



Stochastic Modeling of a Base Station in 5G Wireless Networks for Energy Aspects Using Advanced Sleep Mechanism

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Abstract

Energy saving in the base stations (BSs) is one of the important issues as huge network capacity, higher data speeds, more availability, and a more uniform user experience is promised by 5G cellular networks. Advanced sleep mechanism (ASM) is one of the efficient techniques for saving energy in the base station. This paper introduces three stochastic models for ASM based on system arrivals and user requests (URs): the Markov model, the semi-Markov model, and the Markov regenerative process model for the base station. Closed-form solutions for steady-state system size probabilities are derived for each model. Additionally, performance metrics such as power consumption, power saving factor, and throughput are evaluated. Finally, a sensitivity analysis is conducted to compare the results obtained from the three different proposed models.

Keywords Markov model · Semi-Markov model · Markov generative process · Power consumption · Power saving factor · Throughput

Mathematics Subject Classification (2010) 60J28 · 60K15 · 60K25 · 68M20 · 90B22

1 Introduction

In current years, the staggering growth of mobile network traffic, posing significant challenges such as higher operational costs and sustainability issues for Mobile Network Operators (MNO). The wide spread of Fifth Generation (5G) cellular technology, permits mobile networks to provide incredibly fast network speed and huge bandwidth capacity pushing the process of mobile network densification.

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5G cellular networks are intended to deliver higher data speeds, ultra-low latency, high reliability, huge network capacity, more availability, and a more uniform user experience. To get a more immediate response, 5G cellular networks can provide much lower latency (less than 1 ms). Han et al. (2020) showed that, in 5G cellular networks, with a bandwidth of 100 MHz the power consumption is five times higher than the power consumption in 4G networks, where the bandwidth is 20 MHz, presuming the power spectrum density to be the same. Hence, in 5G networks, high power consumption is one of the major issues to be concerned. To control this issue, energy efficiency has brought much attention to 5G cellular networks. 5G networks contain the following two types of cells: macro cells and small cells. There is a base station (BS) in each cell, and all mobile devices in the same cell communicate with the BS in the same area. Small cell BSs reduce the load of macro cell BSs. To reach more energy efficiency from macro cell or small cell BSs of 5G cellular networks, one of the effective ways is to adopt an Advanced sleep mechanism (ASM).

To bring down mobile network energy consumption, BS sleep modes can be used by deactivating unnecessary radio resources when traffic demand is low. ASM allows the deactivation of the BS to intense sleep modes which correspond to lower power levels but longer reactivation times, by making it possible to obtain a convenient tradeoff between reactivation delay and energy saving. Clearly, ASM can be used in three scenarios, which are expected in 5G classification, where the cell layout is heavy and a few macro-cell BSs supply full coverage which are always active, while various small cell BSs can be activated on demand, whenever needed. The ASM application to 5G networks is shown in Fig. 1. The respective scenarios are Meo et al. (2021):

- Signaling scenario: In this scenario, the BS has no traffic to deliver, but it has to periodically send control signals and synchronization. In this case, Signaling Scenario has occurred. Within this scenario, focusing only on the low traffic demand period, no other type of traffic data is presumed.
- Massive IoT (MIoT) scenario: when the BS is about to forward a few pieces of data collected from time to time in a wireless network, then the MIoT scenario has occurred. In this scenario, many devices are connected to the internet and communicating with each other.
- Standard 5G traffic (S5GT) scenario: This scenario occurs when the BS controls (low) user traffic. Even in this scenario, the traffic load is assumed to be negligible due to signaling.

The ASM captures the behavior of sleep mode 1, sleep mode 2, sleep mode 3, and sleep mode 4. They combine the BS's components having similar transition times.

Because of the ASM strategy, the BS can save more power and reduce power consumption. But, it influences the performance of URs in sleeping cells depending on their BS association criteria, scheduling criteria, channel conditions, and traffic loads at the active BSs. Also, more power saving in the whole network may degrade the bandwidth efficiency of the URs in sleeping cells depending on the scheduling criteria employed and traffic loads at the active BSs. Hence, the multi-user scheduling criteria are used at active BSs which have a direct consequence on the channel access probability and reached bandwidth efficiency of the sleeping cell URs. In this paper, for mathematical tractability, by considering the above scenario and without degrading the quality of service (QoS), we have obtained energy efficiency in the context of throughput. The tradeoff between the power saving factor and power consumption with throughput is presented for all the models to show which model provides the optimum results.

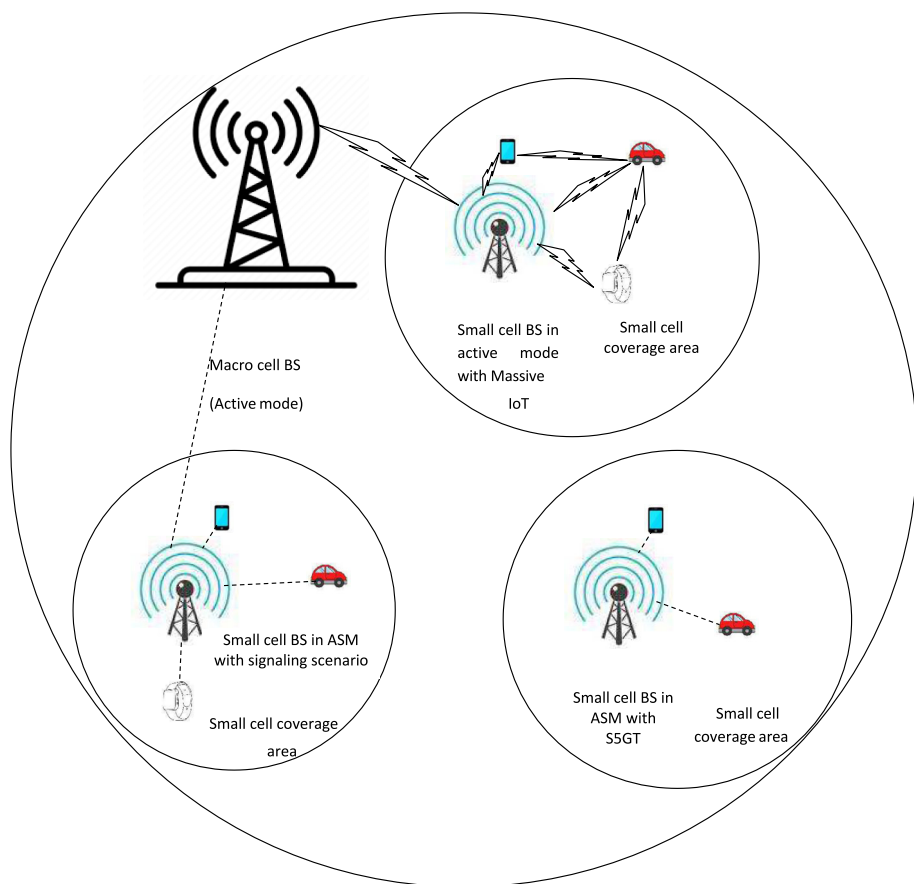


Fig. 1 An overview of ASM in 5G networks

1.1 State of the Art

In the earlier research work, various attempts were made on sleeping strategies in 5G BSs to get maximum energy efficiency. In existing cellular networks, switching off some components of low-utilized BSs is an efficient perspective to conserve energy while maintaining the QoS of mobile users. However, 5G cellular networks work with highly heterogeneous network architecture and new physical layer techniques, new challenges appear in the design of BSs sleep-awake (ON-OFF) switching strategies to get energy efficiency. The energy-saving mechanism of 5G base stations is modeled using an M/G/1 queue with two sleep modes, and the trade-off between power consumption, power saving, and throughput is analyzed to optimize energy efficiency and demonstrate the effectiveness of the sleeping strategy Kalita and Selvamuthu (2023). In Feng et al. (2017), Feng et al. discussed a detailed review of recent advances in switching mechanisms on different application scenarios of 5G networks. One of the necessities for 5G networks is an increase in energy efficiency by at least 10 to 100 times (Pervaiz et al. 2018). To increase energy efficiency, Pervaiz et al. (2018) proposed a multilevel sleep strategy to predict the BSs sleep time in advance. Recently, Deepa et al. (2023) discussed a sleeping strategy to get energy efficiency from small cell BSs in 4G/5G

networks. In this sleeping strategy, two different sleep modes are considered namely, short sleep and long sleep mode. Short sleep mode consists of M number of short sleep.

