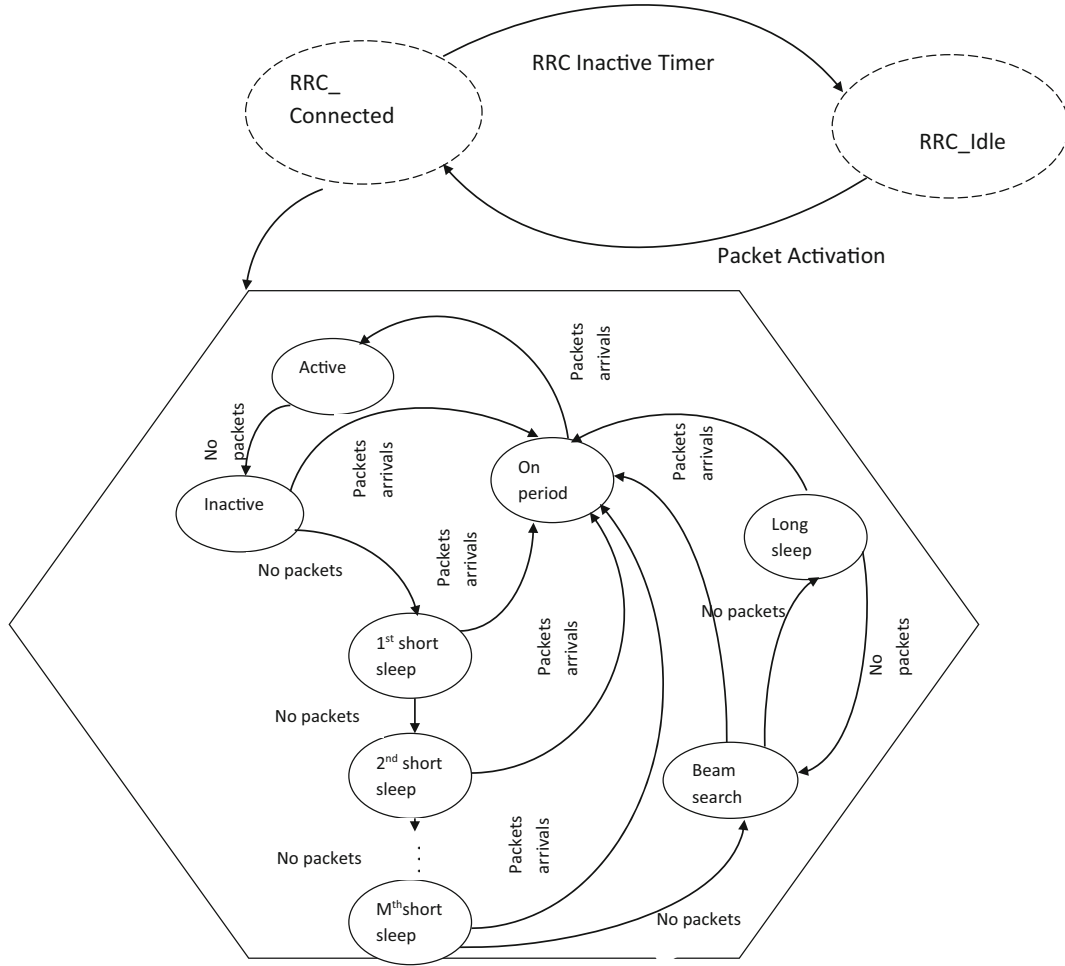


Fig. 4. LTE-DRX in RRC connected for the proposed model as in current 5G system

packets in the system (if any packets is being served, incorporating the one also) and $j \in 0, 1, 2, 3, 4, 5, 6, \dots, M + 4$ symbolizes the states of the system i.e., whether the system lies in Beam searching state (B), or in inactive system (I), or in active state (A), or in long sleep state (L), or in ON state (O), or 1^{st} short sleep, or in 2^{nd} short sleep, ..., or in M^{th} short sleep (which are in between short sleep (S)). Each state in Ω is identified as follows:

- $A_{i,2}$ represents the system lies in the active state with i number of packets.
- $I_{i,1}$ symbolizes the system in the inactive state with i number of packets.
- $B_{i,0}$ represents the system in the beam searching state with i number of packets.
- $L_{i,3}$ symbolizes the system in the long sleep state with i number of packets.
- $O_{i,4}$ represents the system in the ON state with i number of packets.
- $S_{i,j}$ represents the system in the j^{th} short sleep state with i number of packets.

Hence, Ω consists of these state combinations, i.e., Hence, $\Omega = \{B_{i,0}; i = 0, 1, \dots, I_{i,1}; i = 0, 1, \dots, A_{i,2}; i = 1, 2, \dots, L_{i,3}; i = 0, 1, \dots, O_{i,4}; i = 1, 2, \dots, S_{i,j}; i = 0, 1, \dots, j = 5, 6, \dots, M + 4\}$.

