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Stochastic modeling for energy efficiency in modified directional discontinuous reception for LTE-5G networks

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Summary

Fifth-generation (5G) networks deal with high-frequency data rates, ultra-low latency, more reliability, massive network capacity, more availability, and a more uniform user experience. To validate the high-frequency rates, 5G networks engage beam searching operation. By adopting a beam searching state between the short and long sleep, one can reduce the system's delay. The energy consumption of user equipment (UE) in 5G networks is much higher than in the 4G networks. To reduce the energy consumption and increase the energy saving in UE, Long-Term Evolution (LTE)-5G networks adopt the discontinuous reception (DRX) scheme with a fixed number of short sleep. LTE-DRX without beam search operation (i.e., beam alignment) cannot work in 5G networks. Hence, keeping this scenario in mind, we have modeled a new modified directional discontinuous reception (MD-DRX) mechanism for LTE-5G networks. The MD-DRX mechanism captures the behavior of a beam searching, an inactive, an active, a long sleep, an ON, and a short sleep states. The short sleep state consists of a maximum M short sleep. To get the optimal energy saving and energy consumption (i.e., energy efficiency) from the MD-DRX mechanism, it is required to check the system's throughput. The trade-off between energy saving/energy consumption and throughput will provide the system's optimal energy saving and optimal energy consumption. In this paper, we have obtained the system's optimal energy saving and throughput by optimizing the maximum short sleep and short sleep duration. To get the energy efficiency for LTE-5G networks, the trade-off between average energy consumption/energy saving and throughput is shown.

KEYWORDS

delay, energy consumption, LTE-5G, Markov regenerative process, throughput

1 | INTRODUCTION

The Third Generation Partnership Project (3GPP) started one fastest wireless technology, named Long-Term Evolution-fifth generation (LTE-5G). LTE-5G is meant for higher data speed, lower latency, greater capacity, and ubiquitous. The extremely impressive increase in demand for any smart device with its better quality of service (QoS) leads to the standardization of the fifth-generation New Radio (5G-NR) wireless system.

Since 2009, LTE has entered the commercial markets and is accessible in more than 10 countries with fast enlarging users. The expected user throughput was 50 Mbps for uplink and 100 Mbps for downlink, with less than 5 ms user-plane latency.[1] It is concluded that it is important but not straightforward to understand the actual user-centric network performance for LTE networks. Then, in 2013, the LTE network was developed as LTE-Advanced (LTE-A), that is, 4G, which provides a higher bandwidth and a high-speed data rate. Huang et al[2] studied LTE-A networks which can work directly from end users.

Although LTE-A provided a higher bandwidth capacity, high-speed data rate, more power saving, better QoS, and so on, the delay is more. Keeping it in mind, Tseng et al[3] represent an extensive analysis of the average power consumption and average delay of the DRX mechanism.

LTE-DRX is managed by the radio resource control (RRC) layer at eNobe-B (eNB). Through a physical downlink control channel (PDCCH), RRC sends the packets signal to UE. RRC performs in two modes: $RRC_{CONNECTED}$ and RRC_{Idle}. As mentioned in the LTE standard for RRC,[4] a device or UE can be either in $RRC_{CONNECTED}$ or in RRC_{Idle} mode whenever it is attached to the network. In RRC_{Idle} mode, the UE performs only paging cycle functions where it neither transmits nor receives any data packets. On the other hand, all the data interchange between eNB and UE happens during $RRC_{CONNECTED}$ mode. Since all data packet transmission happens during the $RRC_{CONNECTED}$ mode, this mode is only responsible for consuming more energy of UE. Therefore, the main task is to obtain optimal energy saving and energy consumption in the MD-DRX mechanism on $RRC_{CONNECTED}$ mode.

The MD-DRX mechanism captures the operations of a beam searching state, an inactive state, an active state, a long sleep state, an ON state, and a short sleep state. MD-DRX mechanism is the modification of the directional DRX (D-DRX) mechanism. The short sleep consists of a maximum M number of short sleep. Agiwal et al[5] discussed the D-DRX mechanism, which represents a separate beam searching state. The beam searching state allows UE to detect the best transmission–reception beam pair through the beamforming algorithm shown in Figure 1. The beam pair detection is over by using a directional antenna. The detection of the best beam pair (TX–RX) and for sweeping the total transmitted beams by UE beams in matrix form $A \times B$,[6] it needs a beam searching period t_{bs}. Here, A represents the number of transmitted beams (from TX0 to TX(A-1)), and B represents the number of UE beams (from RX0 to RX(B-1)) as shown in Figure 2. Agiwal et al[5] presented a beam searching state in which the UE looks for the best beam pair after every sleep cycle but shows higher energy consumption in UE. To overcome this difficulty, we proposed the MD-DRX mechanism. In the MD-DRX mechanism, the system goes for a beam searching state after taking a maximum number of M short sleep. If it finds no beam pair in beam searching state, it goes for a long sleep. After completing the long sleep, if it finds no beam pair again, it goes for a beam searching state. If it finds the best beam pair in this state, it goes for an ON period for a short duration. After the expiration of the ON period, it goes to an active state.

In the beam searching state, UE measures the reference signal of various transmission beams (A beams) with each one of its receiver beams (B beams).[6,7] The UE must be sweep the total of $A \times B$ beams. After UE sweeps the transmitted beam, a period exists between the UE and the base station (BS) named as feedback period. When UE recognize the best beam pair, it sends the feedback period to BS which is shown in Figure 1.

In beam searching state, one indication occurs, named as power-saving indication (PSI). Based on the PSI, the system will stay in the beam searching state. Philip and Balakrishnan[8] proposed a semi-Markov model (SMM) to show the behavior of the hybrid directional DRX (HD-DRX) mechanism for 5G communication with beam searching state. From the EH-DRX mechanism, they enhanced the power saving at the UE/M2M device up to 16%. They have not considered the packets interchange (i.e., transmits/receive) between UE and BS during reoccurring beam patterns. In the proposed MD-DRX mechanism, we have considered both the data packets interchange between UE and BS and also with eNB.

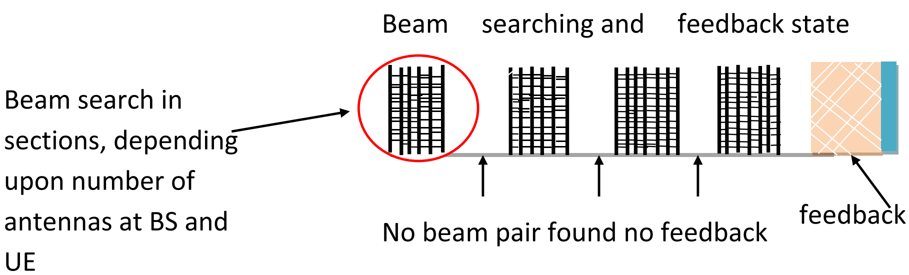

FIGURE 1 Beam searching algorithm

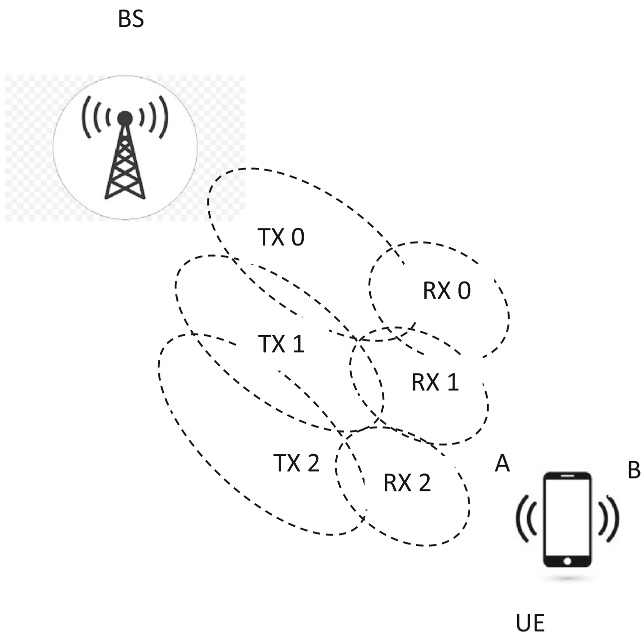

FIGURE 2 Analog beamforming process

LTE-5G networks cannot be imaginable without considering the beam searching state after completing short and long sleep. Thus, to obtain optimal energy saving and reduce energy consumption, there is a need to explore more into the performance analysis of the MD-DRX mechanism for LTE-5G communications. To get the optimal energy saving and energy consumption (i.e., energy efficiency) from the MD-DRX mechanism, we need to check the system's throughput. The trade-off between energy saving/energy consumption and throughput will provide the system's optimal energy saving and energy consumption. In a practical scenario, capturing the energy consumption by the system via stochastic modeling is a difficult task. Based on the mathematical tractable, the stochastic model is developed in this paper, which is almost closer to the practical scenario.

The rest of the paper is presented as follows: Section 2 describes the state of the art of DRX mechanism for LTE/LTE-A/LTE-5G networks. In Section 3, the MD-DRX mechanism is explained. In Section 4, the stochastic model using Markov regenerative process (MRGP) is developed. Section 5 demonstrates the performance measures. Section 6 displays the energy saving and throughput optimization. Numerical illustration is presented in Section 7. Finally, in Section 8, the conclusion and future work are given.

2 | STATE OF THE ART

In the previous research work, various efforts were made on the DRX mechanism in LTE/LTE-A networks to get maximum energy efficiency. In today's world, one of the well-known communication technology is machine-type communication (MTC). To reduce the energy consumption in MTC UE, Balasubramanya et al[9] proposed a modified D-DRX (MD-DRX) mechanism incorporating the quick sleeping indication (QSI), which is an energy-efficient solution for simple and low-mobility MTC UE devices. At the UE receiver, DRX with QSI mechanism provides 45% up-gradation in the energy efficiency and 66% decrement in the computational complexity. In LTE/LTE-A networks, DRX is a popular mechanism for energy saving. Depending on the traffic running at the UE, Gautam et al[10] proposed a switching technique to get the more energy saving in the DRX mechanism. Gautam et al[10] mapped the switching technology in the DRX mechanism with the $M^X/G/1$ vacation queueing system along with N-policy. Another necessary criterion to achieve the maximum energy efficiency in wireless communication is less energy consumption in UE. Because of the high speed of processing signal on LTE-A (4G) UE, the energy consumption is higher and latency is lower. By using the DRX mechanism, one can get the optimal energy consumption and optimal latency.[11] Arunsundar et al[11] studied the effects of a short cycle, long cycle, arrival rate, and short cycle timer value (which are the component of the DRX mechanism) on energy consumption by using Markov chain and semi-Markov modeling. DRX for LTE-A communication saves power, but at the same time, the delay may increase. Various networks have various traffic arrival patterns and require different optimal balances between transmission delay and energy efficiency. Therefore, the trade-off between transmission delay and energy efficiency is necessary to achieve the optimal utilization of the DRX mechanism in a wide range of cases. To get the energy efficiency of the DRX mechanism in LTE-A networks, Wu and Park[12] proposed a new metric named as real power-saving (RPS) factor. In RPS, all the states and state transitions are considered in the DRX specification.

Although LTE-A (4G) reached to fulfill all the demands of customers, the delay is more when the power saving is more. Therefore, to save more power and to get the optimal delay, LTE-A (5G) is configured with extra features like beamforming or beam searching state. Beamforming is the key facilitator, which facilitates the millimeter wave (mmWave)-based directional communication in 5G-NR systems. mmWave in 5G wireless networks can come across the growing demands of mobile subscriptions and data rates. Jasim et al[13] discussed an effective accessible scheme for analog beamforming cascaded codebooks. When the DRX mechanism is used in mmWave-5G networks, it is required to incorporate with the beam searching procedure by making it less effective. By introducing beam searching state in the DRX mechanism, it is sometimes called as HD-DRX or D-DRX mechanism. Philip and Malarkodi[14] proposed an SMM to show the performance of HD-DRX for 5G (4G BS) communication with beam search. Dual connectivity with both LTE eNB (4G BS) and NR NodeB (5G New RAN BS) from UE is possible in HD-DRX system.[6] Sallam et al[6] used semi-Markov process (SMP) to obtain the UE state probability, and by using the probability, they found power-saving factor and average delay. In Maheshwari et al,[7] for directional air interface, a new D-DRX is discussed in mmWave-enabled 5G communications. For directional beam alignment between gNodeBs (gNBs) and UE, the D-DRX mechanism highlights the importance of the beam searching state after every sleep cycle. To reduce the barrier to power saving, Maheshwari et al[7] proposed three new D-DRX mechanisms: integrated D-DRX (ID-DRX), standalone D-DRX (SD-DRX), and cooperative D-DRX (CD-DRX). For all the three different D-DRX mechanisms, they obtained the probabilistic results of UE's delay and power saving. In wireless technology, one of the necessary techniques is artificial intelligence (AI), by which one can understand and predict the packet's arrival-time values from the real wireless traffic. Memon et al[15] discussed a DRX mechanism based on AI (AI-DRX) for energy efficiency in LTE-5G connected devices. To facilitate dynamic short and long sleep cycles in DRX, Memon et al[15] propose an AI-DRX algorithm for multiple beam communications.

Without human interference, another communication technology has received substantial attention nowadays, termed MTC. In MTC, intelligent machines can communicate with each other. Mehmood et al[16] has used the DRX mechanism in MTC devices to deal with energy-saving factors and wake-up latency. For the analytical model, they have used SMM. Maheswari et al[17] proposed a signaling-based DRX mechanism to upgrade UE's energy saving in LTE-5G networks by removing the concept of beam searching. They obtained the probabilistic estimation of delay and power saving by analyzing SMM. Energy harvesting in the DRX mechanism upgrades the performance of the machine-to-machine (M2M) devices by fully utilizing the extra sleep cycles. Philip and Balakrishnan[8] discussed a model combining energy harvesting-DRX (EH-DRX) with a beam-aware mechanism in the M2M mmWave LTE-5G networks to upgrade the energy efficiency.

LTE-A (4G) networks offer much faster peak speeds for downlink and uplink data packet arrivals. Although LTE-A (4G) processes a higher speed of data, has lower latency, and saves power but energy consumption is high, that is, energy efficiency is low. To get more energy efficiency, that is, optimal delay, energy consumption, and energy saving, LTE-5G comes with a new feature named beam searching or beamforming. Beamforming is used to identify the most efficient route for delivering. To save the power in UE, DRX mechanism is used in LTE/LTE-A networks by switching off the radio circuitry. DRX mechanism in LTE-A networks saves the power, but the delay is more. In Wu and Park,[12] it is mentioned that LTE/LTE-A networks with DRX mechanism saves the power, but delay may increase. In previous studies,[14,15,17] it is mentioned that LTE-5G with DRX mechanism (by introducing beam searching state) can reduce the delay in the system and also provide energy efficiency. This leads to the motivation of studying the DRX mechanism for LTE-5G communication.

In the previous research work, many authors analyzed the performance of the DRX mechanism for LTE/LTE-A/LTE-5G networks using the Markov model (MM), queueing model (QM), and SMM. Among them, Wang et al[18] and Tseng et al[3] used the MM to show the performance of the DRX mechanism for LTE and LTE-A networks. Baek and Choi[19] and Gautam et al[10] used the $M/G/1$ queue and $M^X/G/1$ queue with multiple vacations and N-policy to display the performance of the DRX mechanism for LTE and LTE-A networks. Recently, Philip and Malarkodi,[14] Sallam et al.,[6] Mehmood et al.,[16] Maheshwari et al.,[7,17] Philip and Balakrishnan,[8] and Arunsundar et al[11] presented the SMM to get the probabilistic results of the UE's power saving, energy consumption, and delay based on the DRX mechanism for

LTE-A and LTE-5G networks. However, no work has been recorded in literature by focusing on optimize the energy saving and energy consumption with the use of system's throughput and stochastic model using MRGP. Hence, in this study, we develop a stochastic model for the MD-DRX mechanism in LTE-5G networks using MRGP.

2.1 | Comparison with existing work

In this subsection, we compare the performance results of this manuscript with the existing works. To get a brief idea about the comparison of our work with some existing works, Table 1 is provided.

According to the research works mentioned in Table 1, a few analytical models has studied LTE/LTE-A/LTE-5G for the DRX mechanism. Among them, only five articles have presented the SMM for DRX mechanism in LTE-5G communication.[6–8,14,22] Sallam et al[6] focused on the comparative study of different DRX mechanisms considering the performance measures only power saving and delay. Maheshwari et al[7] focused on D-DRX mechanism for mmWave-enabled LTE-5G communication and Maheshwari et al[22] focused on D-DRX mechanism for LTE-5G communication considering performance measures only power saving and delay. For the validation of the model, Sallam et al[6] investigated a comparative study among different DRX mechanisms but only for the power-saving factor. Among these papers, only Maheshwari et al[22] focused on D-DRX mechanism for LTE-5G which is closer to our proposed model. They assumed some simplistic assumptions in their modeling. Hence, the notable research works for comparison is the research works in Maheshwari et al.[22]

Some of the detailed drawbacks of Maheshwari et al[22] and the strength of our paper are as follows:

- *Advantage of D-DRX*: Maheshwari et al[17] proposed a SMM to show the performance of the D-DRX mechanism for LTE-5G networks. In the D-DRX mechanism, after the active state is expired, that is, no packets are waiting in the queue, UE transits to the ON state. If no arrival occurs in ON state, the system switches to short sleep state or long sleep state. In between short sleep state, the UE can repeat n_{ss} times short sleep cycle. In each short sleep cycle, the UE sleeps for short sleep time t_{ss}. At the expiry of each short sleep time t_{ss}, if the system finds empty, the UE transits to the beam search state. If any packets arrive during this state, the UE transits to the active state. After n_{ss} times short sleep cycle, if the system is empty, the UE switches to a long sleep cycle. At the expiry of a long sleep cycle, if the system finds empty, the UE switches to the beam search state.
- *Disadvantage of D-DRX*: In the D-DRX mechanism, after the expiry of each short sleep time t_{ss}, if the system is empty, the UE switches to beam search state. The energy saving will be less because of switching beam search state after each short sleep.

