

## RESEARCH ARTICLE

# Stochastic Modeling of a Base Station in 5G Wireless Networks for Energy Efficiency

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## ABSTRACT

The potential benefits of 5G networks, such as faster data speeds and improved user experiences, come with a critical challenge—efficiently preserving energy in base stations (BSs). Addressing this challenge is crucial, necessitating a focus on maximizing the energy efficiency of these stations. BSs play a vital role in providing coverage and capacity by using different frequency bands adapting to diverse communication needs. The choice of a specific frequency band can impact network performance, data rates, coverage range, and signal propagation. This study emphasizes the crucial challenge of preserving energy in 5G BSs and underscores the significance of strategic frequency band selection for optimizing energy efficiency and network performance. For this, stochastic models are introduced for BSs, specifically the Markov and semi-Markov models, which help choose frequency bands based on system traffic. Closed-form solutions for steady-state system size probabilities in these models are derived. A sensitivity analysis is conducted to assess power consumption in various scenarios. Furthermore, a comparison between analytical findings and simulation outcomes regarding power consumption is presented. The convergence of results in both methodologies emphasizes a significant level of agreement; that is, the introduced stochastic models provide insights and are validated by a close alignment between analytical and discrete-event simulation outcomes. We have shown the behavior of power consumption with respect to three different distributions named deterministic, exponential, and hypo-exponential. This research highlights the importance of strategic frequency band selection for 5G BSs to optimize energy efficiency and meet the demands of evolving communication networks.

## 1 | Introduction

Cellular networks have evolved significantly to meet growing demand, starting with 1G in the late 1970s, which supported only voice calls at 2.4 kbps. The introduction of 2G in 1991 enabled messaging at up to 64 kbps. Later advancements brought 3G in 2000 and 4G in 2008, with 3G offering speeds up to 14 Mbps and 4G up to 100 Mbps, though with slightly higher latency. A study conducted by Al-Falahy and Alani [1] highlighted the distinctions between 5G and previous mobile network generations, enumerating various improvements. These enhancements aimed to minimize latency, attain higher multi-Gbps

peak data speeds, bolster reliability, and establish an expansive network infrastructure. In early 2019, to address these challenges and meet the growing demand for connectivity, 5G cellular networks were introduced. This next generation of networks promised reduced latency, higher multi-Gbps peak data speeds, greater reliability, and massive bandwidth capacities.

The bandwidth of a signal or communication channel is determined by the range of frequencies it encompasses. Chih-Lin et al. [2] showed that, in 5G cellular networks, with a bandwidth of 100 MHz, the power consumption is five times higher than the power consumption in 4G networks, where the bandwidth

is 20 MHz, presuming the power spectrum density to be the same. The deployment of 5G networks poses significant energy challenges due to increased network complexity, densification of infrastructure, and the power-intensive nature of advanced technologies. Addressing environmental implications and ensuring sustainable energy consumption are crucial considerations for the widespread adoption of 5G technology. Cellular base stations (BSs) employ backup batteries for continuous power supply; hence, research [3] investigated how the surplus capacity in 5G BS backup batteries, influenced by varying electricity demand, can be utilized as a flexible asset for power systems. Albert Ayang et al. [4] analyzed the energy consumption in each component of the BS. The study [5] explored energy efficiency within 5G BSs, utilizing an M/G/1 queue model featuring multiple sleep modes and an N-policy. The diverse approaches, which address the aforementioned issue at various scales, include network planning and deployment, switching-off techniques [6], radio resource management optimization, component level augmentation, and the use of renewable energy sources. A strategy based on Q-learning in combination with logarithmic cooling was proposed to optimize the connection of UE with a BS or access point to address energy efficiency issues of device-to-device communications in heterogeneous networks [7]. One of the necessities for 5G networks is an increase in energy efficiency by at least 10 to 100 times [8]. To increase energy efficiency, Pervaiz et al. [8] proposed a multilevel sleep strategy to predict the BSs sleep time in advance. By assuming the high formation density of 5G BSs, the altogether power consumption may extend two times that of 4G BSs [2]. Elsherif et al. [9] developed an MDP-based algorithm for real-time management of BS ON/OFF switching to enhance efficiency. In BS communication, frequency bands are specific radio frequency ranges used for transmitting and receiving signals between BSs and user equipment (UE). These bands, set by regulatory bodies, are crucial for interference-free wireless communication across technologies like 2G, 3G, 4G, and 5G. These bands are further subdivided into segments, each with its own defined frequency range. BSs leverage these diverse frequency bands to provide coverage and capacity, adapting to changing communication requirements. The choice of a particular frequency band can have a significant impact on network performance, data rates, coverage area, and signal propagation characteristics. Therefore, frequency bands play a crucial role in BS communication, facilitating efficient wireless communication within specific geographic areas. In the context of 5G, available allocated frequency bands include 2300, 2100, 1800, and 900 MHz. The selection of a frequency band is directly tied to network traffic considerations.

The surge in mobile network traffic over recent years has led to a rise in operational costs and sustainability concerns. One primary focal point when dealing with heightened traffic loads is the escalating power consumption issue. It is worth noting that there is existing literature addressing this very issue in 5G networks. Energy consumption stands as a critical factor in wireless networks, attracting the attention of numerous researchers, such as those who explored energy-related issues in wireless networks [10, 11].

In the realm of wireless networks, BSs play a pivotal role in communication but are also substantial consumers of energy. Consequently, there exists an extensive body of literature dedicated to the analysis of energy consumption in BSs. Within article

[12], a 5G BS sleeping mechanism has been devised and modeled. The objective is to diminish average power consumption and attain the utmost efficiency in conserving power for 5G BSs. In the manuscript [13], they achieved the system's peak energy conservation by finely tuning the parameters of maximum short sleep and short sleep duration, that is, introduced a mechanism tailored for 5G networks, with a strong emphasis on enhancing energy efficiency. Moreover, to deliver a specified quality of service (QoS) to users while conserving energy in BSs, various strategies have been investigated, including the reduction in the number of BSs as proposed in [14] and network traffic aggregation as discussed in [15]. The energy utilization of BSs has been thoroughly examined, as exemplified by the analysis presented in [16].

Furthermore, the research conducted in [17] systematically scrutinized the energy efficiency of small cell BSs operating in 4G/5G networks. This comprehensive study evaluated the factors driving energy consumption and assessed the overall performance of these stations. The author [18] proposed a mobile robot localization approach based on the Extended Kalman Filter, which is utilized to determine the robot's precise location. To improve network performance within the constraints of limited capacity, the research [19] proposed a method for ensuring QoS for real-time traffic. The paper [20] investigated energy efficiency in wireless sensor networks by employing a semi-Markov model to depict BS availability, considering failures in its components. Additionally, the article [21] concentrated on enhancing BS energy efficiency through the application of the Q-learning algorithm, presenting the intricate trade-off between energy efficiency and QoS. The basic BS switching strategy was investigated in [22] for saving energy in cellular wireless access networks. Research [23] investigated how the density of BSs and the number of transmit antennas impacted the network's throughput and energy efficiency. Deep reinforcement learning is used in order to reduce the amount of energy used by the BS in dense wireless networks in [24], where the challenge of reducing energy consumption was carried out by turning cells on and off in dense wireless networks as a Markov decision process. Linear models are accurate enough to explain BS power consumption in terms of load, while distribution unit power is constant. Remote unit (RU) power is more responsive to changes in load; hence, power reduction was accomplished by network load adaptability and dynamic switching on and off [25]. In [26], energy efficiency for cellular networks with BSs that support full-duplex communications was proposed. The study [27] presented a stochastic model for energy-saving states, employing both Markov process (MP) and semi-Markov process (SMP) analyses to assess steady-state probabilities and enhance energy efficiency. For a concise overview of how our work compares to existing research, please refer to Table 1.

Sending and receiving wireless signals necessitates energy, and the energy consumption is closely tied to the amount of bandwidth employed. Broadening the bandwidth for data transmission typically results in higher energy consumption. Therefore, optimizing the utilization of bandwidth becomes pivotal in reducing energy usage. Therefore, the connection between the capacity of frequency bandwidth and energy efficiency holds great significance in the development and operation of wireless communication systems. With the