The process $\{(N(t), S(t)); t \geq 0\}$ is not a continuous-time Markov chain (CTMC) because the sojourn times in each state are not exponentially distributed. Additionally, it is not a Semi-Markov Process (SMP) since the number of packets can change between two sequential service completion points due to new packet arrivals before the completion of previous packet services. Moreover, the transitions between states (beam search, inactive, active, long sleep, ON, short sleep) do not happen immediately upon new arrivals.

Due to the time remaining when new arrivals occur during the time durations in the beam search state, inactive state, active state, long sleep state, ON state, and short sleep state (including the 1st short sleep, 2nd short sleep, and so on, up to the M^{th} short sleep), all the state completion points become regeneration points. We define a sequence of epochs, denoted as $\{\tau_n, n \geq 0\}$, at which the process $\{(N(t), S(t)); t \geq 0\}$ is observed. These τ_n epochs correspond to the termination of the beam search period, inactive period, service, long sleep period, ON period, and short sleep period. At these regeneration epochs, we denote ζ_n , which represents the state of the system. These elements in the state space Ω are referred to as regeneration epochs because they occur at the completion points of states, including the service completion point, inactive period completion epoch, long sleep period completion epoch, ON period completion epoch, and short sleep completion epoch. As a result, we can assert that the sequence $\{\zeta_n, \tau_n\}$ constitutes an embedded Markov renewal sequence. Consequently, the stochastic process $\{(N(t), S(t)); t \geq 0\}$ is classified as a Markov Regenerative Process (MRGP) [12].

Packet arrivals in the system are modeled as a Poisson process with the parameter λ . The inter-arrival times of packets are independent and follow exponential distributions with the parameter λ . The service time, inactive time, beam searching duration, short sleep duration, long sleep duration, and ON duration all follow deterministic distributions with parameters $\frac{1}{\mu}$, $\frac{1}{\nu}$, $\frac{1}{\zeta}$, $\frac{1}{\beta}$, $\frac{1}{\gamma}$, and $\frac{1}{\alpha}$, respectively. The state transition diagram for the proposed model is depicted in Fig. 5. Table 3 provides explanations for all the cumulative distribution functions (CDFs) along with their corresponding parameters.

Theorem 3. *The steady-state probabilities for the Markov regenerative process model are given as:*

$$\begin{aligned} \phi_{B_{0,0}} &= \frac{1}{G} \left[(e^{-\frac{\lambda}{\beta}})^{M-1} e^{-\lambda(\frac{1}{\mu} + \frac{1}{\nu})} (1 - e^{-\frac{\lambda}{\beta}}) + \frac{e^{-\lambda(\frac{1}{\zeta} + \frac{1}{\nu})} (e^{-\frac{\lambda}{\beta}})^M}{(1 - e^{-\lambda(\frac{1}{\gamma} + \frac{1}{\zeta})})} (1 - e^{-\frac{\lambda}{\gamma}}) e^{-\frac{\lambda}{\mu}} \right], \\ \phi_{I_{0,1}} &= \frac{1}{G} (1 - e^{-\frac{\lambda}{\mu}}), \\ \phi_L &= \frac{1}{G} \frac{e^{-\lambda(\frac{1}{\mu} + \frac{1}{\nu})} (e^{-\frac{\lambda}{\beta}})^M}{(1 - e^{-\lambda(\frac{1}{\gamma} + \frac{1}{\zeta})})} (1 - e^{-\frac{\lambda}{\zeta}}), \end{aligned}$$