In the proposed MD-DRX mechanism, after taking the maximum M number of short sleep in a short sleep state, if the system finds empty, the UE switches to beam searching state. In this way, the UE can save more power.

- *Advantage of Maheshwari et al[22] paper*: They show the behavior of the performance measures (power saving and delay), which varies as the parameter value changes.
- *Disadvantage of Maheshwari et al[22] paper*: They have not mentioned the system's throughput, which is related to obtain the system efficiency. Basically, to obtain the optimal energy saving and energy consumption, one should check the trade-off between the throughput and the performance measures. This will provide the optimal value of the performance measures (like energy saving and energy consumption).

In this research work, all the optimality issues regarding energy saving and throughput, considering the optimization problem and energy consumption are mentioned.

Therefore, based on the above comparison, it is clear that the proposed model may be more precise and closer to making the real-time simulation results.

3 | MD-DRX MECHANISM FOR LTE-5G

In LTE networks, any DRX mechanism is managed by the RRC layer. A device or UE in LTE can be either in $RRC_{CONNECTED}$ mode or RRC_{Idle} mode. In RRC_{Idle} mode, the UE only monitors the paging signals during the paging cycle operation where UE neither receives nor transmits any data packets, whereas all the interchange of data between eNB and UE happens during $RRC_{CONNECTED}$ mode. Therefore, in this paper, we consider mainly the use of MD-DRX mechanism in $RRC_{CONNECTED}$ mode and is shown in Figure 3. The DRX mechanism is meant for power saving by discontinuously monitoring the PDCCH. Here, we explain the MD-DRX mechanism of UE for LTE-5G networks. The

TABLE 1 Comparison related papers with our work

Related papers	LTE	LTE-A	LTE-5G	Energy consumption (E_{cons})	Energy saving (ES)	Delay (Dl)	Throughput	Analytical/ simulation models
Baek and Choi (2011)[19]	√	-	-	√	√	√	-	M/G/1 queue with multiple vacation
Huang et al. (2012)[2]	-	√	-	√	√	√	√	Simulation
Wang et al. (2016)[18]	√	-	-	√	√	√	-	Markov
Tseng et al. (2016)[3]	-	√	-	√	√	√	-	Markov
Corcoran et al. (2017)[20]	-	-	√	√	√	√	-	Simulation
Philip and Malarkodi (2019)[14]	-	-	√	-	√	√	-	Semi-Markov
Sallam et al. (2018)[6]	-	-	√	-	√	√	-	Semi-Markov
Mehmood et al. (2019)[16]	-	√	-	√	√	√	-	Semi-Markov
Verma et al. (2019)[21]	-	-	√	√	-	√	-	Simulation
Memon et al. (2019)[15]	-	-	√	-	√	√	-	Simulation
Maheshwari et al. (2018)[22]	-	-	√	-	√	√	-	Semi-Markov
Maheshwari et al. (2018)[7]	-	-	√	-	√	√	-	Semi-Markov
Gautam et al. (2020)[10]	-	√	-	√	-	√	-	$M^X/G/1$ queue with multiple vacation N-policy
Maheshwari et al. (2021)[17]	-	-	√	√	√	√	-	Semi-Markov
Philip and Balakrishnan (2020)[8]	-	-	√	√	√	√	-	Semi-Markov
Arunsundar et al. (2020)[11]	-	√	-	√	√	-	-	Semi-Markov
Wu and Park (2021)[12]	-	√	-	√	√	√	-	Simulation
Our proposed paper	-	-	√	√	√	-	√	Markov regenerative process

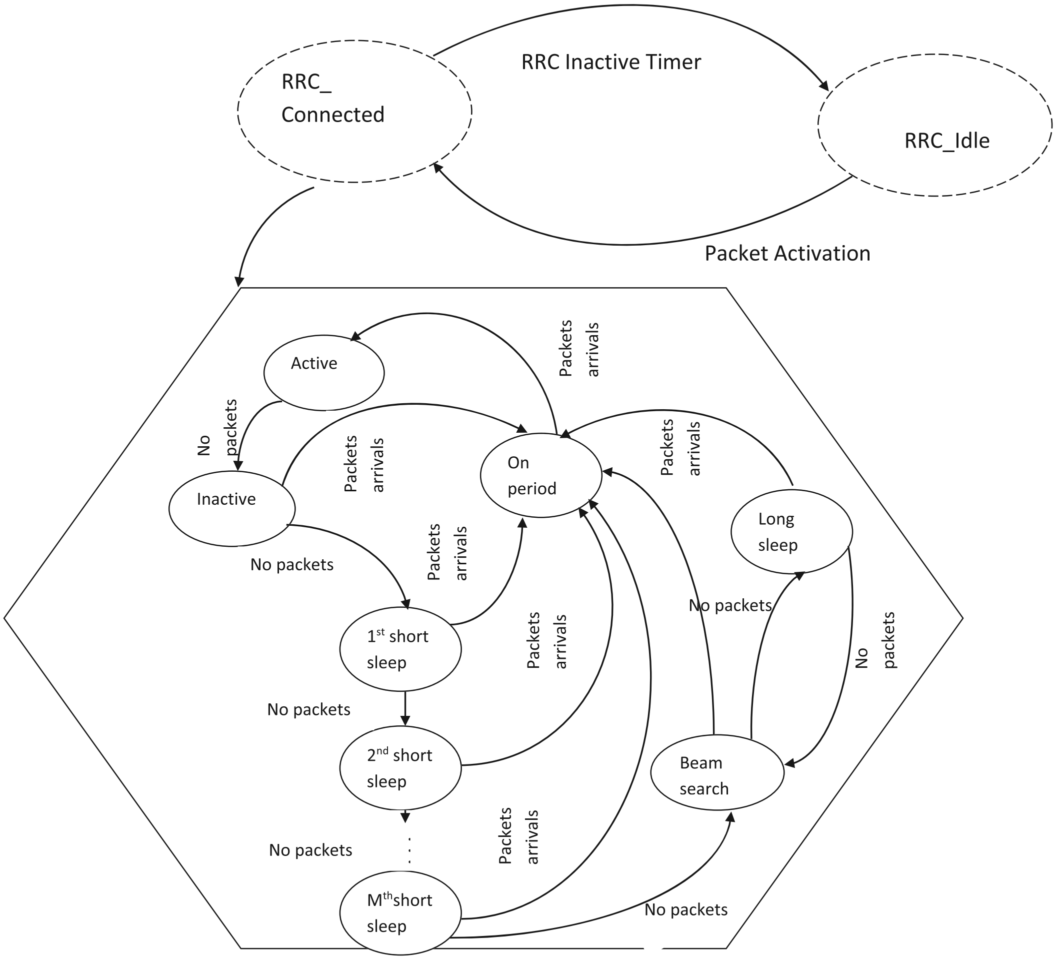

FIGURE 3 LTE-DRX in RRC connected for the proposed model as in current 5G system

proposed MD-DRX mechanism captures the operations of a beam searching state, an inactive state, an active state, a long sleep state, an ON state, and a short sleep state. The short sleep consists of a maximum M number of short sleep. The states are as follows:

- *Active state*: During the active state, the UE receives/transmits the data packets, that is, it is the state where UE is available or active and provides service to each packet.
- *Inactive state*: After the active state, the UE enters the inactive state for a random duration of time. If any packet arrives before expiration of inactive duration, it will wait till the duration is over, then it switches to the ON period. At the completion of inactive duration, if no packets is available in the buffer, the UE switches to short sleep state.
- *Short sleep state*: During short sleep periods, the UE turns off most of its components. The UE takes a maximum M number of short sleep in this state. If no data packets are available after completing each short sleep, the UE takes a maximum M number of short sleep in this state. If no data packets arrive till the Mth short sleep, the UE goes to the beam searching state. If any data packets arrive during the short sleep, it will wait till the duration is over, then it switches to the ON period state.
- *Beam searching state*: UE switches to this state after completion of a maximum M short sleep and long sleep. In this state, the UE measures the reference signal of available beams to align the best beam pair. During this beam alignment process, if the UE receives a data packet, it will wait till the duration is over, then it switches to the ON period state. If no beam pair and data packets are found during the beam searching state, it goes for the long sleep state.
- *Long sleep state*: During the long sleep state, the UE can receives the data packets but cannot served them. At the completion of long sleep duration, if the system finds empty, it goes for the beam searching state. On the other hand,

at the completion of long sleep duration, if the buffer is loaded with minimum one data packet, the UE goes for the ON period.

- *ON period state*: At the completion of inactive state, short sleep state, beam searching state, and long sleep state, if the buffer is loaded with minimum one data packet, the UE enters to the ON period. It is the state where UE takes a very short amount of time before starting the service of packets, that is, before going to the active state.

After service completion of each data packet, the UE enters the inactive state for a random duration of time. At the expiry of the inactive duration, if any data packets are found in the buffer, the UE moves to the ON period. After spending some random amount of time (which is a little amount of time to start the system) in this state, the UE goes for the active state. During the ON period, any packets may arrive, which will be served in the active period. At the completion of an active state, if the system finds empty, it enters the inactive state. If no data packets are found during the inactive state, it goes for the first short sleep for a random amount of time. If no data packets are available during the first short sleep, the UE moves to second short sleep, and this procedure will be continue up to a maximum M number of short sleep. If any data packets are available during the short sleep, the UE enters to the ON period. If no data packets arrive during the short sleep, the UE enters to the beam searching state. If any data packets are found during the beam searching state, it will wait till the duration is over, then it switches to the ON state. If no packet indication is found in the beam searching state, the UE moves to the long sleep state for a random amount of time. If any packets arrive during the long sleep, it has to wait till the long sleep time is over. At the completion of a long sleep duration, it goes for the ON period. After completion of the ON period, the UE moves to the active period. If there is no packet indication during the long sleep, after spending a random amount of time, the UE enters the Beam searching state. In the beam searching state, the UE waits for the first arrival. If any packet is found in the beam searching state, after completing the random duration, it enters the ON period state.

We consider that the service time of each packet in the active state follows a deterministic distribution with parameter $\frac{1}{\mu}$. Also, we assume that the packets' inter-arrival times are independent and are exponentially distributed with parameter λ.

4 | THE MRGP MODEL

The system state bivariate stochastic process for the MD-DRX mechanism is consider as $\{(N(t),S(t));t\geq 0\}$, where $\{N(t);t\geq 0\}$ indicates the number of packets in the system at time t and $\{S(t);t\geq 0\}$ denotes the state of the system at time t with state space Ω. Consider $\Omega\equiv\varsigma_{(i,j)}\in\{0,1,...\}\times\{0,1,2,3,4,5,6,...,M+4\}$. Here, $i\in\{0,1,2,...\}$ denotes the number of packets in the system (if any packets is being served, incorporating the one also) and $j\in 0,1,2,3,4,5,6,...,M+4$ indicates the states of the system, that is, whether the system is in beam searching state (B), or in inactive system (I), or in active state (A), or in long sleep state (L), or in ON state (0), or first short sleep, or in second short sleep, ..., or in Mth short sleep (which are in between short sleep [S]). Let $A_{i,2};i=1,2,...$ indicates the system is in the active state with i number of packets, $I_{i,1};i=0,1,...$ indicates it is in the inactive state with i number of packets, $B_{i,0};i=0,1,...$ indicates it is in the beam searching state with i number of packets, $L_{i,3},i=0,1,2,...$ indicates it is in the long sleep state with i number of packets, $O_{i,4},i=1,2,...$ indicates it is in the ON state with i number of packets, $S_{i,j},i=0,1,2,...,j=5,6,...,M+4$ indicates it is in the first short sleep, second short sleep, ... Mth short sleep with i number of packets. Hence, $\Omega=\{B_{i,0};i=0,1,...,I_{i,1};i=0,1,...,A_{i,2};i=1,2,...,L_{i,3};i=0,1,...,O_{i,4};i=1,2,...,S_{i,j};i=0,1,...,j=5,6,...,M+4\}$. The process $\{(N(t),S(t));t\geq 0\}$ is not a continuous time Markov chain (CTMC) as the sojourn times in each states are not exponentially distributed. Also, $\{(N(t),S(t));t\geq 0\}$ is not even a SMP since between two sequential service completion points (as the service completion points are the regeneration epochs), the system state with number of packets can be changed due to new packets arrival before completing the service of previous packets. Also, the states (beam search state, inactive state, active state long sleep state, ON state, and short sleep state) do not proceed immediately as the new arrivals occur.

Hence, because of the remaining time of the new arrivals during the time duration in beam search state, inactive state, active state, long sleep state, ON state, short sleep state (i.e., in first short sleep, second short sleep, ..., Mth short sleep), all the states completion points are the regeneration points. Assume that the succession of epochs $\{\tau_n,n\geq 0\}$ where the process $\{(N(t),S(t));t\geq 0\}$ is observed. Define $\{\tau_n,n\geq 0\}$ as the regeneration epochs at which beam search period, inactive period, service, long sleep period, ON period, and short sleep period are terminated. Let $\zeta_n\in\Omega$ represent the state of the system at these regeneration epochs. The elements of Ω are called regeneration epochs as they occur at the service completion point, inactive period completion epoch, long sleep period completion epoch, ON period completion epoch, and short sleep completion epoch. Hence, we can say that $\{\zeta_n, n \geq 0\}$ is a DTMC with one-step transition probability matrix $K(\infty)$. By observing the behavior of the stochastic process $\{(N(t), S(t)); t \geq 0\}$ at these epochs, we conclude that the sequence $\{\zeta_n, \tau_n\}$ is an embedded Markov renewal sequence and $\{(N(t), S(t)); t \geq 0\}$ is a MRGP.[23]

4.1 | Description of the proposed model

The proposed model is discussed in detail in this section. Let the packet arrival in the system conforms to a Poisson process with parameter λ. The inter-arrival time of packets are independent and are exponentially distributed with parameter λ. The service time, inactive time, beam searching duration, short sleep duration, long sleep duration, and ON duration are follow deterministic distributions with parameter $\frac{1}{\mu}, \frac{1}{\nu}, \frac{1}{\zeta}, \frac{1}{\beta}, \frac{1}{\gamma}, \frac{1}{\alpha}$ respectively. The state transition diagram for

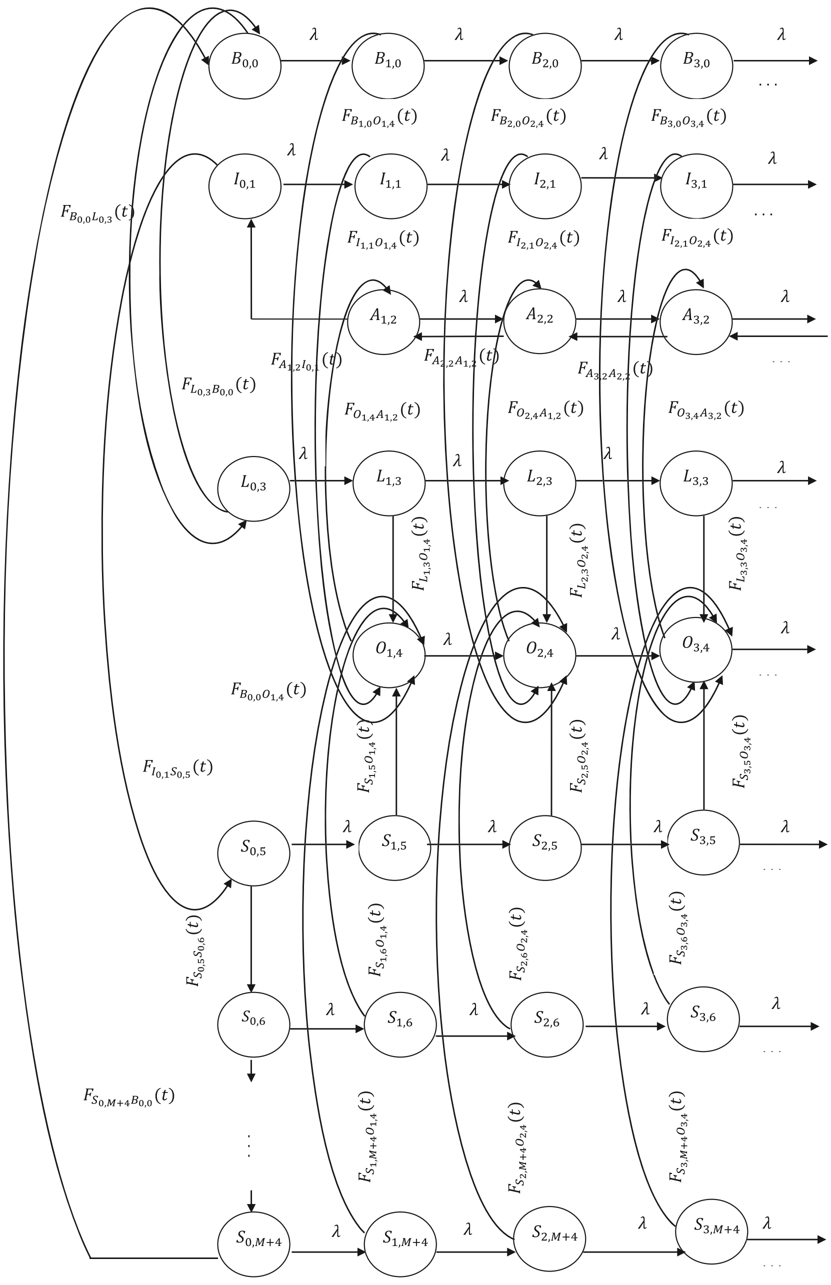

FIGURE 4 State transition diagram for the proposed MRGP model

TABLE 2 Cumulative distribution functions with parameters

CDF	Parameter	CDF	Parameter
$F_{B_{i,j}O_{i,j+4}}(t); i=1,2,...,$ $j=0$	$\frac{1}{\zeta}$	$F_{B_{0,0}L_{0,3}}(t)$	$\frac{1}{\zeta}$
$F_{I_{0,1}S_{0,5}}(t)$	$\frac{1}{\nu}$	$F_{I_{i,j}O_{i,j+3}}(t);$ $i=1,2,...,j=1$	$\frac{1}{\nu}$
$F_{A_{1,2}I_{0,1}}(t)$	$\frac{1}{\mu}$	$F_{S_{i,j}O_{i,j-k}}(t);$ $i=1,2,...,j-k=4$ $j=5,6,...,M+4, k=1,2,...,M$	$\frac{1}{\beta}$
$F_{A_{i,2}A_{i-1,2}}(t); i=2,3,...,$	$\frac{1}{\mu}$	$F_{L_{0,3}B_{0,0}}(t)$	$\frac{1}{\gamma}$
$F_{L_{i,j}O_{i,j+1}}(t); i=1,2,...,$ $j=3$	$\frac{1}{\gamma}$	$F_{O_{i,j}A_{i,j-2}}(t)$	$\frac{1}{\alpha}$
$F_{S_{0,j}S_{0,j+1}}(t); j=5,6,...,M+4$	$\frac{1}{\beta}$	$F_{S_{0,M+4}B_{0,0}}(t)$	$\frac{1}{\beta}$

the proposed model is shown in Figure 4. Table 2 explains all the cumulative distribution functions (CDFs) with the corresponding parameters.