The paper (Aggarwal et al. 2024) introduces stochastic models, including Markov and semi-Markov models, to analyze and optimize energy efficiency in 5G base stations by strategically selecting frequency bands based on system traffic with a focus on power consumption under different traffic distributions. A semi-Markov model for base station energy consumption is analysed by assigning reward rates to base station components based on their energy usage, and throughput is evaluated using an M/G/1 queueing model with server breakdown (Aggarwal and Selvamuthu 2024). An energy consumption model for 5G base stations is presented in Zou et al. (2022), with the impact of data flow, terminal activity, and key factors on energy usage analyzed in distribution systems. A novel analytical methodology is presented in Peesapati et al. (2021) to evaluate the energy consumption of 5G base stations under varying traffic loads, with the impact of deep sleep modes considered for achieving significant energy savings. A Q-learning-based approach for ASMs in 5G base stations is proposed to show the trade-off between energy consumption and QoS by gradually deactivating BS components and managing component transition times (Aggarwal and Selvamuthu 2023). To get more energy efficiency without degrading the QoS, one of the effective techniques used in 5G networks is ASM. The ASM strategy based on shutting down the BS components slowly starting from the lowest level with a short activation delay until the deepest one. In the ASM strategy, the BS has to periodically send control signals and synchronization, even when there is no traffic demand. In Salem et al. (2017), Fatma et al. studied the impact of the signaling, and synchronization periodicity on the performance tradeoffs initiated by the ASM. Salem et al. (2018) proposed a solution for ASM strategy based on the Q-learning approach. One can reduce power consumption or save more power when the traffic load is very low. Focusing on the low traffic load periods, Meo et al. (2021) extensively discussed the potentiality of the ASM strategy-based system in terms of the tradeoff between delay and energy saving under traffic loads and different 5G scenarios. In the literature, various works have been recorded to obtain energy efficiency from 5G BSs by using stochastic models like Markov models, semi-Markov process (SMP) model, and Markov decision process (MDP) model. Among them, Gautam and Dharmaraja (2019) presented the DRX scheme selection using Markov modeling for voice traffic in LTE-A networks. Dharmaraja et al. (2022) presented the Markov model and SMP model for the power consumption in the user equipment (UE). To reduce the power consumption in the DRX, Jain et al. (2023) also presented a stochastic model. CK et al. (2023) presented that the power consumption in the base station can be minimized by modeling the service process of the BS as the MRGP model. Elsherif et al. (2019) designed an algorithm based on MDP to manage the ON/OFF switching of BSs in real-time. Salem et al. (2020) proposed an MDP-based ASM strategy to find the optimal policy by choosing the best sleep level according to the traffic load. However, no work has been recorded where the three stochastic models, i.e., continuous-time Markov model, SMP model, and Markov regenerative process (MRGP) model for ASM strategy in 5G BSs is proposed to show which model provides the optimum results by showing the tradeoff between different measures, like power-saving factor, power consumption, and throughput using the steady-state solutions.

1.2 Motivation and Main Contribution

5G BSs provide higher speed, lower latency, more reliability, and more available data service but it consumes more power about four times than 4G. By assuming the high formation density of 5G BSs, the altogether power consumption may extend 2 times that of 4G BSs

(Han et al. 2020). Hence, the high power consumption in 5G BS is a major issue to be concerned about. There is a BS feature ASM that allows for reducing power consumption and there is no work where different stochastic models are proposed using ASM. Hence, this motivates us to develop a stochastic model with ASM for 5G BSs to get energy efficiency without deteriorating the QoS.

In this research, we propose three stochastic models for 5G BSs that implement an ASM strategy to enhance energy efficiency. The strategy includes several operational states: Active Mode (A) for serving URs, Idle Mode (I) when no URs are in the queue, and four progressively deeper Sleep Modes (S_1 to S_4), each designed to reduce power consumption by turning off more base station components. S_1 is a short sleep mode with partial shutdown, while S_2 and S_3 progressively deactivate more components. S_4 is the deepest sleep mode, with most components powered down, retaining only the wake-up functionality. These modes are dynamically managed based on the arrival of URs, for energy savings without compromising responsiveness. We have obtained the closed-form steady-state solutions for all the models. The tradeoff with different measures is presented to show which models provide better results.

The rest of the paper is ordered as follows. The advanced sleep mechanism strategy is elaborated in Section 2. The illustration of the proposed continuous-time Markov model is presented in Section 3. The semi-Markov model is presented in Section 4. The Markov regenerative model is discussed in Section 5. Performance measures are analyzed in Section 6 for all three models. In Section 7, numerical validation is shown for the proposed models. Finally concluding remarks and future work are given in Section 8.

2 Description of Advanced Sleep Mechanism

The modes in the ASM strategy are defined as follows:

- Active Mode (A): In this mode, the base station is fully operational, receiving and transmitting URs.
- Idle Mode (I): When no URs are present in the buffer after service completion, the system enters the idle mode. If no UR arrives within the idle period, the BS switches to a sleep mode. If a UR arrives during this period, the BS switches back to the active mode.
- Sleep Mode 1 (S_1): This is a short-duration sleep mode where some base station components, including certain digital baseband and analog front-end components, are turned off to save power.
- Sleep Mode 2 (S_2): This mode has a longer duration than S_1 . In addition to the components deactivated in S_1 , additional analog front-end components are turned off to further reduce power consumption.
- Sleep Mode 3 (S_3): A deeper sleep mode than S_2 , where almost all digital baseband components, the power amplifier, and most analog front-end components are deactivated, with only the clock generator remaining active.
- Sleep Mode 4 (S_4): The deepest and longest-duration sleep mode, where the majority of the base station components are powered down. Only the wake-up functionality is maintained to allow the system to reactivate when needed.

The transitions between these states depend on the arrival of URs. After serving all URs in the active mode, if no new requests are in the queue, the system transitions to idle mode. If a UR arrives in any of the sleep modes or idle mode, the system switches back to active mode and starts providing the service. The transitions follow a hierarchical order from S_1 to S_4 , with each mode progressively shutting down more components to save energy. If no

UR arrives during S_4 , it moves to I . Figure 2 presents the transition between states for the ASM-based proposed model.

3 Continuous Time Markov Chain Model

A stochastic model is proposed for the energy aspects of the base station, considering the BS in different states, i.e., active, idle, and sleep. In the proposed stochastic model, sojourn time in each state follows an independent exponential distribution. Hence the underlying stochastic process is a continuous time Markov chain.

Let $\{N(t), t \geq 0\}$ indicates the continuous time Markov chain where $\{N(t), t \geq 0\}$ represents the state of the system at time t with state space $\Omega = \{I, S_1, S_2, S_3, S_4, A_i; i = 1, 2, \dots\}$. The state I indicates that the system is in idle mode with no user request. The state $A_i, i = 1, 2, \dots$ indicates that the system is in active mode providing service to i URs, and $S_j, j = 1, 2, 3, 4$ indicates the system is in j^{th} sleep mode.

Suppose the sojourn time in the idle period is exponentially distributed with parameter γ . Let the UR arrival in the system occurs according to the Poisson process with parameter λ . The service time for users conforms to an exponential distribution with the parameter μ , i.e, if there are i users in the system for the service, the service is provided to them with the same service rate and service rate μ is divided among all the users. Let the sojourn time in $S_j; j = 1, 2, 3, 4$ be exponentially distributed with parameter $\nu_j; j = 1, 2, 3, 4$, respectively.

If any UR arrives during the idle mode, then the BS moves to active mode A_1 , i.e., active mode with one user, and starts delivering the service to the user. Before the service completion of any user, if there is more than one user arrival, then the system proceeds to A_{i+1} , when i users are already getting the service. After the termination of every single service, the system proceeds to state A_{i-1} from state A_i . If there is no UR arrival in the idle mode, then the

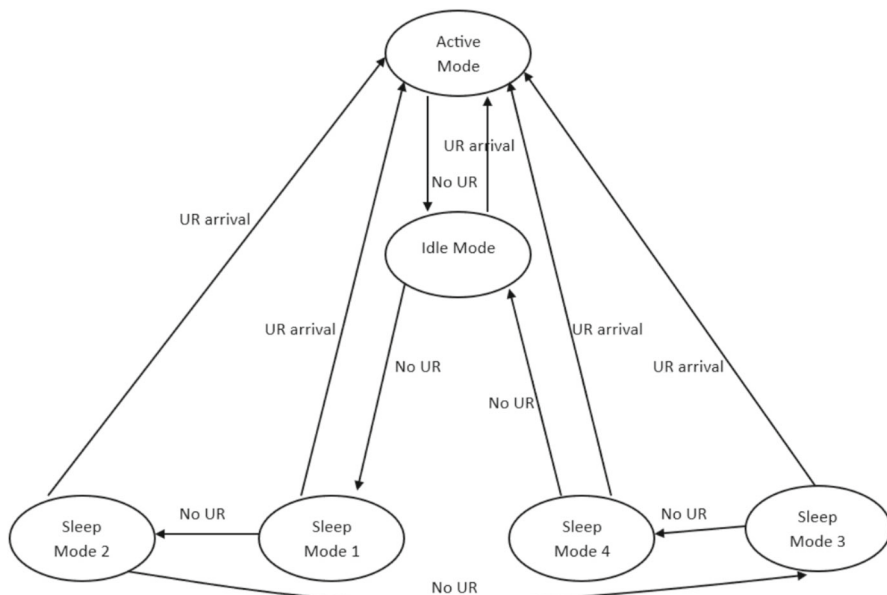


Fig. 2 ASM-based proposed model for the BS in 5G system

system proceeds to the first sleep mode S_1 . It keeps on moving to further sleep modes S_j , if there is no user arrival in sleep mode S_{j-1} ; $j = 2, 3, 4$. If there is a UR arrival in any of the sleep modes S_j , $j = 1, 2, 3, 4$, the system proceeds to active mode A_1 . If there is no user arrival in sleep mode S_4 , then on termination of S_4 , then the system proceeds to idle mode to wait for the URs to arrive. The state transition diagram for the proposed Markov model is presented in Fig. 3.

