
Numerical optimisation of loss system with retrial phenomenon in cellular networks

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Abstract: In this study, we extend upon the model by Haring et al. (2001) by introducing retrial phenomenon in multi-server queueing system. When the most g number of guard channels are available, it allows new calls to join the retrial group. This retrial group is called orbit and can hold a maximum of m retrial calls. The impact of retrial over certain performance measures is numerically investigated. The focus of this work is to construct optimisation problems to determine the optimal number of channels, the optimal number of guard channels and the optimal orbit size. Further, it has been emphasised that the proposed model with retrial phenomenon reduces the blocking probability of new calls in the system.

Keywords: multi-server queueing model; retrial phenomenon; cellular network; blocking probability; optimisation.

Reference to this paper should be made as follows: Jain, V., Raj, R. and Dharmaraja, S. (2023) 'Numerical optimisation of loss system with retrial phenomenon in cellular networks', *Int. J. Operational Research*, Vol. 46, No. 2, pp.210–226.

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1 Introduction

Getting motivation from the previously reported research models, specifically, Gia and Mandjes (1997) and Haring et al. (2001), a multi-server queueing system with retrial phenomenon is studied here. The retrial phenomenon (Dharmaraja et al., 2008) is considered as when new calls are blocked due to non-availability of idle channels and consequently join the orbit for retry. In this work, such blocked new calls in the orbit will be referred as retrial calls. In a cellular network, termination of new calls and handoff calls are crucial factors to determine the performance of the system. Therefore, dropping probability of handoff calls and blocking probability of new calls are the most essential performance measures for any cellular networks. In a realistic scenario, the service provider will always prefer handoff calls over new calls. Henceforth, the service quality of handoff calls might be improved by reserving a group of guard channels (Guerin, 1988) for handoff calls. Blocking probability of new calls has been a very important concept from 2G to today's 5G. Hence, we address the problem of reducing blocking probability of new calls by introducing retrial concept in cellular networks for a multi-server model with guard channels for handoff calls and retrial calls.

A lot of research has been carried out over the customer retrial phenomenon in a cellular network. Some of relevant studies are discussed here. Marsan et al. (2000) proposed a novel approximate multi-server model for the evaluation of call blocking probabilities in mobile cellular network taking into account retrial phenomenon. Another related work is Gia and Mandjes (1997), in which retrial model with guard channel policy was proposed, and a nearly recursive algorithm was derived for providing state probabilities. Parthasarathy and Sudhesh (2007) studied transient analysis of a single-server retrial queueing system. Wang and Wolff (2009) analysed a multi-server retrial queueing system without considering the guard channel policy. They investigated the behaviour of blocking probability and showed that blocking probability is decreasing for retrial queues in case of multi-servers. Additionally, there are several other techniques available in the literature to evaluate the performance of multi-server retrial queueing models (Do, 2011; Trivedi et al., 2002; Madan et al., 2008; Marsan et al., 2001; etc.). Recently, Duc (2014) proposed a multi server retrial queueing model with two types of non-persistent customers and studied their give up behaviour. They developed a numerically stable algorithm to compute the joint stationary distribution.

The most relevant work for our paper is Haring et al. (2001), which derived closed-form expressions for blocking probability of a new call and dropping probability of a handoff call. Haring et al. (2001) derived recursive formulae to compute loss probabilities for a multi-server queueing model and optimised the number of guard channels. In this work, an extension and generalisation of Haring et al. (2001) is proposed. It investigates the impact of a retrial on various performance measures including blocking probability, dropping probability, mean number of busy channels and mean number of retrial calls. In this present study, there is no closed form expression reported due to its complexity. However, the numerical results and optimisation problems presented here consider Haring et al. (2001) as a particular case when there is no retrial.

This remainder of this paper is arranged as follows. Section 2 provides a brief description of the proposed multi-server retrial queueing model and elaborates this model mathematically. Section 3 illustrates the numerical investigation of this multi-server retrial queueing model and explains the impact of various parameters, i.e., orbit size, retrial rate, number of total channels and number of guard channels, call arrival rate, etc., on the given performance measures. Further, Section 4 presents few optimisation problems and their solution algorithms to determine the optimal values of the total number of channels, the total number of guard channels and orbit size. At last, discussion and future directions are provided in Section 5.

2 Mathematical model

This work considers a homogeneous cellular system where each cell is served by a unique base station. Each base station consists of a finite number of total channels, say c , termed as channel pool. This system can be modelled as a multi-server queueing model with retrial phenomena as depicted in Figure 1.