Theorem 1. The steady-state probabilities for the MRGP model are given as

$$\phi_{B_{0,0}} = \frac{1}{G}\left[\left(e^{-\frac{\lambda}{\beta}}\right)^{M-1} e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\nu}\right)}\left(1-e^{-\frac{\lambda}{\beta}}\right) + \frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\left(e^{-\frac{\lambda}{\beta}}\right)^{M}}{\left(1-e^{-\lambda\left(\frac{1}{\gamma}+\frac{1}{\zeta}\right)}\right)}\left(1-e^{-\frac{\lambda}{\gamma}}\right)e^{-\frac{\lambda}{\mu}}\right], \tag{1}$$

$$\phi_{I_{0,1}} = \frac{1}{G}\left(1-e^{-\frac{\lambda}{\mu}}\right), \tag{2}$$

$$\phi_{L} = \frac{1}{G}\frac{e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\nu}\right)}\left(e^{-\frac{\lambda}{\beta}}\right)^{M}}{\left(1-e^{-\lambda\left(\frac{1}{\gamma}+\frac{1}{\zeta}\right)}\right)}\left(1-e^{-\frac{\lambda}{\zeta}}\right), \tag{3}$$

$$\begin{aligned}\phi_{O_{i,4}} = \frac{1}{G}\Bigg[&\left(1-e^{\frac{\lambda}{\nu}}\right)^{i} e^{-\frac{\lambda}{\mu}}\left(1-e^{-\frac{\lambda}{\nu}}\right) + \left(e^{-\frac{\lambda}{\beta}}\right)^{j-5}\left(1-e^{-\frac{\lambda}{\beta}}\right)^{i} e^{-\lambda\left(\frac{1}{\nu}+\frac{1}{\mu}\right)} \\ &\left(1-e^{-\frac{\lambda}{\beta}}\right) + \left(1-e^{-\frac{\lambda}{\zeta}}\right)^{i}\frac{e^{-\frac{\lambda}{\nu}}\left(e^{-\frac{\lambda}{\beta}}\right)^{M}}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}\right)} e^{-\frac{\lambda}{\mu}}\left(1-e^{-\frac{\lambda}{\zeta}}\right)\Bigg]; \\ &i=0,1,...,j=5,6,...,M+4,\end{aligned} \tag{4}$$

$$\phi_{S} = \frac{1}{G}\left[e^{-\frac{\lambda}{\mu}}\left\{\left(1-e^{-\frac{\lambda}{\nu}}\right) + e^{-\frac{\lambda}{\nu}}\left(1-\left(e^{-\frac{\lambda}{\beta}}\right)^{M}\right)\right\}\right], \tag{5}$$

where G is given in Appendix 3. The steady-state probability for $\phi_{A_{i,2}}$ is given in Appendix 3.

Proof. Details of the proof are given in Appendix 3.

Here, the system state probabilities $\phi_n \forall n \in \Omega$ exist as $\lambda < \mu$.

Note: Since we assume that the arrival of packet follows Poisson process, the service time follows deterministic distribution and infinite capacity single server model; hence, it is a $M/D/1$ queue. From the steady-state probabilities of EMC of $M/D/1$ queue mentioned in Medhi,[24], p261 we conclude that $\rho\left(=\frac{\lambda}{\mu}\right)$ should be less than 1 to obtain the stability condition from the results. Therefore, $\lambda < \mu$ is the stability condition for the proposed model.

5 | PERFORMANCE MEASURES

Performance measures for the proposed model are presented in this section. Section 5.1 deals with the system state probabilities of the Markov generative process model.

5.1 | System state probabilities

Let P_L, P_B, P_I, P_S, P_A, P_O be the steady- state probabilities for being in long sleep state, beam searching state, inactive state, short sleep state, active state, and ON state, respectively.

$$P_L = \frac{1}{G}\frac{e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\nu}\right)}\left(e^{-\frac{\lambda}{\beta}}\right)^M}{\left(1-e^{-\lambda\left(\frac{1}{\gamma}+\frac{1}{\zeta}\right)}\right)}\left(1-e^{-\frac{\lambda}{\zeta}}\right), \tag{6}$$

$$P_B = \frac{1}{G}\left[\left(e^{-\frac{\lambda}{\beta}}\right)^{M-1} e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\nu}\right)}\left(1-e^{-\frac{\lambda}{\beta}}\right)+\frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\left(e^{-\frac{\lambda}{\beta}}\right)^M}{\left(1-e^{-\lambda\left(\frac{1}{\gamma}+\frac{1}{\zeta}\right)}\right)}\left(1-e^{-\frac{\lambda}{\gamma}}\right)e^{-\frac{\lambda}{\mu}}\right], \tag{7}$$

$$P_I = \frac{1}{G}\left(1-e^{-\frac{\lambda}{\mu}}\right), \tag{8}$$

$$P_S = \frac{1}{G}\left[e^{-\frac{\lambda}{\mu}}\left\{\left(1-e^{-\frac{\lambda}{\nu}}\right)+e^{-\frac{\lambda}{\nu}}\left(1-\left(e^{-\frac{\lambda}{\beta}}\right)^M\right)\right\}\right], \tag{9}$$

$$P_O = \frac{1}{G}\left[e^{\frac{\lambda}{\nu}}e^{-\frac{\lambda}{\mu}}\left(1-e^{-\frac{\lambda}{\nu}}\right)+e^{\frac{\lambda}{\beta}}\frac{\left(1-\left(e^{-\frac{\lambda}{\beta}}\right)^M\right)}{1-e^{-\frac{\lambda}{\beta}}}e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\nu}\right)}\left(1-e^{-\frac{\lambda}{\beta}}\right)+\frac{e^{-\lambda\left(\frac{1}{\nu}+\frac{1}{\mu}\right)}e^{\frac{\lambda}{\zeta}}\left(e^{-\frac{\lambda}{\beta}}\right)^M}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}\right)}\left(1-e^{-\frac{\lambda}{\zeta}}\right)\right], \tag{10}$$

$$P_A = \sum_{i=6}^{\infty}\phi_{A_{i,2}}. \tag{11}$$

The result of P_A is given in Appendix 3.

5.2 | Energy-saving factor

Now, we obtain the energy-saving factor (ES), which is defined as the percentage of time the device (UE) spends in sleep state, i.e., in short sleep and long sleep. Using the Equations (6) and (9), we obtain

$$ES = (P_L + P_S) \times 100\%. \tag{12}$$

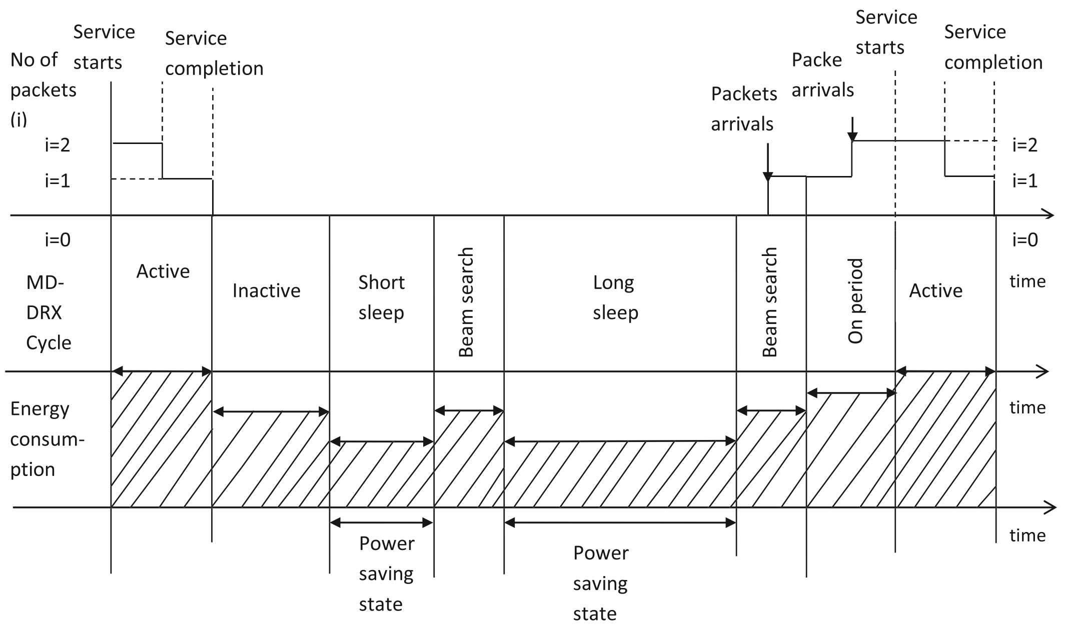

FIGURE 5 Energy consumption in MD-DRX mechanism

5.3 | Average energy consumption

In this subsection, we derive the average energy consumption spent by the UE's transceiver. UE consumes a different amount of power in different states.[17] Let EC_L, EC_S, EC_A, EC_I, EC_B, EC_O be the energy consumed during the long sleep state, short sleep state, active state, inactivity timer, beam searching state, and ON state.

Hence, using Equations (6)–(11), we obtain the average energy consumption by UE:

$$E_{cons} = P_L EC_L + P_S EC_S + P_A EC_A + P_I EC_I + P_B EC_B + P_O EC_O. \tag{13}$$

The energy consumption in the MD-DRX mechanism in LTE-5G is demonstrated in Figure 5.

5.4 | Throughput of the system

In this subsection, we have derived the throughput of the system. To get the optimal energy consumption and energy saving, we need to find out the system's throughput. The maximum throughput is the fraction of time the system is available to customers multiplied by the servicing rate for the infinite capacity model.[25] Therefore, the system's throughput is defined as

$$\text{Throughput} = P_A \times \mu, \tag{14}$$

where P_A is given in Equation (11) and μ is the service rate.

6 | ENERGY SAVING AND THROUGHPUT OPTIMIZATION

In this section, we optimize the energy saving and throughput to get the maximum efficiency from the proposed model. Hence, we choose a multi-objective optimization problem. The decision variables are the short sleep duration $\frac{1}{\beta}$ and a maximum number of short sleep M.

There are many different ways to set up the optimization problem. Among them, we choose M and $\frac{1}{\beta}$ as the objective function such that energy saving and throughput are optimized at the same value, which is imposed as a constraint.[26] That is, we need to obtain optimal values of M and $\frac{1}{\beta}$ such that energy saving and throughput are optimized.

To obtain the optimum energy saving $ES\left(M,\frac{1}{\beta}\right)$ and optimal throughput $throughput\left(M,\frac{1}{\beta}\right)$, the optimal values of the control parameters M and $\frac{1}{\beta}$, namely, M^* and $\frac{1}{\beta}^*$ are determined as in Gautam et al.[10] Depends on the dynamic optimization concept, we calculate the optimal values of $\left(M^*,\frac{1}{\beta}^*\right)$.[27] Following the procedure as Gautam et al,[10] we obtain $\left(M^*,\frac{1}{\beta}^*\right)$ which gives the optimal throughput at the point energy saving is also optimized.Step 1: Set $M=1$. Find

$\frac{1}{\beta}^*(M)=min\left\{\frac{1}{\beta}=[2,5,8,10,16,20,32,40,64,80,128,160,256,320,512,640]|-\right.$

$\left.ES\left(M,\frac{1}{\beta}+c\right)-ES\left(M,\frac{1}{\beta}\right), throughput\left(M,\frac{1}{\beta}+c\right)-throughput\left(M,\frac{1}{\beta}\right)\right\}>0$ and compute $ES\left(M,\frac{1}{\beta}\right)$ and $throughput\left(M,\frac{1}{\beta}\right)$.

Note that this code should run separately for the similar increments between the parameter $\frac{1}{\beta}$ values (the values of $\frac{1}{\beta}$ can be in the specified values in 5G).

Step 2: Compute $\frac{1}{\beta}^*(M+1)$, $ES\left(M+1,\frac{1}{\beta}^*(M+1)\right)$ and $throughput\left(M+1,\frac{1}{\beta}^*(M+1)\right)$.

Step 3: If $ES\left(M+1,\frac{1}{\beta}^*(M+1)\right)>ES\left(M,\frac{1}{\beta}^*(M)\right)$ and $throughput\left(M+1,\frac{1}{\beta}^*(M+1)\right)>throughput\left(M,\frac{1}{\beta}^*(M)\right)$, then stop. The optimal values are $\left(M^*,\frac{1}{\beta}^*\right)=\left(M^*,\frac{1}{\beta}^*(M)\right)$.
Otherwise, go to Step 2.

7 | NUMERICAL ILLUSTRATION

Using MATLAB, we numerically show the trade-off between the performance measures presented in Section 5. On utilization of above-mentioned optimization technique with the parameter values $\lambda=0.004$, $\nu=0.02$, $\gamma=0.0125$, $\mu=0.05$, $\zeta=1.5, \alpha=1$, we obtain the optimal value of $M^*=6$ and $\frac{1}{\beta}^*=2$ for which optimal energy-saving factor $ES\left(M^*,\frac{1}{\beta}^*(M)\right)=32.6340$ and optimal throughput $throughput\left(M^*,\frac{1}{\beta}^*(M)\right)=0.0025$.

To show the trade-off between the performance measures, we have used the parameters as given in Table 3.[20]

7.1 | Trade-off between energy-saving factor and throughput as arrival rate varies

Now, by setting the parameter values as $\nu=0.02$, $\mu=0.05$, $\zeta=0.01, \alpha=1$, $\gamma=0.0125$, and $M^*=6$, the trade-off between energy-saving factor and throughput as arrival rate varies from 0.001 to 0.004 and is shown in Figure 6. From Figure 6, the observations are as follows:

TABLE 3 Possible parameter values for MD-DRX mechanism

Parameter	Possible values
Inactive duration (ms)	1, 2, 3, 4, 5, 6, 8, 10, 20, 30, 40, 50, 60, 80, 100, 750, 1280, 1920, 2560
Short sleep duration (ms)	2, 5, 8, 10, 16, 20, 32, 40, 64, 80, 128, 160, 256, 320, 512, 640
ON duration (ms)	1, 2, 3, 4, 5, 6, 8, 10, 20, 30, 40, 50, 60, 80, 100, 200
Long sleep duration (ms)	10, 20, 32, 40, 64, 80, 128, 160, 256, 320, 512, 640, 1024, 1280, 2048, 2560

- As the arrival rate (λ) varies from 0.001 to 0.004, the energy-saving factor (ES) decreases, and the throughput increases as it should be.
- As the short sleep duration $\left(\frac{1}{\beta}\right)$ increases, the energy-saving factor (ES) increases whereas the throughput decreases. The reason is that, in short sleep duration, only a few components of UE are working, and most of the components are not working. Hence, the longer short sleep duration increases the energy-saving factor (ES) of UE.

7.2 | Trade-off between average energy consumption and throughput as arrival rate varies

By considering the same parameter as in Figure 6, the trade-off between average energy consumption and throughput as arrival rate varies from 0.001 to 0.004 and is presented in Figure 7. The energy consumption in different states are $EC_L = 11.4$, $EC_S = 11.4$, $EC_A = 1680.2$, $EC_I = 1060$, $EC_B = 1060$, $EC_O = 1280.04$.[17] From Figure 7, our observations are as follows:

- As the arrival rate (λ) varies from 0.001 to 0.004, the average energy consumption (E_{cons}) and the throughput increase as they should be.
- As the short sleep duration $\left(\frac{1}{\beta}\right)$ increases, the average energy consumption (E_{cons}) decreases, and hence, the throughput also decreases. This happens because, in short sleep duration, only a few components of UE are working, and most of the components are not working. Hence, the longer short sleep duration decreases the average energy consumption (E_{cons}) of UE.

