Reliability modeling of a flight module of a tethered high-altitude telecommunication platform

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Abstract—In the current paper we study the reliability of the functioning of a flight module of a tethered high-altitude telecommunication platform, utilizing the k-out-of-n-F model. Several variants of such a model are considered, including the ones taking into account the diverse failure logic, i.e., the dependence of system failures on the configuration of failed components. An algorithm has been developed and implemented in software that allows calculating the reliability function of such a system, its mean time to failure and variance of its uptime, as well as the quantiles of the distribution function. In order to verify the practicality of the approach, numerical and graphical reliability analysis for two most common special cases was performed.

Index Terms—tethered high-altitude telecommunication platform, complex k-out-of-n systems, mathematical modeling, reliability function, heterogeneity

I. INTRODUCTION

At present, tethered high-altitude unmanned telecommunication platforms, the long-term operation of which is ensured by transmitting electricity from the ground to the board via a thin cable, have found wide application and have been widely developed [1], [2], [4], [10]. Tethered high-altitude platforms (HAPs) occupy an intermediate position between satellite systems and terrestrial systems, the equipment of which (cellular base stations, radio relay and radar equipment, etc.) is located in high-rise structures. Tethered HAPs are highly economical compared to expensive satellite systems, and surpass the terrestrial telecommunications systems in terms of the vastness of the telecommunications and video coverage area. Given the breadth of practical application of tethered unmanned HAPs, both in civil and defense industries, research centers in the leading countries of the world are intensively working on the design and implementation of a new generation of such platforms.

Tethered platforms of a new generation should be able to lift and hold a payload of significant weight (up to 15 kg) at heights of 100-150 m, for which it is necessary to ensure the transmission of high power energy (up to 10-15kW) from the ground to the unmanned aerial platform. Such platforms, that is especially important for long-term operation without lowering to the ground, must have high reliability characteristics, both of the main components and of the HAP as a whole [5].

Note, that unlike autonomous unmanned aerial vehicles (UAVs), for tethered unmanned HAPs, reliability indicators are of crucial importance due to the fact that the main area of its application is solving problems related to the long-term operation (tens of hours) without lowering the unmanned flight module to the ground.

For the ground-to-air power transmission system, this is achieved by selecting highly reliable electronic components and reserving “bottlenecks” in both ground-based (AC-AC 380V/1000V) and airborne (AC-DC 1000V/48V) voltage converters [6]. In order to increase strength of a thin cable-rope, in a turbulent atmosphere, a Kevlar thread is used along with copper wires and optical fiber. Various measures are envisaged in the event of emergency situations. For example, when power feeding from the ground from an industrial source of 380V / 50Hz is interrupted (even for a short time), the system of accident-free landing of the copter is implemented by switching to a backup on-board battery, which ensures the safety of the unmanned module itself and the payload. When operating in mountainous conditions (or near high-rise buildings), when GPS / GLONASS signals are weakened or completely disappear, a switching to a backup local navigation system with ground beacons [7]–[9] is provided. A description of the hardware and software of the tethered high-altitude
telecommunication platform “Albatros”, which was developed under the guidance and participation of a number of authors of the current book, is given in [10].

High reliability of the unmanned flight module is achieved by selecting propulsion systems with a long time between failures, redundancy of individual elements of the control system and, most importantly, the use of a multi-rotor architecture. A multi-rotor UAV is a system consisting of \( n \) rotors (elements) arranged uniformly in a circle and pairwise symmetrically with respect to the center of the circle [11]. There are various modifications of UAVs. The most common architectures are quad-, hexa-, and octocopters. Obviously, the higher the redundancy ratio, the higher the reliability of the system, therefore, in practice, in the development and implementation of both tethered and autonomous HAPs based on UAVs, flight modules with 6 or 8 rotors are most often used. The failure of the entire system depends not only on the number of the failed engines but on their location. The failure of adjacent engines is more likely to lead to system failure than the failures of distant engines. To study the reliability of this type of complex systems in the world literature, \( k \)-out-of-\( n \) mathematical models are effectively used, which have wide practical applications [12]. Taking into account the features of the functioning of an unmanned HAP significantly complicates the study of the \( k \)-out-of-\( n \) models, considered in this paper, compared to the known ones, and is an urgent task. Adequate mathematical models for flight modules of above configurations are 3-out-of-6 : \( F \) and 4-out-of-8 : \( F \) models. Numerical and graphical reliability analysis for these two special cases will be discussed in detail in subsections 3.1, 3.2, 4.1 and 4.2.