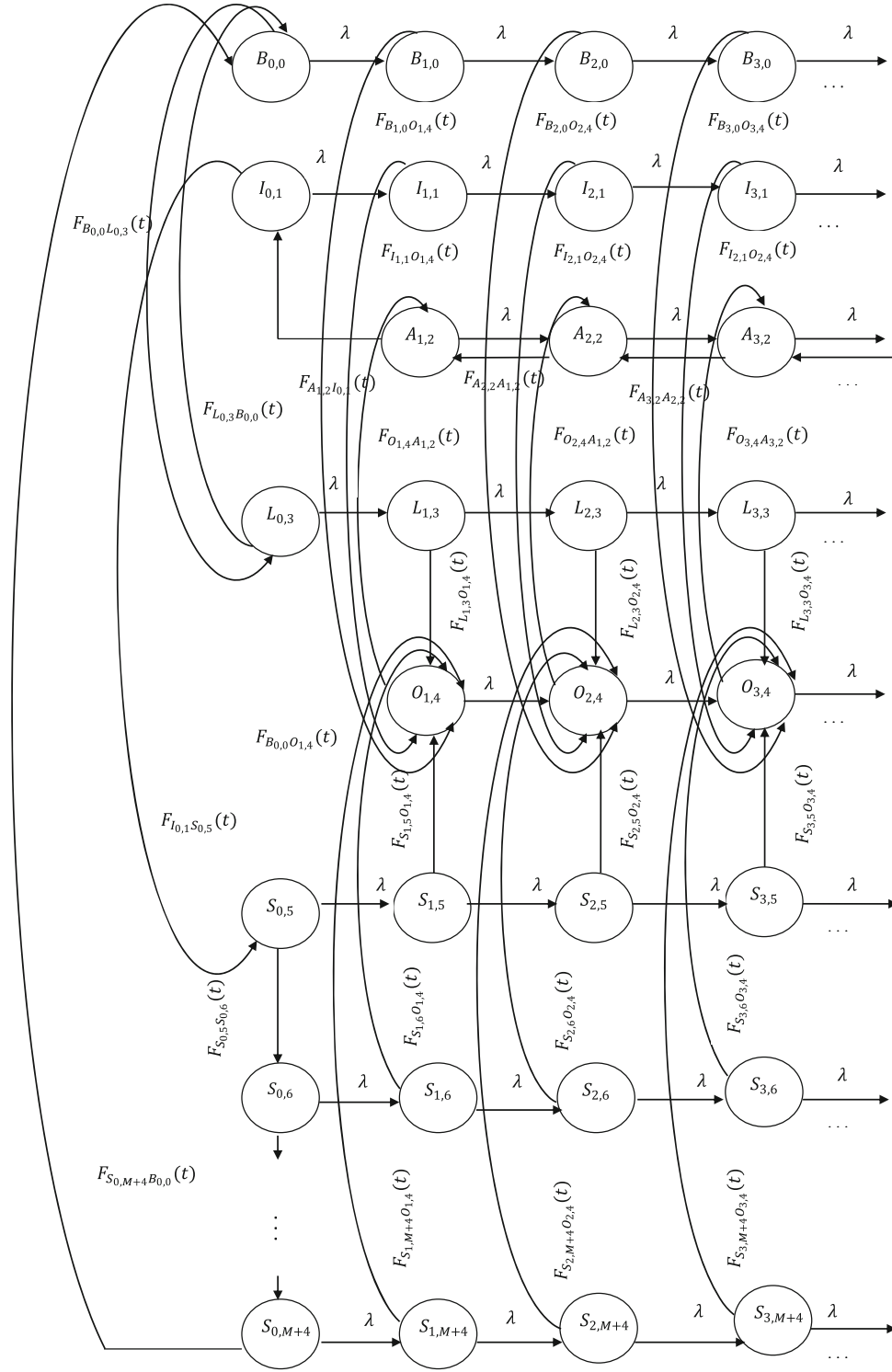


Fig. 5. State transition diagram for the proposed MRGP model

$$\begin{aligned}
 \phi_{O_{i,4}} = & \frac{1}{G} \left[(1 - e^{-\frac{\lambda}{\nu}})^i e^{-\frac{\lambda}{\mu}} (1 - e^{-\frac{\lambda}{\nu}}) + (e^{-\frac{\lambda}{\beta}})^{j-5} (1 - e^{-\frac{\lambda}{\beta}})^i e^{-\lambda(\frac{1}{\nu} + \frac{1}{\mu})} (1 - e^{-\frac{\lambda}{\beta}}) \right. \\
 & \left. + (1 - e^{-\frac{\lambda}{\gamma}})^i \frac{e^{-\frac{\lambda}{\nu}} (e^{-\frac{\lambda}{\beta}})^M}{(1 - e^{-\lambda(\frac{1}{\gamma} + \frac{1}{\gamma})})} e^{-\frac{\lambda}{\mu}} (1 - e^{-\frac{\lambda}{\gamma}}) \right]; i = 0, 1, \dots, j = 5, 6, \dots, M + 4,
 \end{aligned}$$

Table 3. Cumulative distribution functions with parameters

CDF	parameter
$F_{B_{i,j}O_{i,j+4}}(t); i = 1, 2, \dots, j = 0$	$\frac{1}{\zeta}$
$F_{B_{0,0}L_{0,3}}(t)$	$\frac{1}{\zeta}$
$F_{I_{0,1}S_{0,5}}(t)$	$\frac{1}{\nu}$
$F_{I_{i,j}O_{i,j+3}}(t);$ $i = 1, 2, \dots, j = 1$	$\frac{1}{\nu}$
$F_{A_{1,2}I_{0,1}}(t)$	$\frac{1}{\mu}$
$F_{S_{i,j}O_{i,j-k}}(t);$ $i = 1, 2, \dots, j - k = 4$	$\frac{1}{\beta}$
$j = 5, 6, \dots, M + 4, k = 1, 2, \dots, M$	
$F_{A_{i,2}A_{i-1,2}}(t); i = 2, 3, \dots,$	$\frac{1}{\mu}$
$F_{L_{0,3}B_{0,0}}(t)$	$\frac{1}{\gamma}$
$F_{L_{i,j}O_{i,j+1}}(t); i = 1, 2, \dots, j = 3$	$\frac{1}{\gamma}$
$F_{O_{i,j}A_{i,j-2}}(t)$	$\frac{1}{\alpha}$
$F_{S_{0,j}S_{0,j+1}}(t); j = 5, 6, \dots, M + 4$	$\frac{1}{\beta}$
$F_{S_{0,M+4}B_{0,0}}(t)$	$\frac{1}{\beta}$

$$\phi_S = \frac{1}{G} \left[e^{-\frac{\lambda}{\mu}} \{ (1 - e^{-\frac{\lambda}{\nu}}) + e^{-\frac{\lambda}{\nu}} (1 - (e^{-\frac{\lambda}{\beta}})^M) \} \right],$$

for the value of G and the steady state probability $\phi_{A_{i,2}}$, refer [23].