**TABLE 1** | Comparison of related papers with our work.

| Related papers                           | LTE-A | LTE-5G | Energy consumption |   | Strategy                                           | Analytical/simulation models                        |
|------------------------------------------|-------|--------|--------------------|---|----------------------------------------------------|-----------------------------------------------------|
|                                          |       |        | $(E_{cons})$       |   |                                                    |                                                     |
| Liu et al.<br>(2016) [28]                | ✓     | —      | ✓                  | ✓ | Sleeping<br>Strategy                               | Simulation                                          |
| P. Bhuvaneswari<br>et al.<br>(2018) [29] | ✓     | —      | ✓                  | ✓ | Backhaul<br>Traffic<br>in cooperation<br>technique | Simulation                                          |
| Chang et al.<br>(2020) [30]              | —     | ✓      | ✓                  | ✓ | BS sleep<br>scheme                                 | Simulation                                          |
| Meo et al.<br>(2021) [31]                | —     | ✓      | ✓                  | ✓ | Advanced<br>sleep<br>mechanism                     | CTMC<br>Model                                       |
| Wu et al.<br>(2021) [32]                 | —     | ✓      | ✓                  | ✓ | data-driven<br>learning<br>approach                | Markov<br>Decision process                          |
| Zhang et al.<br>(2021) [33]              | —     | ✓      | ✓                  | ✓ | Shutoff BS<br>parts                                | Machine<br>learning                                 |
| Pessapati et al.<br>(2021) [34]          | —     | ✓      | ✓                  | ✓ | Sleep modes<br>of BS                               | Machine<br>learning                                 |
| Omoniwa et al.<br>(2022) [35]            | —     | ✓      | ✓                  | ✓ | BS equipped<br>with<br>autonomous<br>agent         | Machine<br>learning                                 |
| Jong et al.<br>(2022) [36]               | ✓     | —      | ✓                  | ✓ | Resource block<br>allocation<br>method             | Simulation                                          |
| Kalita et al.<br>(2023) [37]             | —     | ✓      | ✓                  | ✓ | Sleeping<br>Strategy                               | M/G/1 queue<br>with two types<br>of vacation        |
| Deepa et al.<br>(2023) [17]              | —     | ✓      | ✓                  | ✓ | Sleeping<br>Strategy                               | M/G/1 queue<br>with two types<br>of sleep modes     |
| Deena et al.<br>(2023) [5]               | —     | ✓      | ✓                  | ✓ | N-policy<br>to awake BS<br>from multiple<br>sleeps | M/G/1 queue<br>with close down,<br>sleep, and setup |
| Our proposed<br>paper                    | —     | ✓      | ✓                  | ✓ | Different<br>frequency<br>bands                    | CTMC and<br>SMP and<br>simulation                   |

persistent rise in the demand for greater data speeds, discovering inventive approaches to maximize bandwidth usage while minimizing energy consumption remains imperative for establishing sustainable and highly efficient wireless networks. Striking this equilibrium is essential in meeting the demands of contemporary communication while addressing energy and environmental considerations.

The model aims to reduce energy consumption in wireless networks, aligning with the increasing demand for data speeds. It employs diverse stochastic models to assess power consumption in BSs with multiple resource units across various frequency bands, seeking an efficient power-saving strategy.

The novelty and contribution of this work is listed below:

- Investigated stochastic models with multiple RUs at a single BS using different frequency bands to address power consumption issues.
- Proposed a novel wireless network architecture designed to minimize power consumption in BSs.
- Developed a stochastic model to assess the feasibility of this architecture, considering a broad range of sojourn time distributions, including nonexponential distributions.
- Created a stochastic model using continuous-time Markov chain (CTMC) and SMPs to analyze state transitions and determine steady-state probabilities.
- Analyzed the impact of various factors on power consumption and compared results from MPs and SMP for the stochastic model.
- Employed a discrete-event simulation approach to verify the accuracy of the presented model.

In the CTMC, state transitions follow exponential distributions. Conversely, in the SMP, mean sojourn times across all states follow a general distribution, encompassing deterministic and hypo-exponential scenarios. A two-stage approach is utilized to analyze SMP dynamics and determine the steady-state probabilities of the system. Our numerical results illustrate the influence of various factors on power consumption and provide a comparison between the results obtained using the MP and SMP for the stochastic model.

The rest of the paper is structured as follows: Section 2 provides a system description outlining the proposed model. The subsequent sections delve into the analysis of the proposed stochastic models—the CTMC model and SMP model are discussed in Section 3, while Section 4 presents numerical results for these models, offering a comparison between analytical and simulation findings. Lastly, Section 5 offers concluding remarks and outlines areas for future research.

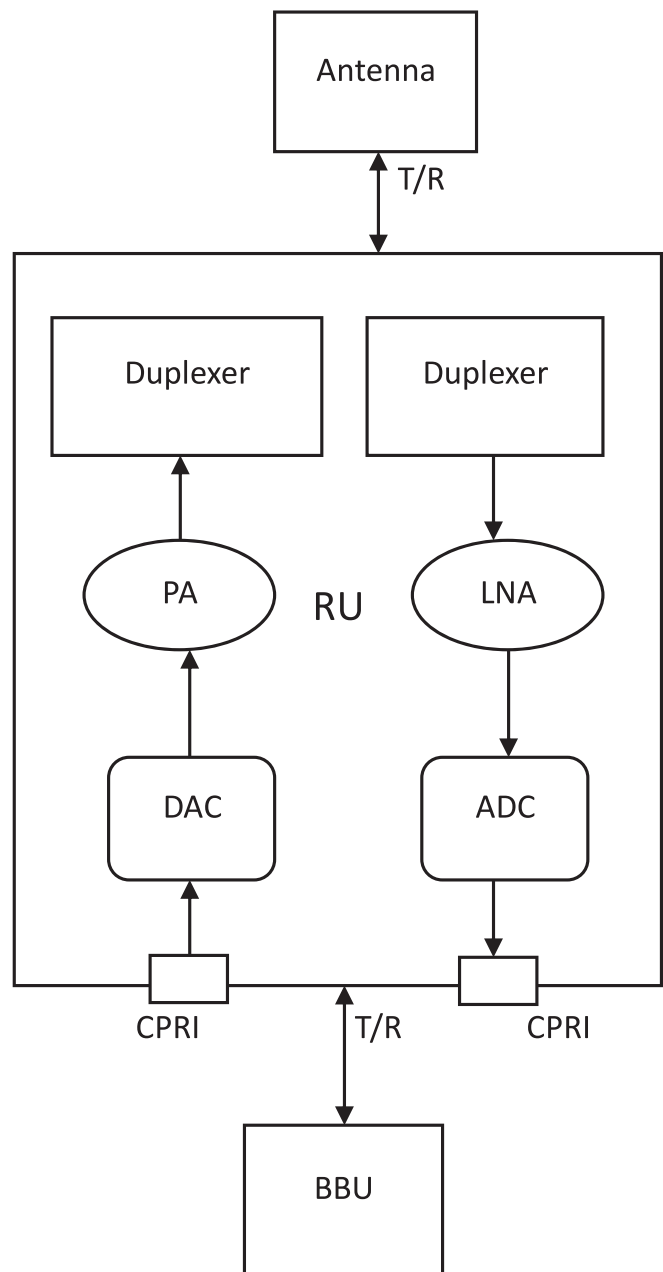
## 2 | System Description

Certainly, let us delve deeper into the intricacies of this model. The primary objective of this study is to enhance the energy efficiency of BSs, with a specific focus on the RU. The RU is a

crucial component within a BS responsible for the transmission and reception of wireless signals. To achieve this goal, it is imperative to understand the architecture and functions of the RU. In Figure 1, a detailed representation of the RU's architecture is presented. It consists of two main parts: the transmission path and the reception path, each with its set of vital components.

### Transmission Path:

- Digital to analog converter (DAC): This component takes digital signals from the baseband unit (BBU) and converts them into analog signals, which can be further processed for transmission.
- Power amplifier (PA): The analog signal is then passed through the PA, which amplifies it to a level suitable for wireless transmission.



**FIGURE 1** | An overview of the architecture of remote unit.

- **Filters (Duplexer):** These filters ensure that the transmitted signal is confined within the desired frequency band to prevent interference with other signals. This is especially crucial in a shared spectrum environment.

### Reception Path:

- **Filter:** Incoming signals from the antenna are initially filtered to eliminate unwanted noise and interference.
- **Low noise amplifier (LNA):** The filtered signal is then amplified by the LNA to boost its strength while minimizing additional noise.
- **Analog to digital converter (ADC):** After amplification, the signal is converted back into digital form, making it suitable for further processing.

Both the transmission and reception paths are connected to the BBU, where more advanced processing and management of these signals occur.

In this model, a noteworthy feature is the allocation of specific frequency bands for handling the transmission and reception of requests. The choice of frequency bands, such as 2300 carrier A, 2300 carrier B, 2100, 1800, and 900, is based on the number of resource blocks required to efficiently manage users connected via the radio resource control (RRC) protocol. By optimizing the energy usage in the RU, especially in its transmission and reception components, this model aims to contribute to the broader goal of reducing the overall power consumption in BSs. This is of paramount importance given the increasing demand for wireless communication and the need for sustainable and energy-efficient network operations.

## 2.1 | System Model

Consider a BS where we have different RUs with different frequency bands to provide the service based on the traffic activity. Frequency bands are 2300 carrier A, 2300 carrier B, 2100, 1800, and 900. A model is proposed based on the frequency bands required to handle the traffic load in the BS. However, the reallocation of frequency bands to different users always takes place based on the current load and the service required. The states for the proposed model as defined as follows:

- **Full service mode ( $S_1$ ):** All frequency bands are active to provide the service.
- **Reduced carrier A ( $S_2$ ):** All frequency bands are active to provide the service except frequency band 2300 with carrier A.
- **Limited carriers A and B ( $S_3$ ):** All frequency bands are active to provide the service except frequency bands 2300 with carriers A and B.
- **Selective band activation ( $S_4$ ):** Only frequency bands 1800 and 900 are active to provide the service.
- **Minimum service mode ( $S_5$ ):** Only frequency band 900 is active to provide the service.