In some of our previous works (see, for example [13], [14]) analytical and numerical methods were used to study the sensitivity analysis of the reliability characteristics of the \( k \)-out-of-\( n \) type models to the shape of their components lifetime and repair distribution functions.

The current paper is a continuation of previous works of the authors, and it is aimed at reliability study of some non-repairable \( k \)-out-of-\( n \) models, calculating their reliability characteristics and analyzing their dependency on the model parameters, such as the coefficient of variation of the elements’ lifetime and various distributions of their lifetime. Section III presents reliability measures of a \( k \)-out-of-\( n \) model, which failure depends only on the number of its failed components. A complication of this model is considered in section IV. Within this model, the failure of the system depends not only on the number of failed elements, but also on their location relative to each other. Moreover, it is assumed that the components’ lifetime distributions are different.

II. PRELIMINARIES

One of the ways to increase a system’s reliability is its components redundancy. We consider a complex redundant system that consists of \( n \) components in hot standby, that fails when \( k \) of them fail. As a model of this system, we propose a complex circular \( k \)-out-of-\( n : F \) model, whose failure depends not only on the number of failed components, but also on their position in it. Further, for brevity, we will omit the symbol \( F \) from the notation. We will consider the operation of the system until its first failure and denote by \( A_i \) the random time of failure-free operation of an \( i \)-th component, and by \( A_i(t) = \text{Pr}(A_i(t) \leq t) \) its cumulative distribution function (c.d.f.). Random variables (r.v.’s) \( A_i, (i = 1, 2, \ldots) \) are supposed to be independent identically distributed (i.i.d.).

In order to calculate the reliability of such a model, we use the following vector notation for the system components’ states: \( \mathbf{j} = (j_1, j_2, \ldots, j_n) \), where \( j_i = 1 \), if an \( i \)-th component is in a state of failure (“DOWN” state), and \( j_i = 0 \), if an \( i \)-th component is in an operating state (“UP” state). In this case \( j = \sum_{1 \leq i \leq n} j_i \) is the number of failed components. Further, denote by

\[
E = \{ \mathbf{j} = (j_1, j_2, \ldots, j_n) : (j_i \in (0, 1)) \}
\]

the system’s set of states, consisting of disjoint subsets \( E_0 \) and \( \bar{E}_0 \) which denote the subsets of its “DOWN” and “UP” states, correspondingly. Note that the description of these subsets is a separate problem depending on the specifics of the system architecture, its operating conditions and other factors, and must be solved in each specific case.

Define a stochastic process \( \mathbf{J} = \{ \mathbf{J}(t) : t \geq 0 \} \) over the set of states \( E \) by the following relation:

\[
\mathbf{J}(t) = \mathbf{j}, \text{ if at time } t \text{ the system is in state } \mathbf{j} \in E
\]

and denote by \( T \) the system’s lifetime,

\[
T = \inf \{ t : \mathbf{J}(t) \in E_0 \} = \sup \{ t : \mathbf{J}(t) \in \bar{E}_0 \}.
\]

In the current paper we are interested in calculating the following reliability characteristics of the system:

- reliability function of the system

\[
R(t) = 1 - F(t) = \text{Pr}(T > t | \mathbf{J}(0) \in \bar{E}_0), \quad (1)
\]

- mean time to failure (MTTF) of the system:

\[
\mathbb{E}[T] = \int_{0}^{\infty} R(t)dt, \quad (2)
\]