In this context, the system state probabilities ϕ_n for all states n in the state space Ω exist when $\lambda < \mu$. Please note: Given that we assume packet arrivals follow a Poisson process, service times follow a deterministic distribution, and the system operates as an infinite capacity single-server model, it conforms to an $M/D/1$ queue. Referring to the steady-state probabilities of the Erlang's Model C ($M/D/1$) queue, as mentioned in [10], page 261, we can conclude that the condition for system stability requires that the traffic intensity ρ (which is equal to $\frac{\lambda}{\mu}$) should be less than 1. Therefore, the stability condition for the proposed model is $\lambda < \mu$.

3 Performance Measures

3.1 Markov Model

We use the notations A_M , S_M , and L_M to denote the steady-state probabilities of the system being in the active state, short sleep state, and long sleep state, respectively, for the Markov model. Then,

$$A_M = \sum_{n=0}^{\infty} \pi_{n,0} = \left\{ 1 - \left[\frac{\lambda}{(\mu-\lambda)} + \frac{\beta((\alpha-\lambda)^N - \alpha^N)}{\alpha^N(\mu-\lambda)} \right] + \frac{\beta(\alpha-\lambda)^N}{(\mu-\lambda)\alpha^N} \left[\frac{(\nu-\lambda)b^4}{(\nu b^2 - \lambda)(1-b^2)} - \frac{\lambda^2}{\nu(\nu b^2 - \lambda)} - \frac{b^2}{1-b^2} + \frac{\lambda^2}{\nu(\nu-\lambda)} \right] \right\} \pi_{0,0},$$

$$S_M = \sum_{j=1}^N \pi_{0,j} = \frac{\beta(\alpha^N - (\alpha - \lambda)^N)}{\lambda\alpha^N} \pi_{0,0},$$

$$L_M = \sum_{n=0}^{\infty} \pi_{n,N+1} = \frac{\beta(\alpha - \lambda)^N}{(\nu - \lambda)\alpha^N} \pi_{0,0}.$$

Let $E[N_M]$ denote the mean number of packets in the system, and let $E[N_{A_M}]$ and $E[N_{S_M}]$ denote the mean number of packets in the active mode and sleep mode, respectively. Then,

$$E[N_M] = E[N_{A_M}] + E[N_{S_M}],$$

where

$$E[N_{A_M}] = \sum_{n=0}^{\infty} n\pi_{n,0} = \left\{ \frac{\mu}{(\lambda - \mu)} \left[\frac{\lambda}{(\mu - \lambda)} + \frac{\beta((\alpha - \lambda)^N - \alpha^N)}{\alpha^N(\mu - \lambda)} \right] + \frac{\beta(\alpha - \lambda)^N}{(\mu - \lambda)\alpha^N} \left[\frac{(\nu - \lambda)b^4}{(\nu b^2 - \lambda)(1 - b^2)^2} - \frac{\lambda^2}{(\nu - \lambda)(\nu b^2 - \lambda)} - \frac{b^2}{(1 - b^2)^2} + \frac{\lambda^2}{(\nu - \lambda)^2} \right] \right\} \pi_{0,0},$$

$$E[N_{S_M}] = \sum_{n=0}^{\infty} n\pi_{n,N+1} = \frac{\beta\lambda(\alpha - \lambda)^N}{(\nu - \lambda)^2\alpha^N} \pi_{0,0}.$$

The energy-saving factor is determined by calculating the percentage of time the UE remains in the sleep state, which involves comparing the time spent in a sleep period to the overall time spent in all states. To compute this energy-saving factor, we utilize steady-state probabilities denoted as $\pi_{i,j}$ for values of $i = 0, 1, 2, \dots$ and $j = 1, 2, \dots, N$. The formula for calculating the energy-saving factor E_S can be found in [9, 11]:

$$E_S = \sum_{j=1}^N \pi_{0,j} + \sum_{i=0}^{\infty} \pi_{i,N+1}$$

$$= \left\{ \frac{\beta(\alpha^N - (\alpha - \lambda)^N)}{\lambda\alpha^N} + \frac{\beta(\alpha - \lambda)^N}{(\nu - \lambda)\alpha^N} \right\} \pi_{0,0}.$$