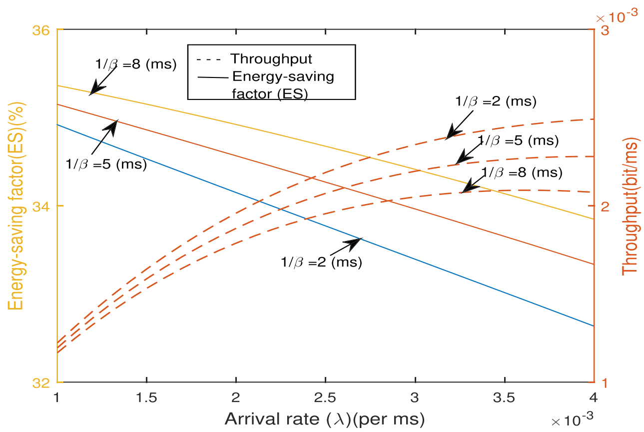

FIGURE 6 Trade-off between energy-saving factor and throughput as arrival rate varies

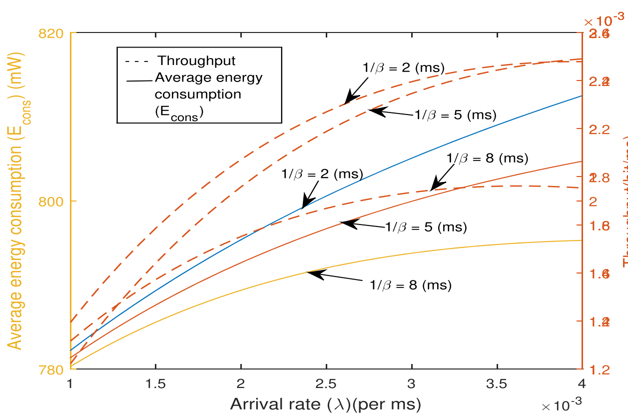

FIGURE 7 Trade-off between average energy consumption and throughput as arrival rate varies

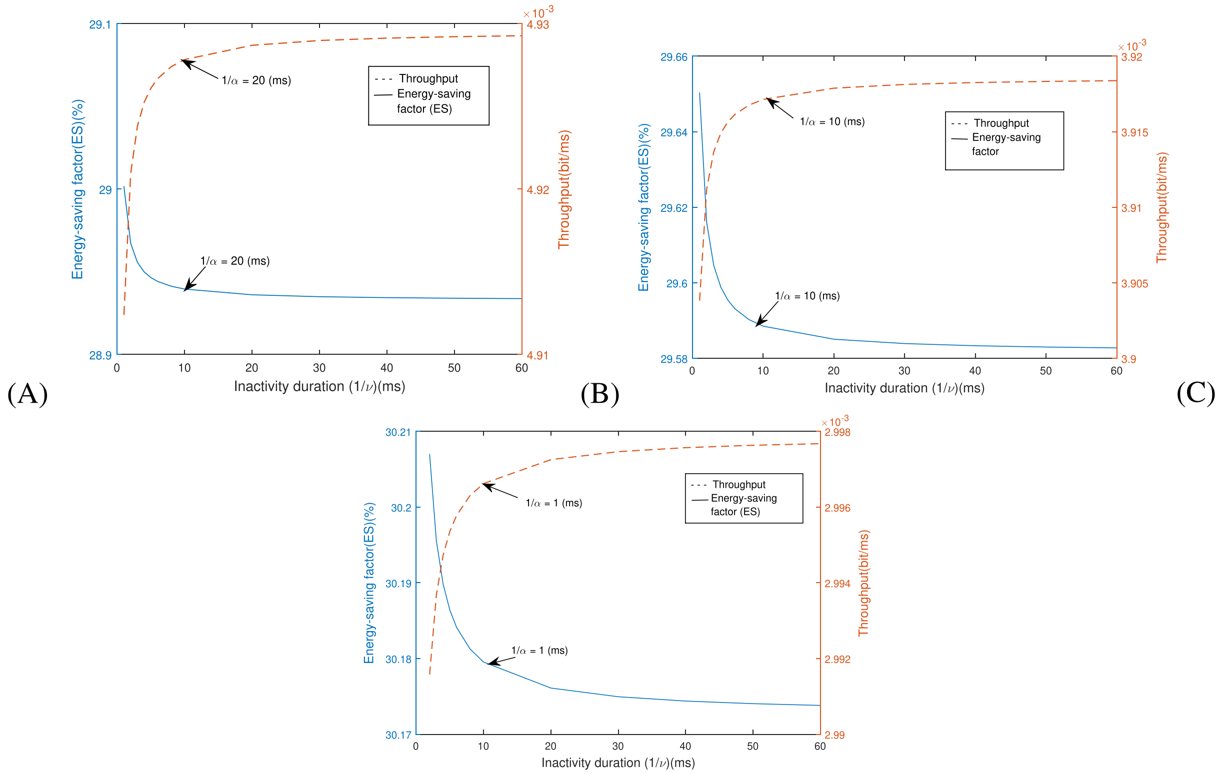

FIGURE 8 Trade-off between energy-saving factor and throughput as inactivity duration varies

7.3 | Trade-off between energy-saving factor and throughput as inactivity duration varies

Presuming the parameter as $\lambda = 0.004$, $\mu = 0.05$, $\zeta = 0.01, \gamma = 0.0125$, $\frac{1}{\beta}^* = 2$, and $M^* = 6$, the trade-off between energy-saving factor (ES) and throughput as inactivity duration $\left(\frac{1}{\nu}\right)$ varies in between $[1,2,3,4,5,6,8,10,20,30,40,50,60]$ is shown in Figure 8.

It is observed from Figure 8A-C that

- As the inactivity duration $\left(\frac{1}{\nu}\right)$ varies in between $[1,2,3,4,5,6,8,10,20,30,40,50,60]$, the energy-saving factor (ES) decreases whereas the throughput increases. The reason is that, as the inactive duration of the UE increases, the components of UE will be active for a longer period, which will increase the average energy consumption and decrease the energy-saving factor.
- As the ON duration $\left(\frac{1}{\alpha}\right)$ increases, the energy-saving factor (ES) decreases whereas throughput increases. This happens because, as the time duration in the ON state is higher, the components of UE will be active for a longer period, which will increase average energy consumption and decrease energy-saving factors.

7.4 | Trade-off between average energy consumption and throughput as inactivity duration varies

Considering the parameter as $\lambda = 0.004$, $\mu = 0.05$, $\zeta = 0.01, \gamma = 0.0125$, $\frac{1}{\beta}^* = 2$, and $M^* = 6$, the trade-off between average energy consumption and throughput as inactivity duration $\left(\frac{1}{\nu}\right)$ varies in between $[1,2,3,4,5,6,8,10,20,30,40,50,60]$ is shown in Figure 9.

From Figure 9A-C, the observation are as follows:

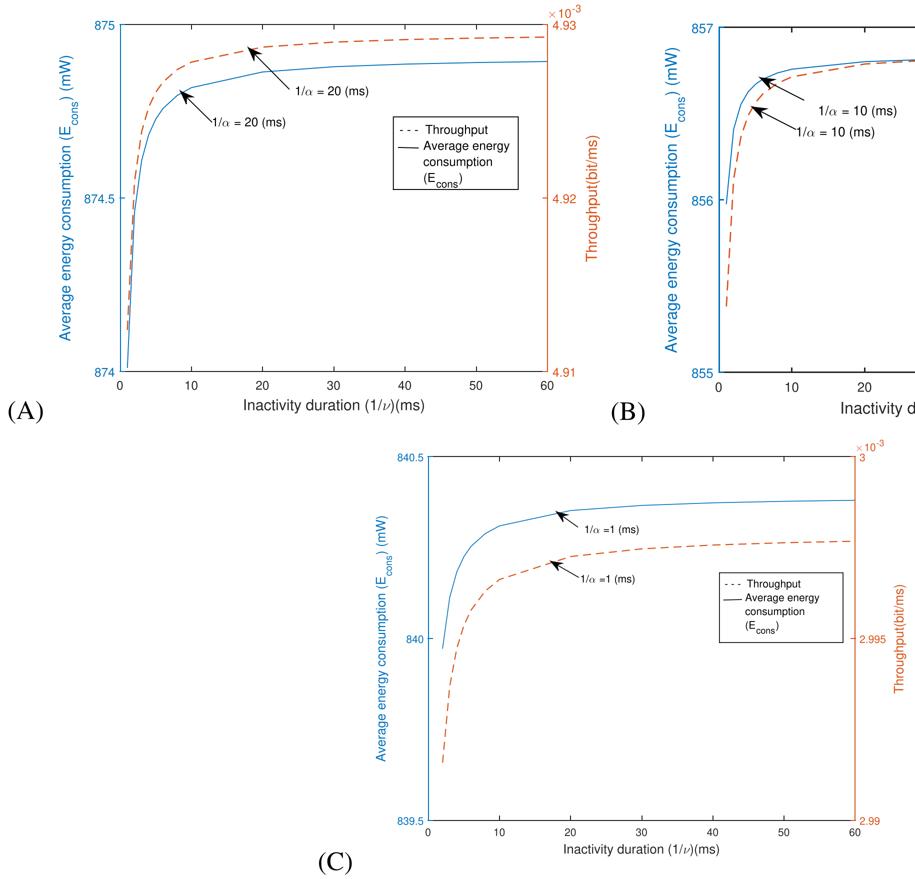

FIGURE 9 Trade-off between average energy consumption and throughput as inactivity duration varies

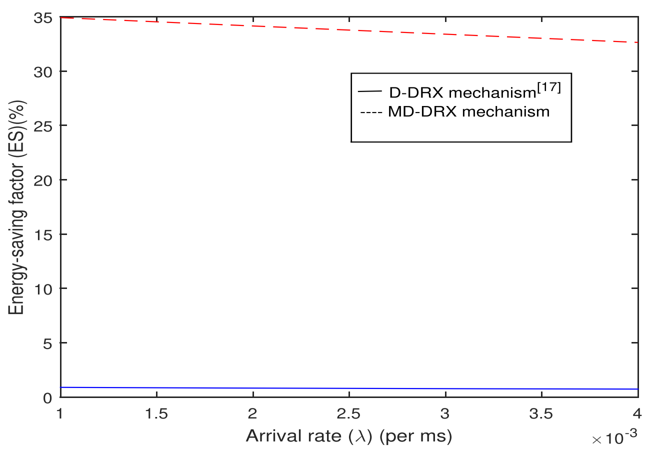

FIGURE 10 Comparison of energy-saving factor as arrival rate varies

- As the inactivity duration $\left(\frac{1}{\nu}\right)$ varies in between $[1,2,3,4,5,6,8,10,20,30,40,50,60]$, the average energy consumption (E_{cons}) and throughput increase. The reason is that, as the inactive duration of the UE increases, the components of UE will be active for a longer period which consumes more energy and this leads to an increase in average energy consumption (E_{cons}). On the other hand, if the components of UE are active, it means the probability that packets get served increases, which leads to an increase in the system's throughput.
- As the ON duration $\left(\frac{1}{\alpha}\right)$ increases, the average energy consumption (E_{cons}) and throughput increase. The reason is similar to Figure 8.

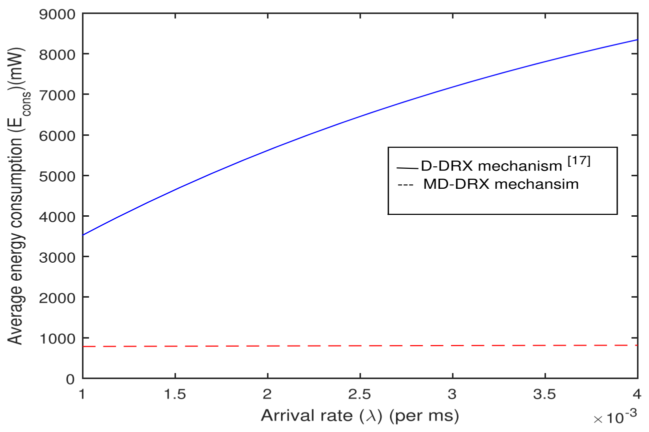

FIGURE 11 Comparison of energy-saving factor as arrival rate varies

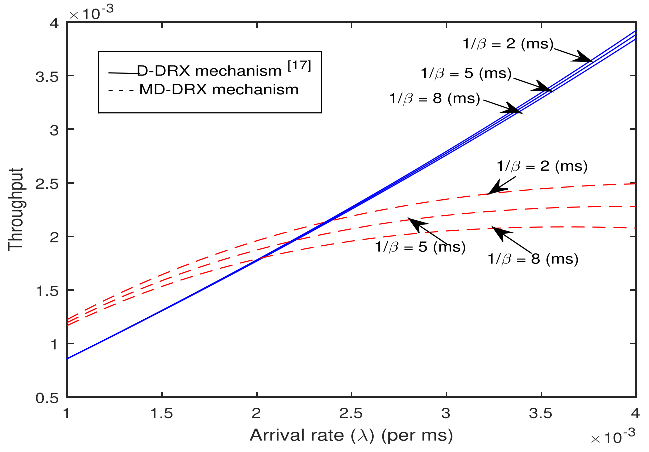

FIGURE 12 Comparison of energy-saving factor as arrival rate varies

7.5 | Numerical comparison with D-DRX mechanism

In this subsection, the numerical comparison of an energy-saving factor, average energy consumption, and throughput for D-DRX[22] and the proposed MD-DRX mechanism are shown in Figures 10–12. Here, we presume the parameter values as $\nu(1/t_I)=0.02$, $\mu=0.05$, $\zeta(1/t_{bs})=0.01$, $\alpha(1/t_{on})=1$, $\gamma(1/t_{ls})=0.0125$, and $M(n_{ss})^*=6$, $\frac{1}{\beta}^*(t_{ss})=2$. Only for Figure 12, the parameter values for $\frac{1}{\beta}(t_{ss})$ are not fixed. From Figure 10, the observations are as follows:

- As the arrival rate (λ) varies from 0.001 to 0.004, the energy-saving factor (ES) for the D-DRX and the proposed MD-DRX mechanism decreases as it should be.
- As the arrival rate (λ) varies from 0.001 to 0.004, the energy-saving factor (ES) for the D-DRX mechanism is less than the proposed MD-DRX mechanism.

From Figure 11, the observation are as follows:

- As the arrival rate (λ) varies from 0.001 to 0.004, the average energy consumption (E_{cons}) for the D-DRX and the proposed MD-DRX mechanism increases as it should be.
- As the arrival rate (λ) varies from 0.001 to 0.004, the average energy consumption (E_{cons}) for the D-DRX mechanism is higher than the proposed MD-DRX mechanism.

From Figure 12, the observation are as follows:

- As the arrival rate (λ) varies from 0.001 to 0.004, the throughput for the D-DRX and the proposed MD-DRX mechanism increases as it should be.
- As the arrival rate (λ) varies from 0.001 to 0.004, initially, the throughput for the D-DRX mechanism is less than the proposed MD-DRX mechanism. After the arrival rate (λ) 0.002, the throughput for the D-DRX is more than the proposed MD-DRX mechanism.
- As the short sleep duration $\left(\frac{1}{\beta}\right)$ increases, the throughput decreases. The reason is that, in short sleep duration, only a few components of UE are working, and most of the components are not working. Hence, the longer short sleep duration decreases the system's throughput.

8 | CONCLUSION AND FUTURE WORK

In this research work, we have analyzed an MRGP model for the MD-DRX mechanism to reach the demand for high-frequency data rates of LTE-5G networks. Observing the behavior of the performance measures, it is noticed that as the inactivity duration increases, average energy consumption (E_{cons}) increases, and the energy-saving factor (ES) decreases but the throughput increases. To get optimal energy saving and energy consumption, we have shown the trade-off between energy saving/energy consumption and throughput. It is also predicted that the proposed model may help system engineers in the design and operation of next-generation wireless communication systems. Here, we have compared the proposed MRGP model with the existing analytical model, which shows the strength of the proposed MRGP model and the drawbacks of the existing model. Also, we have numerically compared the proposed MD-DRX mechanism with the exiting D-DRX mechanism. The comparison makes it clear that the proposed MRGP model for the MD-DRX mechanism is expected to be more precise, better, and closer to making the real-time simulation results.

Further, the comparative study of the MD-DRX mechanism can be achieved with the other existing LTE-DRX mechanisms (D-DRX, HD-DRX, etc.) by using the simulation model. Also, for the validation of model in the real scenario, the time series data can be used to analyze the number of users connected and sleep mode switching. The proposed MRGP model for the MD-DRX mechanism may be incorporated with LTE-6G networks with its updated features.

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DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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APPENDIX A: STEADY-STATE SOLUTION OF THE MRGP MODEL

In this section, the system state probabilities (in steady state) in different states are obtained. To find out these performance measures, the expressions for global kernel $K(t)$ and local kernel $E(t)$ are needed.