- variance of the time to failure of the system:

\[
\text{Var}[T] = \int_{0}^{\infty} (t - \mathbb{E}[T])^2 dF(t), \quad (3)
\]

- time to failure of the system with a given probability \( \text{Pr}(T < t) = \gamma \), i.e. the \( \gamma \)-quantile of its distribution function:

\[
t_{1-\gamma} = R^{-1}(1 - \gamma). \quad (4)
\]

III. RELIABILITY MODELLING OF A COMPLEX HOMOGENEOUS SYSTEM

Consider a complex homogeneous system (with identical components, i.e. when \( A_i(t) = A(t), i = 1, \ldots, n \)), which failure depends only on the number of its failed components. Denote by \( D^i_n(t) \) an event that exactly \( i \) from \( n \) components of
the system are in “DOWN” state up to time \( t \). For the homogeneous case its probability is

\[
P\{D^i_n(t)\} = \binom{n}{i} (1 - A(t))^{n-i} A(t)^i,
\]

and the corresponding system reliability function is

\[
R(t) = P\{T > t\} = \sum_{i=0}^{k-1} \binom{n}{i} (1 - A(t))^{n-i} A(t)^i,
\]

and the mean time to first system failure (MTTF) is

\[
E[T] = \sum_{i=0}^{k-1} \binom{n}{i} \int_0^\infty (1 - A(t))^{n-i} A(t)^i dt.
\]

A. Example 1: homogeneous 3-out-of-6 model

If we don’t take into account the location of the failing components, this system is operational until 3 out of 6 rotors fail. For the considered example, we restrict ourselves to the case of exponential distribution of the uptime of the system’s components \( A(t) = 1 - e^{-\alpha t}, t \geq 0 \), under which the distributions of the residual and initial uptime of the components coincide. This makes it possible to obtain not only explicit analytical expressions for the reliability characteristics of the system, but also the final numerical results. Namely, according to (1)–(3):

- the system reliability function takes the form:
  \[
  R(t) = \sum_{i=0}^{2} \frac{6!}{i!(6-i)!} e^{-(6-i)\alpha t}(1 - e^{-\alpha t})^i = 15e^{-4\alpha t} - 24e^{-5\alpha t} + 10e^{-6\alpha t},
  \]
- the MTTF of the system equals
  \[
  E[T] = \int_0^\infty (15e^{-4\alpha t} - 24e^{-5\alpha t} + 10e^{-6\alpha t}) dt = \frac{37}{60\alpha},
  \]
- the variance of the time to failure of the system equals
  \[
  Var[T] = \frac{469}{3600\alpha^2}.
  \]

In a particular case, with the value of the mean time of failure-free operation of the components \( \alpha^{-1} = 1 \) and the value \( \gamma = 0.1 \), we get:

- \( R(t) = 15e^{-4t} - 24e^{-5t} + 10e^{-6t} \)
- \( E[T] \approx 0.6167 \)
- \( Var[T] \approx 0.1303 \)
- \( t_{1-\gamma} = 0.2243 \)

Fig. 1 shows the dependence of the system reliability function on time and the value of \( t_{1-\gamma} \) in this particular case.

B. Example 2: homogeneous 4-out-of-8 model

Consider a homogeneous 4-out-of-8 model, whose identical components again have exponentially distributed lifetimes, \( A(t) \sim \text{Exp}(\alpha) \), with the same mean value \( E[A] = \alpha^{-1} \). For this example it holds

\[
R(t) = 56e^{-5\alpha t} - 140e^{-6\alpha t} + 120e^{-7\alpha t} - 35e^{-8\alpha t},
\]

Fig. 2 shows the reliability function \( R(t) \) curves for the distributions above, and Table II represents the calculated values of the MTTF for the considered model. Black, red and blue colors indicate the graphs of the reliability function for \( \Gamma \), GW and L\(\text{N} \) distributions, respectively. The behavior of the curves of the reliability function corresponds to the values of the MTTF. The lower \( v \), the higher the reliability \( R(t) \).
of the model. Conversely, the lowest reliability falls on the highest \( v \). Moreover, as \( v < 1 \), the reliability function \( R(t) \) is asymptotically insensitive to the shape of lifetime distribution, but sensitive to its coefficient of variation. As \( t \approx 2 \), the model is fully unreliable.