3.2 Semi-Markov Model

Let A_{SM} , S_{SM} , and L_{SM} represent the steady-state probabilities of being in the active state, short sleep, and long sleep, respectively, for the semi-Markov model in the long run. Then

$$A_{SM} = \pi_{I_0} + \sum_{k=1}^{\infty} \pi_{W_k} = \frac{1}{M} \left[\frac{\mu}{\lambda(\mu - \lambda)} - \frac{e^{-\left(\frac{\lambda}{\beta} + \frac{\lambda N}{\alpha} + \frac{\lambda}{\nu}\right)}}{\mu - \lambda} - \frac{e^{-\frac{\lambda}{\beta}}}{\lambda} - \frac{\lambda}{\mu} \frac{e^{-\left(\frac{\lambda N}{\alpha} + \frac{\lambda}{\beta}\right)} \left(1 - e^{-\frac{\lambda}{\nu}}\right)^2}{\left(\frac{\lambda}{\mu} - 1 + e^{-\frac{\lambda}{\nu}}\right)} \left[\frac{e^{\frac{\lambda}{\nu}}}{\mu} - \frac{1}{\mu - \lambda} \right] \right],$$

$$S_{SM} = \sum_{j=1}^N \pi_{S_j} = \frac{1}{M} \left[\frac{e^{-\frac{\lambda}{\beta}} (1 - e^{-\frac{\lambda N}{\alpha}})}{\lambda} \right],$$

$$L_{SM} = \sum_{i=0}^{\infty} \pi_{L_i} = \frac{1}{M} \left[\frac{e^{-\left(\frac{\lambda N}{\alpha} + \frac{\lambda}{\beta}\right)} (e^{\frac{\lambda}{\nu}} - 1)}{\lambda} \right].$$

Let $E[N_{SM}]$ denote the mean number of packets in the system, and let $E[N_{A_{SM}}]$ and $E[N_{S_{SM}}]$ represent the mean number of packets in active mode and sleep mode, respectively. Then,

$$E[N_{SM}] = E[N_{A_{SM}}] + E[N_{S_{SM}}],$$

where

$$E[N_{A_{SM}}] = \sum_{n=1}^{\infty} n \pi_{W_n} = \frac{1}{M} \left[\frac{\mu (1 - e^{-\left(\frac{\lambda}{\beta} + \frac{\lambda N}{\alpha} + \frac{\lambda}{\nu}\right)})}{(\mu - \lambda)^2} \right. \\ \left. + \frac{\lambda}{\mu^2} \frac{e^{-\left(\frac{\lambda N}{\alpha} + \frac{\lambda}{\beta}\right)} (1 - e^{-\frac{\lambda}{\nu}})}{\left(\frac{\lambda}{\mu} - 1 + e^{-\frac{\lambda}{\nu}}\right)} \left[1 - e^{\frac{2\lambda}{\nu}} - \frac{(1 - e^{-\frac{\lambda}{\nu}})(2 - \frac{\lambda}{\mu})}{(1 - \frac{\lambda}{\mu})^2} \right] \right],$$

$$E[N_{S_{SM}}] = \sum_{n=0}^{\infty} n \pi_{L_n} = \frac{1}{M} \left[\frac{e^{-\left(\frac{\lambda N}{\alpha} + \frac{\lambda}{\beta}\right)} e^{\frac{2\lambda}{\nu}} (1 - e^{\frac{\lambda}{\nu}})^2}{\lambda} \right].$$

The energy saving factor is represented as the percentage of time the UE spends in the sleep state, which is the ratio of the time the UE spends in a sleep period to the total time across all the states. Therefore, using steady-state probabilities π_m for $m \in \Omega$, the energy saving factor E_S is computed as:

$$E_S = \sum_{j=1}^N \pi_{S_j} + \sum_{i=0}^{\infty} \pi_{L_i} \\ = \frac{1}{M} \left[\frac{e^{-\frac{\lambda}{\beta}} (1 - e^{-\frac{\lambda N}{\alpha}})}{\lambda} + \frac{e^{-\left(\frac{\lambda N}{\alpha} + \frac{\lambda}{\beta}\right)} (e^{\frac{\lambda}{\nu}} - 1)}{\lambda} \right].$$

3.3 Markov Regenerative Process Model

Let P_L , P_B , P_I , P_S , P_A , and P_O represent the steady-state probabilities for being in the long sleep state, beam searching state, inactive state, short sleep state, active state, and ON state, respectively.

$$P_L = \frac{1}{G} \frac{e^{-\lambda(\frac{1}{\mu} + \frac{1}{\nu})} (e^{-\frac{\lambda}{\beta}})^M}{(1 - e^{-\lambda(\frac{1}{\gamma} + \frac{1}{\xi})})} (1 - e^{-\frac{\lambda}{\xi}}),$$

$$P_B = \frac{1}{G} \left[(e^{-\frac{\lambda}{\beta}})^{M-1} e^{-\lambda(\frac{1}{\mu} + \frac{1}{\nu})} (1 - e^{-\frac{\lambda}{\beta}}) + \frac{e^{-\lambda(\frac{1}{\xi} + \frac{1}{\nu})} (e^{-\frac{\lambda}{\beta}})^M}{(1 - e^{-\lambda(\frac{1}{\gamma} + \frac{1}{\xi})})} (1 - e^{-\frac{\lambda}{\gamma}}) e^{-\frac{\lambda}{\mu}} \right],$$