Figure 1 A multi-server queueing model with retrial phenomenon

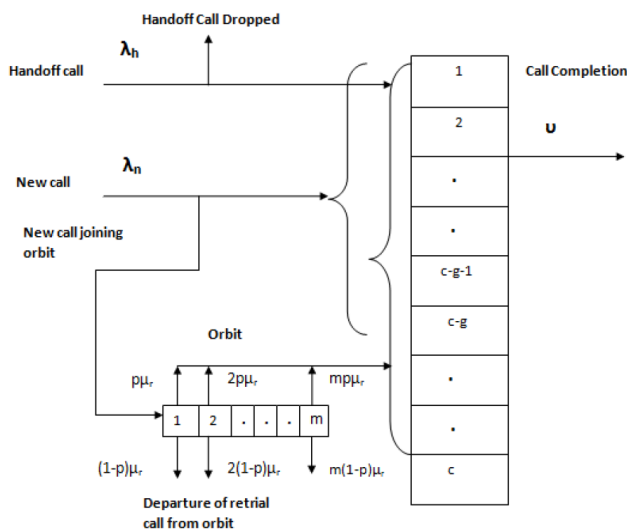
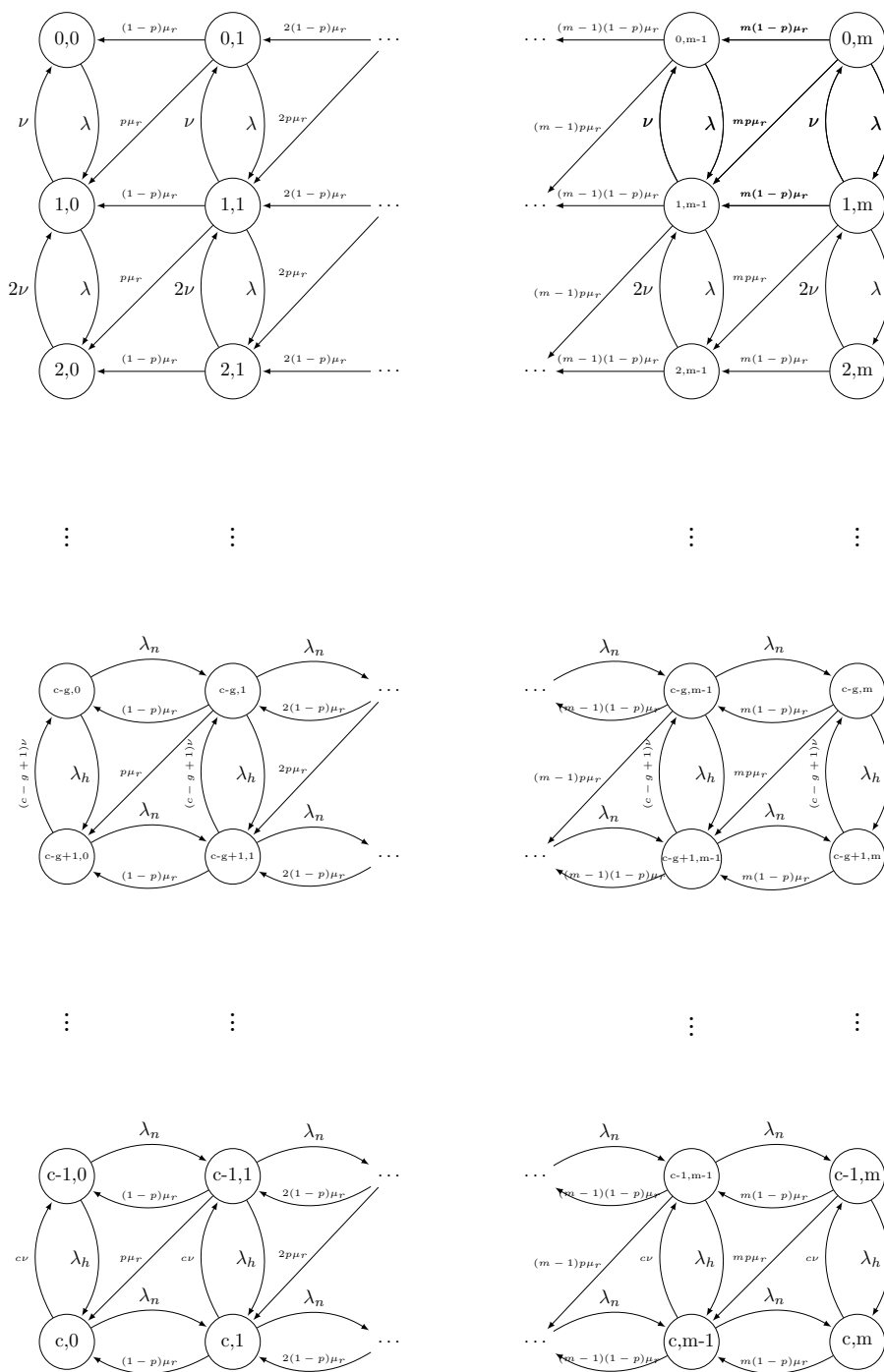


Figure 2 State transition diagram



Assume that the arrival pattern of the new calls and the handoff calls follows the independent Poisson process with the arrival rate λ_n and λ_h , respectively. Define the total arrival rate $\lambda = \lambda_n + \lambda_h$. The call duration of new calls and handoff calls are i.i.d. random variables, each follows independent exponential distribution with the rate ν . Note that a handoff call is dropped when all channels are busy in the channel pool, whereas a new call joins the retrial group when at least $c - g$ channels are busy. The new call, which joins the retrial group, is referred as retrial call in this work. Here, it is assumed that this retrial group, called orbit, has a capacity of finite size, say m . Once the orbit is occupied by m retrial calls, the new call will not be able to enter into the system and therefore, it will be blocked. In this model, a very well-known guard channel policy is considered. Under this policy, g number of channels are reserved for handoff calls and retrial calls. In the proposed model, it is considered that a retrial call retries at random intervals and in random order. It obtains the service with probability p or leaves the system forever without obtaining the service with probability $1 - p$. The inter-retrial time between the retrial calls is exponentially distributed with rate μ_r .

Let us analyse the underlying stochastic process for the proposed queueing model. Suppose $\Xi(t)$ represents the number of busy channels at time t and $\Theta(t)$ defines the number of retrial calls at time t . Based on the model description, the stochastic process $\{(\Xi(t), \Theta(t)) : t \geq 0\}$ can be modelled as a quasi-birth-and-death (QBD) process with a state space $S = \{(j, k) : 0 \leq j \leq c; 0 \leq k \leq m\}$. Here j denotes the number of busy channels at time t and k denotes the number of retrial calls in the orbit at time t . Figure 2 presents the state transition diagram for the proposed model.