A.1 | Global kernel

The elements of the global kernel matrix $K(t)$ is denoted by $K_{m,n}(t)$, where $K_{m,n}(t)$ is the probability that the system will be in state n at time of next regeneration epoch which happen on or before time t, condition that just after the previous regeneration epoch, the system was in state m. Hence, the nonzero elements of the global kernel matrix are as follows:

$$K_{B_{0,0}B_{1,0}}(t)=\int_0^t(1-F_{B_{0,0}L_{0,3}}(x))dF_{B_{0,0}B_{1,0}}(x), \tag{B1}$$

$$K_{B_{i,0}B_{i+1,0}}(t)=\int_0^t(1-F_{B_{i,0}O_{i,4}}(x))dF_{B_{i,0}B_{i+1,0}}(x), i=1,2,..., \tag{B2}$$

$$K_{B_{0,0}B_{1,0}}(t)=K_{B_{i,0}B_{i+1,0}}(t)=\begin{cases}1-e^{-\lambda t} & \text{if } t<\dfrac{1}{\zeta},\\ 1-e^{-\frac{\lambda}{\zeta}} & \text{if } t\geq\dfrac{1}{\zeta},\end{cases} \tag{B3}$$

$$K_{L_{0,3}B_{0,0}}(t)=\int_0^t(1-F_{L_{0,3}L_{1,3}}(x))dF_{L_{0,3}B_{0,0}}(x), \tag{B4}$$

$$K_{L_{0,3}B_{0,0}}(t)=\begin{cases}e^{-\frac{\lambda}{\gamma}} & \text{if } t\geq\dfrac{1}{\gamma},\\ 0 & \text{if } t<\dfrac{1}{\gamma},\end{cases} \tag{B5}$$

$$K_{B_{0,0}L_{0,3}}(t)=\int_0^t(1-F_{B_{0,0}B_{1,0}}(x))dF_{B_{0,0}L_{0,3}}(x), \tag{B6}$$

$$K_{B_{i,0}O_{i,4}}(t)=\int_0^t(1-F_{B_{i,0}B_{i+1,0}}(x))dF_{B_{i,0}O_{i,4}}(x), i=1,2,...,j=0, \tag{B7}$$

$$K_{B_{0,0}L_{0,3}}(t)=K_{B_{i,0}O_{i,4}}(t)=\begin{cases}e^{-\frac{\lambda}{\zeta}} & \text{if } t\geq\dfrac{1}{\zeta},\\ 0 & \text{if } t<\dfrac{1}{\zeta},\end{cases} \tag{B8}$$

$$K_{I_{0,1}I_{1,1}}(t)=\int_0^t(1-F_{I_{0,1}S_{0,5}}(x))dF_{I_{0,1}I_{1,1}}(x), \tag{B9}$$

$$K_{I_{i,1}I_{i+1,1}}(t)=\int_0^t(1-F_{I_{i,j}O_{i,j+3}}(x))dF_{I_{i,j}I_{i+1,j}}(x), i=1,2,..., \tag{B10}$$

$$K_{I_{0,1}I_{1,1}}(t) = K_{I_{i,1}I_{i+1,1}}(t) = \begin{cases} 1-e^{-\lambda t} & \text{if } t < \frac{1}{\nu}, \\ 1-e^{-\frac{\lambda}{\nu}} & \text{if } t \geq \frac{1}{\nu}, \end{cases} \tag{B11}$$

$$K_{I_{i,1}O_{i,4}}(t) = \int_0^t (1-F_{I_{i,1}I_{i+1,1}}(x))dF_{I_{i,1}O_{i,4}}(x), i = 1,2,..., \tag{B12}$$

$$K_{I_{i,1}O_{i,4}}(t) = \begin{cases} e^{-\frac{\lambda}{\nu}} & \text{if } t \geq \frac{1}{\nu}, \\ 0 & \text{if } t < \frac{1}{\nu}, \end{cases} \tag{B13}$$

$$K_{A_{1,2}A_{2,2}}(t) = \int_0^t (1-F_{A_{1,2}I_{0,1}}(x))dF_{A_{1,2}A_{2,2}}(x), \tag{B14}$$

$$K_{A_{i,2}A_{i+1,2}}(t) = \int_0^t (1-F_{A_{i,2}A_{i-1,2}}(x))dF_{A_{i,2}A_{i+1,2}}(x), i = 2,3,..., \tag{B15}$$

$$K_{A_{1,2}A_{2,2}}(t) = K_{A_{i,2}A_{i+1,2}}(t) = \begin{cases} 1-e^{-\lambda t} & \text{if } t < \frac{1}{\mu}, \\ 1-e^{-\frac{\lambda}{\mu}} & \text{if } t \geq \frac{1}{\mu}, \end{cases} \tag{B16}$$

$$K_{L_{0,3}L_{1,3}}(t) = \int_0^t (1-F_{L_{0,3}B_{0,0}}(x))dF_{L_{0,3}L_{1,3}}(x), \tag{B17}$$

$$K_{L_{i,3}L_{i+1,3}}(t) = \int_0^t (1-F_{L_{i,3}O_{i,4}}(x))dF_{L_{i,3}L_{i+1,3}}(x), i = 1,2,..., \tag{B18}$$

$$K_{L_{0,3}L_{1,3}}(t) = K_{L_{i,3}L_{i+1,3}}(t) = \begin{cases} 1-e^{-\lambda t} & \text{if } t < \frac{1}{\gamma}, \\ 1-e^{-\frac{\lambda}{\gamma}} & \text{if } t \geq \frac{1}{\gamma}, \end{cases} \tag{B19}$$

$$K_{S_{i,j}S_{i+1,j}}(t) = \int_0^t (1-F_{S_{i,j}O_{i,j-k}}(x))dF_{S_{i,j}S_{i+1,j}}(x), i = 1,2,...,j-k = 4, j = 5,6,...,M+4, k = 1,2,...,M, \tag{B20}$$

$$K_{S_{0,j}S_{1,j}}(t) = \int_0^t (1-F_{S_{0,j}S_{0,j+1}}(x))dF_{S_{0,j}S_{1,j}}(x), j = 5,6,...,M+3, \tag{B21}$$

$$K_{S_{0,M+4}S_{1,M+4}}(t)=\int_{0}^{t}(1-F_{S_{0,M+4}B_{0,0}}(x))dF_{S_{0,M+4}S_{1,M+4}}(x), \tag{B22}$$

$$K_{S_{i,j}S_{i+1,j}}(t)=K_{S_{0,j}S_{1,j}}(t),K_{S_{0,M+4}S_{1,M+4}}(t)=\begin{cases}1-e^{-\lambda t} & \text{if } t<\frac{1}{\beta},\\ 1-e^{-\frac{\lambda}{\beta}} & \text{if } t\geq\frac{1}{\beta},\end{cases} \tag{B23}$$

$$K_{O_{i,4}A_{i,2}}(t)=\int_{0}^{t}(1-F_{O_{i,4}O_{i+1,4}}(x))dF_{O_{i,4}A_{i,2}}(x), i=1,2,..., \tag{B24}$$

$$K_{O_{i,4}A_{i,2}}(t)=\begin{cases}e^{-\frac{\lambda}{\alpha}} & \text{if } t\geq\frac{1}{\alpha},\\ 0 & \text{if } t<\frac{1}{\alpha},\end{cases} \tag{B25}$$

$$K_{I_{0,1}S_{0,5}}(t)=\int_{0}^{t}(1-F_{I_{0,1}I_{1,1}}(x))dF_{I_{0,1}S_{0,5}}(x), \tag{B26}$$

$$K_{I_{0,1}S_{0,5}}(t)=\begin{cases}e^{-\frac{\lambda}{\nu}} & \text{if } t\geq\frac{1}{\nu},\\ 0 & \text{if } t<\frac{1}{\nu},\end{cases} \tag{B27}$$

$$K_{S_{i,j}O_{i,j-k}}(t)=\int_{0}^{t}(1-F_{S_{i,j}S_{i+1,j}}(x))dF_{S_{i,j}O_{i,j-k}}(x), i=1,2,...,j-k=4, j=5,6,...,M+4, k=1,2,...,M, \tag{B28}$$

$$K_{S_{0,M+4}B_{0,0}}(t)=\int_{0}^{t}(1-F_{S_{0,M+4}S_{1,M+4}}(x))dF_{S_{0,M+4}B_{0,0}}(x), \tag{B29}$$

$$K_{S_{0,j}S_{0,j+1}}(t)=\int_{0}^{t}(1-F_{S_{0,j}S_{1,j}}(x))dF_{S_{0,j}S_{0,j+1}}(x), j=5,...,M+3, \tag{B30}$$

$$K_{S_{i,j}O_{i,j-k}}(t)=K_{S_{0,M+4}B_{0,0}}(t),K_{S_{0,j}S_{0,j+1}}(t)=\begin{cases}e^{-\frac{\lambda}{\beta}} & \text{if } t\geq\frac{1}{\beta},\\ 0 & \text{if } t<\frac{1}{\beta},\end{cases} \tag{B31}$$

$$K_{A_{i,2}A_{i-1,2}}(t)=\int_{0}^{t}(1-F_{A_{i,j}A_{i+1,j}}(x))dF_{A_{i,j}A_{i-1,j}}(x), i=2,3,..., \tag{B32}$$

$$K_{A_{1,2}I_{0,1}}(t) = \int_0^t (1 - F_{A_{1,2}A_{2,2}}(x))dF_{A_{1,2}I_{0,1}}(x), \tag{B33}$$

$$K_{A_{i,2}A_{i-1,2}}(t) = K_{A_{1,2}I_{0,1}}(t) = \begin{cases} e^{-\frac{\lambda}{\mu}} & \text{if } t \geq \frac{1}{\mu}, \\ 0 & \text{if } t < \frac{1}{\mu}, \end{cases} \tag{B34}$$

$$K_{O_{i,4}O_{i+1,4}}(t) = \int_0^t (1 - F_{0_{i,j}A_{i,j-2}}(x))dF_{O_{i,4}O_{i+1,4}}(x), i = 1,2,..., \tag{B35}$$

$$K_{O_{i,4}O_{i+1,4}}(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t < \frac{1}{\alpha}, \\ 1 - e^{-\frac{\lambda}{\alpha}} & \text{if } t \geq \frac{1}{\alpha}, \end{cases} \tag{B36}$$

$$K_{L_{i,3}O_{i,4}}(t) = \int_0^t (1 - F_{L_{i,j}L_{i+1,j}}(x))dF_{L_{i,j}O_{i,j+1}}(x), i = 1,2,..., \tag{B37}$$

$$K_{L_{i,3}O_{i,4}}(t) = \begin{cases} e^{-\frac{\lambda}{\gamma}} & \text{if } t \geq \frac{1}{\gamma}, \\ 0 & \text{if } t < \frac{1}{\gamma}. \end{cases} \tag{B38}$$

Define $K_{m,n}(\infty) = \lim_{t \to \infty} K_{m,n}(t)$, where $m, n \in \Omega$. Hence, the global kernel matrix $K(t)$ at $t \to \infty$ with its element $K_{m,n}(\infty)$ gives the one-step transition probability for the underlying model, and the row sum of $K(\infty)$ matrix is 1. Therefore, by using Equations (B1)–(B38), the one-step transition probabilities for the proposed model are obtained in Table A.1 as

TABLE A.1 The one-step transition probabilities for the proposed model

$K_{B_{i,0}B_{i+1,0}}(\infty) = 1 - e^{-\frac{\lambda}{\zeta}}; i = 0,1,...,$	$K_{I_{i,1}I_{i+1,1}}(\infty) = 1 - e^{-\frac{\lambda}{\nu}}; i = 0,1,...,$	$K_{L_{0,3}B_{0,0}}(\infty) = e^{-\frac{\lambda}{\gamma}}$
$K_{A_{i,2}A_{i+1,2}}(\infty) = 1 - e^{-\frac{\lambda}{\mu}}; i = 1,2,...,$	$K_{L_{i,3}L_{i+1,3}}(\infty) = 1 - e^{-\frac{\lambda}{\gamma}}; i = 0,1,...,$	$K_{B_{0,0}L_{0,3}}(\infty) = e^{-\frac{\lambda}{\zeta}}$
$K_{O_{i,4}0_{i+1,4}}(\infty) = 1 - e^{-\frac{\lambda}{\alpha}}; i = 1,2,...,$	$K_{S_{0,j}S_{1,j}}(\infty) = 1 - e^{-\frac{\lambda}{\beta}}; j = 5,6,...,M+4,$	$K_{B_{i,0}O_{i,4}}(\infty) = e^{-\frac{\lambda}{\zeta}};$ $i = 1,2,...,$
$K_{I_{i,1}0_{i,4}}(\infty) = e^{-\frac{\lambda}{\nu}}; i = 1,2,...,$	$K_{O_{i,4}A_{i,2}}(\infty) = e^{-\frac{\lambda}{\alpha}}; i = 1,2,...,$	$K_{I_{0,1}S_{0,5}}(\infty) = e^{-\frac{\lambda}{\nu}}$
$K_{S_{i,j}0_{i,j-k}}(\infty) = e^{-\frac{\lambda}{\beta}}; i = 1,2,...,$	$K_{A_{i,2}A_{i-1,2}}(\infty) = e^{-\frac{\lambda}{\mu}}; i = 2,...,$	$K_{S_{0,M+4}B_{0,0}}(\infty) = e^{-\frac{\lambda}{\beta}}$
$j - k = 4, j = 5,6,...,M+4,$ $k = 1,2,...,M,$	$K_{S_{0,j}S_{0,j+1}}(\infty) = e^{-\frac{\lambda}{\beta}};$ $j = 5,...,M+3,$	$K_{L_{i,3}O_{i,4}}(\infty) = e^{-\frac{\lambda}{\gamma}};$ $i = 1,2,...,$
$K_{A_{1,2}I_{0,1}}(\infty) = e^{-\frac{\lambda}{\mu}}$	$K_{S_{i,j}S_{i+1,j}}(\infty) = 1 - e^{-\frac{\lambda}{\beta}};$ $i = 1,2,...,$ $j = 5,...,M+4,$	

Let $\pi=[\pi_{B_{i,0}};i=0,1,...,\pi_{I_{i,1}};i=0,1,...,\pi_{A_{i,2}};i=1,2,...,\pi_{L_{i,3}};i=0,1,...,\pi_{O_{i,4}};i=1,2,...,\pi_{S_{i,j}};i=0,1,...,j=5,6,...,M+4]$ be the steady-state probability vector of the embedded Markov chain and $\pi_{B_{i,0}};i=0,1,...,\pi_{I_{i,1}};i=0,1,...,\pi_{A_{i,2}};i=1,2,...,\pi_{L_{i,3}};i=0,1,...,\pi_{O_{i,4}};i=1,2,...,\pi_{S_{i,j}};i=0,1,...,j=5,6,...,M+4$ are its corresponding probabilities, which can be obtained by solving

$$\pi=\pi K(\infty),\sum_{m\in\Omega}\pi_m=1.$$

Hence, by using $\pi=\pi K(\infty)$ and the element of the matrix $K(\infty)$ as mention in Table A.1, the system of linear equations is obtained as follows:

$$\pi_{B_{0,0}}=e^{-\frac{\lambda}{\beta}}\pi_{S_{0,M+4}}+e^{-\frac{\lambda}{\gamma}}\pi_{L_{0,3}}, \tag{B39}$$

$$\pi_{B_{i+1,0}}=\left(1-e^{-\frac{\lambda}{\zeta}}\right)\pi_{B_{i,j}};i=0,1,..., \tag{B40}$$

$$\pi_{I_{0,1}}=e^{-\frac{\lambda}{\mu}}\pi_{A_{1,2}}, \tag{B41}$$

$$\pi_{I_{i+1,1}}=\left(1-e^{-\frac{\lambda}{\nu}}\right)\pi_{I_{i,j}};i=0,1,..., \tag{B42}$$

$$\pi_{A_{i-1,2}}=e^{-\frac{\lambda}{\alpha}}\pi_{O_{i-1,j+2}}+e^{-\frac{\lambda}{\mu}}\pi_{A_{i,j}};i=2,3,..., \tag{B43}$$

$$\pi_{A_{i+1,2}}=\left(1-e^{-\frac{\lambda}{\mu}}\right)\pi_{A_{i,j}};i=1,2,..., \tag{B44}$$

$$\pi_{L_{0,3}}=e^{-\frac{\lambda}{\zeta}}\pi_{B_{0,0}}, \tag{B45}$$

$$\pi_{L_{i+1,3}}=\left(1-e^{-\frac{\lambda}{\gamma}}\right)\pi_{L_{i,j}};i=0,1,..., \tag{B46}$$

$$\pi_{O_{1,4}}=e^{-\frac{\lambda}{\zeta}}\pi_{B_{1,0}}+e^{-\frac{\lambda}{\gamma}}\pi_{L_{1,3}}+e^{-\frac{\lambda}{\beta}}(\pi_{S_{1,5}}+\pi_{S_{1,6}}+...+\pi_{S_{1,M+4}})+e^{-\frac{\lambda}{\nu}}\pi_{I_{1,1}}, \tag{B47}$$

$$\pi_{O_{i+1,4}}=\left(1-e^{-\frac{\lambda}{\alpha}}\right)\pi_{O_{i,4}}+e^{-\frac{\lambda}{\beta}}(\pi_{S_{i+1,5}}+\pi_{S_{i+1,6}}+...+\pi_{S_{i+1,M+4}})+e^{-\frac{\lambda}{\zeta}}\pi_{B_{i+1,0}}+e^{-\frac{\lambda}{\gamma}}\pi_{L_{i+1,3}}+e^{-\frac{\lambda}{\nu}}\pi_{I_{i+1,1}};i=1,2,..., \tag{B48}$$

$$\pi_{S_{0,5}}=e^{-\frac{\lambda}{\nu}}\pi_{I_{0,1}}, \tag{B49}$$

$$\pi_{S_{i+1,j}}=\left(1-e^{-\frac{\lambda}{\beta}}\right)\pi_{S_{i,j}};i=0,1,...,j=5,6,...,M+4, \tag{B50}$$

$$\pi_{S_{0,j+1}}=e^{-\frac{\lambda}{\beta}}\pi_{S_{0,j}};j=5,6,...,M+3. \tag{B51}$$