The transition from one state to another is shown in the flow-chart given in Figure 2. Here, RRC defines the RRC-connected users to the BS, and PRB defines the physical resource blocks (PRBs) required to handle the current connected load. Let us assume the BS is in state  $S_1$ , that is, the traffic load is high, the number of PRBs required to handle the traffic is greater than 55%, that is, more than 55% of the total resource blocks in the BS are required active to handle the connected users and RRC connected users is almost 100%, that is, most of the users require the service. Hence, at this time, all the frequency bands are required to provide the service. If the number of resource blocks required to handle the traffic is not greater than 55%, and other frequency bands like 2300 carrier B, 2100, 1800, and 900 have enough PRBs that are not utilized in providing the service and are ready to share a traffic load of frequency 2300 carrier A, then the system moves to state  $S_2$ . Similarly, now, if the number of resource blocks required to handle the traffic is less than 50%, and RRC-connected users are less than 98%, and other frequency bands like 2100, 1800, and 900 have enough resource blocks that are not utilized in providing the service and are ready to share a traffic load of frequency 2300 carrier B, then the system moves to state  $S_3$ . Similarly, transition takes place based on the number of PRBs required and RRC-connected users. If enough PRBs are available to share the load, that is, the lower frequency bands acknowledge that they can share the load only then the transition can take place. After the transition, the reallocation of frequency bands to the users takes place based on the current traffic load.

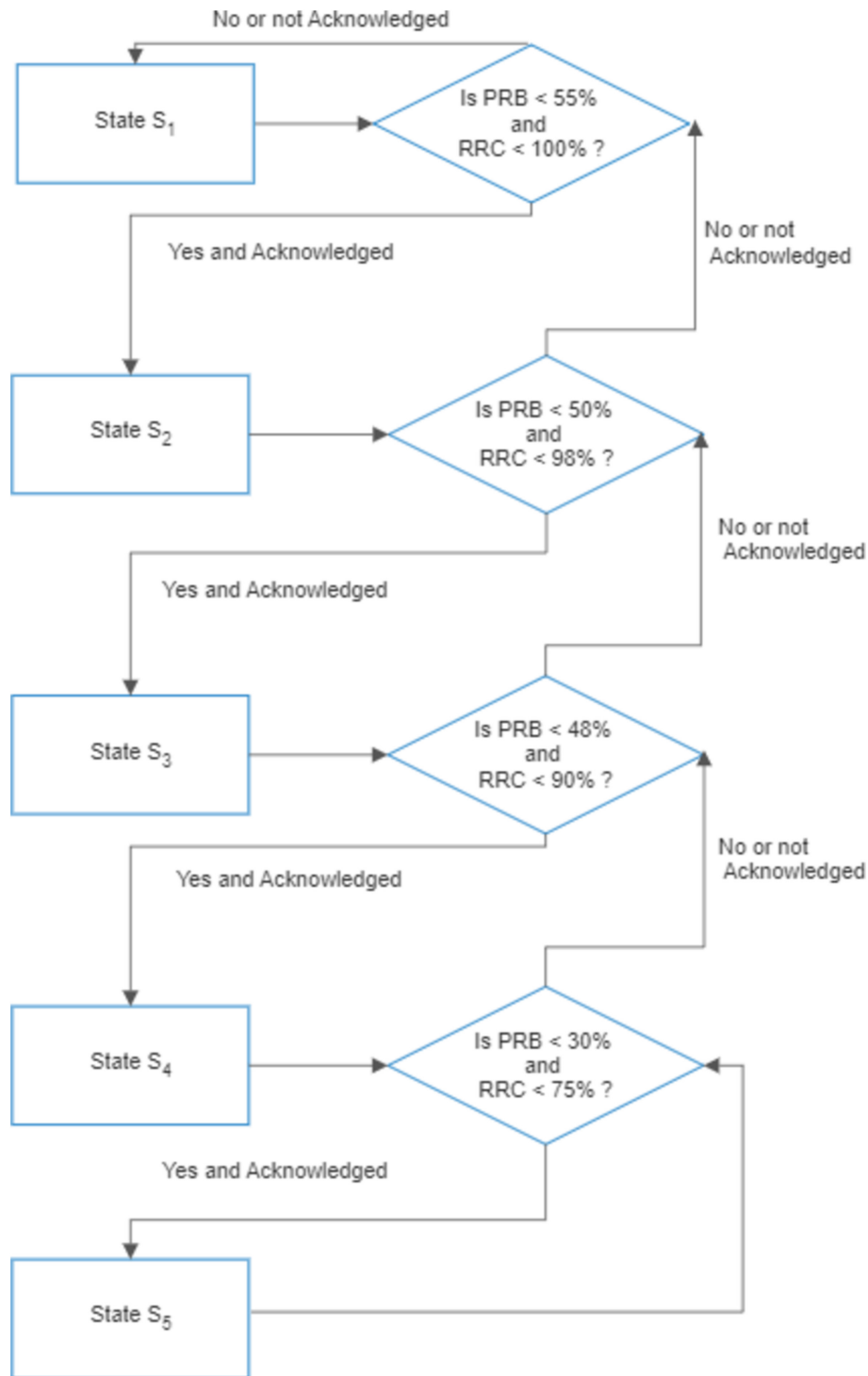
## 3 | Steady-State Analysis

In this section, we have presented the steady-state analysis for both stochastic models. The steady-state for the SMP model and CTMC model is presented in Sections 3.1 and 3.2, respectively. In SMP, the sojourn times follow general distribution, which is considered deterministic in Section 3.1.1 and hypo-exponential in Section 3.1.2.

### 3.1 | SMP

In this section, the analysis of the model in its steady-state will be explored. The journey begins by examining the underlying system state process, which we denote as  $\{X(t), t \geq 0\}$ . This representation of the system's state at any given time  $t$  is encapsulated by the sequence of system states denoted as  $\Omega = \{S_1, S_2, S_3, S_4, S_5\}$ , which corresponds to specific time instances  $T = \{t_0, t_1, t_2, \dots\}$ . Together, this sequence and its associated time instances form an embedded discrete-time Markov chain—a mathematical model that characterizes a series of events or states within a system. It operates on the fundamental principle that the probability of transitioning to the next state depends solely on the current state, entirely independent of previous states.

In the realm of a discrete-time Markov chain, if we denote the system's state at time  $t_n$  as  $X(t_n)$ , we can express the transition probabilities as  $P\{X(t_n + 1) = j | X(t_n) = i\}$ , where  $i$  and  $j$  signify distinct system states. Given the presence of a general distribution governing the mean sojourn time of states and the adherence to Markov attributes during the time epochs  $T$ , it becomes

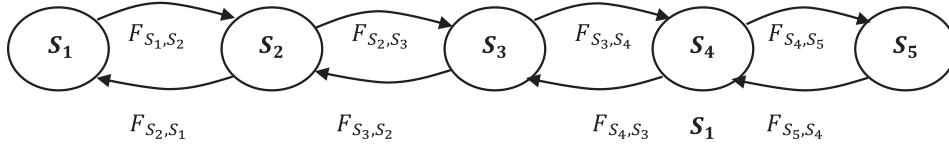


**FIGURE 2** | Flow chart for the proposed model.

suitable to represent the underlying stochastic process using a SMP [38]. The state space for this foundational stochastic process is defined as  $\Omega = \{S_1, S_2, S_3, S_4, S_5\}$ .

For a visual representation of state transitions within the proposed semi-Markov model, please refer to Figure 3. Furthermore, the cumulative distribution function (CDF) for the sojourn time in various states of the semi-Markov model is detailed in Table 2.

To calculate these probabilities, a two-stage method is used. In this approach, the focus is on the components of the kernel matrix  $K(t)$ , which is defined as  $K_{m,n}(t) = P\{Y_1 = n, t_1 \leq t | Y_0 = m\}$ , where  $m, n \in \Omega$ , and  $\{(Y_n, t_n), n \geq 0\}$  represents the underlying Markov renewal sequence of random variables. Alternatively,  $K_{m,n}(t)$  can be interpreted as the likelihood that, when the SMP has just entered state  $m$ , the subsequent transition will take place within time  $t$ , leading to state  $n$ . We can denote  $F_{m,n}$  as the CDF governing the transition from state  $m$  to state  $n$  within the