Since the underlying stochastic process is ergodic, the stationary distribution of the system exists and is independent of initial distribution. Let $P_{j,k}$ be defined as

$$P_{j,k} = \lim_{t \rightarrow \infty} P(\Xi(t) = j, \Theta(t) = k); \quad 0 \leq j \leq c, \quad 0 \leq k \leq m.$$

Then, the steady state equations can be written as follows:

$$\lambda P_{0,0} = \nu P_{1,0} + (1 - p)\mu_r P_{0,1}, \quad (1)$$

$$(\lambda + k\mu_r)P_{0,k} = \nu P_{1,k} + (k + 1)(1 - p)\mu_r P_{0,k+1}; \quad 1 \leq k \leq m - 1, \quad (2)$$

$$(\lambda + m\mu_r)P_{0,m} = \nu P_{1,m}, \quad (3)$$

$$\begin{aligned} (\lambda + k\mu_r + j\nu)P_{j,k} &= \lambda P_{j-1,k} + (j + 1)\nu P_{j+1,k} + (k + 1)p\mu_r P_{j-1,k+1} \\ &\quad + (k + 1)(1 - p)\mu_r P_{j,k+1}; \\ 1 \leq j \leq c - g - 1, \quad 0 \leq k \leq m - 1, \end{aligned} \quad (4)$$

$$(\lambda + m\mu_r + j\nu)P_{j,m} = \lambda P_{j-1,m} + (j + 1)\nu P_{j+1,m}; \quad 1 \leq j \leq c - g - 1, \quad (5)$$

$$\begin{aligned} (\lambda + (c - g)\nu)P_{c-g,0} &= \lambda P_{c-g-1,0} + (c - g + 1)\nu P_{c-g+1,0} \\ &\quad + p\mu_r P_{c-g-1,1} + (1 - p)\mu_r P_{c-g,1}, \end{aligned} \quad (6)$$

$$\begin{aligned} (\lambda + k\mu_r + (c - g)\nu)P_{c-g,k} &= (c - g + 1)\nu P_{c-g+1,k} \\ &\quad + \lambda P_{c-g-1,k} + (k + 1)p\mu_r P_{c-g-1,k+1} \\ &\quad + (k + 1)(1 - p)\mu_r P_{c-g,k+1}; \quad 1 \leq k \leq m - 1, \end{aligned} \quad (7)$$

$$(\lambda_h + m\mu_r + (c - g)\nu)P_{c-g,m} = \lambda P_{c-g-1,m} + (c - g + 1)\nu P_{c-g+1,m}, \quad (8)$$

$$(\lambda + j\nu)P_{j,0} = \lambda_h P_{j-1,0} + (j+1)\nu P_{j+1,0} + p\mu_r P_{j-1,1} \\ + (1-p)\mu_r P_{j,1}; \quad c-g+1 \leq j \leq c-1, \quad (9)$$

$$(\lambda + k\mu_r + j\nu)P_{j,k} = \lambda_h P_{j-1,k} + \lambda_n P_{j-1,k-1} + (j+1)\nu P_{j+1,k} \\ + (k+1)p\mu_r P_{j-1,k+1} + (k+1)(1-p)\mu_r P_{j,k+1}; \quad (10) \\ c-g+1 \leq j \leq c-1, \quad 1 \leq k \leq m-1,$$

$$(\lambda_h + m\mu_r + j\nu)P_{j,m} = \lambda_h P_{j-1,m} + \lambda_n P_{j-1,m-1} + (j+1)\nu P_{j+1,m}; \\ c-g+1 \leq j \leq c-1, \quad (11)$$

$$(\lambda_n + c\nu)P_{c,0} = \lambda_h P_{c-1,0} + p\mu_r P_{c-1,1} + (1-p)\mu_r P_{c,1}, \quad (12)$$

$$(\lambda_n + k(1-p)\mu_r + c\nu)P_{c,k} = \lambda_h P_{c-1,k} + \lambda_n P_{c,k-1} \\ + (k+1)p\mu_r P_{c-1,k+1} \\ + (k+1)(1-p)\mu_r P_{c,k+1}; \quad 1 \leq k \leq m-1, \quad (13)$$

$$(m(1-p)\mu_r + c\nu)P_{c,m} = \lambda_h P_{c-1,m} + \lambda_n P_{c,m-1}. \quad (14)$$

Further, with these steady state equations, the infinitesimal generator matrix Q for the QBD $\{(\Xi(t), \Theta(t)) : t \geq 0\}$ is constructed as follows:

$$Q = \begin{pmatrix} Q_{0,0} & Q_{0,1} & 0 & & & \\ Q_{1,0} & Q_{1,1} & Q_{1,2} & & & \\ 0 & Q_{2,1} & Q_{2,2} & & & \\ 0 & 0 & Q_{3,2} & & & \\ 0 & 0 & 0 & & & \\ & & \ddots & \ddots & \ddots & \\ & & & & Q_{c-1,c} & \\ & & & & Q_{c,c-1} & Q_{c,c} \end{pmatrix}.$$

Each element of matrix Q is a block matrix of order $(m+1) \times (m+1)$. More specifically, this tridiagonal matrix Q is described as:

- The elements of the upper diagonal, denoted as block matrix $Q_{l,l+1}$ where $0 \leq l \leq c-1$, shows the transition due to the arrival of a (handoff or new) call or joining the orbit by a new call for retrial. Therefore, structure of $Q_{l,l+1}$ is as follows:

$$Q_{l,l+1} = \begin{pmatrix} \Lambda & 0 & 0 & & & \\ p\mu_r & \Lambda & 0 & & & \\ 0 & 2p\mu_r & \Lambda & & & \\ 0 & 0 & 3p\mu_r & & & \\ & & \ddots & \ddots & & \\ & & & mp\mu_r & \Lambda & \end{pmatrix}$$

$$\text{where } \Lambda = \begin{cases} \lambda = \lambda_h + \lambda_n, & 0 \leq l \leq c-g-1, \\ \lambda_h, & c-g \leq l \leq c-1. \end{cases}$$

- The elements of the main diagonal, denoted as $Q_{l,l}$ where $0 \leq l \leq c$, represents the transitions either due to the departure of a call after its completion or arrival of a new call or departure of a retrial call without getting the connection. Consequently, the block matrix $Q_{l,l}$ is constructed as

$$Q_{l,l} = \begin{pmatrix} b_0 & \Lambda' & 0 & 0 \\ d_1 & b_1 & \Lambda' & 0 \\ 0 & d_2 & b_2 & \Lambda' \\ & & \ddots & \ddots & \ddots \\ & & & \ddots & \ddots & \Lambda' \\ & & & & d_m & b_m \end{pmatrix}$$

$$\text{where } \Lambda' = \begin{cases} 0, & 0 \leq l \leq c - g - 1, \\ \lambda_n, & c - g \leq l \leq c. \end{cases}$$

$$b_k = -(\lambda + l\nu + k\mu_r); 0 \leq k \leq m,$$

and

$$d_k = k(1 - p)\mu_r; 1 \leq k \leq m.$$

- The elements of the lower diagonal, denoted as $Q_{l,l-1}$ where $1 \leq l \leq c$, exhibits the transitions due to the completion of a call (new or handoff). Hence, these block matrices $Q_{l,l-1}$ are presented as

$$Q_{l,l-1} = \begin{pmatrix} l\nu & 0 & & \\ 0 & l\nu & & \\ & & \ddots & \\ & & & l\nu \end{pmatrix}.$$

Finally, the steady state equations (1)-(14) can be expressed in matrix form as $\Pi Q = 0$, where

$$\Pi = (\Pi_0, \Pi_1, \Pi_2, \dots, \Pi_c)$$

and

$$\Pi_j = (P_{j,0}, P_{j,1}, \dots, P_{j,m}), \quad 0 \leq j \leq c,$$

with the normalisation condition $\Pi e = 1$ where e is a unit vector. Here Q is a complicated and highly structured matrix, therefore, it is difficult to obtain a compact analytical form for Π . This system is solved by applying the direct method to compute steady state probabilities.

2.1 Performance measures

With the steady state probabilities, the relevant performance measures for the proposed model are represented in this section.

- The probability of new calls being blocked is computed by:

$$P_b = \sum_{j=c-g}^c P_{j,m}.$$

- The probability of handoff call being dropped can be calculated as:

$$P_d = \sum_{k=0}^m P_{c,k}.$$

- The mean number of busy channels in the channel pool can be evaluated by:

$$M_b = \sum_{j=1}^c \sum_{k=0}^m j P_{j,k}.$$

- The mean number of repeated retrial calls in the orbit can be obtained by:

$$M_o = \sum_{j=1}^c \sum_{k=0}^m k P_{j,k}.$$

- The mean number of new calls and handoff calls in the system can be evaluated by:

$$M_s = M_b + M_o.$$

3 Numerical illustration

The goal of this section is to analyse numerical results received after implementing the mathematical model presented in Section 3. This analysis has two main objectives: first objective is to present that this work is an extension of the work provided by Haring et al. (2001), and second objective is to examine how the retrial phenomenon affects the system performance. Therefore, for illustration purpose, we set the parameters as $\lambda = \lambda_h + \lambda_n = 80$, $\lambda_n = 40$, $\nu = 1$, $p = 0.8$ and $\mu_r = 0.5$. Here, values of all these parameters are same as mentioned in Haring et al. (2001) except retrial probability p and retrial rate μ_r . It is important to note that though the number of channels c , number of guard channels g and orbit size m are integers, yet these parameters are considered here as real numbers in order to plot figures and to analyse the results.

After performing the numerical illustration for $g = 1, 2, 3$ and $m = 0$, Figures 3(a) and 3(b) plot loss probabilities (the blocking probability P_b and the dropping probability P_d) with respect to c . We observe by these figures that for no retrial, i.e., for $m = 0$, P_b and P_d decreases with c for a fixed value of g , P_b increases with g for a fixed value of c , and P_d decreases with g for a fixed value of c . In this regard, Figure 4 clearly displays the dependence of loss probabilities over c and g simultaneously for $m = 0$. These properties of P_b and P_d match well with the properties provided by Haring et al. (2001). Therefore, we declare that, in case of no retrial, this work provides same properties of P_b and P_d as reported by Haring et al. (2001).