3.1 Steady State Analysis

Let $P_i(t)$ be the probability that the system is in state i , $i \in \Omega$ at any time t and π_i be the steady state probability when the system is in state i . Using forward Kolmogorov equations and $\pi_i = \lim_{t \rightarrow \infty} P_i(t)$ and $\lim_{t \rightarrow \infty} P'_i(t) = 0$, $i \in \Omega$, we have:

$$0 = -(\gamma + \lambda)\pi_I + \mu\pi_{A_1} + v_4\pi_{S_4} \quad (1)$$

$$0 = -(\nu_1 + \lambda)\pi_{S_1} + \gamma\pi_I \quad (2)$$

$$0 = -(\nu_2 + \lambda)\pi_{S_2} + \nu_1\pi_{S_1} \quad (3)$$

$$0 = -(\nu_3 + \lambda)\pi_{S_3} + \nu_2\pi_{S_2} \quad (4)$$

$$0 = -(\nu_4 + \lambda)\pi_{S_4} + \nu_3\pi_{S_3} \quad (5)$$

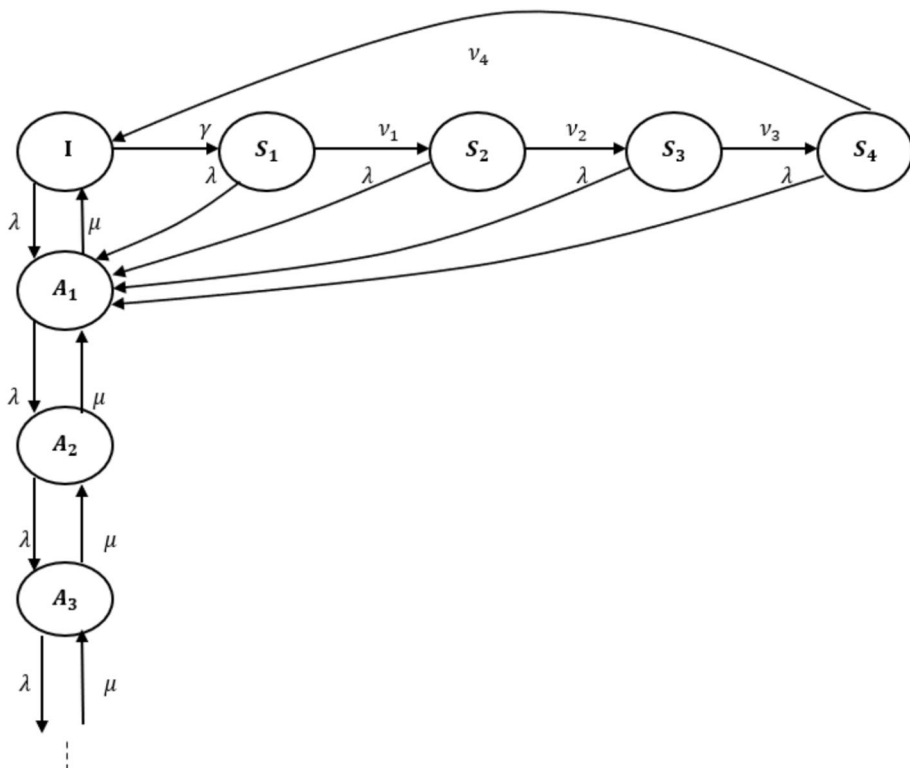


Fig. 3 State transition diagram for the proposed CTMC model

$$0 = \lambda(\pi_I + \pi_{S_1} + \pi_{S_2} + \pi_{S_3} + \pi_{S_4}) - (\lambda + \mu)\pi_{A_1} + \mu\pi_{A_2} \quad (6)$$

$$0 = \lambda\pi_{A_{i-1}} + \mu\pi_{A_{i+1}} - (\lambda + \mu)\pi_{A_i}, i = 2, 3, 4, \dots \quad (7)$$

From Eq. 2, we get

$$\pi_{S_1} = \frac{\gamma}{v_1 + \lambda} \pi_I. \quad (8)$$

From Eq. 3, we get

$$\pi_{S_2} = \frac{v_1}{v_2 + \lambda} \pi_{S_1},$$

Substituting the value of π_{S_1} obtained in Eq. 8, we get,

$$\pi_{S_2} = \frac{v_1 \gamma}{(v_2 + \lambda)(v_1 + \lambda)} \pi_I. \quad (9)$$

From Eq. 4, we have

$$\pi_{S_3} = \frac{v_2}{v_3 + \lambda} \pi_{S_2},$$

Substituting value of π_{S_2} obtained in Eq. 9, we get,

$$\pi_{S_3} = \frac{v_2 v_1 \gamma}{(v_3 + \lambda)(v_2 + \lambda)(v_1 + \lambda)} \pi_I. \quad (10)$$

From Eq. 5, we have

$$\pi_{S_4} = \frac{v_3}{v_4 + \lambda} \pi_{S_3},$$

Using the value of π_{S_3} obtained in Eq. 10, we get,

$$\pi_{S_4} = \frac{v_3 v_2 v_1 \gamma}{(v_4 + \lambda)(v_3 + \lambda)(v_2 + \lambda)(v_1 + \lambda)} \pi_I. \quad (11)$$

From Eq. 1 and using the value of π_{S_4} obtained in Eq. 11, we obtain

$$\pi_{A_1} = \frac{1}{\mu} \left[\gamma + \lambda - \frac{v_4 v_3 v_2 v_1 \gamma}{(v_4 + \lambda)(v_3 + \lambda)(v_2 + \lambda)(v_1 + \lambda)} \right] \pi_I. \quad (12)$$

Solving (1)-(6) simultaneously, we have

$$\pi_{A_2} = \frac{\lambda}{\mu} \pi_{A_1}. \quad (13)$$

For $i = 2, 3, 4, \dots$, using Eq. 7, we have

$$\pi_{A_{i+1}} - \pi_{A_i} = \frac{\lambda}{\mu} (\pi_{A_i} - \pi_{A_{i-1}}),$$

which recursively yields,

$$\pi_{A_{i+1}} - \pi_{A_i} = \left(\frac{\lambda}{\mu} \right)^{i-1} (\pi_{A_2} - \pi_{A_1}), \quad i = 2, 3, 4, \dots$$

Adding this equation for $i = 2, 3, 4, \dots$, we obtain

$$\pi_{A_{i+1}} - \pi_{A_2} = \sum_{n=1}^{i-1} \left(\frac{\lambda}{\mu} \right)^n (\pi_{A_2} - \pi_{A_1}),$$

Substituting the value of π_{A_2} obtained in Eq. 13, we get,

$$\pi_{A_{i+1}} = \left(\frac{\lambda}{\mu}\right)^i \pi_{A_1}. \quad (14)$$

As we know,

$$\sum_{n=1}^{\infty} \pi_{A_{n+1}} + \pi_{A_1} + \pi_{S_1} + \pi_{S_2} + \pi_{S_3} + \pi_{S_4} + \pi_I = 1.$$

Using value of A_n obtained in Eq. 14, for $n = 2, 3, 4, \dots$ and assuming that $\frac{\lambda}{\mu} < 1$, we obtain

$$\frac{\mu}{\mu - \lambda} \pi_{A_1} + \pi_{S_1} + \pi_{S_2} + \pi_{S_3} + \pi_{S_4} + \pi_I = 1.$$

Hence, using (9)-(12),

$$\pi_I = \frac{1}{M}$$

where,

$$M = 1 + \frac{\gamma + \lambda}{\mu - \lambda} + \frac{\gamma}{v_1 + \lambda} + \frac{v_1 \gamma}{(v_2 + \lambda)(v_1 + \lambda)} + \frac{v_2 v_1 \gamma}{(v_3 + \lambda)(v_2 + \lambda)(v_1 + \lambda)} \\ + \frac{v_3 v_2 v_1 \gamma}{(v_4 + \lambda)(v_3 + \lambda)(v_2 + \lambda)(v_1 + \lambda)} - \frac{v_4 v_3 v_2 v_1 \gamma}{(v_4 + \lambda)(v_3 + \lambda)(v_2 + \lambda)(v_1 + \lambda)(\mu - \lambda)}.$$

Using the value of π_I , we can obtain the steady-state probabilities for all the states of the CTMC model for the BS.

4 Semi Markov Model

The proposed CTMC model and the proposed semi-Markov model share the same state space. The way the system operates is exactly similar. The sojourn time in idle and the sleep mode, however, are not distributed exponentially.

4.1 Description of the Model

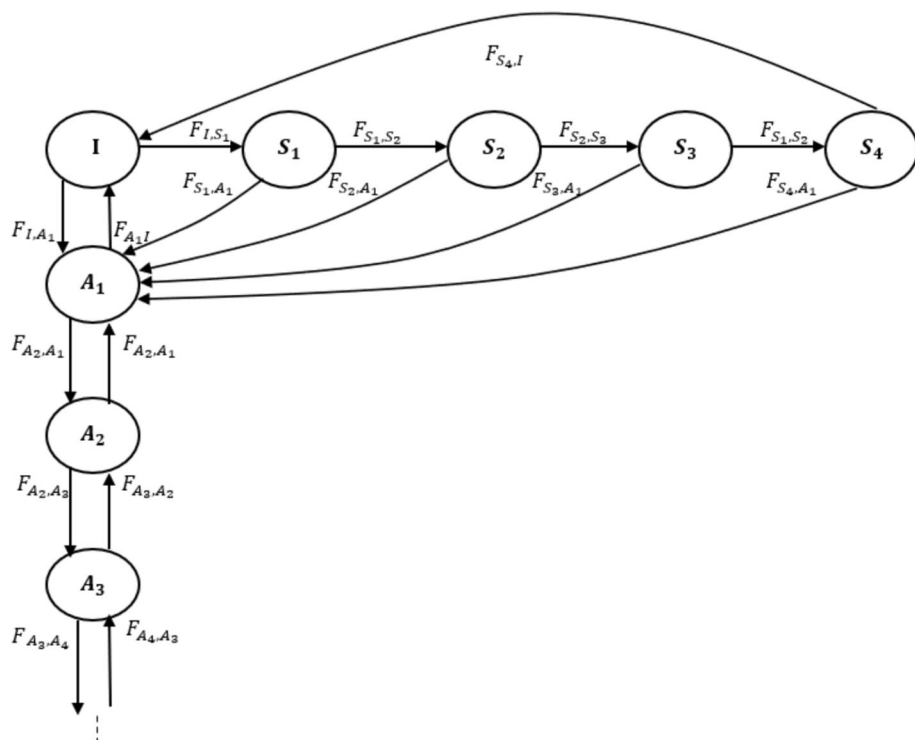
The space of possible states is the same as in the Markov model i.e., Ω . Let $F_{m,n}(t)$ stand for the cumulative distribution functions (CDF) for moving from state m to state n , where $m, n \in \Omega$. The stay time distribution with CDF for the different states of the semi-Markov model is shown in Table 1.