**FIGURE 3** | State transition diagram of the proposed model.

**TABLE 2** | Distributions of sojourn times for different states of the proposed semi-Markov model.

| CDF              | Distribution | Mean            |
|------------------|--------------|-----------------|
| $F_{S_1,S_2}(t)$ | General      | $\frac{1}{v_1}$ |
| $F_{S_2,S_3}(t)$ | General      | $\frac{1}{v_2}$ |
| $F_{S_3,S_4}(t)$ | General      | $\frac{1}{v_3}$ |
| $F_{S_4,S_5}(t)$ | General      | $\frac{1}{v_4}$ |
| $F_{S_2,S_1}(t)$ | General      | $\frac{1}{a_1}$ |
| $F_{S_3,S_2}(t)$ | General      | $\frac{1}{a_2}$ |
| $F_{S_4,S_3}(t)$ | General      | $\frac{1}{a_3}$ |
| $F_{S_5,S_4}(t)$ | General      | $\frac{1}{a_4}$ |

state space. The nonzero elements within the kernel matrix are as follows:

$$K_{m,n}(t) = P(X_1 = n, T_1 \leq t/X_0 = m).$$

$$K_{S_1,S_2}(t) = \int_0^t d(F_{S_1,S_2}(x)), \quad (1)$$

$$K_{S_2,S_1}(t) = \int_0^t (1 - F_{S_2,S_3}(x)) d(F_{S_2,S_1}(x)), \quad (2)$$

$$K_{S_2,S_3}(t) = \int_0^t (1 - F_{S_2,S_1}(x)) d(F_{S_2,S_3}(x)), \quad (3)$$

$$K_{S_3,S_4}(t) = \int_0^t (1 - F_{S_3,S_2}(x)) d(F_{S_3,S_4}(x)), \quad (4)$$

$$K_{S_3,S_2}(t) = \int_0^t (1 - F_{S_3,S_4}(x)) d(F_{S_3,S_2}(x)), \quad (5)$$

$$K_{S_4,S_5}(t) = \int_0^t (1 - F_{S_4,S_3}(x)) d(F_{S_4,S_5}(x)), \quad (6)$$

$$K_{S_4,S_3}(t) = \int_0^t (1 - F_{S_4,S_5}(x)) d(F_{S_4,S_3}(x)), \quad (7)$$

$$K_{S_5,S_4}(t) = \int_0^t d(F_{S_5,S_4}(x)). \quad (8)$$

Using the two-stage technique for the semi-Markov model,  $K_{m,n}(\infty) = \lim_{t \rightarrow \infty} K_{m,n}(t)$ , where  $m, n \in \Omega$ . Here, matrix  $K(\infty)$

with entries given as  $K_{m,n}(\infty)$  denotes the one-step transition probabilities for the EMC (embedded Markov chain) of the SMP model. The row sum of the  $K(\infty)$  matrix is 1 for all the rows. The steady-state probabilities for states of EMC, denoted by vector  $V$  and are given as  $(V_{S_1}, V_{S_2}, V_{S_3}, V_{S_4}, V_{S_5})$ , can be obtained by solving the following:

$$V = VK(\infty), \quad \sum_{m \in \Omega} V_m = 1. \quad (9)$$

Let us assume that  $T_m$  denotes the sojourn time in any state  $m \in \Omega$ . Since the EMC obtained is irreducible, aperiodic, and positive recurrent, that is, ergodic, hence, for this model, the steady-state probabilities for each state  $m \in \Omega$  of the semi-Markov model can be obtained as follows:

$$\pi_m = \frac{V_m E[T_m]}{\sum_{n \in \Omega} V_n E[T_n]}, \quad m \in \Omega, \quad (10)$$

where  $E[T_m]$  denotes the expected sojourn time in any state  $m \in \Omega$ .

$$E[T_{S_1}] = \int_0^{\infty} (1 - F_{S_1,S_2}(t)) dt, \quad (11)$$

$$E[T_{S_2}] = \int_0^{\infty} (1 - F_{S_2,S_1}(t))(1 - F_{S_2,S_3}(t)) dt, \quad (12)$$

$$E[T_{S_3}] = \int_0^{\infty} (1 - F_{S_3,S_2}(t))(1 - F_{S_3,S_4}(t)) dt, \quad (13)$$

$$E[T_{S_4}] = \int_0^{\infty} (1 - F_{S_4,S_3}(t))(1 - F_{S_4,S_5}(t)) dt, \quad (14)$$

$$E[T_{S_5}] = \int_0^{\infty} (1 - F_{S_5,S_4}(t)) dt. \quad (15)$$

### 3.1.1 | Deterministic Distribution

The transitions from one state to another depend only upon the current state based on the traffic load not on the past states, and hence, the proposed model is visualized as a SMP if the sojourn time in different states is considered as deterministic distributed with a given mean sojourn time as a parameter in Table 2. This assumption is grounded in the reality that, in real-time scenarios, there might not be a specific distribution that accurately characterizes traffic variability. The possible distributions for general distribution is shown in Table 3. For the sake of obtaining results, we have adopted the use of a deterministic distribution as an approximation, offering a way to gain insights into the system's behavior. The state space is given as  $\Omega = \{S_1, S_2, S_3, S_4, S_5\}$ .

**TABLE 3** | Possible distributions for general distribution.

| General distribution | CDF                                                                                                                              |
|----------------------|----------------------------------------------------------------------------------------------------------------------------------|
| Deterministic        | $F(x) \begin{cases} 1 & x \leq 1/k \\ 0 & x > 1/k \end{cases}$                                                                   |
| Hypo-exponential     | $F(x) = 1 - \frac{\lambda_2 e^{-\lambda_1 x}}{\lambda_2 - \lambda_1} + \frac{\lambda_1 e^{-\lambda_2 x}}{\lambda_2 - \lambda_1}$ |

The nonzero elements of the Kernel matrix are obtained using Equations (1–8) which are given as follows:

$$K_{S_1, S_2}(t) = \begin{cases} 0 & \text{if } t < \frac{1}{v_1} \\ 1 & \text{if } t \geq \frac{1}{v_1} \end{cases}.$$

If  $\frac{1}{v_2} < \frac{1}{\alpha_1}$ , then

$$K_{S_2, S_3}(t) = \begin{cases} 0 & \text{if } t < \frac{1}{v_2} \\ 1 & \text{if } t \geq \frac{1}{v_2} \end{cases}.$$

$$K_{S_2, S_1}(t) = 0.$$

If  $\frac{1}{v_2} > \frac{1}{\alpha_1}$ , then

$$K_{S_2, S_1}(t) = \begin{cases} 0 & \text{if } t < \frac{1}{\alpha_1} \\ 1 & \text{if } t \geq \frac{1}{\alpha_1} \end{cases}.$$

$$K_{S_2, S_3}(t) = 0.$$

If  $\frac{1}{v_3} < \frac{1}{\alpha_2}$ , then

$$K_{S_3, S_4}(t) = \begin{cases} 0 & \text{if } t < \frac{1}{v_3} \\ 1 & \text{if } t \geq \frac{1}{v_3} \end{cases}.$$

$$K_{S_3, S_2}(t) = 0.$$

If  $\frac{1}{v_3} > \frac{1}{\alpha_2}$ , then

$$K_{S_3, S_2}(t) = \begin{cases} 0 & \text{if } t < \frac{1}{\alpha_2} \\ 1 & \text{if } t \geq \frac{1}{\alpha_2} \end{cases}.$$

$$K_{S_3, S_4}(t) = 0.$$

If  $\frac{1}{v_4} < \frac{1}{\alpha_3}$ , then

$$K_{S_4, S_5}(t) = \begin{cases} 0 & \text{if } t < \frac{1}{v_4} \\ 1 & \text{if } t \geq \frac{1}{v_4} \end{cases}.$$

$$K_{S_4, S_3}(t) = 0.$$

If  $\frac{1}{v_4} > \frac{1}{\alpha_3}$ , then

$$K_{S_4, S_3}(t) = \begin{cases} 0 & \text{if } t < \frac{1}{\alpha_3} \\ 1 & \text{if } t \geq \frac{1}{\alpha_3} \end{cases}.$$

$$K_{S_4, S_5}(t) = 0.$$

$$K_{S_5, S_4}(t) = \begin{cases} 0 & \text{if } t < \frac{1}{\alpha_4} \\ 1 & \text{if } t \geq \frac{1}{\alpha_4} \end{cases}.$$

Here, based on values of whether  $\frac{1}{v_{i+1}}$  is greater or lesser than  $\frac{1}{\alpha_i}$ ,  $i = 1, 2, 3$ , we can have eight possible cases.

**Case 1:**  $\frac{1}{v_2} < \frac{1}{\alpha_1}$ ,  $\frac{1}{v_3} < \frac{1}{\alpha_2}$ ,  $\frac{1}{v_4} < \frac{1}{\alpha_3}$

The one-step transition probabilities for the model are given as follows:

$$K(\infty) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Therefore, using  $V = VK(\infty)$  from Equation (9) and the entries of kernel matrix  $K(\infty)$  [38], we get the system of linear equations, given as follows:

$$V_{S_1} = 0; \quad V_{S_2} = V_{S_1}; \quad V_{S_3} = V_{S_2};$$

$$V_{S_4} = V_{S_5} + V_{S_3}; \quad V_{S_5} = V_{S_4}.$$

Solving this system of equations and using  $\sum_{m \in \Omega} V_m = 1$  from Equation (10). The steady-state probabilities of each state,  $V_m, m \in \Omega$ , are obtained as follows:

$$V_{S_1} = 0; \quad V_{S_2} = 0; \quad V_{S_3} = 0; \quad V_{S_4} = \frac{1}{2}; \quad V_{S_5} = \frac{1}{2}.$$

Then, the expected sojourn time spent in each state can be obtained using Equations (11–15) given as follows:

$$E[T_{S_1}] = \frac{1}{v_1}.$$

$$E[T_{S_2}] = \min\left(\frac{1}{\alpha_1}, \frac{1}{v_2}\right).$$

$$E[T_{S_3}] = \min\left(\frac{1}{\alpha_2}, \frac{1}{v_3}\right).$$

$$E[T_{S_4}] = \min\left(\frac{1}{\alpha_3}, \frac{1}{v_4}\right).$$

$$E[T_{S_5}] = \frac{1}{\alpha_4}.$$

Hence, the steady-state probabilities  $\pi_i, i \in \Omega$  using Equation (10), for the semi-Markov model are given by the following:  $(0, 0, 0, \frac{\alpha_4}{v_4 + \alpha_4}, \frac{v_4}{v_4 + \alpha_4})$ .