Figure 3 Dependence of loss probabilities over c for $m = 0$, (a) dependence of P_b over c for $m = 0$ (b) dependence of P_d over c for $m = 0$ (see online version for colours)

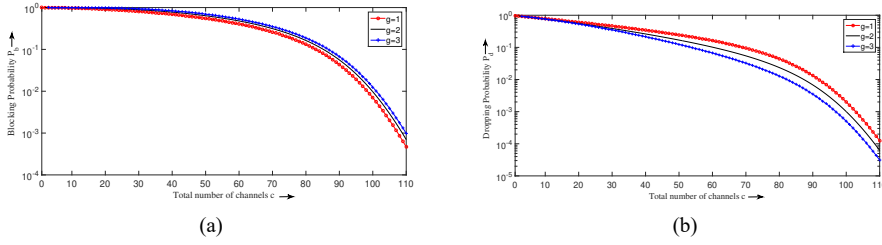
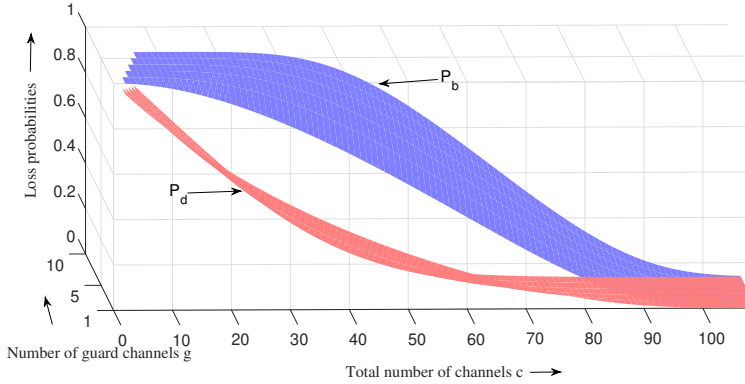


Figure 4 Dependence of P_b and P_d over c and g for $m = 0$ (see online version for colours)



The remaining part of this section demonstrates the effect of retrial phenomenon over the various performance measures given in previous section. For $m > 0$, Figures 5(a) and 5(b) exhibit that $P_b(g)$ is an increasing function and $P_d(g)$ is a decreasing function for a fixed value of c . Figures 6(a) and 6(b) explore that by increasing the orbit size, i.e., m , P_b decreases and P_d increases. Similar results can be obtained by examining Figures 7(a) and 7(b).

Figure 5 Dependence of loss probabilities over g for $c = 100$, (a) dependence of P_b over g for $c = 100$ (b) dependence of P_d over g for $c = 100$ (see online version for colours)

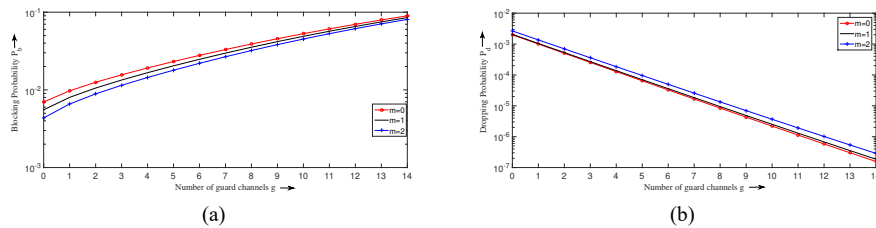


Figure 6 Dependence of loss probabilities over c and g for $m = 1, 4, 10, 20$, (a) dependence of P_b over c and g for $m = 1, 4, 10, 20$ (b) dependence of P_d over c and g for $m = 1, 4, 10, 20$ (see online version for colours)

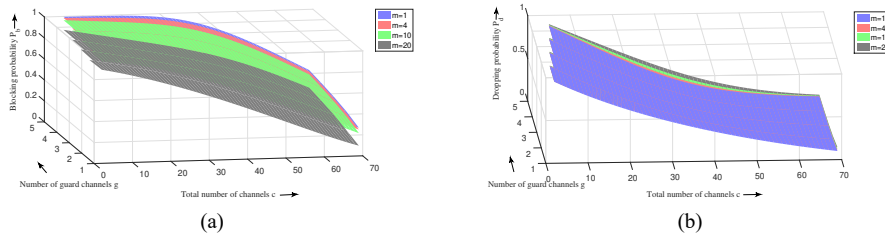


Figure 7 Dependence of loss probabilities over m for $c = 100$, (a) dependence of P_b over m for $c = 100$ (b) dependence of P_d over m for $c = 100$ (see online version for colours)

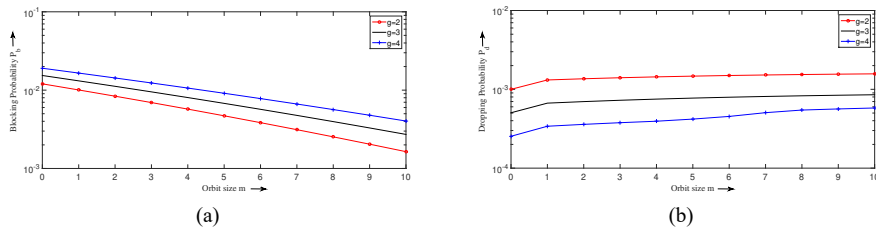
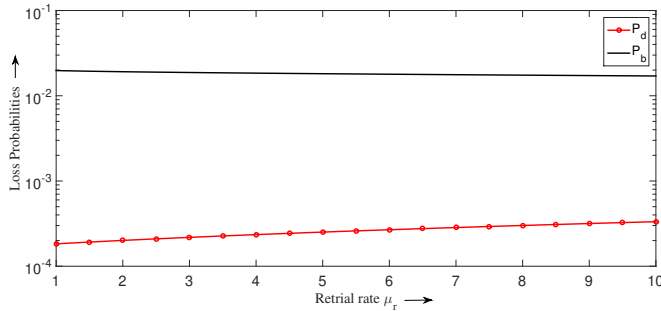


Figure 8 reflects the impact of retrial rate on the loss probabilities. It is obvious that with the increased retrial rate, the blocking probability is gradually reduced as more retrial calls get connected. With the similar reasoning, increment in retrial rate will increase the dropping probability P_d .