The transition from one state to another depends on many things, like traffic load and user arrivals, therefore it indicates randomness. Some states sojourn times follow the deterministic distribution, and all transition time epochs have the Markov property. Thus, this random process can be pictured as an SMP (Semi-Markov Process). Figure 4 shows the state transition diagram of the indicated semi-Markov model.

The kernel matrix describes the two-stage method, which is used to solve the SMP model. Let $K_{m,n}(t)$ be the elements of the kernel matrix $K(t)$, where $K_{m,n}(t)$ is the probability that the system is in state m and will move to the state n in the next transition in time less than t .

Table 1 Cumulative distribution functions with parameters

CDF	Distribution	Parameter
$F_{I,S_1}(t)$	Deterministic	$\frac{1}{\gamma}$
$F_{S_1,S_2}(t)$	Deterministic	$\frac{1}{v_1}$
$F_{S_2,S_3}(t)$	Deterministic	$\frac{1}{v_2}$
$F_{S_3,S_4}(t)$	Deterministic	$\frac{1}{v_3}$
$F_{S_4,I}(t)$	Deterministic	$\frac{1}{v_4}$
$F_{A_k,A_{k+1}}(t)$	Exponential	λ
$F_{A_1,I}(t)$	Exponential	μ
$F_{A_k,A_{k-1}}(t)$	Exponential	μ
$F_{I,A_1}(t)$	Exponential	λ
$F_{S_1,A_1}(t)$	Exponential	λ
$F_{S_2,A_1}(t)$	Exponential	λ
$F_{S_3,A_1}(t)$	Exponential	λ
$F_{S_4,A_1}(t)$	Exponential	λ

**Fig. 4** State transition diagram for the proposed SMP model

In this section, we examine the steady-state analysis for the semi-Markov model. These are the non-zero elements of the kernel matrix given as:

$$K_{I,S_1}(t) = \int_0^t (1 - F_{I,A_1}(x)) dF_{I,S_1}(x)$$

$$K_{I,S_1}(t) = \begin{cases} 0 & \text{if } t < \frac{1}{\gamma} \\ e^{-\frac{\lambda}{\gamma}t} & \text{if } t \geq \frac{1}{\gamma} \end{cases}$$

$$K_{I,A_1}(t) = \int_0^t (1 - F_{I,S_1}(x)) dF_{I,A_1}(x)$$

$$K_{I,A_1}(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t < \frac{1}{\gamma} \\ 1 - e^{-\frac{\lambda}{\gamma}t} & \text{if } t \geq \frac{1}{\gamma} \end{cases}$$

$$K_{S_1,S_2}(t) = \int_0^t (1 - F_{S_1,A_1}(x)) dF_{S_1,S_2}(x)$$

$$K_{S_1,S_2}(t) = \begin{cases} 0 & \text{if } t < \frac{1}{v_1} \\ e^{-\frac{\lambda}{v_1}t} & \text{if } t \geq \frac{1}{v_1} \end{cases}.$$

$$K_{S_2,S_3}(t) = \int_0^t (1 - F_{S_2,A_1}(x)) dF_{S_2,S_3}(x)$$

$$K_{S_2,S_3}(t) = \begin{cases} 0 & \text{if } t < \frac{1}{v_2} \\ e^{-\frac{\lambda}{v_2}t} & \text{if } t \geq \frac{1}{v_2} \end{cases}.$$

$$K_{S_3,S_4}(t) = \int_0^t (1 - F_{S_3,A_1}(x)) dF_{S_3,S_4}(x)$$

$$K_{S_3,S_4}(t) = \begin{cases} 0 & \text{if } t < \frac{1}{v_3} \\ e^{-\frac{\lambda}{v_3}t} & \text{if } t \geq \frac{1}{v_3} \end{cases}.$$

$$K_{S_4,I}(t) = \int_0^t (1 - F_{S_4,A_1}(x)) dF_{S_4,I}(x)$$

$$K_{S_4,I}(t) = \begin{cases} 0 & \text{if } t < \frac{1}{v_4} \\ e^{-\frac{\lambda}{v_4}t} & \text{if } t \geq \frac{1}{v_4} \end{cases}.$$

$$K_{S_1,A_1}(t) = \int_0^t (1 - F_{S_1,S_2}(x)) dF_{S_1,A_1}(x)$$

$$K_{S_1,A_1}(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t < \frac{1}{v_1} \\ 1 - e^{-\frac{\lambda}{v_1}t} & \text{if } t \geq \frac{1}{v_1} \end{cases}$$

$$K_{S_2,A_1}(t) = \int_0^t (1 - F_{S_2,S_3}(x)) dF_{S_2,A_1}(x)$$

$$K_{S_2,A_1}(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t < \frac{1}{v_2} \\ 1 - e^{-\frac{\lambda}{v_2}t} & \text{if } t \geq \frac{1}{v_2} \end{cases}$$

$$K_{S_3,A_1}(t) = \int_0^t (1 - F_{S_3,S_4}(x)) dF_{S_3,A_1}(x)$$

$$\begin{aligned}
K_{S_3, A_1}(t) &= \begin{cases} 1 - e^{-\lambda t} & \text{if } t < \frac{1}{v_3} \\ 1 - e^{-\frac{\lambda}{v_3}} & \text{if } t \geq \frac{1}{v_3} \end{cases} \\
K_{S_4, A_1}(t) &= \int_0^t (1 - F_{S_4, I}(x)) dF_{S_4, A_1}(x) \\
K_{S_4, A_1}(t) &= \begin{cases} 1 - e^{-\lambda t} & \text{if } t < \frac{1}{v_4} \\ 1 - e^{-\frac{\lambda}{v_4}} & \text{if } t \geq \frac{1}{v_4} \end{cases} \\
K_{A_1, I}(t) &= \int_0^t (1 - F_{A_1, A_2}(x)) dF_{A_1, I}(x) \\
K_{A_1, I}(t) &= \frac{\mu}{\lambda + \mu} (1 - e^{-(\lambda + \mu)t}) \\
K_{A_k, A_{k+1}}(t) &= \int_0^t (1 - F_{A_k, A_{k+1}}(x)) dF_{A_k, A_{k+1}}(x) \\
K_{A_k, A_{k+1}}(t) &= \frac{\lambda}{\lambda + \mu} (1 - e^{-(\lambda + \mu)t}), k = 1, 2, \dots \\
K_{A_k, A_{k-1}}(t) &= \int_0^t (1 - F_{A_k, A_{k+1}}(x)) dF_{A_k, A_{k-1}}(x) \\
K_{A_k, A_{k-1}}(t) &= \frac{\mu}{\lambda + \mu} (1 - e^{-(\lambda + \mu)t}), k = 2, 3, 4, \dots
\end{aligned}$$

Using the two-stage method for the semi-Markov model, $K_{m,n}(\infty) = \lim_{t \rightarrow \infty} K_{m,n}(t)$, where $m, n \in \Omega$. Here, the one-step transition probabilities for the EMC (Embedded Markov Chain) of the SMP model are shown by the matrix $K(\infty)$, whose values are written as $K_{m,n}(\infty)$. The sum of each row in a $K(\infty)$ matrix is 1. Therefore, the probabilities for the model's one-step transition, given as:

$$\begin{aligned}
K_{I, S_1}(\infty) &= e^{-\frac{\lambda}{\gamma}} & K_{I, A_1}(\infty) &= 1 - e^{-\frac{\lambda}{\gamma}} \\
K_{S_1, S_2}(\infty) &= e^{-\frac{\lambda}{v_1}} & K_{S_1, A_1}(\infty) &= 1 - e^{-\frac{\lambda}{v_1}} \\
K_{S_2, S_3}(\infty) &= e^{-\frac{\lambda}{v_2}} & K_{S_2, A_1}(\infty) &= 1 - e^{-\frac{\lambda}{v_2}} \\
K_{S_3, S_4}(\infty) &= e^{-\frac{\lambda}{v_3}} & K_{S_3, A_1}(\infty) &= 1 - e^{-\frac{\lambda}{v_3}} \\
K_{S_4, I}(\infty) &= e^{-\frac{\lambda}{v_4}} & K_{S_4, A_1}(\infty) &= 1 - e^{-\frac{\lambda}{v_4}} \\
K_{A_1, I}(\infty) &= \frac{\mu}{\lambda + \mu} & K_{A_i, A_{i-1}}(\infty) &= \frac{\mu}{\lambda + \mu}, i = 2, 3, \dots \\
K_{A_i, A_{i+1}}(\infty) &= \frac{\lambda}{\lambda + \mu}, i = 1, 2, 3, \dots
\end{aligned}$$