$$P_I = \frac{1}{G} (1 - e^{-\frac{\lambda}{\mu}}),$$

$$\begin{aligned}
P_S &= \frac{1}{G} \left[e^{-\frac{\lambda}{\mu}} \{ (1 - e^{-\frac{\lambda}{\nu}}) + e^{-\frac{\lambda}{\nu}} (1 - (e^{-\frac{\lambda}{\beta}})^M) \} \right], \\
P_O &= \frac{1}{G} \text{bigl} [e^{\frac{\lambda}{\nu}} e^{-\frac{\lambda}{\mu}} (1 - e^{-\frac{\lambda}{\nu}}) + e^{\frac{\lambda}{\beta}} \frac{(1 - (e^{-\frac{\lambda}{\beta}})^M)}{1 - e^{-\frac{\lambda}{\beta}}} e^{-\lambda(\frac{1}{\mu} + \frac{1}{\nu})} (1 - e^{-\frac{\lambda}{\beta}}) \\
&\quad + \frac{e^{-\lambda(\frac{1}{\nu} + \frac{1}{\mu})} e^{\frac{\lambda}{\zeta}} (e^{-\frac{\lambda}{\beta}})^M}{(1 - e^{-\lambda(\frac{1}{\zeta} + \frac{1}{\gamma})})} (1 - e^{-\frac{\lambda}{\zeta}})], \\
P_A &= \sum_{i=6}^{\infty} \phi_{A_{i,2}}
\end{aligned}$$

Now, we calculate the energy-saving factor (ES), defined as the percentage of time the device (UE) spends in the sleep state, including short sleep and long sleep periods.

$$ES = (P_L + P_S) \times 100\%.$$

The average energy consumption is the power consumed by the UE's transceiver, and it varies in different states [7]. Let EC_L , EC_S , EC_A , EC_I , EC_B , and EC_O represent the energy consumed during the long sleep state, short sleep state, active state, inactivity timer, beam searching state, and ON state, respectively.

Hence, we obtain the average energy consumption by UE

$$E_{cons} = P_L EC_L + P_S EC_S + P_A EC_A + P_I EC_I + P_B EC_B + P_O EC_O.$$

The energy consumption in the MD-DRX mechanism in LTE-5G is demonstrated in Fig. 6.

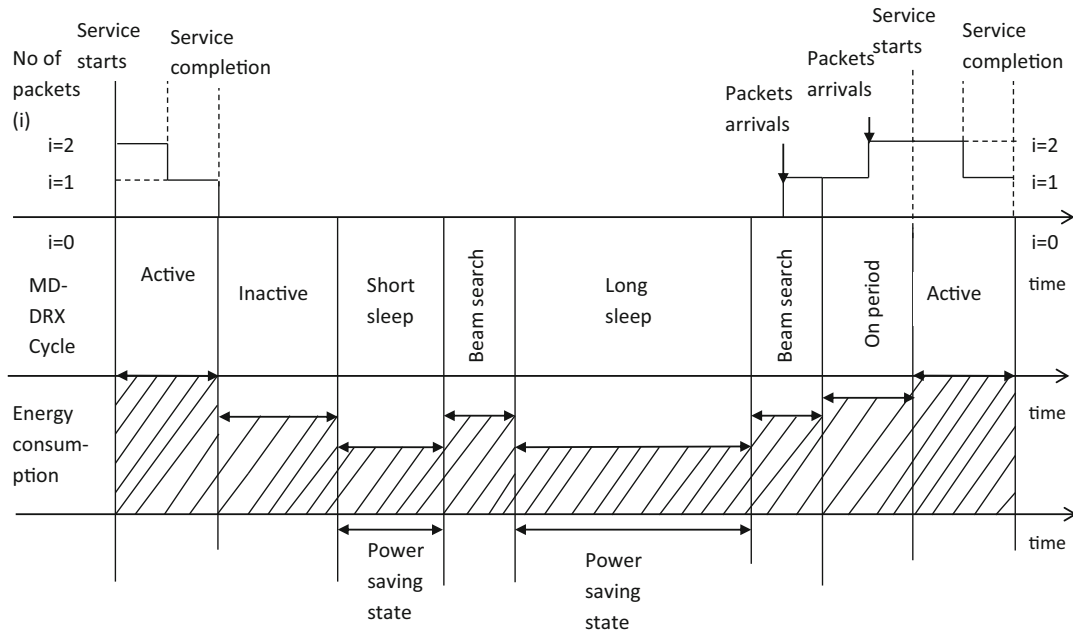


Fig. 6. Energy consumption in MD-DRX mechanism

The throughput of the system is a crucial factor in determining the optimal energy consumption and energy-saving strategies. It represents the efficiency of the system in serving customers. To calculate the system's throughput, we consider the fraction of time the system is available to customers, multiplied by the servicing rate in the case of an infinite capacity model [8]. Therefore, the system's throughput is defined as follows:

$$\text{Throughput} = P_A \times \mu,$$

where μ is the service rate.