Figure 8 Loss probabilities as a function of retrial rate μ_r for $c = 100$, $m = 5$ and $g = 5$ (see online version for colours)



Additionally, in case of retrial phenomenon, we can intuitively perceive that the increment of handoff call arrival rate will increase P_b and the increment of new call arrival rate will increase P_d for a fixed value of g . It can also be realised that for larger values of g , i.e., almost equal to c , the arrival rate of new calls should have a negligible

impact over P_d . Such effect of λ_h on P_b and λ_n on P_d are displayed by Figure 9(a) and Figure 9(b).

Figure 9 Dependence of loss probabilities over call arrival rates for $c = 100$ and $m = 5$, (a) dependence of P_b over λ_h for $c = 100$ and $m = 5$ (b) dependence of P_d over λ_n for $c = 100$ and $m = 5$ (see online version for colours)

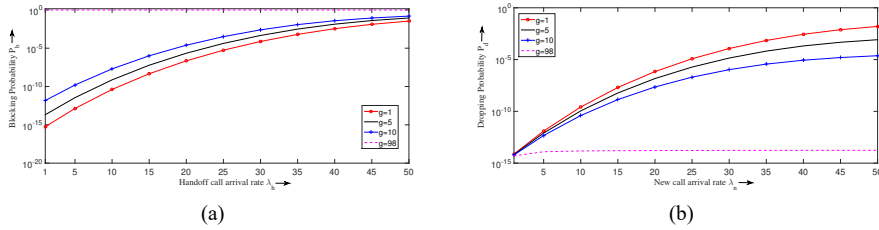


Figure 10 Dependence of M_b and M_o over c for $m = 5$ and $g = 5$, (a) dependence of M_b over c for $m = 5$ and $g = 5$ (b) dependence of M_o over c for $m = 5$ and $g = 5$ (see online version for colours)

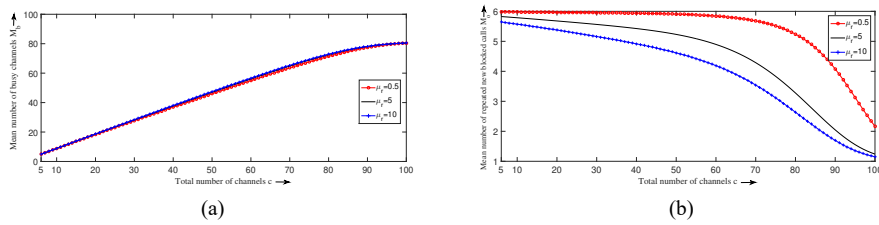
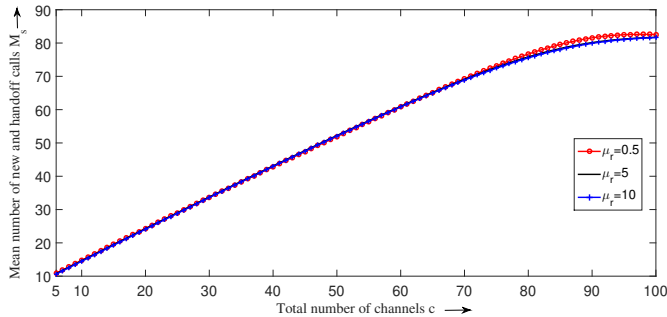


Figure 11 Dependence of M_s over c for $m = 5$ and $g = 5$ (see online version for colours)



Further, the other performance measures are discussed as follows. Variation of performance measures, i.e., mean number of busy channels M_b and mean number of repeated calls in the orbit M_o , with respect to c are shown in Figures 10(a) and 10(b), respectively. It is observed that, for fixed values of m and g , M_b increases with c and M_o decreases with c . It can also be explored that smaller values of retrial rate μ_r have a negligible impact on the mean number of busy channels M_b . On the other side, M_o is a decreasing function of retrial rate μ_r . Finally, Figure 11 exhibits the impact of c on mean number of calls M_s . For fixed values of m and g , M_s is an increasing function

of c . It can also be observed that for smaller values of μ_r , there is a negligible change in the values of M_s .

4 Optimisation problem

In the proposed model, loss probabilities are very important performance measures. From the customer's perspective, these performance measures need to be minimised. In this section, by taking such factor into account, a few optimisation problems are proposed to optimise total channels, guard channels and orbit size. Specifically, either blocking probability P_b or dropping probability P_d may be minimised and the other one may be constrained. Thus, few optimisation problems for a multi-server retrial queueing model are presented as follows. Note that values of parameters λ_h , λ_n , ν , p and μ_r are considered as provided in Section 4.

4.1 ($O1_r$)

Given c , λ_h , λ_n , ν , μ_r , p ; calculate the optimal integer value of m and g such that

$$\begin{aligned} &\min P_b(m, g) \\ &\text{subject to,} \\ &P_d(m, g) \leq P_{d0}. \end{aligned}$$

Here the constant P_{d0} is a pre-defined value.

This optimisation problem ($O1_r$) can be optimised by two different algorithms, provided below.