The vector V represents the steady-state probabilities for states of EMC, and are defined as $(V_I, V_{A_1}, \dots, V_{S_1}, V_{S_2}, V_{S_3}, V_{S_4})$ can be obtained by solving:

$$V = VK(\infty), \sum_{m \in \Omega} V_m = 1. \quad (15)$$

Hence, using $V = VK(\infty)$ and the entries of matrix $K(\infty)$, we obtain the system of linear equations, i.e.,

$$\begin{aligned}
V_I &= \frac{\mu}{\lambda + \mu} V_{A_1} + e^{-\frac{\lambda}{v_4}} V_{S_4} \\
V_{S_1} &= e^{-\frac{\lambda}{\gamma}} V_I \\
V_{S_2} &= e^{-\frac{\lambda}{v_1}} V_{S_1} \\
V_{S_3} &= e^{-\frac{\lambda}{v_2}} V_{S_2}
\end{aligned}$$

$$\begin{aligned}
 V_{S_4} &= e^{-\frac{\lambda}{v_3}} V_{S_3} \\
 V_{A_1} &= (1 - e^{-\frac{\lambda}{\gamma}}) V_I + \frac{\mu}{\lambda + \mu} V_{A_2} + (1 - e^{-\frac{\lambda}{v_1}}) V_{S_1} + (1 - e^{-\frac{\lambda}{v_2}}) V_{S_2} + (1 - e^{-\frac{\lambda}{v_3}}) V_{S_3} + \\
 &\quad (1 - e^{-\frac{\lambda}{v_4}}) V_{S_4} \\
 V_{A_i} &= \frac{\lambda}{\lambda + \mu} V_{A_{i-1}} + \frac{\mu}{\lambda + \mu} V_{A_{i+1}}, i = 2, \dots
 \end{aligned}$$

Solving this system of equations, we get $V_m, m \in \Omega$, the steady-state probabilities of each state in terms of V_I , i.e.,

$$\begin{aligned}
 V_{S_1} &= e^{-\frac{\lambda}{\gamma}} V_I \\
 V_{S_2} &= e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1})} V_I \\
 V_{S_3} &= e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2})} V_I \\
 V_{S_4} &= e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2} + \frac{\lambda}{v_3})} V_I \\
 V_{A_i} &= \left(\frac{\lambda + \mu}{\mu}\right) \left(\frac{\lambda}{\mu}\right)^{i-1} (1 - e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2} + \frac{\lambda}{v_3} + \frac{\lambda}{v_4})}) V_I, i = 1, 2, \dots
 \end{aligned}$$

Using equation $\sum_{m \in \Omega} V_m = 1$ and the probabilities, $V_m, m \in \Omega$ in terms of V_I , we obtain

$$V_I = \frac{1}{Sum},$$

where,

$$\begin{aligned}
 Sum &= 1 + e^{-\frac{\lambda}{\gamma}} + e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1})} + e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2})} \\
 &\quad + e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2} + \frac{\lambda}{v_3})} + \left(\frac{\lambda + \mu}{\mu - \lambda}\right) (1 - e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2} + \frac{\lambda}{v_3} + \frac{\lambda}{v_4})}).
 \end{aligned}$$

Let T_m denotes the sojourn time in state $m \in \Omega$. As the EMC for this SMP model is irreducible, non-periodic, and positive recurrent, hence ergodic. Therefore, this means that the steady-state probabilities of each state $m \in \Omega$ can be determined as follows:

$$\pi_m = \frac{V_m E[T_m]}{\sum_{n \in \Omega} V_n E[T_n]}, m \in \Omega,$$

where $E[T_m]$ denotes the expected sojourn time in state $m \in \Omega$. The expected sojourn time spent in each state can be obtained as:

$$\begin{aligned}
 E[T_I] &= \int_0^\infty (1 - F_{I, S_1}(t))(1 - F_{I, A_1}(t)) dt \\
 E[T_I] &= \frac{1 - e^{-\frac{\lambda}{\gamma}}}{\lambda} \\
 E[T_{A_i}] &= \int_0^\infty (1 - F_{A_i, A_{i-1}}(t))(1 - F_{A_i, A_{i+1}}(t)) dt \\
 E[T_{A_i}] &= \frac{1}{\lambda + \mu}, i = 1, 2, \dots \\
 E[T_{S_1}] &= \int_0^\infty (1 - F_{S_1, A_1}(t))(1 - F_{S_1, S_2}(t)) dt \\
 E[T_{S_1}] &= \frac{1 - e^{-\frac{\lambda}{v_1}}}{\lambda} \\
 E[T_{S_2}] &= \int_0^\infty (1 - F_{S_2, A_1}(t))(1 - F_{S_2, S_3}(t)) dt
 \end{aligned}$$

$$E[T_{S_2}] = \frac{1 - e^{-\frac{\lambda}{v_2}}}{\lambda}$$

$$E[T_{S_3}] = \int_0^\infty (1 - F_{S_3, A_1}(t))(1 - F_{S_3, S_4}(t))dt$$

$$E[T_{S_3}] = \frac{1 - e^{-\frac{\lambda}{v_3}}}{\lambda}$$

$$E[T_{S_4}] = \int_0^\infty (1 - F_{S_4, A_1}(t))(1 - F_{S_4, I}(t))dt$$

$$E[T_{S_4}] = \frac{1 - e^{-\frac{\lambda}{v_4}}}{\lambda}$$

The stability conditions for the existence of steady-state probabilities of the semi-Markov model are given as $\lambda < \mu$, $\lambda \leq v_1$, $\lambda < v_2$, $\lambda < v_3$ and $\lambda < v_4$. Hence, under stability conditions using expected sojourn times and the steady-state probabilities of EMC, we get

$$\sum_{n \in \Omega} V_n E[T_n] = L V_I,$$

where

$$L = \left(\frac{\mu}{\lambda(\mu - \lambda)} \right) \left(1 - e^{-\left(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2} + \frac{\lambda}{v_3} + \frac{\lambda}{v_4} \right)} \right).$$

Therefore, the steady-state probabilities for the semi-Markov model are given as:

$$\pi_I = \frac{1 - e^{-\frac{\lambda}{\gamma}}}{\lambda L},$$

$$\pi_{A_i} = \frac{\left(\frac{1}{\mu} \right) \left(\frac{\lambda}{\mu} \right)^{i-1} \left(1 - e^{-\left(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2} + \frac{\lambda}{v_3} + \frac{\lambda}{v_4} \right)} \right)}{L}, i = 1, 2, \dots,$$

$$\begin{aligned} \pi_{S_1} &= \frac{e^{-\frac{\lambda}{\gamma}} \left(1 - e^{-\frac{\lambda}{v_1}} \right)}{\lambda L}, \\ \pi_{S_2} &= \frac{e^{-\left(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} \right)} \left(1 - e^{-\frac{\lambda}{v_2}} \right)}{\lambda L}, \\ \pi_{S_3} &= \frac{e^{-\left(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2} \right)} \left(1 - e^{-\frac{\lambda}{v_3}} \right)}{\lambda L}, \\ \pi_{S_4} &= \frac{e^{-\left(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2} + \frac{\lambda}{v_3} \right)} \left(1 - e^{-\frac{\lambda}{v_4}} \right)}{\lambda L}. \end{aligned}$$

The steady-state probabilities $\pi_m \forall m \in S$ exist under stability conditions given as $\lambda < \mu$, $\lambda < v_1$, $\lambda < v_2$, $\lambda < v_3$ and $\lambda < v_4$.

5 The Markov Regenerative Process Model

The system state bivariate stochastic process for the ASM is considered as $\{(N(t), S(t)); t \geq 0\}$, where $\{N(t); t \geq 0\}$ indicates the number of URs in the system at time t and $\{S(t); t \geq 0\}$ denotes the state of the system at time t with state space Ω . Consider $\Omega \equiv \zeta_{(i,j)} \in \{0, 1, \dots\} \times \{0, 1, 2, 3, 4, 5\}$. Here, $i \in \{0, 1, 2, \dots\}$ denotes the number of URs in the system (if any URs are being served, comprising the one also) and $j \in 0, 1, 2, 3, 4, 5$ indicates the states of the system i.e., whether the system is in idle mode (I), or in the active mode (A), or in sleep mode 1 (S_1), or in sleep mode 2 (S_2), or in sleep mode 3 (S_3), or in sleep mode 4 (S_4). Let $A_{i,1}; i = 1, 2, \dots$ indicates the system is in the active mode with i number of URs, $I_{i,0}; i = 0, 1, \dots$ denotes it is in the idle mode with i number of URs, $S_{1,i,2}; i = 0, 1, \dots$ denotes it is in the sleep mode 1 with i number of URs, $S_{2,i,3}; i = 0, 1, \dots$ denotes it is in the sleep mode 2 with i number of URs, $S_{3,i,4}; i = 0, 1, \dots$ denotes it is in the sleep mode 3 with i number of URs, $S_{4,i,5}; i = 0, 1, \dots$ denotes it is in the sleep mode 4 with i number of URs.

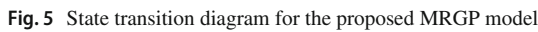
As the sojourn times in each state are not exponentially distributed, the process $\{(N(t), S(t)); t \geq 0\}$ is not a CTMC. Also, $\{(N(t), S(t)); t \geq 0\}$ is not even an SMP since between two consecutive service termination points (as the service termination points are the regeneration epochs), the system state with a number of URs can be switched due to new URs arrival before terminating the service of previous URs. Also, the states (idle mode, active mode, sleep mode 1, sleep mode 2, sleep mode 3, sleep mode 4) do not move immediately as the new arrivals occur. Hence, as the arrivals are independent and the remaining time does not impact in idle mode, active mode, sleep mode 1, sleep mode 2, sleep mode 3, and sleep mode 4, all the states termination points are the regeneration points.

By observing the behavior of all transition time points in an active mode, it is declared that the underlying process satisfies the Markov property at each service completion point. Assume the sequence of points $\{\tau_n, n \geq 0\}$ at which the process $\{(Z(t), S(t)); t \geq 0\}$ is noticed. Let us define $\{\tau_n, n \geq 0\}$ as the regeneration points at which service is finished or idle period is finished or sleep mode 1 is finished or sleep mode 2 is finished or sleep mode 3 is finished or sleep mode 4 is finished. Let $\xi_n \in \Omega$ denote the state of the system at these regeneration points. Here, the point of Ω are called regeneration points and they occur at the service completion point, idle period completion point, sleep mode 1 completion point, sleep mode 2 completion point, sleep mode 3 completion point, and sleep mode 4 completion point. Hence, we can affirm that $\{\xi_n, n \geq 0\}$ is a DTMC with one-step transition probability matrix $K(\infty)$. Hence, by perceiving the stochastic process $\{(N(t), S(t)); t \geq 0\}$ at these points, we see that the sequence $\{\xi_n, \tau_n\}$ is an embedded Markov renewal sequence and $\{(N(t), S(t)); t \geq 0\}$ is a MRGP.