![Reliability function](image)

**Fig. 2. Reliability function \( R(t) \) for the homogeneous 4-out-of-8 model**

**Table II**

<table>
<thead>
<tr>
<th>( v )</th>
<th>( \Gamma )</th>
<th>( GW )</th>
<th>( LaN )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v = 0.1 )</td>
<td>0.9819</td>
<td>0.9947</td>
<td>0.9808</td>
</tr>
<tr>
<td>( v = 0.5 )</td>
<td>0.8631</td>
<td>0.8806</td>
<td>0.8496</td>
</tr>
<tr>
<td>( v = 1 )</td>
<td>0.6345</td>
<td>0.6345</td>
<td>0.6614</td>
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<tr>
<td>( v = 5 )</td>
<td>0.0035</td>
<td>0.0617</td>
<td>0.2007</td>
</tr>
<tr>
<td>( v = 10 )</td>
<td>0.0001</td>
<td>0.0157</td>
<td>0.1114</td>
</tr>
</tbody>
</table>

**IV. RELIABILITY MODELLING OF A COMPLEX HETEROGENEOUS SYSTEM**

For the reliability study of a complex heterogeneous system, which failure depends on the position of the failed components in the system, and the lifetimes c.d.f’s \( A_i(t) \) are different, denote by

\[
p_j(t) = \prod_{1 \leq i \leq n} (1 - A_i(t))^{1-j_i} A_i(t)^{j_i}
\]

the probability of its state \( j = (j_1, \ldots, j_n) \) at time \( t \). At that the probabilities of “UP” and “DOWN” states of the system to the time \( t \) are equal correspondingly to

\[
P(UP) = \sum_{j \in E_0} p_j(t), \quad P(DOWN) = \sum_{j \in E_0} p_j(t),
\]

The corresponding reliability function of the system equals to

\[
R(t) = \sum_{j \in E_0} \prod_{1 \leq i \leq n} (1 - A_i(t))^{1-j_i} A_i(t)^{j_i}.
\]  

**A. Example 3: \((2,3)\)-out-of-6 model**

Continue the study of the 3-out-of-6 model, but now we’ll complicate it assuming that the system is operational until either two adjacent engines fail, or any three engines fail. We will designate such a model as \((2,3)\)-out-of-6. Suppose that the system is homogeneous with exponential distributions of the uptime of its components with parameter \( \alpha \).

For convenience, we renumber the states of the system in binary code, i.e., assign the state \( j = (j_1, j_2, \ldots, j_n) \) to its number according to the following formula:

\[
j = \sum_{1 \leq i \leq n} (1 - j_i)2^{n-i}.
\]

Then the set of “UP” states \( E_0 \) consists of states with the following numbers:

\[E_0 = \{0, 1, 2, 4, 5, 8, 9, 10, 16, 17, 18, 20, 32, 34, 36, 40\}.

Thus, the reliability function has the following form:

\[
R(t) = \sum_{j \in E_0} \prod_{1 \leq i \leq n} e^{-\alpha t} (1 - e^{-\alpha t})^{1-j_i} = 9e^{-4\alpha t} - 12e^{-5\alpha t} + 4e^{-6\alpha t}.
\]

And, according to (1)–(3), \( E[T] = 31/\alpha \), \( Var[T] = 433/\alpha^2 \).

Then, in a particular case, with the value of the MTTF of the components \( \alpha^{-1} = 1 \) and the value \( \gamma = 0.1 \), we get:

- \( R(t) = 9e^{-4t} - 12e^{-5t} + 4e^{-6t} \)
- \( E[T] \approx 0.5167 \)
- \( Var[T] \approx 0.1203 \)
- \( t_{1-\gamma} = 0.1476 \) (which is expectedly less than that in example 1)

Fig. 3 shows the reliability function curve and the value of \( t_{1-\gamma} \) in this particular case.