Algorithm 1: to find optimal orbit size

- Step 1 Set the value of c .
- Step 2 Set $g = \lceil (x\%)(c) \rceil$, where x can be assumed on the basis of system requirement.
- Step 3 For such g , find m for which $P_d(m, g) \leq P_{d0}$.
- Step 4 The largest value of m , say m^* will be declared as the optimal solution of ($O1_r$). The optimal value of ($O1_r$) will be $P_b(m^*, g)$.

Optimal values for the orbit size, m^* , for different pre-defined values of P_{d0} are provided in Table 1.

Table 1 Result of optimisation problem ($O1_r$) by applying Algorithm 1 for $c = 100$ and $x = 5$

P_{d0}	g	m^*	$P_d(m^*, g)$	$P_b(m^*, g)$
10^{-2}	5	0	0.00016136	0.02313149
10^{-3}	5	0	0.00016136	0.02313149
10^{-4}	5	69	0.00009624	0.04732208

Algorithm 2: to find optimal number of guard channels

- Step 1 Set the value of c .
- Step 2 Set $m = \lceil (x\%)(c) \rceil$, where x can be defined based on system requirement.
- Step 3 For such m , find g for which $P_d(m, g) \leq P_{d0}$.
- Step 4 The smallest value of g , say g^* will be declared as the optimal solution of $(O1_r)$. The optimal value will be $P_b(m, g^*)$.

Optimal values of g^* for different values of P_{d0} are mentioned in Table 2.

Table 2 Result of optimisation problem $(O1_r)$ by applying Algorithm 2 for $c = 100$ and $x = 5$

P_{d0}	m	g^*	$P_d(m, g^*)$	$P_b(m, g^*)$
10^{-2}	5	0	0.000786833	0.000784093
10^{-3}	5	1	0.000786833	0.000784093
10^{-4}	5	5	0.0000572980	0.00378360

It is remarkable that if blocked new calls do not join the orbit for retrial, i.e., $m = 0$, the optimisation problem $(O1_r)$ will be transformed to $(O1)$ as presented by Haring et al. (2001). Moreover, Algorithm 2 will be similar to the algorithm proposed by Haring et al. (2001) to solve their proposed optimisation problem $O1$. Therefore, in case of no retrial, we can clearly observe that the numerical results, summarised in Table 3, match well with results provided by Haring et al. (2001).

Table 3 Result of optimisation problem $(O1_r)$ by applying Algorithm 2 for $c = 100$ and $x = 0$

P_{d0}	g^*	$P_d(g^*)$	$P_b(g^*)$
10^{-2}	0	0.003992	0.003992
10^{-3}	3	0.000504	0.012528
10^{-4}	6	0.000065	0.023195
10^{-5}	9	0.000008	0.038967

4.2 $(O2_r)$

Given $c, \lambda_h, \lambda_n, \nu, \mu_r, p$; find the optimal integer value of m and g such that

$$\begin{aligned} &\min P_d(m, g) \\ &\text{subject to,} \\ &P_b(m, g) \leq P_{b0}. \end{aligned}$$

Here the constant P_{b0} is a pre-defined value. The optimisation problem $(O2_r)$ can be optimised by the Algorithm 3.

Algorithm 3: to find optimal orbit size

- Step 1 Set the value of c .
- Step 2 Set $g = \lceil (x\%)(c) \rceil$, where x can be defined based on system requirement.
- Step 3 For such g , find all m for which $P_b(m, g) \leq P_{b0}$.
- Step 4 The smallest value of m , say m^* , will be declared as the optimal solution of $(O2_r)$. The optimal value will be $P_d(m^*, g)$.

Table 4 summarises the optimal values of m for different values of P_{b0} .

Table 4 Result of optimisation problem $(O2_r)$ by applying Algorithm 3 for $c = 105$ and $x = 5$

P_{b0}	g	m^*	$P_b(m^*, g)$	$P_d(m^*, g)$
10^{-2}	5	0	0.0082	0.000046
10^{-3}	5	12	0.00086	0.000067
10^{-4}	5	22	0.000083	0.000070

4.3 $(O3_r)$

Given $\lambda_h, \lambda_n, \nu, \mu_r, p$; determine the optimal integer value of c, m and g such that

$$\begin{aligned} &\min c \\ &\text{subject to,} \\ &P_d(c, m, g) \leq P_{d0}, \\ &P_b(c, m, g) \leq P_{b0}. \end{aligned}$$

Here constants P_{b0} and P_{d0} are pre-defined values.

Feasible region for $(O3_r)$ will contain all those values of (c, m, g) that satisfy both the constraints $P_d(c, m, g) \leq P_{d0}$ and $P_b(c, m, g) \leq P_{b0}$ simultaneously. This optimisation problem $(O3_r)$ can be optimised by the algorithm provided below.

Algorithm 4: to find optimal orbit size, optimal number of channels and optimal guard channels

- Step 1 Set $m = 0$.
- Step 2 $g := 0$. Determine c_{d0} and c_{b0} , where c_{d0} is the smallest value of c for which $P_d(c, m, g) \leq P_{d0}$ and c_{b0} is the smallest value of c for which $P_b(c, m, g) \leq P_{b0}$. To obtain minimum value of c , define $c_{mid} = \lceil \frac{c_{d0} + c_{b0}}{2} \rceil$.
- Step 3 For the value c_{mid} , find $g_{\max}$ and $g_{\min}$, where $g_{\max}$ is the maximum value of g for which $P_b(c_{mid}, 0, g) \leq P_{b0}$ and $g_{\min}$ is the minimum value of g for which $P_d(c_{mid}, 0, g) \leq P_{d0}$.