4 Numerical Illustration

To create the graphs, we've established fixed parameter values that adhere to the stability conditions of our proposed models, which are $\frac{\lambda}{\mu} < 1$, $\frac{\lambda}{\nu} < 1$, and $\alpha \geq \lambda$, with $\beta > 0$ and $\lambda > 0$. The specific parameter values are as follows: $\alpha = 1$, $\beta = 1.2$, $\lambda = 0.5$, $\mu = 1.5$, and $\nu = 1.8$. In order to present the results of the Markov model, we've labeled it as *MM* with varying parameter values, and for the Semi-Markov model, we've denoted it as *SMM* with the corresponding parameter variations. The energy-saving factor is plotted against the values of N in Fig. 7, keeping all other parameter values constant. Notably, in the case of the Semi-Markov model, the power-saving factor is lower compared to the Markov model, as the steady-state probability of the UE being in the active state is higher. However, this factor stabilizes as the value of N increases.

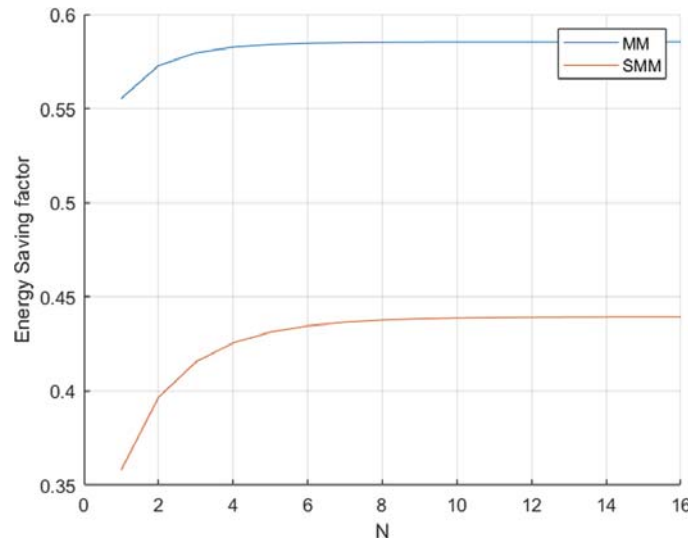


Fig. 7. Energy Saving factor varying number of short cycles

The percentage difference in measurements between both models is calculated as the measured value reaches its steady-state value. To facilitate comparison, we've

used the following parameter values for both models: $\alpha = 1$, $\beta = 1.2$, $\lambda = 0.5$, $\mu = 1.5$, and $\nu = 1.8$. The comparison between the energy-saving factors for both models is presented in Table 3. Our observations from Table 4 reveal that the energy-saving factor approaches its steady-state value for the Markov model when $N \geq 12$ and for the semi-Markov model when $N \geq 16$. Furthermore, we've noted that the energy-saving factor is at least 33.19% higher for the Markov model compared to the semi-Markov model. Hence, the maximization of energy savings in the UE can be achieved through the Markov modeling of the DRX mechanism.

Table 4. Values for energy saving factor by varying number of short cycles

N	Markov model (MM)	Semi-Markov model (SMM)	% Difference between MM and SMM
8	0.5852	0.4377	33.69
9	0.5853	0.4384	33.50
10	0.5853	0.4388	33.38
11	0.5853	0.4391	33.29
12	0.5854	0.4393	33.25
13	0.5854	0.4393	33.25
14	0.5854	0.4394	33.22
15	0.5854	0.4394	33.22
16	0.5854	0.4395	33.19
17	0.5854	0.4395	33.19
18	0.5854	0.4395	33.19

Using parameter values of $\nu = 0.02$, $\mu = 0.05$, $\zeta = 0.01$, $\alpha = 1$, $\gamma = 0.0125$, and $M^* = 6$, we have generated a plot illustrating the trade-off between the energy-saving factor and throughput. This plot varies the arrival rate from 0.001 to 0.004 for the MRGP model. Key observations from Fig. 8 include:

- As the arrival rate (λ) ranges from 0.001 to 0.004, it's observed that the energy-saving factor (ES) decreases, while the throughput increases, which aligns with the expected behavior.
- As the short sleep duration ($\frac{1}{\beta}$) increases, it is observed that the energy-saving factor (ES) also increases, while the throughput decreases. This behavior can be attributed to the fact that during longer short sleep durations, fewer components of the UE remain active, resulting in increased energy savings for the UE.