Solving the linear equations in Equations (B39)–(B51), the steady-state probabilities of each state, π_m, $m \in \Omega$ in terms of $\pi_{A_{1,2}}$, are obtained as

$$\pi_{L_{i,3}} = \left(1-e^{-\frac{\lambda}{\gamma}}\right)^i \frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\left(e^{-\frac{\lambda}{\beta}}\right)^M}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}\right)} e^{-\frac{\lambda}{\mu}}\pi_{A_{1,2}}; i=0,1,\ldots, \tag{B52}$$

$$\pi_{B_{i,0}} = \left(1-e^{-\frac{\lambda}{\zeta}}\right)^i \frac{e^{-\frac{\lambda}{\nu}}\left(e^{-\frac{\lambda}{\beta}}\right)^M}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}\right)} e^{-\frac{\lambda}{\mu}}\pi_{A_{1,2}}; i=0,1,\ldots, \tag{B53}$$

$$\pi_{S_{i,j}} = \left(1-e^{-\frac{\lambda}{\beta}}\right)^i \left(e^{-\frac{\lambda}{\beta}}\right)^{j-5} e^{-\frac{\lambda}{\nu}} e^{-\frac{\lambda}{\mu}}\pi_{A_{1,2}}; i=0,1,\ldots, j=5,6,\ldots,M+4, \tag{B54}$$

$$\pi_{I_{i,1}} = \left(1-e^{-\frac{\lambda}{\nu}}\right)^i e^{-\frac{\lambda}{\mu}}\pi_{A_{1,2}}; i=0,1,\ldots, \tag{B55}$$

$$\begin{aligned}
\pi_{O_{i,4}} = \Bigg[& e^{-\frac{\lambda}{\zeta}}\left(1-e^{-\frac{\lambda}{\zeta}}\right)\frac{e^{-\frac{\lambda}{\nu}}\left(e^{-\frac{\lambda}{\beta}}\right)^M}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}\right)}\sum_{k=1}^{i}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\zeta}}\right)^{(k-1)} \\
& +e^{-\frac{\lambda}{\gamma}}\left(1-e^{-\frac{\lambda}{\gamma}}\right)\frac{\left(e^{-\frac{\lambda}{\beta}}\right)^M e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\right)}\sum_{k=1}^{i}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{(k-1)} \\
& +e^{-\frac{\lambda}{\beta}}\left(1-\left(e^{-\frac{\lambda}{\beta}}\right)^M\right)e^{-\frac{\lambda}{\nu}}\sum_{k=1}^{i}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\beta}}\right)^{(k-1)} \\
& +e^{-\frac{\lambda}{\nu}}\left(1-e^{-\frac{\lambda}{\nu}}\right)\sum_{k=1}^{i}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\nu}}\right)^{(k-1)}\Bigg] e^{-\frac{\lambda}{\mu}}\pi_{A_{1,2}}, i=1,2,\ldots,
\end{aligned} \tag{B56}$$

$$\pi_{A_{2,2}} = \left(1-e^{-\frac{\lambda}{\mu}}\right)\pi_{A_{1,2}}, \tag{B57}$$

$$\begin{aligned}
\pi_{A_{3,2}} = \Bigg\{ & -e^{-\frac{\lambda}{\mu}}+e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\Bigg[e^{-\frac{\lambda}{\zeta}}\left(1-e^{-\frac{\lambda}{\zeta}}\right)\frac{e^{-\frac{\lambda}{\nu}}\left(e^{-\frac{\lambda}{\beta}}\right)^M}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}\right)} \\
& +e^{-\frac{\lambda}{\gamma}}\left(1-e^{-\frac{\lambda}{\gamma}}\right)\frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\left(e^{-\frac{\lambda}{\beta}}\right)^M}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\right)}+e^{-\frac{\lambda}{\beta}}\left(1-\left(e^{-\frac{\lambda}{\beta}}\right)^M\right)e^{-\frac{\lambda}{\nu}}+e^{-\frac{\lambda}{\nu}}\left(1-e^{-\frac{\lambda}{\nu}}\right)\Bigg]\Bigg\}\pi_{A_{1,2}},
\end{aligned} \tag{B58}$$

$$\begin{aligned}
\pi_{A_{4,2}} = \Bigg\{ & e^{-\frac{\lambda}{\mu}}e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\zeta}}\frac{\left(1-e^{-\frac{\lambda}{\zeta}}\right)e^{-\frac{\lambda}{\nu}}\left(e^{-\frac{\lambda}{\beta}}\right)^M}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}\right)}\left(1+\left(1-e^{-\frac{\lambda}{\alpha}}\right)+\left(1-e^{-\frac{\lambda}{\zeta}}\right)\right)+e^{-\frac{\lambda}{\mu}}e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\gamma}}\left(1-e^{-\frac{\lambda}{\gamma}}\right)\frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\left(e^{-\frac{\lambda}{\beta}}\right)^M}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\right)} \\
& \left(1+\left(1-e^{-\frac{\lambda}{\alpha}}\right)+\left(1-e^{-\frac{\lambda}{\gamma}}\right)\right)+e^{-\frac{\lambda}{\mu}}e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\beta}}\left(1-\left(e^{-\frac{\lambda}{\beta}}\right)^M\right)e^{-\frac{\lambda}{\nu}}\left(1+\left(1-e^{-\frac{\lambda}{\alpha}}\right)+\left(1-e^{-\frac{\lambda}{\beta}}\right)+e^{-\frac{\lambda}{\mu}}e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\nu}}\left(1-e^{-\frac{\lambda}{\nu}}\right)\right. \\
& \left(1+\left(1-e^{-\frac{\lambda}{\alpha}}\right)+\left(1-e^{-\frac{\lambda}{\nu}}\right)\right)-1\Bigg\}\pi_{A_{1,2}},
\end{aligned} \tag{B59}$$

$$
\begin{aligned}
\pi_{A_{5,2}} =& \Bigg\{ e^{-\frac{\lambda}{\mu}} e^{-\frac{\lambda}{\alpha}} e^{-\frac{\lambda}{\zeta}} \left(1-e^{-\frac{\lambda}{\zeta}}\right) \frac{e^{-\frac{\lambda}{\nu}}\left(e^{-\frac{\lambda}{\beta}}\right)^M}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}\right)} \\
&\left(\left(1-e^{-\frac{\lambda}{\alpha}}\right)+\left(1-e^{-\frac{\lambda}{\zeta}}\right)+\left(1-e^{-\frac{\lambda}{\alpha}}\right)^2+\left(1-e^{-\frac{\lambda}{\alpha}}\right)\left(1-e^{-\frac{\lambda}{\zeta}}\right)+\left(1-e^{-\frac{\lambda}{\zeta}}\right)^2\right) \\
&+e^{-\frac{\lambda}{\mu}} e^{-\frac{\lambda}{\alpha}} e^{-\frac{\lambda}{\gamma}} \left(1-e^{-\frac{\lambda}{\gamma}}\right) \frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\left(e^{-\frac{\lambda}{\beta}}\right)^M}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\right)} \\
&\left(\left(1-e^{-\frac{\lambda}{\alpha}}\right)+\left(1-e^{-\frac{\lambda}{\gamma}}\right)+\left(1-e^{-\frac{\lambda}{\alpha}}\right)^2+\left(1-e^{-\frac{\lambda}{\alpha}}\right)\left(1-e^{-\frac{\lambda}{\gamma}}\right)+\left(1-e^{-\frac{\lambda}{\gamma}}\right)^2\right) \\
&+e^{-\frac{\lambda}{\mu}} e^{-\frac{\lambda}{\alpha}} e^{-\frac{\lambda}{\beta}} \left(1-\left(e^{-\frac{\lambda}{\beta}}\right)^M\right) e^{-\frac{\lambda}{\nu}} \left(\left(1-e^{-\frac{\lambda}{\alpha}}\right)+\left(1-e^{-\frac{\lambda}{\beta}}\right)+\left(1-e^{-\frac{\lambda}{\alpha}}\right)^2+\left(1-e^{-\frac{\lambda}{\alpha}}\right)\left(1-e^{-\frac{\lambda}{\beta}}\right)+\left(1-e^{-\frac{\lambda}{\beta}}\right)^2\right) \\
&+e^{-\frac{\lambda}{\mu}} e^{-\frac{\lambda}{\alpha}} e^{-\frac{\lambda}{\nu}} \left(1-e^{-\frac{\lambda}{\nu}}\right) \left(\left(1-e^{-\frac{\lambda}{\alpha}}\right)+\left(1-e^{-\frac{\lambda}{\nu}}\right)+\left(1-e^{-\frac{\lambda}{\alpha}}\right)^2\right. \\
&\left.+\left(1-e^{-\frac{\lambda}{\alpha}}\right)\left(1-e^{-\frac{\lambda}{\nu}}\right)+\left(1-e^{-\frac{\lambda}{\nu}}\right)^2\right)-\left(1-e^{-\frac{\lambda}{\mu}}\right)\Bigg\} \pi_{A_{1,2}},
\end{aligned}
\tag{B60}
$$

$$
\begin{aligned}
\pi_{A_{i,2}} =& \Bigg[e^{-\frac{\lambda}{\mu}} e^{-\frac{\lambda}{\alpha}} e^{-\frac{\lambda}{\zeta}} \left(1-e^{-\frac{\lambda}{\zeta}}\right) \frac{e^{-\frac{\lambda}{\nu}}\left(e^{-\frac{\lambda}{\beta}}\right)^M}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}\right)} \\
&\left(\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}+\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\zeta}}\right)^{(i-k)}-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\zeta}}\right)^{(i+1-k)}\right. \\
&\left.+\sum_{k=4}^{(i-1)}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\zeta}}\right)^{(k-3)}+\sum_{l=5}^{i}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i+1-l)}\left(1-e^{-\frac{\lambda}{\zeta}}\right)^{(l-4)}+1\right) \\
&+e^{-\frac{\lambda}{\mu}} e^{-\frac{\lambda}{\alpha}} e^{-\frac{\lambda}{\gamma}} \left(1-e^{-\frac{\lambda}{\gamma}}\right) \frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\left(e^{-\frac{\lambda}{\beta}}\right)^M}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\right)} \\
&\left(\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}+\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{(i-k)}\right. \\
&-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{(i+1-k)}+\sum_{k=4}^{(i-1)}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{(k-3)} \\
&\left.+\sum_{l=5}^{i}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i+1-l)}\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{(l-4)}+1\right)+e^{-\frac{\lambda}{\mu}} e^{-\frac{\lambda}{\alpha}} e^{-\frac{\lambda}{\beta}} e^{-\frac{\lambda}{\nu}}\left(1-\left(e^{-\frac{\lambda}{\beta}}\right)^M\right)\left(\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\right. \\
&+\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\beta}}\right)^{(i-k)}-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\beta}}\right)^{(i+1-k)}+\sum_{k=4}^{(i-1)}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\beta}}\right)^{(k-3)} \\
&\left.+\sum_{l=5}^{i}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i+1-l)}\left(1-e^{-\frac{\lambda}{\beta}}\right)^{(l-4)}+1\right) \\
&+e^{-\frac{\lambda}{\mu}} e^{-\frac{\lambda}{\alpha}} e^{-\frac{\lambda}{\nu}} \left(1-e^{-\frac{\lambda}{\nu}}\right)\left(\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}+\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\nu}}\right)^{(i-k)}-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\nu}}\right)^{(i+1-k)}\right. \\
&\left.+\sum_{k=4}^{(i-1)}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\nu}}\right)^{(k-3)}+\sum_{l=5}^{i}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i+1-l)}\left(1-e^{-\frac{\lambda}{\nu}}\right)^{(l-4)}+1\right)\Bigg] \pi_{A_{1,2}}; \\
&i=6,7,\ldots,(i-k)\geq 0,(i+1-k)>0.
\end{aligned}
\tag{B61}
$$

Using equation $\sum_{m\in\Omega}\pi_m=1$ and the probabilities, $\pi_m, m\in\Omega\setminus A_{1,2}$ in terms of $\pi_{A_{1,2}}$, we obtain

$$
\pi_{A_{1,2}}=\frac{1}{C},
$$

where

$$
\begin{aligned}
C =& \frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\left(e^{-\frac{\lambda}{\beta}}\right)^{M}}{\left(1-e^{-\lambda\left(\frac{1}{\gamma}+\frac{1}{\zeta}\right)}\right)}e^{-\frac{\lambda}{\mu}}\Bigg\{e^{\frac{\lambda}{\gamma}}+e^{\frac{2\lambda}{\zeta}}+\left(1-e^{-\frac{\lambda}{\zeta}}\right)\Bigg[\frac{1}{\left(e^{-\frac{\lambda}{\zeta}}-e^{-\frac{\lambda}{\alpha}}\right)}\left(\frac{1-e^{-\frac{\lambda}{\alpha}}}{e^{-\frac{\lambda}{\alpha}}}-\frac{1-e^{-\frac{\lambda}{\zeta}}}{e^{-\frac{\lambda}{\zeta}}}\right)\\
&+e^{-\frac{\lambda}{\alpha}}\Bigg[2+\left(1-e^{-\frac{\lambda}{\alpha}}\right)\Bigg(2+\left(1-e^{-\frac{\lambda}{\alpha}}\right)\left(1+\frac{2-e^{-\frac{\lambda}{\alpha}}}{e^{-\frac{\lambda}{\alpha}}}\right)\\
&+\left(1-e^{-\frac{\lambda}{\zeta}}\right)\left(1+\frac{2}{\left(e^{-\frac{\lambda}{\zeta}}-e^{-\frac{\lambda}{\alpha}}\right)}\left(\frac{\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{2}}{e^{-\frac{\lambda}{\alpha}}}-\frac{\left(1-e^{-\frac{\lambda}{\zeta}}\right)^{2}}{e^{-\frac{\lambda}{\zeta}}}\right)\right)\Bigg)+1\\
&+\left(1-e^{-\frac{\lambda}{\zeta}}\right)\left(2+\left(1-e^{-\frac{\lambda}{\zeta}}\right)\left(1+\frac{2-e^{-\frac{\lambda}{\zeta}}}{e^{-\frac{\lambda}{\zeta}}}\right)\right)-\frac{2}{e^{-\frac{\lambda}{\alpha}}}-\frac{2-e^{-\frac{\lambda}{\zeta}}}{e^{-\frac{\lambda}{\zeta}}}+1\Bigg]\Bigg]\Bigg\}\\
&+e^{-\frac{\lambda}{\beta}}\left(1-\left(e^{-\frac{\lambda}{\beta}}\right)^{M}\right)e^{-\frac{\lambda}{\nu}}e^{-\frac{\lambda}{\mu}}\Bigg\{\frac{e^{\frac{2\lambda}{\beta}}}{\left(1-e^{-\frac{\lambda}{\beta}}\right)}+\frac{1}{\left(e^{-\frac{\lambda}{\beta}}-e^{-\frac{\lambda}{\alpha}}\right)}\left[\frac{\left(1-e^{-\frac{\lambda}{\alpha}}\right)}{e^{-\frac{\lambda}{\alpha}}}-\frac{\left(1-e^{-\frac{\lambda}{\beta}}\right)}{e^{-\frac{\lambda}{\beta}}}\right]+2e^{-\frac{\lambda}{\alpha}}+e^{-\frac{\lambda}{\alpha}}\\
&\left(1-e^{-\frac{\lambda}{\alpha}}\right)\left(2+\left(1-e^{-\frac{\lambda}{\alpha}}\right)+\left(1-e^{-\frac{\lambda}{\beta}}\right)\right)+e^{-\frac{\lambda}{\alpha}}\left(1-e^{-\frac{\lambda}{\beta}}\right)\left(2+\left(1-e^{-\frac{\lambda}{\beta}}\right)\left(1+\frac{2-e^{-\frac{\lambda}{\beta}}}{e^{-\frac{\lambda}{\beta}}}\right)\right)\\
&+2+\left(1-e^{-\frac{\lambda}{\alpha}}\right)\left(\left(1-e^{-\frac{\lambda}{\alpha}}\right)\left(2-e^{-\frac{\lambda}{\alpha}}\right)+\frac{2e^{-\frac{\lambda}{\alpha}}\left(1-e^{-\frac{\lambda}{\beta}}\right)}{\left(e^{-\frac{\lambda}{\beta}}-e^{-\frac{\lambda}{\alpha}}\right)}\left[\frac{\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{2}}{e^{-\frac{\lambda}{\alpha}}}-\frac{\left(1-e^{-\frac{\lambda}{\beta}}\right)^{2}}{e^{-\frac{\lambda}{\beta}}}\right]\right)-e^{-\frac{\lambda}{\alpha}}\left(\frac{2-e^{-\frac{\lambda}{\beta}}}{e^{-\frac{\lambda}{\beta}}}\right)\Bigg\}\\
&+e^{-\frac{\lambda}{\mu}}e^{\frac{\lambda}{\nu}}-e^{-\frac{\lambda}{\mu}}-1+e^{-\frac{\lambda}{\mu}}e^{-\frac{\lambda}{\gamma}}\left(1-e^{-\frac{\lambda}{\gamma}}\right)\frac{\left(e^{-\frac{\lambda}{\beta}}\right)^{M}e^{-\lambda\left(\frac{1}{\nu}+\frac{1}{\zeta}\right)}}{\left(1-e^{-\lambda\left(\frac{1}{\nu}+\frac{1}{\zeta}\right)}\right)}\Bigg\{\frac{1}{\left(e^{-\frac{\lambda}{\gamma}}-e^{-\frac{\lambda}{\alpha}}\right)}\left[\frac{\left(1-e^{-\frac{\lambda}{\alpha}}\right)}{e^{-\frac{\lambda}{\alpha}}}-\frac{\left(1-e^{-\frac{\lambda}{\gamma}}\right)}{e^{-\frac{\lambda}{\gamma}}}\right]\\
&+2e^{-\frac{\lambda}{\alpha}}\left(1+\left(1-e^{-\frac{\lambda}{\alpha}}\right)+\left(1-e^{-\frac{\lambda}{\gamma}}\right)\right)+e^{-\frac{\lambda}{\alpha}}\left(\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{2}+\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{2}\right)\\
&+\left(1-e^{-\frac{\lambda}{\alpha}}\right)\left(e^{-\frac{\lambda}{\alpha}}\left(1-e^{-\frac{\lambda}{\gamma}}\right)+\left(1-e^{-\frac{\lambda}{\alpha}}\right)\left(2-e^{-\frac{\lambda}{\alpha}}\right)\right)-2+\frac{e^{-\frac{\lambda}{\alpha}}\left(2-e^{-\frac{\lambda}{\gamma}}\right)}{e^{-\frac{\lambda}{\gamma}}}\left(\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{2}-1\right)\\
&+e^{-\frac{\lambda}{\alpha}}\left(\frac{2\left(1-e^{-\frac{\lambda}{\gamma}}\right)\left(1-e^{-\frac{\lambda}{\alpha}}\right)}{\left(e^{-\frac{\lambda}{\gamma}}-e^{-\frac{\lambda}{\alpha}}.\right)}\left[\frac{\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{2}}{e^{-\frac{\lambda}{\alpha}}}-\frac{\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{2}}{e^{-\frac{\lambda}{\gamma}}}\right]+1\right)\Bigg\}\\
&+e^{-\frac{\lambda}{\nu}}\left(1-e^{-\frac{\lambda}{\nu}}\right)e^{-\frac{\lambda}{\mu}}\Bigg\{\frac{1}{\left(e^{-\frac{\lambda}{\nu}}-e^{-\frac{\lambda}{\alpha}}\right)}\left[\frac{\left(1-e^{-\frac{\lambda}{\alpha}}\right)}{e^{-\frac{\lambda}{\alpha}}}-\frac{\left(1-e^{-\frac{\lambda}{\nu}}\right)}{e^{-\frac{\lambda}{\nu}}}\right]\\
&+2e^{-\frac{\lambda}{\alpha}}\left(1+\left(1-e^{-\frac{\lambda}{\alpha}}\right)+\left(1-e^{-\frac{\lambda}{\nu}}\right)\right)+e^{-\frac{\lambda}{\alpha}}\left(1-e^{-\frac{\lambda}{\alpha}}\right)\left(\left(1-e^{-\frac{\lambda}{\alpha}}\right)+\left(1-e^{-\frac{\lambda}{\nu}}\right)\right)\\
&+e^{-\frac{\lambda}{\alpha}}\left(1-e^{-\frac{\lambda}{\nu}}\right)^{2}\left(1+\frac{2-e^{-\frac{\lambda}{\nu}}}{e^{-\frac{\lambda}{\nu}}}+\frac{\left(2-e^{-\frac{\lambda}{\nu}}\right)}{e^{-\frac{\lambda}{\nu}}}\right)+\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{2}\left(2-e^{-\frac{\lambda}{\alpha}}\right)\\
&-e^{-\frac{\lambda}{\alpha}}\left(\frac{2-e^{-\frac{\lambda}{\nu}}}{e^{-\frac{\lambda}{\nu}}}-\frac{2\left(1-e^{-\frac{\lambda}{\nu}}\right)\left(1-e^{-\frac{\lambda}{\alpha}}\right)}{\left(e^{-\frac{\lambda}{\nu}}-e^{-\frac{\lambda}{\alpha}}\right)}\left[\frac{\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{2}}{e^{-\frac{\lambda}{\alpha}}}-\frac{\left(1-e^{-\frac{\lambda}{\nu}}\right)^{2}}{e^{-\frac{\lambda}{\nu}}}\right]\right)-2+e^{-\frac{\lambda}{\alpha}}\Bigg\}.
\end{aligned}
\tag{B62}
$$