![Reliability function](image)
B. Example 4: heterogeneous 4-out-of-8 and (3,4)-out-of-8 models

Consider the 4-out-of-8 model, whose components’ lifetimes have the same distribution with different mean values. Suppose that the system fails if any 4 components fail or any 3 adjacent components fail. According to the notation introduced above this is a (3,4)-out-of-8 model. Let $A_i(t) \sim Exp(\alpha_i)$, where $\alpha_i$, means the failure intensity of an $i$-th component, $i = 0, 8$.

Again, we number the system states using formula (11) and obtain the expression for the reliability function using formula (10):

\[
R(t) = e^{-\sum_{i=1}^{8} \alpha_i t} + (1 - e^{-\alpha t})^{e^{-\sum_{i=1}^{7} \alpha_i t}} + 1 - e^{-\alpha t} \left( \sum_{i=1}^{5} \alpha_i \right)^{t} + ... + (1 - e^{-\alpha t})(1 - e^{-\alpha t})^{e^{-\sum_{i=1}^{3} \alpha_i t}} + ... + (1 - e^{-\alpha t})(1 - e^{-\alpha t})^{e^{-\sum_{i=1}^{3} \alpha_i t}} + (1 - e^{-\alpha t})^{e^{-\sum_{i=1}^{3} \alpha_i t}} \cdot (1 - e^{-\alpha t})^{e^{-\sum_{i=1}^{3} \alpha_i t}}.
\]

The obtained complete analytical expressions for $R(t)$ and $E[T]$ are not given in the article due to their cumbersome nature. But substituting $\alpha_i = \alpha$ in these expressions we obtain a homogeneous case, and formulas for $R(t)$ and $E[T]$ take the following form:

\[
R(t) = 48e^{-5\alpha t} - 116e^{-6\alpha t} + 96e^{-7\alpha t} - 27e^{-8\alpha t}.
\]

\[
E[T] = \frac{509}{840\alpha} = 0.605952\alpha^{-1}.
\]

If the system’s failure doesn’t depend on the adjacency of the failed components and $\alpha_i = \alpha$, these reliability measures turn to expressions (8)–(9) as expected.

In order to consider these results graphically (Fig. 4), let $\alpha_i = 1 + 0.2 \cdot i$, $i = 1, 8$, which in practice can be due to different initial elapsed lifetimes of the rotors, or to the fact that they were produced by different manufacturers. Here Case 1 corresponds to the reliability function (13) for the heterogeneous (3,4)-out-of-8 model, and Case 2 shows the reliability function for the heterogeneous 4-out-of-8 model which doesn’t take into account the adjacency of the failed components. Case 3 curve is given just for the reference and corresponds to the homogeneous 4-out-of-8 model.

This figure shows that despite different scenarios of the system’s failure, the reliability function curves are very close to each other. MTTF values also confirms this fact (Table III). The curve for Case 1 is expectedly steeper than the one for Case 2, because Case 1 additionally takes into account system failures due to failure of adjacent components.

V. CONCLUSION

The current study is focused on the reliability investigation of some non-repairable $k$-out-of-$n$ models, which are adequate mathematical models for flight modules of UAV-based tethered telecommunication platforms. Analytical expressions and algorithms are presented for calculating the reliability function, the mean time to the first system failure and other reliability measures. The results are accompanied by numerical examples to verify the practicality of the approach. In addition to taking into account the number of failed elements and the proximity of failed elements, an analysis was carried out of the influence on the system reliability function of such factors as the coefficient of variation of the lifetime of elements and various (non-exponential) distributions of their lifetime.

In the further work, the proposed approach for reliability analysis of tethered high-altitude unmanned telecommunication platforms will be extended in certain aspects, such as the influence of a multi-state random external environment on the reliability characteristics and the increase in the functional load on the rest of the operable elements.

REFERENCES


Fig. 4. Reliability function for the heterogeneous models ($A_i(t) \sim \text{Exp}(\alpha_i)$)