**Case 2:**  $\frac{1}{v_2} < \frac{1}{\alpha_1}, \frac{1}{v_3} < \frac{1}{\alpha_2}, \frac{1}{v_4} > \frac{1}{\alpha_3}$

Using the similar approach as used in Case 1, the steady-state probabilities  $\pi_i, i \in \Omega$  of the semi-Markov model are given by the following:  $(0, 0, \frac{\alpha_3}{v_3 + \alpha_3}, \frac{v_3}{v_3 + \alpha_3}, 0)$ .

**Case 3:**  $\frac{1}{v_2} < \frac{1}{\alpha_1}, \frac{1}{v_3} > \frac{1}{\alpha_2}, \frac{1}{v_4} > \frac{1}{\alpha_3}$

Using the similar approach as used in Case 1, the steady-state probabilities  $\pi_i, i \in \Omega$  of the semi-Markov model are given by the following:  $(0, \frac{\alpha_2}{v_2 + \alpha_2}, \frac{v_2}{v_2 + \alpha_2}, 0, 0)$ .

**Case 4:**  $\frac{1}{v_2} > \frac{1}{\alpha_1}, \frac{1}{v_3} > \frac{1}{\alpha_2}, \frac{1}{v_4} > \frac{1}{\alpha_3}$

Using the similar approach as used in Case 1, the steady-state probabilities  $\pi_i, i \in \Omega$  of the semi-Markov model are given by the following:  $(\frac{\alpha_1}{v_1 + \alpha_1}, \frac{v_1}{v_1 + \alpha_1}, 0, 0, 0)$ .

The other possible cases cannot be considered as they are not a stable system to be solved. Hence, consider the above-mentioned four possible cases and obtain the steady-state solutions for them.

### 3.1.2 | Hypo-Exponential Distribution

Similarly, the proposed model is visualized as a SMP if the sojourn time in different states is considered as hypo-exponential distributed with a given mean sojourn time as a parameter in Table 2. This assumption is also based on the practical consideration that real-time situations often lack a precise distribution to represent traffic fluctuations. To facilitate result generation,

we have chosen to employ a hypo-exponential distribution as an approximation. This approach provides a means to gain valuable insights into the system's behavior. The state space is given as  $\Omega = \{S_1, S_2, S_3, S_4, S_5\}$ . The CDF of hypo-exponential distribution is given as follows:

$$F(x) = 1 - \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_1 x} + \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 x}.$$

Therefore, consider the CDF of transition of a system from one state to another state given as follows:

$$\begin{aligned} F_{S_1, S_2}(x) &= 1 - \frac{v_{12}}{v_{12} - v_{11}} e^{-v_{11}x} + \frac{v_{11}}{v_{12} - v_{11}} e^{-v_{12}x}, \\ F_{S_2, S_3}(x) &= 1 - \frac{v_{22}}{v_{22} - v_{21}} e^{-v_{21}x} + \frac{v_{21}}{v_{22} - v_{21}} e^{-v_{22}x}, \\ F_{S_3, S_4}(x) &= 1 - \frac{v_{32}}{v_{32} - v_{31}} e^{-v_{31}x} + \frac{v_{31}}{v_{32} - v_{31}} e^{-v_{32}x}, \\ F_{S_4, S_5}(x) &= 1 - \frac{v_{42}}{v_{42} - v_{41}} e^{-v_{41}x} + \frac{v_{41}}{v_{42} - v_{41}} e^{-v_{42}x}, \\ F_{S_2, S_1}(x) &= 1 - \frac{\alpha_{12}}{\alpha_{12} - \alpha_{11}} e^{-\alpha_{11}x} + \frac{\alpha_{11}}{\alpha_{12} - \alpha_{11}} e^{-\alpha_{12}x}, \\ F_{S_3, S_2}(x) &= 1 - \frac{\alpha_{22}}{\alpha_{22} - \alpha_{21}} e^{-\alpha_{21}x} + \frac{\alpha_{21}}{\alpha_{22} - \alpha_{21}} e^{-\alpha_{22}x}, \\ F_{S_4, S_3}(x) &= 1 - \frac{\alpha_{32}}{\alpha_{32} - \alpha_{31}} e^{-\alpha_{31}x} + \frac{\alpha_{31}}{\alpha_{32} - \alpha_{31}} e^{-\alpha_{32}x}, \\ F_{S_5, S_4}(x) &= 1 - \frac{\alpha_{42}}{\alpha_{42} - \alpha_{41}} e^{-\alpha_{41}x} + \frac{\alpha_{41}}{\alpha_{42} - \alpha_{41}} e^{-\alpha_{42}x}. \end{aligned}$$

The parameters  $v_{ij}$  and  $\alpha_{ij}, i = 1, 2, 3, 4$  and  $j = 1, 2$  are chosen such that

$$\frac{1}{v_i} = \frac{1}{v_{i1}} + \frac{1}{v_{i2}}, i = 1, 2, 3, 4.$$

$$\frac{1}{\alpha_i} = \frac{1}{\alpha_{i1}} + \frac{1}{\alpha_{i2}}, i = 1, 2, 3, 4.$$

Using Equations (1–8), the entries of the kernel matrix are computed and displayed as follows.

$$\begin{aligned} K_{S_1, S_2}(t) &= 1 - \frac{v_{12}}{v_{12} - v_{11}} e^{-v_{11}t} + \frac{v_{11}}{v_{12} - v_{11}} e^{-v_{12}t} \\ K_{S_2, S_1}(t) &= \frac{\alpha_{11}\alpha_{12}}{(v_{22} - v_{21})(\alpha_{12} - \alpha_{11})} \left[ \frac{v_{22}(1 - e^{-(v_{21} + \alpha_{11})t})}{v_{21} + \alpha_{11}} - \frac{v_{22}(1 - e^{-(v_{21} + \alpha_{11})t})}{v_{21} + \alpha_{12}} - \frac{v_{21}(1 - e^{-(v_{22} + \alpha_{11})t})}{v_{22} + \alpha_{11}} + \frac{v_{21}(1 - e^{-(v_{22} + \alpha_{12})t})}{v_{22} + \alpha_{12}} \right] \\ K_{S_2, S_3}(t) &= \frac{v_{21}v_{22}}{(\alpha_{12} - \alpha_{11})(v_{22} - v_{21})} \left[ \frac{\alpha_{12}(1 - e^{-(\alpha_{11} + v_{21})t})}{\alpha_{11} + v_{21}} - \frac{\alpha_{12}(1 - e^{-(\alpha_{11} + v_{21})t})}{\alpha_{11} + v_{22}} - \frac{\alpha_{11}(1 - e^{-(\alpha_{12} + v_{21})t})}{\alpha_{12} + v_{21}} + \frac{\alpha_{11}(1 - e^{-(\alpha_{12} + v_{22})t})}{\alpha_{12} + v_{22}} \right] \\ K_{S_3, S_2}(t) &= \frac{\alpha_{21}\alpha_{22}}{(v_{32} - v_{31})(\alpha_{22} - \alpha_{21})} \left[ \frac{v_{32}(1 - e^{-(v_{31} + \alpha_{21})t})}{v_{31} + \alpha_{21}} - \frac{v_{32}(1 - e^{-(v_{31} + \alpha_{21})t})}{v_{31} + \alpha_{22}} - \frac{v_{31}(1 - e^{-(v_{32} + \alpha_{21})t})}{v_{32} + \alpha_{21}} + \frac{v_{31}(1 - e^{-(v_{32} + \alpha_{22})t})}{v_{32} + \alpha_{22}} \right] \\ K_{S_3, S_4}(t) &= \frac{v_{31}v_{32}}{(\alpha_{22} - \alpha_{21})(v_{32} - v_{31})} \left[ \frac{\alpha_{22}(1 - e^{-(\alpha_{21} + v_{31})t})}{\alpha_{21} + v_{31}} - \frac{\alpha_{22}(1 - e^{-(\alpha_{21} + v_{31})t})}{\alpha_{21} + v_{32}} - \frac{\alpha_{21}(1 - e^{-(\alpha_{22} + v_{31})t})}{\alpha_{22} + v_{31}} + \frac{\alpha_{21}(1 - e^{-(\alpha_{22} + v_{32})t})}{\alpha_{22} + v_{32}} \right] \\ K_{S_4, S_3}(t) &= \frac{\alpha_{31}\alpha_{32}}{(v_{42} - v_{41})(\alpha_{32} - \alpha_{31})} \left[ \frac{v_{42}(1 - e^{-(v_{41} + \alpha_{31})t})}{v_{41} + \alpha_{31}} - \frac{v_{42}(1 - e^{-(v_{41} + \alpha_{31})t})}{v_{41} + \alpha_{32}} - \frac{v_{41}(1 - e^{-(v_{42} + \alpha_{31})t})}{v_{42} + \alpha_{31}} + \frac{v_{41}(1 - e^{-(v_{42} + \alpha_{32})t})}{v_{42} + \alpha_{32}} \right] \\ K_{S_4, S_5}(t) &= \frac{v_{41}v_{42}}{(\alpha_{32} - \alpha_{31})(v_{42} - v_{41})} \left[ \frac{\alpha_{32}(1 - e^{-(\alpha_{31} + v_{41})t})}{\alpha_{31} + v_{41}} - \frac{\alpha_{32}(1 - e^{-(\alpha_{31} + v_{41})t})}{\alpha_{31} + v_{42}} - \frac{\alpha_{31}(1 - e^{-(\alpha_{32} + v_{41})t})}{\alpha_{32} + v_{41}} + \frac{\alpha_{31}(1 - e^{-(\alpha_{32} + v_{42})t})}{\alpha_{32} + v_{42}} \right] \\ K_{S_5, S_4}(t) &= 1 - \frac{\alpha_{42}}{\alpha_{42} - \alpha_{41}} e^{-\alpha_{41}t} + \frac{\alpha_{41}}{\alpha_{42} - \alpha_{41}} e^{-\alpha_{42}t}. \end{aligned}$$