- Step 4 If $g_{\max} = g_{\min} = g^*$, then c_{mid} and g^* will be optimal value of total number of channels and optimal number of guard channels for $m = 0$.
- Step 5 If $g_{\max} \neq g_{\min}$, define $c_{mid} = c_{mid} + 1$.
- Step 6 Proceed with Step 2 until both constraints $P_d(c_{mid}, 0, g) \leq P_{d0}$ and $P_b(c_{mid}, 0, g) \leq P_{b0}$ are satisfied at c_{mid} and g^* .

This procedure can be further proceeded for $m > 0$ from the Step 2.

Table 5 exhibits optimal orbit size m^* , optimal number of channels c^* and optimal guard channels g^* for different combinations of P_{d0} and P_{b0} by applying Algorithm 4. We emphasise that, for $m = 0$, $(O3_r)$ is converted the optimisation problem O_2 provided by Haring et al. (2001) and Table 5 exhibits exactly same results presented by Haring et al. (2001).

Table 5 Result of optimisation problem $(O3_r)$ by applying Algorithm 4

P_{d0}	P_{b0}	c^*	m^*	g^*	$P_b(c^*, m^*, g^*)$	$P_d(c^*, m^*, g^*)$
10^{-2}	10^{-1}	87	0	3	0.09089	0.00127
10^{-3}	10^{-2}	101	0	2	0.0077859	0.000791455
10^{-4}	10^{-3}	109	0	2	0.0009482	0.00008555
10^{-5}	10^{-4}	116	0	2	0.0000933	0.000007625
10^{-6}	10^{-5}	122	0	2	0.0000091	0.000000687

4.4 $(O4_r)$

Given $\lambda_h, \lambda_n, \nu, \mu_r, p$; determine the optimal integer value of c, m and g such that

$$\begin{aligned}
 &\min m \\
 &\text{subject to,} \\
 &P_d(c, m, g) \leq P_{d0}, \\
 &P_b(c, m, g) \leq P_{b0}.
 \end{aligned}$$

Here constants P_{b0} and P_{d0} are pre-defined values. The optimisation problem $(O4_r)$ can be optimised by the following algorithm.

Algorithm 5: To find optimal orbit size for given number of total channels and guard channels.

- Step 1 Start with $c = 2$.
- Step 2 Set $g = \lceil (x\%)(c) \rceil$, where x can be defined based on system requirement.
- Step 3 Find feasible region for m for $m = 0$ to $c/2$ such that $P_d(c, m, g) \leq P_{d0}$ and $P_b(c, m, g) \leq P_{b0}$.
- Step 4 Minimum value of m is declared as the optimal orbit size m^* . Also, declare corresponding c and g as the optimal number of total channels and the optimal number of guard channels respectively.

Step 5 This procedure can be further proceeded for $c > 2$.

After solving $(O4_r)$ by Algorithm 5, we obtain the optimal orbit size m^* corresponding to fix c and g for different combinations of P_{d0} and P_{b0} given in the Table 6.

Table 6 Result of optimisation problem $(O4_r)$ by applying Algorithm 5 for $x = 5$

P_{d0}	P_{b0}	c	g	m^*	$P_b(c, m^*, g)$	$P_d(c, m^*, g)$
10^{-2}	10^{-1}	90	4	3	0.09089	0.00127
10^{-3}	10^{-2}	103	5	3	0.009173	0.000089
10^{-4}	10^{-3}	112	5	3	0.0007501	0.0000061
10^{-5}	10^{-4}	118	5	3	0.0000949	0.00000071

In practice, the total available bandwidth, distributed for new calls and handoff calls, is limited. By introducing the retrial phenomenon, this limited bandwidth will be shared among c , g and m . Consequently, increment of orbit size m will decrease the total number of channels and guard channels. This scenario is summarised by Table 7.

Table 7 Impact of retrial on bandwidth sharing by c and g

m	P_{d0}	P_{b0}	g	c	P_b	P_d
0	10^{-1}	10^{-2}	3	87	0.096834	0.00544
1	10^{-1}	10^{-2}	3	87	0.092030	0.005744
10	10^{-1}	10^{-2}	2	83	0.027259	0.089082

5 Conclusions and future remarks

The main focus of this study is to extend the work presented by Haring et al. (2001). We then formulate optimisation problems to compute the optimal number of channels, optimal number of guard channels and the optimal orbit size. More specifically, it is reported that, for no retrial, these optimisation problems and their optimal solutions are exactly similar to the results obtained by Haring et al. (2001). Further, we investigate numerically the impact of retrial over the system performance. The blocking probability of new calls, a very important concept, has still not been well addressed. Irrespective of the type of the network, i.e., 3G, 4G or 5G, this work addresses the problem of reducing the blocking probability of new calls in cellular networks for a multi-server model with retrial phenomenon.

We next discuss some possible extensions of this proposed model. A closed form expression for the proposed retrial model could be explored in future. Additionally, the direct method is adapted here to solve Q -matrix and to obtain steady state transition probabilities. This methodology can be compared with the other methods and algorithms, e.g., matrix geometric method. Another possible extension is to consider different type of customers' arrival and service pattern. Furthermore, it is important to consider situations that could bring the system into the inactive state. Dharmaraja et al. (2017) could be a useful direction in order to study the system with such catastrophe.

Acknowledgements

Authors are thankful to the editor and two anonymous reviewers for their valuable suggestions and constructive comments on an earlier draft. One of the authors, S. Dharmaraja, gratefully acknowledges the financial support received from the Department of Telecommunications (DoT), India.

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