5.1 Description of the Proposed Model

The proposed model is described mathematically in this section. Let the UR arrival in the system conforms to a Poisson process with parameter λ . The inter-arrival time of URs are independent and are exponentially distributed with parameter λ . The service time conforms to an exponential distribution with parameter μ , idle time, sleep mode 1 time, sleep mode 2 time, sleep mode 3 time, and sleep mode 4 time all are follows deterministic distributions with parameters $\frac{1}{\gamma}, \frac{1}{\nu_1}, \frac{1}{\nu_2}, \frac{1}{\nu_3}, \frac{1}{\nu_4}$ respectively. The state transition diagram for the proposed model is presented in Fig. 5. Table 2 illustrates all the CDFs with the corresponding parameters.

To solve the MRGP model, the expression for global kernel $K(t)$ and local kernel $E(t)$ are needed.



The components of the global kernel matrix $K(t)$ are denoted by $K_{m,n}(t)$, where $K_{m,n}(t)$ symbolize the probability that the system will be in state n at time of next regeneration epoch

CDF	parameter	CDF	parameter
$F_{A_{1,1}I_{0,0}}(t);$	μ	$FS_{4_{0,5}}I_{0,0}(t)$	$\frac{1}{v_4}$
$F_{I_{0,0}S_{1_{0,2}}}(t)$	$\frac{1}{\gamma}$	$FS_{1_{i,j}}A_{i,j-1}(t);$ $i = 1, 2, \dots, j = 2$	$\frac{1}{v_1}$
$F_{I_{0,0}S_{1_{0,2}}}(t)$	$\frac{1}{\gamma}$	$FS_{2_{i,j}}A_{i,j-2}(t);$ $i = 1, 2, \dots, j = 3$	$\frac{1}{v_2}$
$F_{A_{i,1}A_{i-1,1}}(t); i = 2, 3, \dots,$	μ	$FS_{3_{i,j}}A_{i,j-2}(t);$ $i = 1, 2, \dots, j = 4$	$\frac{1}{v_3}$
$FS_{1_{0,2}}S_{2_{0,3}}(t);$	$\frac{1}{v_1}$	$FS_{4_{i,j}}A_{i,j-4}(t);$ $i = 1, 2, \dots, j = 5$	$\frac{1}{v_4}$
$FS_{2_{0,3}}S_{3_{0,4}}(t);$	$\frac{1}{v_2}$	$FS_{3_{0,4}}S_{4_{0,5}}(t);$	$\frac{1}{v_3}$
$FS_{4_{0,5}}I_{0,0}(t);$	$\frac{1}{v_4}$	$FI_{i,j}A_{i,j+1}(t);$ $i = 1, 2, \dots, j = 0$	$\frac{1}{\gamma}$

which happens on or before time t , a condition that just after the preceding regeneration epoch, the system was in state m . Hence the non-zero components of the global kernel matrix are as follows:

$$\begin{aligned}
 K_{A_{1,1}I_{0,0}}(t) &= \int_0^t (1 - F_{A_{1,1}A_{2,1}}(x)) dF_{A_{1,1}I_{0,0}}(x) \\
 K_{A_{1,1}I_{0,0}}(t) &= \frac{\mu}{\lambda + \mu} \left(1 - e^{-(\lambda + \mu)t}\right) \\
 K_{I_{0,0}I_{1,0}}(t) &= \int_0^t (1 - F_{I_{0,0}S_{10,2}}(x)) dF_{I_{0,0}I_{1,0}}(x) \\
 K_{I_{0,0}I_{1,0}}(t) &= \begin{cases} 1 - e^{-\lambda t} & \text{if } t < \frac{1}{\gamma} \\ 1 - e^{-\frac{\lambda}{\gamma}} & \text{if } t \geq \frac{1}{\gamma} \end{cases} \\
 K_{I_{i,0}I_{i+1,0}}(t) &= \int_0^t (1 - F_{I_{i,0}A_{i+1,1}}(x)) dF_{I_{i,0}I_{i+1,0}}(x) \\
 K_{I_{i,0}I_{i+1,0}}(t) &= \begin{cases} 1 - e^{-\lambda t} & \text{if } t < \frac{1}{\gamma} \\ 1 - e^{-\frac{\lambda}{\gamma}} & \text{if } t \geq \frac{1}{\gamma}, i = 1, 2, \dots \end{cases} \\
 K_{I_{0,0}S_{10,2}}(t) &= \int_0^t (1 - F_{I_{0,0}I_{1,0}}(x)) dF_{I_{0,0}S_{10,2}}(x) \\
 K_{I_{0,0}S_{10,2}}(t) &= \begin{cases} e^{-\frac{\lambda}{\gamma}} & \text{if } t \geq \frac{1}{\gamma} \\ 0 & \text{if } t < \frac{1}{\gamma} \end{cases} \\
 K_{A_{i,1}A_{i-1,1}}(t) &= \int_0^t (1 - F_{A_{i,1}A_{i+1,1}}(x)) dF_{A_{i,1}A_{i-1,1}}(x) \\
 K_{A_{i,1}A_{i-1,1}}(t) &= \frac{\mu}{\lambda + \mu} \left(1 - e^{-(\lambda + \mu)t}\right); i = 2, 3, \dots \\
 K_{A_{1,1}A_{2,1}}(t) &= \int_0^t (1 - F_{A_{1,1}I_{0,0}}(x)) dF_{A_{1,1}A_{2,1}}(x) \\
 K_{A_{1,1}A_{2,1}}(t) &= \frac{\lambda}{\lambda + \mu} \left(1 - e^{-(\lambda + \mu)t}\right) \\
 K_{A_{i,1}A_{i+1,1}}(t) &= \int_0^t (1 - F_{A_{i,1}A_{i-1,1}}(x)) dF_{A_{i,1}A_{i+1,1}}(x) \\
 K_{A_{i,1}A_{i+1,1}}(t) &= \frac{\lambda}{\lambda + \mu} \left(1 - e^{-(\lambda + \mu)t}\right); i = 2, 3, \dots \\
 K_{S_{10,2}S_{20,3}}(t) &= \int_0^t (1 - F_{S_{10,2}S_{1,2}}(x)) dF_{S_{10,2}S_{20,3}}(x) \\
 K_{S_{1i,j}A_{i,j-1}}(t) &= \int_0^t (1 - F_{S_{1i,j}S_{1i+1,j}}(x)) dF_{S_{1i,j}A_{i,j-1}}(x) \\
 K_{S_{10,2}S_{20,3}}(t) = K_{S_{1i,j}A_{i,j-1}}(t) &= \begin{cases} e^{-\frac{\lambda}{v_1}} & \text{if } t \geq \frac{1}{v_1} \\ 0 & \text{if } t < \frac{1}{v_1}, i = 1, 2, \dots, j = 2 \end{cases} \\
 K_{S_{20,3}S_{30,4}}(t) &= \int_0^t (1 - F_{S_{20,3}S_{2,3}}(x)) dF_{S_{20,3}S_{30,4}}(x)
 \end{aligned}$$