Then, the expected sojourn time spent in each state can be obtained from Equations (11–15), which are as follows:

$$\begin{aligned} E[T_{S_1}] &= \frac{1}{v_1}. \\ E[T_{S_2}] &= \frac{1}{(\alpha_{12} - \alpha_{11})(v_{22} - v_{21})} \\ &\quad \left[ \frac{\alpha_{12}v_{22}}{\alpha_{11} + v_{21}} - \frac{\alpha_{12}v_{21}}{\alpha_{11} + v_{22}} - \frac{\alpha_{11}v_{22}}{\alpha_{12} + v_{21}} + \frac{\alpha_{11}v_{21}}{\alpha_{12} + v_{22}} \right]. \\ E[T_{S_3}] &= \frac{1}{(\alpha_{22} - \alpha_{21})(v_{32} - v_{31})} \\ &\quad \left[ \frac{\alpha_{22}v_{32}}{\alpha_{21} + v_{31}} - \frac{\alpha_{22}v_{31}}{\alpha_{21} + v_{32}} - \frac{\alpha_{21}v_{32}}{\alpha_{22} + v_{31}} + \frac{\alpha_{21}v_{31}}{\alpha_{22} + v_{32}} \right]. \\ E[T_{S_4}] &= \frac{1}{(\alpha_{32} - \alpha_{31})(v_{42} - v_{41})} \\ &\quad \left[ \frac{\alpha_{32}v_{42}}{\alpha_{31} + v_{41}} - \frac{\alpha_{32}v_{41}}{\alpha_{31} + v_{42}} - \frac{\alpha_{31}v_{42}}{\alpha_{32} + v_{41}} + \frac{\alpha_{31}v_{41}}{\alpha_{32} + v_{42}} \right]. \\ E[T_{S_5}] &= \frac{1}{\alpha_4}. \end{aligned}$$

Using these Kernel matrix elements and expected sojourn rates, we obtained the steady-state probabilities of this proposed SMP model.

### 3.2 | Continuous Time Markov Chain Model

Let  $\{X(t), t \geq 0\}$  be the stochastic process where  $X(t)$  denotes the state of the system at any time  $t \geq 0$ . The state transition diagram for the presented model mirrors the depiction in Figure 3. The state space for the underlying process is represented by  $\Omega$ .

Let  $P_{S_i}(t) = P[X(t) = S_i], i = 1, 2, 3, 4, 5$  be the probability that at any time  $t$  the system is in  $i$  state. Then, the steady-state probabilities for the system are given as  $\lim_{t \rightarrow \infty} P_{S_i}(t) = \pi_{S_i}, i = 1, 2, 3, 4, 5$ , where  $\pi_{S_i}$  is called the steady-state probability of the system. The infinitesimal generator matrix ( $Q$ ) for the Markov model is given by the following:

$$Q = \begin{pmatrix} -v_1 & v_1 & 0 & 0 & 0 \\ \alpha_1 & -(\alpha_1 + v_2) & v_2 & 0 & 0 \\ 0 & \alpha_2 & -(\alpha_2 + v_3) & v_3 & 0 \\ 0 & 0 & \alpha_3 & -(\alpha_3 + v_4) & v_4 \\ 0 & 0 & 0 & \alpha_4 & -\alpha_4 \end{pmatrix}$$

To derive the steady-state probabilities of the system, we formulate the steady-state equations employing the expression  $\pi Q = 0$ , which are represented as follows:

$$\begin{aligned} v_1 \pi_{S_1} &= \alpha_1 \pi_{S_2}, \\ v_1 \pi_{S_1} + \alpha_2 \pi_{S_3} &= (\alpha_1 + v_2) \pi_{S_2}, \\ v_2 \pi_{S_2} + \alpha_3 \pi_{S_4} &= (\alpha_2 + v_3) \pi_{S_3}, \\ v_3 \pi_{S_3} + \alpha_4 \pi_{S_5} &= (\alpha_3 + v_4) \pi_{S_4}, \\ v_4 \pi_{S_4} &= \alpha_4 \pi_{S_5}. \end{aligned}$$

Solving these equations, the steady-state probabilities in terms of  $\pi_{S_1}$  are as follows:

$$\pi_{S_2} = \frac{v_1}{\alpha_1} \pi_{S_1}.$$

$$\pi_{S_3} = \frac{v_2 v_1}{\alpha_2 \alpha_1} \pi_{S_1}.$$

$$\pi_{S_4} = \frac{v_3 v_2 v_1}{\alpha_3 \alpha_2 \alpha_1} \pi_{S_1}.$$

$$\pi_{S_5} = \frac{v_4 v_3 v_2 v_1}{\alpha_4 \alpha_3 \alpha_2 \alpha_1} \pi_{S_1}.$$

Using  $\sum_{m \in \Omega} \pi_m = 1$ , we obtain the following:

$$\pi_{S_1} = \frac{1}{\left(1 + \frac{v_1}{\alpha_1} + \frac{v_2 v_1}{\alpha_2 \alpha_1} + \frac{v_3 v_2 v_1}{\alpha_3 \alpha_2 \alpha_1} + \frac{v_4 v_3 v_2 v_1}{\alpha_4 \alpha_3 \alpha_2 \alpha_1}\right)}.$$

## 4 | Numerical Illustration

In this section, we present the numerical results. In the stochastic models presented in this paper, the transition from one state to another takes place following general distribution, but the sojourn time in each state is decided at the time instant the system enters the particular state. Hence, the power consumption is directly proportional to the sojourn time in different states. The power consumption value is different for different states. Consider the power consumption values given in Table 4 to obtain the results. The power consumption  $PC$  in the system is given by the following:

$$PC = PC_{S_1} \pi_{S_1} + PC_{S_2} \pi_{S_2} + PC_{S_3} \pi_{S_3} + PC_{S_4} \pi_{S_4} + PC_{S_5} \pi_{S_5},$$

where  $PC_i, i \in \Omega$  is power consumption in state  $i$ .

In Section 3.1.1, four cases are considered based on different values of the parameters. The steady-state probabilities obtained are used to get the power consumption in the system.

We have obtained the results for power consumption with varying values of different parameters for all the cases. The parametric values are considered for sensitivity analysis based on the parametric conditions.

### 4.1 | Comparison of Analytical and Simulated Results

In this experiment, we establish a comparison between the theoretical predictions and the outcomes derived from simulations for power consumption. The parameters considered for calculating analytical values remain consistent and are applied within a MATLAB program for simulation purposes. The simulation is iterated 10,000 times under identical parameter

**TABLE 4** | Power consumption in different states.

| State             | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ |
|-------------------|-------|-------|-------|-------|-------|
| Power consumption | 250 W | 230 W | 200 W | 180 W | 100 W |

settings, generating a series of 10,000 metric values. To assess the reliability of the simulation outcomes, the  $t$ -distribution test is employed.

The normal distribution is particularly useful for large sample sizes, accounting for the inherent uncertainty in the sample's standard deviation. Subsequently, we compute the mean and standard deviation of the 10,000 generated values and ascertain a 95% confidence interval (CI) for the accuracy of the results. This CI can be computed using the formula:

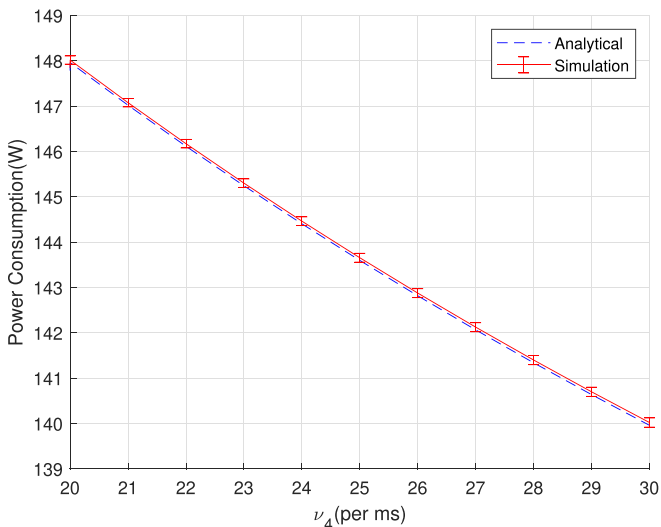
$$\text{C.I.} = \bar{X} \pm \frac{s * Z_{0.025}}{\sqrt{k}}$$

where  $\bar{X}$  represents the sample mean,  $s$  denotes the standard deviation,  $Z_{0.025}$  is the tabulated value for a 95% confidence interval, while  $k$  stands for the sample size.