Using the same parameters as in Fig. 8, we have plotted the trade-off between average energy consumption and throughput, varying the arrival rate from 0.001

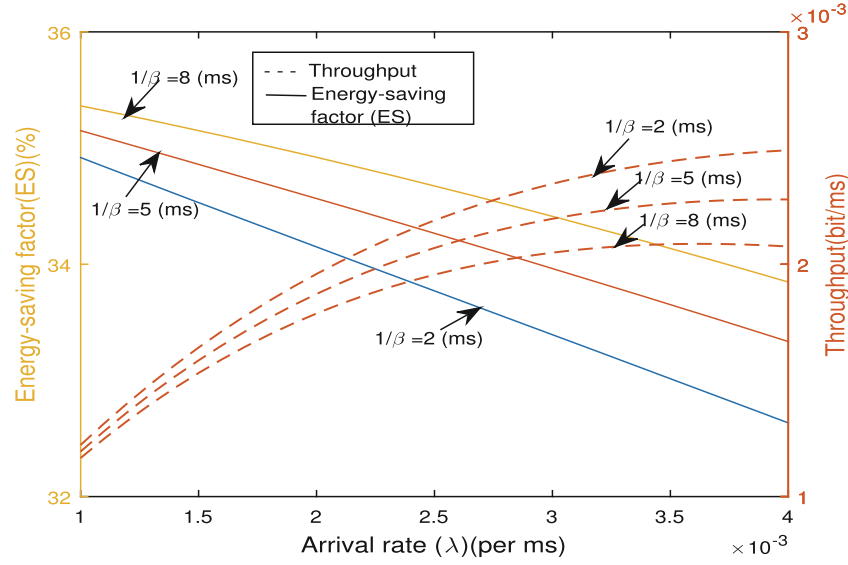


Fig. 8. Trade-off between Energy-saving factor and throughput as arrival rate varies

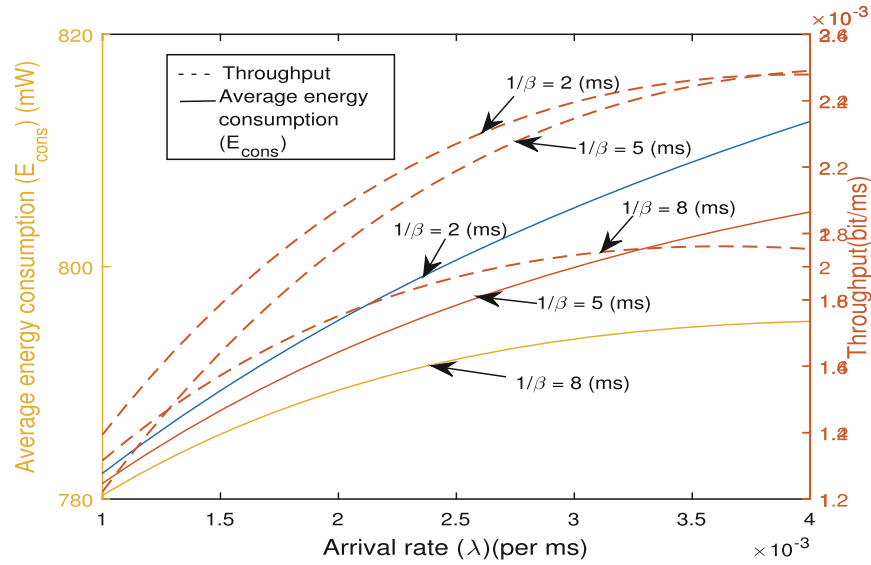


Fig. 9. Trade-off between average energy consumption and throughput as arrival rate varies

to 0.004 for the MRGP model. The energy consumption values in different states are as follows: $EC_L = 11.4$, $EC_S = 11.4$, $EC_A = 1680.2$, $EC_I = 1060$, $EC_B = 1060$, $EC_O = 1280.04$ [7]. From Fig. 9, the following observations can be made:

- As the arrival rate (λ) varies from 0.001 to 0.004, both the average energy consumption (E_{cons}) and throughput increase as expected.
- As the short sleep duration ($\frac{1}{\beta}$) increases, the average energy consumption (E_{cons}) decreases, resulting in a decrease in throughput. This occurs because, in a longer short sleep duration, only a few components of the UE are active,

while most of the components remain inactive. As a result, the longer short sleep duration reduces the average energy consumption (E_{cons}) of the UE, but it also decreases the throughput.

5 Conclusion and Future Work

In this study, we delve into the exploration of stochastic models designed to investigate User Equipment (UE) energy-saving mechanisms, with a particular emphasis on the DRX (Discontinuous Reception) mechanism. Our research introduces a variety of continuous-time models grounded in different assumptions and DRX states. We conduct analyses centered on energy savings within the UE, making use of steady-state probabilities and expected sojourn times as key metrics. Our results indicate that the Markov model outperforms the semi-Markov model in terms of overall performance.

Furthermore, we present an MRGP (Markov Regenerative Process) model tailored specifically for MD-DRX, providing insights into the intricate trade-offs between energy-saving, energy consumption, and throughput. We carry out a comparative evaluation of our proposed MRGP model against existing models, underlining its superior accuracy and capability for real-time simulations.

Our future research endeavors will focus on potential improvements by exploring different state sojourn time distributions and extending the modeling framework to encompass LTE-A, 5G, and NR networks. Additionally, we plan to develop a validation testbed for the DRX mechanism and conduct comparative studies with other LTE-DRX mechanisms, with the ultimate goal of integrating our findings into the evolving landscape of LTE-6G networks.

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