$$\begin{aligned}
K_{S_{2i,j} A_{i,j-2}}(t) &= \int_0^t \left(1 - F_{S_{2i,j} S_{2i+1,j}}(x)\right) dF_{S_{2i,j} A_{i,j-2}}(x) \\
K_{S_{20,3} S_{30,4}}(t) &= K_{S_{2i,j} A_{i,j-2}}(t) = \begin{cases} e^{-\frac{\lambda}{v_2}} & \text{if } t \geq \frac{1}{v_2} \\ 0 & \text{if } t < \frac{1}{v_2}, i = 1, 2, \dots, j = 3 \end{cases} \\
K_{S_{30,4} S_{40,5}}(t) &= \int_0^t \left(1 - F_{S_{30,4} S_{31,4}}(x)\right) dF_{S_{30,4} S_{40,5}}(x) \\
K_{S_{3i,j} A_{i,j-3}}(t) &= \int_0^t \left(1 - F_{S_{3i,j} S_{3i+1,j}}(x)\right) dF_{S_{3i,j} A_{i,j-3}}(x) \\
K_{S_{30,4} S_{40,5}}(t) &= K_{S_{3i,j} A_{i,j-3}}(t) = \begin{cases} e^{-\frac{\lambda}{v_3}} & \text{if } t \geq \frac{1}{v_3}, i = 1, 2, \dots, j = 4 \\ 0 & \text{if } t < \frac{1}{v_3}, \end{cases} \\
K_{S_{40,5} I_{0,0}}(t) &= \int_0^t \left(1 - F_{S_{40,5} S_{41,5}}(x)\right) dF_{S_{40,5} I_{0,0}}(x) \\
K_{S_{4i,j} A_{i,j-4}}(t) &= \int_0^t \left(1 - F_{S_{4i,j} S_{4i+1,j}}(x)\right) dF_{S_{4i,j} A_{i,j-4}}(x) \\
K_{S_{40,5} I_{0,0}}(t) &= K_{S_{4i,j} A_{i,j-4}}(t) = \begin{cases} e^{-\frac{\lambda}{v_4}} & \text{if } t \geq \frac{1}{v_4} \\ 0 & \text{if } t < \frac{1}{v_4}, i = 1, 2, \dots, j = 5, \end{cases} \\
K_{S_{10,2} S_{11,2}}(t) &= \int_0^t \left(1 - F_{S_{10,2} S_{20,3}}(x)\right) dF_{S_{10,2} S_{11,2}}(x) \\
K_{S_{20,3} S_{21,3}}(t) &= \int_0^t \left(1 - F_{S_{20,3} S_{30,4}}(x)\right) dF_{S_{20,3} S_{21,3}}(x) \\
K_{S_{30,4} S_{31,4}}(t) &= \int_0^t \left(1 - F_{S_{30,4} S_{40,5}}(x)\right) dF_{S_{30,4} S_{31,4}}(x) \\
K_{S_{40,5} S_{41,5}}(t) &= \int_0^t \left(1 - F_{S_{40,5} I_{0,0}}(x)\right) dF_{S_{40,5} S_{41,5}}(x) \\
K_{S_{1i,j} S_{1i+1,j}}(t) &= \int_0^t \left(1 - F_{S_{1i,j} A_{i,j-1}}(x)\right) dF_{S_{1i,j} S_{1i+1,j}}(x) \\
K_{S_{1i,j} S_{1i+1,j}}(t) &= K_{S_{10,2} S_{11,2}}(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t < \frac{1}{v_1} \\ 1 - e^{-\frac{\lambda}{v_1}} & \text{if } t \geq \frac{1}{v_1}, i = 1, 2, \dots, j = 2, \end{cases} \\
K_{S_{2i,j} S_{2i+1,j}}(t) &= \int_0^t \left(1 - F_{S_{2i,j} A_{i,j-2}}(x)\right) dF_{S_{2i,j} S_{2i+1,j}}(x) \\
K_{S_{2i,j} S_{2i+1,j}}(t) &= K_{S_{20,3} S_{21,3}}(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t < \frac{1}{v_2} \\ 1 - e^{-\frac{\lambda}{v_2}} & \text{if } t \geq \frac{1}{v_2}, i = 1, 2, \dots, j = 3, \end{cases} \\
K_{S_{3i,j} S_{3i+1,j}}(t) &= \int_0^t \left(1 - F_{S_{3i,j} A_{i,j-3}}(x)\right) dF_{S_{3i,j} S_{3i+1,j}}(x) \\
K_{S_{3i,j} S_{3i+1,j}}(t) &= K_{S_{30,4} S_{31,4}}(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t < \frac{1}{v_3} \\ 1 - e^{-\frac{\lambda}{v_3}} & \text{if } t \geq \frac{1}{v_3}, i = 1, 2, \dots, j = 4, \end{cases} \\
K_{S_{4i,j} S_{4i+1,j}}(t) &= \int_0^t \left(1 - F_{S_{4i,j} A_{i,j-4}}(x)\right) dF_{S_{4i,j} S_{4i+1,j}}(x)
\end{aligned}$$

$$\begin{aligned}
 K_{S_{4i,j}S_{4i+1,j}}(t) &= K_{S_{40,5}S_{41,5}}(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t < \frac{1}{v_4} \\ 1 - e^{-\frac{\lambda}{v_4}t} & \text{if } t \geq \frac{1}{v_4}, \end{cases} i = 1, 2, \dots, j = 5, \\
 K_{I_{i,j}A_{i,j+1}}(t) &= \int_0^t (1 - F_{I_{i,j}I_{i+1,j}}(x)) dF_{I_{i,j}A_{i,j+1}}(x) \\
 K_{I_{i,j}A_{i,j+1}}(t) &= \begin{cases} e^{-\frac{\lambda}{\gamma}t} & \text{if } t \geq \frac{1}{\gamma} \\ 0 & \text{if } t < \frac{1}{\gamma}, \end{cases} i = 1, 2, \dots, j = 0.
 \end{aligned}$$

Now, the one-step transition probabilities for the proposed model are obtained in Table 3 as

Now, the steady-state probabilities of the EMC are defined as $H_{I_{i,0}}; i = 0, 1, \dots, H_{A_{i,1}}; i = 1, 2, \dots, H_{S_{1i,2}}; i = 0, 1, \dots, H_{S_{2i,3}}; i = 0, 1, \dots, H_{S_{3i,4}}; i = 0, 1, \dots, H_{S_{4i,5}}; i = 0, 1, \dots$, which can be obtained by solving. Hence, by using $H = HK(\infty)$ and the component of the matrix $K(\infty)$ as mention in Table 3, the system of linear equations are found as follows:

$$\begin{aligned}
 H_{I_{0,0}} &= \frac{\mu}{\lambda + \mu} H_{A_{1,1}} + e^{-\frac{\lambda}{v_4}} H_{S_{40,5}} \\
 H_{I_{i+1,0}} &= (1 - e^{-\frac{\lambda}{v_4}}) H_{I_{i,0}}; i = 0, 1, \dots \\
 H_{A_{i,1}} &= e^{-\frac{\lambda}{v_1}} H_{S_{1i,2}} + e^{-\frac{\lambda}{v_2}} H_{S_{2i,3}} + e^{-\frac{\lambda}{v_3}} H_{S_{3i,4}} + e^{-\frac{\lambda}{v_4}} H_{S_{4i,5}} + \frac{\mu}{\lambda + \mu} H_{A_{i+1,1}} + e^{-\frac{\lambda}{\gamma}} H_{I_{i,0}} \\
 H_{A_{i+1,1}} &= \frac{\mu}{\lambda + \mu} H_{A_{i,1}}; i = 1, 2, \dots \\
 H_{S_{10,2}} &= e^{-\frac{\lambda}{\gamma}} H_{I_{0,0}} \\
 H_{S_{20,3}} &= e^{-\frac{\lambda}{v_1}} H_{S_{10,2}} \\
 H_{S_{30,4}} &= e^{-\frac{\lambda}{v_2}} H_{S_{20,3}}
 \end{aligned}$$

Table 3 The one-step transition probabilities for the proposed model

$K_{A_{i,1}A_{i+1,1}}(\infty) = \frac{\lambda}{\lambda + \mu}; i = 1, \dots,$	$K_{A_{1,1}I_{0,0}}(\infty) = \frac{\lambda}{\lambda + \mu},$
$K_{I_{i,0}I_{i+1,0}}(\infty) = 1 - e^{-\frac{\lambda}{\gamma}}; i = 0, 1, \dots,$	$K_{I_{0,0}S_{10,3}}(\infty) = e^{-\frac{\lambda}{\gamma}}$
$K_{I_{i,0}A_{i,1}}(\infty) = e^{-\frac{\lambda}{\gamma}}, i = 1, 2, \dots,$	$K_{A_{i,1}A_{i-1,1}}(\infty) = \frac{\mu}{\lambda + \mu};$ $i = 2, 3, \dots,$
$K_{S_{10,2}S_{20,3}}(\infty) = e^{-\frac{\lambda}{v_1}},$	$K_{S_{20,3}S_{30,4}}(\infty) = e^{-\frac{\lambda}{v_2}}$
$K_{S_{30,4}S_{40,5}}(\infty) = e^{-\frac{\lambda}{v_3}},$	$K_{S_{40,5}I_{0,0}}(\infty) = e^{-\frac{\lambda}{v_4}},$
$K_{S_{1i,2}S_{1i+1,2}}(\infty) = 1 - e^{-\frac{\lambda}{v_1}}, i = 0, 1, \dots,$	$K_{S_{2i,3}S_{2i+1,3}}(\infty) = 1 - e^{-\frac{\lambda}{v_2}},$ $i = 0, 1, \dots,$
$K_{S_{3i,4}S_{3i+1,4}}(\infty) = 1 - e^{-\frac{\lambda}{v_3}}, i = 0, 1, \dots,$	$K_{S_{4i,5}S_{4i+1,5}}(\infty) = 1 - e^{-\frac{\lambda}{v_4}},$ $i = 0, 1, \dots,$

$$\begin{aligned}
H_{S_{40,5}} &= e^{-\frac{\lambda}{v_3}} H_{S_{30,4}} \\
H_{S_{1i+1,2}} &= (1 - e^{-\frac{\lambda}{v_1}}) H_{S_{1i,2}}; i = 1, 2, \dots \\
H_{S_{2i+1,3}} &= (1 - e^{-\frac{\lambda}{v_2}}) H_{S_{2i,3}}; i = 1, 2, \dots \\
H_{S_{3i+1,4}} &= (1 - e^{-\frac{\lambda}{v_1}}) H_{S_{3i,4}}; i = 1, 2, \dots \\
H_{S_{4i+1,5}} &= (1 - e^{-\frac{\lambda}{v_1}}) H_{S_{4i,5}}; i = 1, 2, \dots
\end{aligned}$$