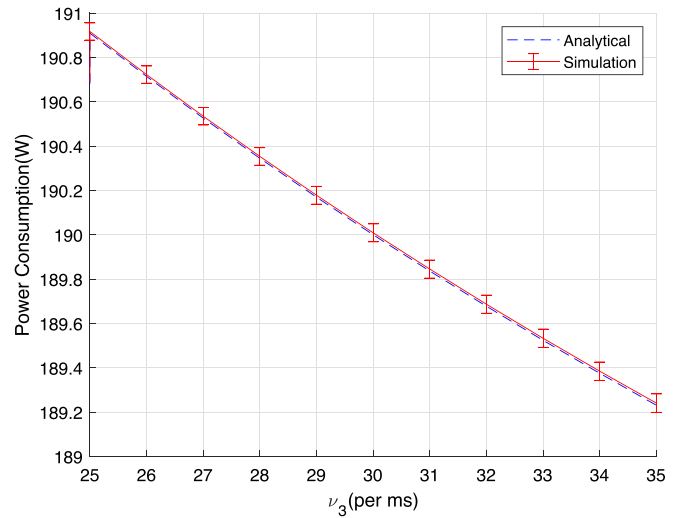
The graphical representations illustrate the findings from the simulation analysis as solid lines with 95% confidence intervals, while analytical results are depicted using dotted lines. In Section 3.1.1, four cases are considered based on different values of the parameters. The steady-state probabilities obtained are used to get the power consumption in the system. Figures 4–7 offer an extensive comparative analysis of power consumption outcomes for both analytical and simulated approaches across all four cases with sojourn times following deterministic distribution. Evidently, these methods yield convergent results, demonstrating a notable level of agreement.

The parameter values for plotting Figure 4 are based on parametric conditions considered in Case 1. It can be observed that the power consumption decreases with the increase in value of  $\nu_4$  as with an increase in value of  $\nu_4$  the system enters state  $S_5$  early; hence, power consumption will be reduced. From this figure, it can be observed that the analytical results match with the simulated ones.

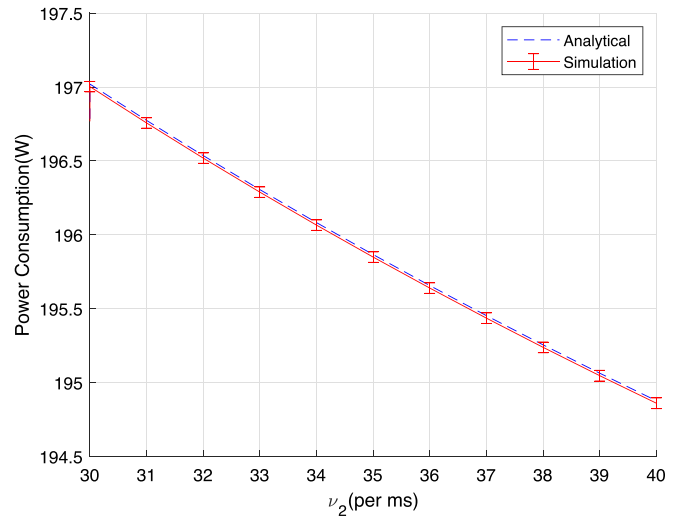
Similarly, the parameter values used to generate Figures 5–7 are derived from the parametric conditions examined in Cases 2–4,



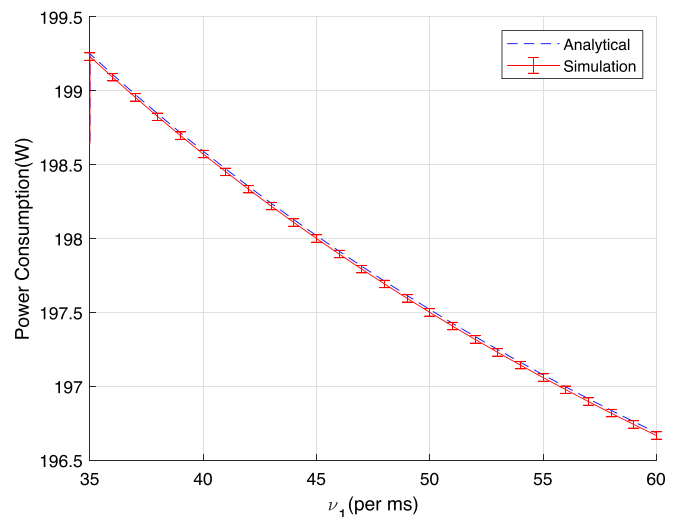
**FIGURE 4** | Comparison of analytical and simulated results with varying parameters values  $\nu_4$  for sojourn times deterministic distributed.



**FIGURE 5** | Comparison of analytical and simulated results with varying parameters values  $\nu_3$  for sojourn times deterministic distributed.



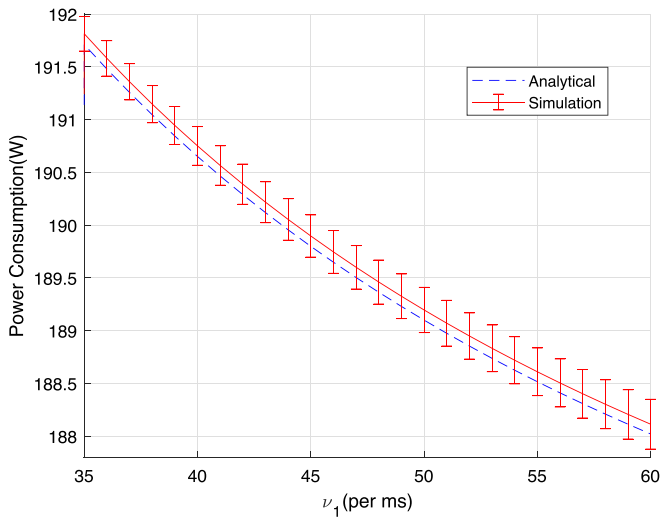
**FIGURE 6** | Comparison of analytical and simulated results with varying parameters values  $\nu_2$  for sojourn times deterministic distributed.



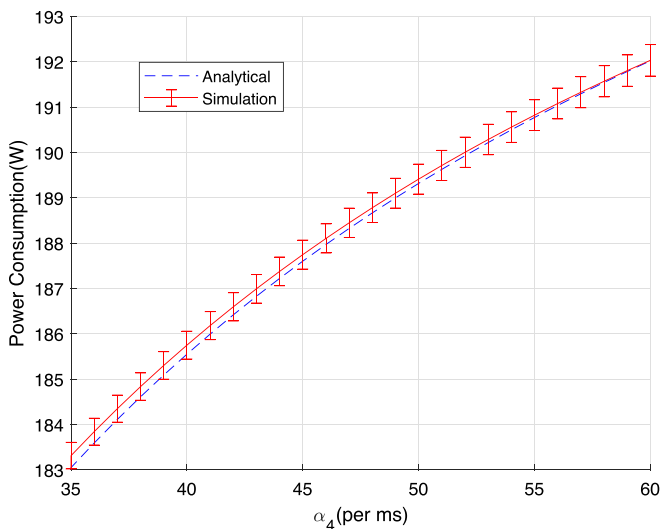
**FIGURE 7** | Comparison of analytical and simulated results with varying parameters values  $\nu_1$  for sojourn times deterministic distributed.

respectively. It is evident from Figure 5 that as the value of  $\nu_3$  increases, there is a corresponding decrease in power consumption. This reduction in power consumption can be attributed to the fact that the system spends less time in a state with more frequency bands providing the service, leading to a reduction in power consumption. Similar results are visible in Figures 6 and 7. This pattern can be attributed to the extended duration the system spends in states with fewer frequency bands active, leading to a reduction in power consumption. From these figures, it can be observed that the analytical results match the simulated ones.

Figures 8 and 9 exhibit the analytical and simulated observations of power consumption for sojourn time following hypo-exponential distribution with respect to  $\nu_1$  and  $\alpha_4$ . The analytical and simulated results for the hypo-exponential



**FIGURE 8** | Comparison of analytical and simulated results with varying parameters values  $\nu_1$  for sojourn times hypo-exponential distributed.



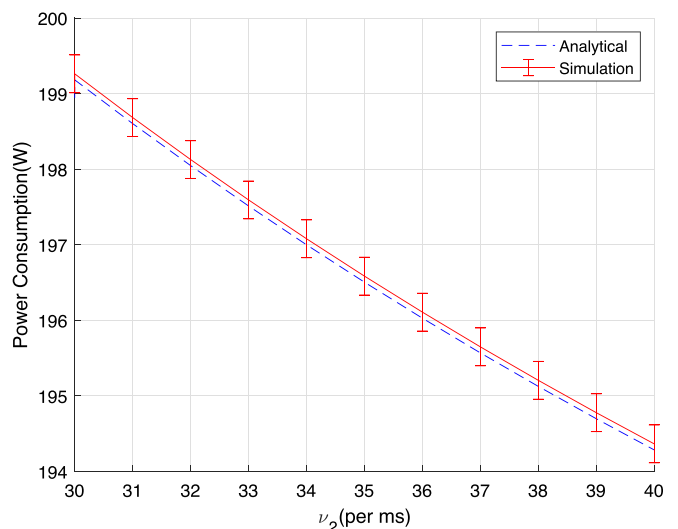
**FIGURE 9** | Comparison of analytical and simulated results with varying parameters values  $\alpha_4$  for sojourn times hypo-exponential distributed.

distribution case exhibit a close alignment, indicating a high level of agreement. In the case of hypo-exponential distribution, there are not any notable differences between the values of the analytical and simulated results; that is, the results from both methods converge and demonstrate a significant level of concurrence.