A.2 | Local kernel

The local kernel $E(t)$ narrates the behavior of the process during the time between two consecutive regeneration points starting from a state of the system just after the next regeneration points occur. Hence, the elements of the local kernel matrix are as follows:

$$E_{m,n}(t)=\begin{cases} E_{A_{i,j}A_{i-1,j}}(t)=(1-F_{A_{i,j}A_{i-1,j}}(t))(1-F_{A_{i,j}A_{i+1,j}}(t)), i=2,\ldots,j=2, \\ E_{A_{i,2}I_{0,1}}(t)=(1-F_{A_{1,2}A_{2,2}}(t))(1-F_{A_{1,2}I_{0,1}}(t)), \\ E_{I_{0,1}S_{0,5}}(t)=(1-F_{I_{0,1}I_{1,1}}(t))(1-(F_{I_{0,1}S_{0,5}}(t)), \\ E_{B_{0,0}L_{0,3}}(t)=(1-F_{B_{0,0}B_{1,0}}(t))(1-F_{B_{0,0}L_{0,3}}(t)), \\ E_{B_{i,0}O_{i,4}}(t)=(1-F_{B_{i,0}O_{i,4}}(t))(1-F_{B_{i,0}B_{i+1,0}}(t)), i=1,\ldots, \\ E_{O_{i,4}A_{i,2}}(t)=(1-F_{O_{i,4}A_{i,2}}(t))(1-F_{O_{i,4}O_{i+1,4}}(t)), i=1,2,\ldots, \\ E_{L_{i,3}O_{i,4}}(t)=(1-F_{L_{i,3}O_{i,4}}(t))(1-F_{L_{i,3}L_{i+1,3}}(t)), i=1,2,\ldots, \\ E_{I_{i,1}O_{i,4}}(t)=(1-F_{I_{i,1}O_{i,4}}(t))(1-F_{I_{i,1}I_{i+1,1}}(t)), i=1,2,\ldots, \\ E_{S_{i,j}O_{i,j-k}}(t)=(1-F_{S_{i,j}O_{i,j-k}}(t))(1-F_{S_{i,j}S_{i+1,j}}(t)), i=1,2,\ldots, \\ j-k=4, j=5,6,\ldots,M+4, k=1,2,\ldots,M. \end{cases}$$

To obtain performance measures, we consider a new variable $\psi_{m,n}$ which is the average amount of time the MRGP spends in state n between two sequential regeneration epochs condition that it was in state m after the last regeneration:

$$\psi_{m,n}=\int_0^\infty E_{m,n}(t)dt.$$

For different values of m and n, we have

$$\psi_{m,n}=\begin{cases} \frac{1-e^{-\frac{\lambda}{\alpha}}}{\lambda}, m=O_{i,j}, n=A_{i,j-2}, i=1,\ldots,j=4, \\ \frac{1-e^{-\frac{\lambda}{\gamma}}}{\lambda}, m=L_{i,j}, n=O_{i,j+1}, i=1,\ldots,j=3, \\ \frac{1-e^{-\frac{\lambda}{\beta}}}{\lambda}, m=S_{i,j}, n=O_{i,j-k}, i=1,2,\ldots,j-k=4, j=5,6,\ldots,M+4, k=1,2,\ldots,M, \\ \frac{1-e^{-\frac{\lambda}{\nu}}}{\lambda}, m=I_{0,1}, n=O_{1,4}, \\ \frac{1-e^{-\frac{\lambda}{\beta}}}{\lambda}, m=S_{0,M+4}, n=B_{0,0}, \\ \frac{1-e^{-\frac{\lambda}{\mu}}}{\lambda}, m=A_{1,2}, n=I_{0,1} \\ \frac{1-e^{-\frac{\lambda}{\gamma}}}{\lambda}, m=L_{0,3}, n=B_{0,0}, \\ \frac{1-e^{-\frac{\lambda}{\nu}}}{\lambda}, m=I_{0,1}, n=S_{0,5}, \\ \frac{1-e^{-\frac{\lambda}{\zeta}}}{\lambda}, m=B_{i,0}, n=O_{i,4}, \\ \frac{1-e^{-\frac{\lambda}{\zeta}}}{\lambda}, m=B_{0,0}, n=L_{0,3}. \end{cases}$$

In order to get the energy efficiency from the model, we need to find the system state probabilities. The system state probabilities are defined as follows:

$$\phi_n=\frac{\sum_{k\in\Omega}\pi_k\psi_{k,n}}{\sum_{k\in\Omega}\pi_k\delta_k}, n\in\Omega, \tag{B63}$$

where $\delta_k=\sum_{l\in\Omega}\psi_{kl}$. The stability conditions for the existence of steady-state probabilities of the MRGP model is $\lambda<\mu$. Hence, under stability condition using δ_k and steady-state probabilities probabilities of EMC, we get

$$\sum_{k\in\Omega}\pi_k\delta_k=\frac{G}{\lambda C},$$

where

$$\begin{aligned}
G =\ & \Big(2-e^{-\frac{\lambda}{\alpha}}+\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2\Big)\Big(1-e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\Big)\Bigg\{e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\zeta}+\frac{1}{\nu}\right)}\Big(e^{-\frac{\lambda}{\beta}}\Big)^M\Bigg(\frac{\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)}{\Big(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}\Big)}+\frac{e^{-\frac{\lambda}{\gamma}}\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)}{\Big(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\Big)}\Bigg)\\
& +e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\nu}\right)}\Big(e^{-\frac{\lambda}{\beta}}-\Big(e^{-\frac{\lambda}{\beta}}\Big)^{M+1}+1-e^{-\frac{\lambda}{\nu}}\Big)\Bigg\}+\frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\mu}+\frac{1}{\gamma}\right)}\Big(e^{-\frac{\lambda}{\beta}}\Big)^M}{\Big(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}\Big)}\Bigg[\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)^2\Big(3-e^{-\frac{\lambda}{\alpha}}+e^{-\frac{\lambda}{\zeta}}\Big)\Big(1-e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\Big)\\
& +\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)\Bigg[\Big(2-e^{-\frac{\lambda}{\alpha}}\Big)\Big\{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2+\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)+\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)^2\Big)\Big\}\\
& +\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)^3\Big(3-e^{-\frac{\lambda}{\zeta}}-e^{-\frac{\lambda}{\alpha}}\Big)\Big)+\frac{1}{\Big(e^{-\frac{\lambda}{\zeta}}-e^{-\frac{\lambda}{\alpha}}\Big)}\Bigg[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^6}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)^6}{e^{-\frac{\lambda}{\zeta}}}\Bigg]\Bigg)\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\\
& +e^{-\frac{\lambda}{\alpha}}\Bigg(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\zeta}}-5+4e^{-\frac{\lambda}{\alpha}}-\Big(e^{-\frac{\lambda}{\alpha}}\Big)^2+\frac{\Big(2-e^{-\frac{\lambda}{\zeta}}\Big)\Big(\Big(e^{-\frac{\lambda}{\zeta}}\Big)^2-2e^{-\frac{\lambda}{\zeta}}\Big)}{e^{-\frac{\lambda}{\zeta}}}\\
& +2\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)}{\Big(e^{-\frac{\lambda}{\zeta}}-e^{-\frac{\lambda}{\alpha}}\Big)}\Bigg[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)^2}{e^{-\frac{\lambda}{\zeta}}}\Bigg]+1\Bigg)\Big(1-e^{-\frac{\lambda}{\mu}}\Big)\Bigg]\\
& +\Big(1-e^{-\frac{\lambda}{\mu}}\Big)^2\Bigg(2-e^{-\frac{\lambda}{\gamma}}+\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^2+\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^3+\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^4+\frac{\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^5}{e^{-\frac{\lambda}{\gamma}}}\Bigg)\Bigg]\\
& +\frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\mu}+\frac{1}{\gamma}+\frac{1}{\nu}\right)}\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)\Big(e^{-\frac{\lambda}{\beta}}\Big)^M}{\Big(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\Big)}\Bigg\{\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)\Big(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\gamma}}\Big)\Big(1-e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\Big)\\
& +\Big(\Big(2-e^{-\frac{\lambda}{\alpha}}\Big)\Big\{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2+\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\\
& \Big(1-e^{-\frac{\lambda}{\gamma}}\Big)+\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^2\Big)\Big\}+\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^3\Big(3-e^{-\frac{\lambda}{\gamma}}-e^{-\frac{\lambda}{\alpha}}\Big)\\
& +\frac{1}{\Big(e^{-\frac{\lambda}{\gamma}}-e^{-\frac{\lambda}{\alpha}}\Big)}\Bigg[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^6}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^6}{e^{-\frac{\lambda}{\gamma}}}\Bigg]\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)+e^{-\frac{\lambda}{\alpha}}\Big(3-e^{-\frac{\lambda}{\alpha}}\\
& -e^{-\frac{\lambda}{\gamma}}-5+4e^{-\frac{\lambda}{\alpha}}-\Big(e^{-\frac{\lambda}{\alpha}}\Big)^2+\frac{\Big(2-e^{-\frac{\lambda}{\gamma}}\Big)\Big(\Big(e^{-\frac{\lambda}{\gamma}}\Big)^2-2e^{-\frac{\lambda}{\gamma}}\Big)}{e^{-\frac{\lambda}{\gamma}}}\\
& +2\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)}{\Big(e^{-\frac{\lambda}{\gamma}}-e^{-\frac{\lambda}{\alpha}}\Big)}\Bigg[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^2}{e^{-\frac{\lambda}{\gamma}}}\Bigg]+1\Bigg)\Big(1-e^{-\frac{\lambda}{\mu}}\Big)\Bigg\}\\
& +e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\beta}+\frac{1}{\nu}\right)}\Big(1-\Big(e^{-\frac{\lambda}{\beta}}\Big)^M\Big)\Big\{\Big(1-e^{-\frac{\lambda}{\beta}}\Big)\Big(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\beta}}\Big)\Big(1-e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\Big)\\
& +\Big(\Big(2-e^{-\frac{\lambda}{\alpha}}\Big)\Big\{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2+\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)
\end{aligned}\tag{B64}$$

$$\Big(1-e^{-\frac{\lambda}{\beta}}\Big)+\Big(1-e^{-\frac{\lambda}{\beta}}\Big)^2\Big)\Big\}+\Big(1-e^{-\frac{\lambda}{\beta}}\Big)^3\Big(3-e^{-\frac{\lambda}{\beta}}-e^{-\frac{\lambda}{\alpha}}\Big)$$

$$+\frac{1}{\Big(e^{-\frac{\lambda}{\beta}}-e^{-\frac{\lambda}{\alpha}}\Big)}\left[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^6}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\beta}}\Big)^6}{e^{-\frac{\lambda}{\beta}}}\right]\Bigg)\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)$$

$$+e^{-\frac{\lambda}{\alpha}}\Bigg(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\beta}}-5+4e^{-\frac{\lambda}{\alpha}}-\Big(e^{-\frac{\lambda}{\alpha}}\Big)^2+\frac{\Big(2-e^{-\frac{\lambda}{\beta}}\Big)\Big(\Big(e^{-\frac{\lambda}{\beta}}\Big)^2-2e^{-\frac{\lambda}{\beta}}\Big)}{e^{-\frac{\lambda}{\beta}}}$$

$$+2\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(1-e^{-\frac{\lambda}{\beta}}\Big)}{\Big(e^{-\frac{\lambda}{\beta}}-e^{-\frac{\lambda}{\alpha}}\Big)}\left[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\beta}}\Big)^2}{e^{-\frac{\lambda}{\beta}}}\right]+1\Bigg)$$

$$\Big(1-e^{-\frac{\lambda}{\mu}}\Big)\Big\}+e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\nu}\right)}\Big(1-e^{-\frac{\lambda}{\nu}}\Big)\Big\{\Big(1-e^{-\frac{\lambda}{\nu}}\Big)\Big(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\nu}}\Big)\Big(1-e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\Big)$$

$$+\Big(\Big(2-e^{-\frac{\lambda}{\alpha}}\Big)\Big\{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2$$

$$+\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(1-e^{-\frac{\lambda}{\nu}}\Big)+\Big(1-e^{-\frac{\lambda}{\nu}}\Big)^2\Big)\Big\}+\Big(1-e^{-\frac{\lambda}{\nu}}\Big)^3\Big(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\nu}}\Big)$$

$$+\frac{1}{\Big(e^{-\frac{\lambda}{\nu}}-e^{-\frac{\lambda}{\alpha}}\Big)}\left[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^6}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\nu}}\Big)^6}{e^{-\frac{\lambda}{\nu}}}\right]\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)$$

$$+e^{-\frac{\lambda}{\alpha}}\Bigg(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\nu}}-5+4e^{-\frac{\lambda}{\alpha}}-\Big(e^{-\frac{\lambda}{\alpha}}\Big)^2+\frac{\Big(2-e^{-\frac{\lambda}{\nu}}\Big)\Big(\Big(e^{-\frac{\lambda}{\nu}}\Big)^2-2e^{-\frac{\lambda}{\nu}}\Big)}{e^{-\frac{\lambda}{\nu}}}$$

$$+2\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(1-e^{-\frac{\lambda}{\nu}}\Big)}{\Big(e^{-\frac{\lambda}{\nu}}-e^{-\frac{\lambda}{\alpha}}\Big)}\left[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\beta}}\Big)^2}{e^{-\frac{\lambda}{\beta}}}\right]+1\Bigg)$$

$$\Big(1-e^{-\frac{\lambda}{\mu}}\Big)\Big\}+\frac{e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\nu}\right)}\Big(e^{-\frac{\lambda}{\beta}}\Big)^M\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)}{1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}}\Big(1+e^{\frac{\lambda}{\zeta}}\Big)$$

$$+\Big(e^{-\frac{\lambda}{\beta}}\Big)^{M-1}e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\nu}\right)}\Big(1-e^{-\frac{\lambda}{\beta}}\Big)+\frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\Big(e^{-\frac{\lambda}{\beta}}\Big)^M}{\Big(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}\Big)}e^{-\frac{\lambda}{\mu}}\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)$$

$$+\Big(1-e^{-\frac{\lambda}{\mu}}\Big)+e^{-\frac{\lambda}{\mu}}\Big[\Big(1-e^{-\frac{\lambda}{\nu}}\Big)+e^{-\frac{\lambda}{\nu}}\Big(1-\Big(e^{-\frac{\lambda}{\beta}}\Big)^M\Big)\Big]$$

$$+e^{\frac{\lambda}{\nu}}e^{-\frac{\lambda}{\mu}}\Big(1-e^{-\frac{\lambda}{\nu}}\Big)+e^{\frac{\lambda}{\beta}}\frac{\Big(1-\Big(e^{-\frac{\lambda}{\beta}}\Big)^M\Big)}{1-e^{-\frac{\lambda}{\beta}}}e^{-\frac{\lambda}{\nu}}e^{-\frac{\lambda}{\mu}}\Big(1-e^{-\frac{\lambda}{\beta}}\Big).$$