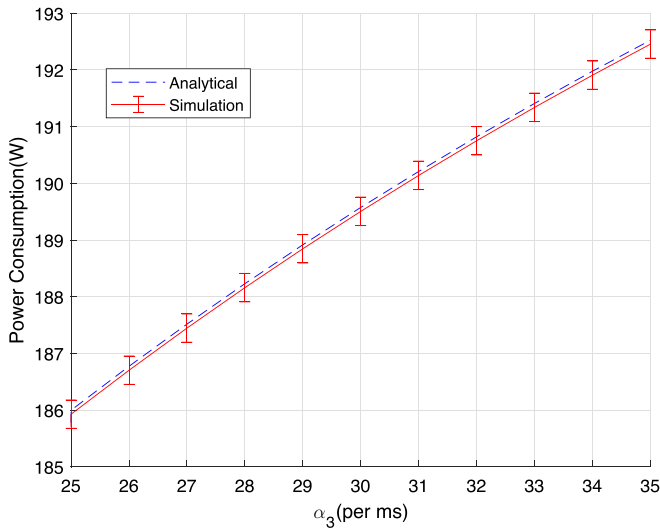
The parameter values for expected sojourn rates in different states are considered similar to the deterministic distribution. It can be observed from Figure 8 that the power consumption decreases with the increase in value of  $\nu_1$  as with an increase in value of  $\nu_1$  the system enters the state  $S_2$  from  $S_1$  in less time; hence, power consumption will be reduced. In Figure 9, clearly, with an elevation in the  $\alpha_4$  value, the power consumption also escalates. This pattern can be attributed to the phenomenon that when  $\alpha_4$  is increased, the system tends to occupy a state characterized by more frequency bands offering services, ultimately resulting in heightened power consumption.

Similarly, Figures 10 and 11 illustrate the analytical and simulated outcomes concerning power consumption in the Markov model, where the sojourn time follows an exponential distribution. These figures reveal a substantial alignment between the analytical and simulated results. The graphical representations clearly indicate a high degree of consistency across all methodologies, underscoring the approximate accuracy of the proposed model and substantiating the validity of the hypothesized model.

The parameter values for expected sojourn rates in different states are assumed to be similar to those for the other distributions. Figure 10 illustrates that power consumption decreases as the value of  $\nu_2$  increases. This reduction is because a higher  $\nu_2$  value keeps the system in state  $S_2$  less often and enters state  $S_3$  fast, leading to reduced power consumption. In Figure 9, it is evident that an increase in the  $\alpha_3$  value leads to an escalation in power consumption. This phenomenon can be explained by the fact that when  $\alpha_3$  is increased, the system tends to occupy a state characterized by more frequency bands offering services, ultimately resulting in higher power consumption.



**FIGURE 10** | Comparison of analytical and simulated results with varying parameters values  $\nu_2$  for CTMC model.



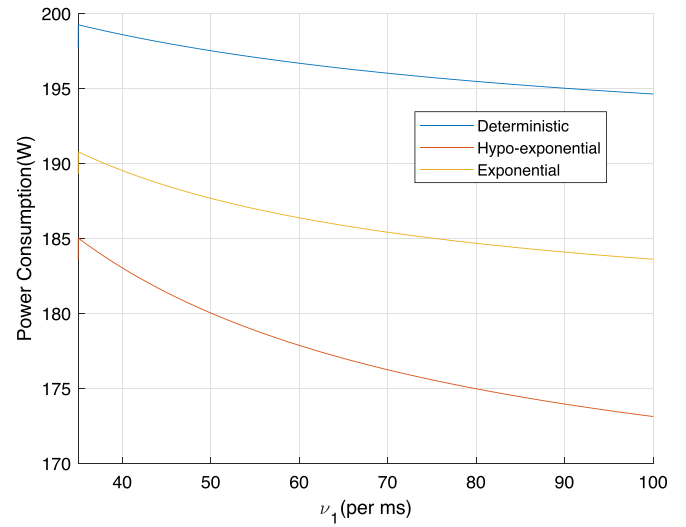
**FIGURE 11** | Comparison of analytical and simulated results with varying parameters values  $\alpha_3$  for CTMC model.

## 4.2 | Effect of Different Distributions on Power Consumption

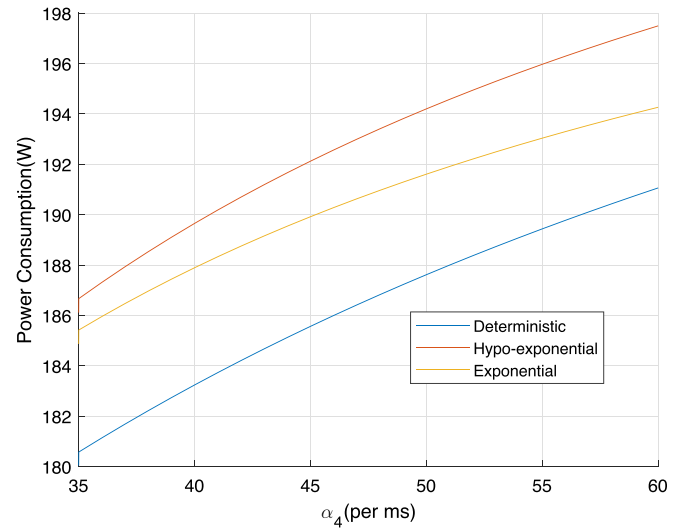
This experiment involves a comparative analysis of the analytical outcomes pertaining to power consumption in scenarios where the states follow exponential, deterministic, and hypo-exponential distributions. Figure 12 illustrates how power consumption behaves concerning  $\nu_1$  across all three distributions. Notably, in deterministic distributed cases, power consumption exhibits higher values when contrasted with hypo-exponential and exponential cases. Specifically, the results reveal that power consumption is approximately 5%–6% higher in the deterministic case compared to the hypo-exponential case and approximately 8%–9% higher than in the exponential case. The hypo-exponential case consistently yields the lowest power consumption values.

Figure 13 depicts the behavior of power consumption with respect to  $\alpha_4$ . The results exhibit that the value of power consumption is almost 2%–3% and 3%–4% more for the hypo-exponential case in comparison to the exponential and deterministic case, respectively. The lowest value of power consumption is obtained for the deterministic case.

The fundamental purpose of considering various distributions for the states is to endeavor toward a more realistic representation through modeling. The experiment vividly demonstrates that the choice of a distribution has a substantial impact on power consumption values, primarily due to the differing transition rates associated with these distributions. The significant disparities observed in the results underscore the substantial influence of the selected distribution type on each analyzed metric. The determination of distribution parameters is contingent on the specific requirements of the system. In scenarios where the system's temporal structure adheres to a deterministic pattern but an exponential distribution is utilized for modeling, the outcomes tend to be inflated. Conversely, when time periods align with hypo-exponential distributions but results are computed based on an exponential distribution, performance metrics have a tendency to be either underestimated or overestimated.



**FIGURE 12** | Comparison of analytical results with varying parameters values  $\nu_1$  for all the models.



**FIGURE 13** | Comparison of analytical results with varying parameters values  $\alpha_4$  for all the models.

Consequently, the meticulous selection of the most appropriate distribution becomes paramount when assessing the system's performance.

## 5 | Conclusion and Future Work

In this paper, we have proposed stochastic models, namely, Markov and semi-Markov models using many RUs with different frequency bands to show how power consumption can be reduced by switching between different frequency bands based on the PRBs required and RRC-connected users. The presented stochastic model explores three distinct scenarios for the mean sojourn times of states: exponential, deterministic, and hypo-exponential distributions. The central objective in including and exploring these various distributions is to strive for a more accurate depiction of states conducive to energy conservation. Analytical outcomes have been derived using the two-stage method for the SMP model. We have considered the four cases

based on the parameter values for the semi-Markov model with sojourn time following exponential distribution. A comprehensive sensitivity analysis was carried out to assess power consumption across diverse scenarios. Additionally, we compare analytical and simulation outcomes for system power consumption, demonstrating a convergence of results that underscores a significant level of agreement.

In the future, this work can be extended by considering different sojourn time distributions, and performance measures can be evaluated for that model. Further, several enhancements can be considered to build on the existing study. A comprehensive sensitivity analysis can be performed to examine the influence of various parameters, including traffic load, frequency band allocation, and user mobility, on energy usage and system stability. Additionally, the study can address potential security and privacy risks associated with implementing these models, ensuring data protection and user privacy in the context of sensitive user information and critical communication networks. Future research can assess the models' scalability to effectively manage large-scale 5G networks involving thousands of BSs and millions of users.

### Conflicts of Interest

The authors declare no conflicts of interest.

### Data Availability Statement

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

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