The steady-state probability for $\phi_{A_{i,2}}$ is as follows:

$$
\begin{aligned}
\phi_{A_{i,2}} =& \frac{1}{G}\Bigg[\frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\mu}+\frac{1}{\nu}\right)}\left(e^{-\frac{\lambda}{\beta}}\right)^{M}}{\left(1-e^{-\lambda\left(\frac{1}{\gamma}+\frac{1}{\zeta}\right)}\right)}\Big\{\left(1-e^{-\frac{\lambda}{\zeta}}\right)\Big(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\zeta}}+\left(1-e^{-\frac{\lambda}{\zeta}}\right)\left(1-e^{-\frac{\lambda}{\alpha}}\right)\\
&+\left(1-e^{-\frac{\lambda}{\zeta}}\right)^{2}+\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{2}\Big)\left(1-e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\right)+\left(1-e^{-\frac{\lambda}{\zeta}}\right)\Big[\Big(\sum_{k=1}^{4}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(4-k)}\left(1-e^{-\frac{\lambda}{\zeta}}\right)^{(k-1)}\\
&+\sum_{k=1}^{5}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(5-k)}\left(1-e^{-\frac{\lambda}{\zeta}}\right)^{(k-1)}+\sum_{k=1}^{i}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\zeta}}\right)^{(k-1)}\Big)\\
&\left(1-e^{-\frac{\lambda}{\alpha}}\right)+e^{-\frac{\lambda}{\alpha}}\Big(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\zeta}}+\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\\
&-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}+\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\zeta}}\right)^{(i-k)}-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\zeta}}\right)^{(i+1-k)}\\
&+\sum_{k=4}^{(i-1)}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\zeta}}\right)^{(k-3)}+\sum_{l=5}^{(i)}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i+1-l)}\left(1-e^{-\frac{\lambda}{\zeta}}\right)^{(l-4)}+1\Big)\left(1-e^{-\frac{\lambda}{\mu}}\right)\Big]\\
&+\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{2}\Big(1+\left(1-e^{-\frac{\lambda}{\gamma}}\right)+\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{2}+\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{3}+\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{4}+\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{(i-1)}\Big)\Big\}\\
&+e^{-\frac{\lambda}{\mu}}e^{-\frac{\lambda}{\gamma}}\left(1-e^{-\frac{\lambda}{\gamma}}\right)\frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\left(e^{-\frac{\lambda}{\beta}}\right)^{M}}{\left(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\right)}+\Big\{\Big(3-e^{-\frac{\lambda}{\gamma}}-e^{-\frac{\lambda}{\alpha}}+\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{2}+\left(1-e^{-\frac{\lambda}{\alpha}}\right)\left(1-e^{-\frac{\lambda}{\gamma}}\right)\\
&+\left(\left(1-e^{-\frac{\lambda}{\gamma}}\right)\right)^{2}\Big)\left(1-e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\right)+\Big(\sum_{k=1}^{4}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(4-k)}\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{(k-1)}+\sum_{k=1}^{5}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(5-k)}\\
&\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{(k-1)}+\sum_{k=1}^{i}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{(k-1)}\Big)\left(1-e^{-\frac{\lambda}{\alpha}}\right)\\
&+e^{-\frac{\lambda}{\alpha}}\Big(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\gamma}}+\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\\
&+\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{(i-k)}-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{(i+1-k)}+\sum_{k=4}^{i-1}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{(k-3)}\\
&+\sum_{l=5}^{i}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i+1-l)}\left(1-e^{-\frac{\lambda}{\gamma}}\right)^{(l-4)}+1\Big)\\
&\left(1-e^{-\frac{\lambda}{\mu}}\right)\Big\}+e^{-\frac{\lambda}{\mu}}e^{-\frac{\lambda}{\beta}}\Big(1-\left(e^{-\frac{\lambda}{\beta}}\right)^{M}\Big)e^{-\frac{\lambda}{\nu}}+\Big\{\Big(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\beta}}+\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{2}\\
&+\left(1-e^{-\frac{\lambda}{\beta}}\right)e^{-\frac{\lambda}{\alpha}}+\left(e^{-\frac{\lambda}{\beta}}\right)^{2}\Big)\left(1-e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\right)\\
&+\Big(\sum_{k=1}^{4}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(4-k)}\left(1-e^{-\frac{\lambda}{\beta}}\right)^{(k-1)}+\sum_{k=1}^{5}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(5-k)}\left(1-e^{-\frac{\lambda}{\beta}}\right)^{(k-1)}\\
&+\sum_{k=1}^{i}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\beta}}\right)^{(k-1)}\Big)\left(1-e^{-\frac{\lambda}{\alpha}}\right)\\
&+e^{-\frac{\lambda}{\alpha}}\Big(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\beta}}+\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}+\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\beta}}\right)^{(i-k)}\\
&-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\beta}}\right)^{(i+1-k)}+\sum_{k=4}^{i-1}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}
\end{aligned}
\tag{B65}
$$

$$\left(1-e^{-\frac{\lambda}{\beta}}\right)^{(k-3)}+\sum_{l=5}^{i}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i+1-l)}\left(1-e^{-\frac{\lambda}{\beta}}\right)^{(l-4)}+1\right)\left(1-e^{-\frac{\lambda}{\mu}}\right)\Bigg\}$$

$$+e^{-\frac{\lambda}{\mu}}e^{-\frac{\lambda}{\nu}}\left(1-e^{-\frac{\lambda}{\nu}}\right)\Bigg\{\Bigg(\left(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\nu}}+\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{2}\right.$$

$$\left.+\left(1-e^{-\frac{\lambda}{\alpha}}\right)\left(1-e^{-\frac{\lambda}{\nu}}\right)+\left(1-e^{-\frac{\lambda}{\nu}}\right)^{2}\right)\left(1-e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\right)$$

$$+\left(\sum_{k=1}^{4}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(4-k)}\left(1-e^{-\frac{\lambda}{\nu}}\right)^{(k-1)}+\sum_{k=1}^{5}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(5-k)}\left(1-e^{-\frac{\lambda}{\nu}}\right)^{(k-1)}\right.$$

$$\left.+\sum_{k=1}^{i}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\nu}}\right)^{(k-1)}\right)\left(1-e^{-\frac{\lambda}{\alpha}}\right)+e^{-\frac{\lambda}{\alpha}}\Big(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\nu}}$$

$$+\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}$$

$$+\sum_{k=3}^{4}\left(1-e^{-\frac{\lambda}{\nu}}\right)^{(i-k)}-\sum_{k=6}^{7}\left(1-e^{-\frac{\lambda}{\nu}}\right)^{(i+1-k)}+\sum_{k=4}^{(i-1)}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i-k)}\left(1-e^{-\frac{\lambda}{\nu}}\right)^{(k-3)}$$

$$+\sum_{l=5}^{(i)}\left(1-e^{-\frac{\lambda}{\alpha}}\right)^{(i+1-l)}\left(1-e^{-\frac{\lambda}{\nu}}\right)^{(l-4)}+1\Big)$$

$$\left(1-e^{-\frac{\lambda}{\mu}}\right)\Bigg\};i=6,7,....$$

The steady-state probability for being the system in active state is

$$
\begin{aligned}
P_A &= \sum\nolimits_{i=6}^{\infty}\phi_{A_{i,2}} = \frac{1}{G}\Bigg[\bigg(\Big(2-e^{-\frac{\lambda}{\alpha}}+\big(1-e^{-\frac{\lambda}{\alpha}}\big)^2\Big)\Big(1-e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\Big)\bigg\{e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\zeta}+\frac{1}{\nu}\right)}\Big(e^{-\frac{\lambda}{\beta}}\Big)^M\bigg(\frac{\big(1-e^{-\frac{\lambda}{\zeta}}\big)}{\Big(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}\Big)}+\frac{e^{-\frac{\lambda}{\gamma}}\big(1-e^{-\frac{\lambda}{\gamma}}\big)}{\Big(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\Big)}\bigg)+e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\nu}\right)} \\
&\quad \Big(e^{-\frac{\lambda}{\beta}}-\Big(e^{-\frac{\lambda}{\beta}}\Big)^{M+1}+1-e^{-\frac{\lambda}{\nu}}\Big)\bigg\}+\frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\mu}+\frac{1}{\gamma}\right)}\Big(e^{-\frac{\lambda}{\beta}}\Big)^M}{\Big(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\gamma}\right)}\Big)}\bigg[\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)^2\Big(3-e^{-\frac{\lambda}{\alpha}}+e^{-\frac{\lambda}{\zeta}}\Big)\Big(1-e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\Big)+\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)\bigg[\Big(2-e^{-\frac{\lambda}{\alpha}}\Big) \\
&\quad \bigg\{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2+\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)+\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)^2\Big)\bigg\}+\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)^3\Big(3-e^{-\frac{\lambda}{\zeta}}-e^{-\frac{\lambda}{\alpha}}\Big)\Big)+\frac{1}{\Big(e^{-\frac{\lambda}{\zeta}}-e^{-\frac{\lambda}{\alpha}}\Big)}\Bigg[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^6}{e^{-\frac{\lambda}{\alpha}}} \\
&\quad -\frac{\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)^6}{e^{-\frac{\lambda}{\zeta}}}\Bigg]\bigg)\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)+e^{-\frac{\lambda}{\alpha}}\Bigg(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\zeta}}-5+4e^{-\frac{\lambda}{\alpha}}-\Big(e^{-\frac{\lambda}{\alpha}}\Big)^2+\frac{\Big(2-e^{-\frac{\lambda}{\zeta}}\Big)\Big(\Big(e^{-\frac{\lambda}{\zeta}}\Big)^2-2e^{-\frac{\lambda}{\zeta}}\Big)}{e^{-\frac{\lambda}{\zeta}}}+2\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)}{\Big(e^{-\frac{\lambda}{\zeta}}-e^{-\frac{\lambda}{\alpha}}\Big)} \\
&\quad \Bigg[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\zeta}}\Big)^2}{e^{-\frac{\lambda}{\zeta}}}\Bigg]+1\Bigg)\Big(1-e^{-\frac{\lambda}{\mu}}\Big)\Bigg]+\Big(1-e^{-\frac{\lambda}{\mu}}\Big)^2\Bigg(2-e^{-\frac{\lambda}{\gamma}}+\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^2+\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^3+\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^4+\frac{\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^5}{e^{-\frac{\lambda}{\gamma}}}\Bigg)\Bigg] \\
&\quad +\frac{e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\mu}+\frac{1}{\gamma}+\frac{1}{\nu}\right)}\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)\Big(e^{-\frac{\lambda}{\beta}}\Big)^M}{\Big(1-e^{-\lambda\left(\frac{1}{\zeta}+\frac{1}{\nu}\right)}\Big)}\bigg\{\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)\Big(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\gamma}}\Big)\Big(1-e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\Big)+\Big(\Big(2-e^{-\frac{\lambda}{\alpha}}\Big)\bigg\{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2+\Big(1-e^{-\frac{\lambda}{\alpha}}\Big) \\
&\quad \Big(1-e^{-\frac{\lambda}{\gamma}}\Big)+\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^2\Big)\bigg\}+\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^3\Big(3-e^{-\frac{\lambda}{\gamma}}-e^{-\frac{\lambda}{\alpha}}\Big)+\frac{1}{\Big(e^{-\frac{\lambda}{\gamma}}-e^{-\frac{\lambda}{\alpha}}\Big)}\Bigg[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^6}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^6}{e^{-\frac{\lambda}{\gamma}}}\Bigg]\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)+e^{-\frac{\lambda}{\alpha}}\Big(3-e^{-\frac{\lambda}{\alpha}} \\
&\quad -e^{-\frac{\lambda}{\gamma}}-5+4e^{-\frac{\lambda}{\alpha}}-\Big(e^{-\frac{\lambda}{\alpha}}\Big)^2+\frac{\Big(2-e^{-\frac{\lambda}{\gamma}}\Big)\Big(\Big(e^{-\frac{\lambda}{\gamma}}\Big)^2-2e^{-\frac{\lambda}{\gamma}}\Big)}{e^{-\frac{\lambda}{\gamma}}}+2\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)}{\Big(e^{-\frac{\lambda}{\gamma}}-e^{-\frac{\lambda}{\alpha}}\Big)}\Bigg[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\gamma}}\Big)^2}{e^{-\frac{\lambda}{\gamma}}}\Bigg]+1\Bigg)\Big(1-e^{-\frac{\lambda}{\mu}}\Big)\Big\} \\
&\quad +e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\beta}+\frac{1}{\nu}\right)}\Big(1-\Big(e^{-\frac{\lambda}{\beta}}\Big)^M\Big)\bigg\{\Big(1-e^{-\frac{\lambda}{\beta}}\Big)\Big(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\beta}}\Big)\Big(1-e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\Big)+\Big(\Big(2-e^{-\frac{\lambda}{\alpha}}\Big)\bigg\{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2+\Big(1-e^{-\frac{\lambda}{\alpha}}\Big) \\
&\quad \Big(1-e^{-\frac{\lambda}{\beta}}\Big)+\Big(1-e^{-\frac{\lambda}{\beta}}\Big)^2\Big)\bigg\}+\Big(1-e^{-\frac{\lambda}{\beta}}\Big)^3\Big(3-e^{-\frac{\lambda}{\beta}}-e^{-\frac{\lambda}{\alpha}}\Big)+\frac{1}{\Big(e^{-\frac{\lambda}{\beta}}-e^{-\frac{\lambda}{\alpha}}\Big)}\Bigg[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^6}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\beta}}\Big)^6}{e^{-\frac{\lambda}{\beta}}}\Bigg]\Bigg)\Big(1-e^{-\frac{\lambda}{\alpha}}\Big) \\
&\quad +e^{-\frac{\lambda}{\alpha}}\Bigg(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\beta}}-5+4e^{-\frac{\lambda}{\alpha}}-\Big(e^{-\frac{\lambda}{\alpha}}\Big)^2+\frac{\Big(2-e^{-\frac{\lambda}{\beta}}\Big)\Big(\Big(e^{-\frac{\lambda}{\beta}}\Big)^2-2e^{-\frac{\lambda}{\beta}}\Big)}{e^{-\frac{\lambda}{\beta}}}+2\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(1-e^{-\frac{\lambda}{\beta}}\Big)}{\Big(e^{-\frac{\lambda}{\beta}}-e^{-\frac{\lambda}{\alpha}}\Big)}\Bigg[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\beta}}\Big)^2}{e^{-\frac{\lambda}{\beta}}}\Bigg]+1\Bigg) \\
&\quad \Big(1-e^{-\frac{\lambda}{\mu}}\Big)\bigg\}+e^{-\lambda\left(\frac{1}{\mu}+\frac{1}{\nu}\right)}\Big(1-e^{-\frac{\lambda}{\nu}}\Big)\bigg\{\Big(1-e^{-\frac{\lambda}{\nu}}\Big)\Big(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\nu}}\Big)\Big(1-e^{-\frac{\lambda}{\alpha}}e^{-\frac{\lambda}{\mu}}\Big)+\Big(\Big(2-e^{-\frac{\lambda}{\alpha}}\Big)\bigg\{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2 \\
&\quad +\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(1-e^{-\frac{\lambda}{\nu}}\Big)+\Big(1-e^{-\frac{\lambda}{\nu}}\Big)^2)\}+\Big(1-e^{-\frac{\lambda}{\nu}}\Big)^3\Big(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\nu}}\Big)+\frac{1}{\Big(e^{-\frac{\lambda}{\nu}}-e^{-\frac{\lambda}{\alpha}}\Big)}\Bigg[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^6}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\nu}}\Big)^6}{e^{-\frac{\lambda}{\nu}}}\Bigg]\Big(1-e^{-\frac{\lambda}{\alpha}}\Big) \\
&\quad +e^{-\frac{\lambda}{\alpha}}\Bigg(3-e^{-\frac{\lambda}{\alpha}}-e^{-\frac{\lambda}{\nu}}-5+4e^{-\frac{\lambda}{\alpha}}-\Big(e^{-\frac{\lambda}{\alpha}}\Big)^2+\frac{\Big(2-e^{-\frac{\lambda}{\nu}}\Big)\Big(\Big(e^{-\frac{\lambda}{\nu}}\Big)^2-2e^{-\frac{\lambda}{\nu}}\Big)}{e^{-\frac{\lambda}{\nu}}}+2\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)\Big(1-e^{-\frac{\lambda}{\nu}}\Big)}{\Big(e^{-\frac{\lambda}{\nu}}-e^{-\frac{\lambda}{\alpha}}\Big)}\Bigg[\frac{\Big(1-e^{-\frac{\lambda}{\alpha}}\Big)^2}{e^{-\frac{\lambda}{\alpha}}}-\frac{\Big(1-e^{-\frac{\lambda}{\beta}}\Big)^2}{e^{-\frac{\lambda}{\beta}}}\Bigg]+1\Bigg) \\
&\quad \Big(1-e^{-\frac{\lambda}{\mu}}\Big)\bigg\}\Bigg].
\end{aligned} \tag{B66}
$$