Solving this system of equations, we obtain the steady-state probabilities of each state, $H_m, m \in \Omega$ in terms of $H_{A_{1,1}}$, i.e.,

$$\begin{aligned}
H_{I_{i,0}} &= (1 - e^{-\frac{\lambda}{\gamma}})^i \left(1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})}\right)^{-1} \frac{\mu}{\lambda + \mu} H_{A_{1,1}}; i = 0, 1, \dots \\
H_{S_{1i,2}} &= (1 - e^{-\frac{\lambda}{v_1}})^i e^{-\frac{\lambda}{\gamma}} \left(1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})}\right)^{-1} \frac{\mu}{\lambda + \mu} H_{A_{1,1}}; i = 0, 1, \dots \\
H_{S_{2i,3}} &= (1 - e^{-\frac{\lambda}{v_2}})^i e^{-\lambda(\frac{1}{v_1} + \frac{1}{\gamma})} \left(1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})}\right)^{-1} \frac{\mu}{\lambda + \mu} H_{A_{1,1}}; i = 0, 1, \dots \\
H_{S_{3i,4}} &= (1 - e^{-\frac{\lambda}{v_3}})^i e^{-\lambda(\frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})} \left(1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})}\right)^{-1} \frac{\mu}{\lambda + \mu} H_{A_{1,1}}; i = 0, 1, \dots \\
H_{S_{4i,5}} &= (1 - e^{-\frac{\lambda}{v_4}})^i e^{-\lambda(\frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})} \left(1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})}\right)^{-1} \frac{\mu}{\lambda + \mu} H_{A_{1,1}}; i = 0, 1, \dots \\
H_{A_{i,1}} &= H_{A_{1,1}} (1 - \frac{\lambda\mu}{(\lambda + \mu)^2})^{-1} \left[\frac{\lambda\mu}{(\lambda + \mu)^2} e^{-\frac{\lambda}{\gamma}} e^{-\frac{\lambda}{v_1}} \left(1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})}\right)^{-1} \{(1 - e^{-\frac{\lambda}{v_1}})^{i-1} + e^{-\frac{\lambda}{v_2}} (1 - e^{-\frac{\lambda}{v_2}})^{i-1} + e^{-\frac{\lambda}{v_3}} (1 - e^{-\frac{\lambda}{v_3}})^{i-1} e^{-\frac{\lambda}{v_2}} + (1 - e^{-\frac{\lambda}{v_4}})^{i-1} e^{-\lambda(\frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_4})} + (1 - e^{-\frac{\lambda}{\gamma}})^{i-1} e^{-\frac{\lambda}{v_1}}\} \right]; i = 1, 2, \dots
\end{aligned}$$

Using equation $\sum_{m \in \Omega} H_m = 1$ and the probabilities, $H_m, m \in \Omega$ in terms of $H_{A_{1,1}}$, we obtain

$$H_{A_{1,1}} = \frac{1}{C},$$

where,

$$\begin{aligned}
C &= e^{\frac{\lambda}{\gamma}} + e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1})} + e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2})} + e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2} + \frac{\lambda}{v_3})} + e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2} + \frac{\lambda}{v_3} + \frac{\lambda}{v_4})} \\
&\quad \times \left(\frac{\mu}{\mu + \lambda} \right) \left(1 - e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2} + \frac{\lambda}{v_3} + \frac{\lambda}{v_4})}\right)^{-1} + \left(1 - \left(\frac{\lambda\mu}{(\mu + \lambda)^2} \right)\right)^{-1} \\
&\quad \left[\frac{\lambda\mu}{(\lambda + \mu)^2} e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1})} \left(1 - e^{-(\frac{\lambda}{\gamma} + \frac{\lambda}{v_1} + \frac{\lambda}{v_2} + \frac{\lambda}{v_3} + \frac{\lambda}{v_4})}\right)^{-1} \{e^{\frac{\lambda}{v_1}} (1 - e^{-\frac{\lambda}{v_1}}) \right. \\
&\quad \left. + (1 - e^{-\frac{\lambda}{v_2}}) + (1 - e^{-\frac{\lambda}{v_3}}) e^{-\frac{\lambda}{v_2}} + (1 - e^{-\frac{\lambda}{v_4}}) e^{-(\frac{\lambda}{v_2} + \frac{\lambda}{v_3})} + e^{\frac{\lambda}{v_1}} e^{\frac{\lambda}{\gamma}} (1 - e^{-\frac{\lambda}{\gamma}}) \} \right]
\end{aligned}$$

5.1.2 Local kernel

The local kernel $E(t)$ explains the nature of the process during the time between two consecutive regeneration points starting from a state which is just after the next regeneration points. Hence, the components of the local kernel matrix are as follows:

$$E_{m,n}(t) = \begin{cases} E_{A_{i,j} A_{i-1,j}}(t) = (1 - F_{A_{i,j} A_{i-1,j}}(t))(1 - F_{A_{i,j} A_{i+1,j}}(t)), \\ i = 2, \dots, j = 1, \\ E_{A_{1,1} I_{0,0}}(t) = (1 - F_{A_{1,2} A_{2,1}}(t))(1 - F_{A_{1,1} I_{0,0}}(t)), \\ E_{S_{1i,j} A_{i,j-1}}(t) = (1 - F_{S_{1i,j} A_{i,j-1}}(t))(1 - (F_{S_{1i,j} S_{1i+1,j}}(t))), \\ i = 1, 2, \dots, j = 2, \\ E_{S_{2i,j} A_{i,j-2}}(t) = (1 - F_{S_{2i,j} A_{i,j-2}}(t))(1 - F_{S_{2i,j} S_{2i+1,j}}(t)), \\ i = 1, 2, \dots, j = 3, \\ E_{S_{3i,j} A_{i,j-3}}(t) = (1 - F_{S_{3i,j} A_{i,j-3}}(t))(1 - F_{S_{3i,j} S_{3i+1,j}}(t)), \\ i = 1, 2, \dots, j = 4, \\ E_{S_{4i,j} A_{i,j-4}}(t) = (1 - F_{S_{4i,j} A_{i,j-4}}(t))(1 - F_{S_{4i,j} S_{4i+1,j}}(t)), \\ i = 1, \dots, j = 5, \\ E_{S_{10,2} S_{20,3}}(t) = (1 - F_{S_{10,2} S_{20,3}}(t))(1 - F_{S_{10,2} S_{20,3}}(t)), \\ i = 1, 2, \dots, \\ E_{S_{20,3} S_{30,4}}(t) = (1 - F_{S_{20,3} S_{30,4}}(t))(1 - F_{S_{20,3} S_{21,3}}(t)), \\ i = 1, 2, \dots, \\ E_{S_{30,4} S_{40,5}}(t) = (1 - F_{S_{30,4} S_{40,5}}(t))(1 - F_{S_{30,4} S_{31,4}}(t)), \\ i = 1, 2, \dots, \\ E_{S_{40,5} I_{0,0}}(t) = (1 - F_{S_{40,5} I_{0,0}}(t))(1 - F_{S_{40,5} S_{41,5}}(t)) \\ E_{I_{0,0} S_{10,2}}(t) = (1 - F_{I_{0,0} S_{10,2}}(t))(1 - F_{I_{0,0} I_{1,0}}(t)) \\ E_{I_{i,j} A_{i,j+1}}(t) = (1 - F_{I_{i,j} A_{i,j+1}}(t))(1 - F_{I_{i,j} I_{i+1,j}}(t)) \end{cases}$$

To obtain the performance measures, we presume a new variable $\psi_{m,n}$ which is the average amount of time the MRGP spends in state n between two sequential regeneration epochs condition that it was in state m after the last regeneration:

$$\psi_{m,n} = \int_0^\infty E_{m,n}(t) dt.$$

For different values of m and n , we have

$$\psi_{m,n} = \begin{cases} \frac{1}{\lambda + \mu}, m = A_{i,j}, n = A_{i-1,j}, i = 2, 3, \dots, j = 1, \\ \frac{1}{\lambda + \mu}, m = A_{1,1}, n = I_{0,0}, \\ \frac{1 - e^{-\frac{\lambda}{v_1}}}{\lambda}, m = S_{1,i,j}, n = A_{i,j-1}, i = 1, 2, \dots, j = 2, \\ \frac{1 - e^{-\frac{\lambda}{v_2}}}{\lambda}, m = S_{2,i,j}, n = A_{i,j-2}, i = 1, \dots, j = 3, \\ \frac{1 - e^{-\frac{\lambda}{v_3}}}{\lambda}, m = S_{3,i,j}, n = A_{i,j-3}, i = 1, 2, \dots, j = 4, \\ \frac{1 - e^{-\frac{\lambda}{v_4}}}{\lambda}, m = S_{4,i,j}, n = A_{i,j-4}, i = 1, 2, \dots, j = 5, \\ \frac{1 - e^{-\frac{\lambda}{v_1}}}{\lambda}, m = S_{10,2}, n = S_{20,3}, \\ \frac{1 - e^{-\frac{\lambda}{v_2}}}{\lambda}, m = S_{20,3}, n = S_{30,4}, \\ \frac{1 - e^{-\frac{\lambda}{v_3}}}{\lambda}, m = S_{30,4}, n = S_{40,5}, \\ \frac{1 - e^{-\frac{\lambda}{v_4}}}{\lambda}, m = S_{40,5}, n = I_{0,0}, \\ \frac{1 - e^{-\frac{\lambda}{\gamma}}}{\lambda}, m = I_{0,0}, n = S_{10,2}, \\ \frac{1 - e^{-\frac{\lambda}{\gamma}}}{\lambda}, m = I_{i,j}, n = A_{i,j+1}, i = 1, 2, \dots, j = 0. \end{cases}$$

In order to acquire the energy efficiency from the model we require to obtain the system state probabilities. The system state probabilities are defined as follows:

$$\phi_n = \frac{\sum_{k \in \Omega} H_k \psi_{k,n}}{\sum_{k \in \Omega} H_k \delta_k}, \quad n \in \Omega, \quad (16)$$

where, $\delta_k = \sum_{l \in \Omega} \psi_{kl}$. The stability conditions for the existence of steady-state probabilities of the MRGP model is $\lambda < \mu$. Hence, under stability conditions using δ_k and steady-state probabilities of EMC, we get

$$\sum_{k \in \Omega} H_k \delta_k = \frac{G}{C},$$

where,

$$\begin{aligned} G = & \frac{1}{(\lambda + \mu)} \left[\left(1 - \frac{\lambda \mu}{(\lambda + \mu)^2} \right)^{-1} \frac{\lambda \mu}{(\lambda + \mu)^2} e^{-\lambda(\frac{1}{\gamma} + \frac{1}{v_1})} (1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})})^{-1} \right. \\ & \{ e^{\frac{\lambda}{v_1}} (1 - e^{-\frac{\lambda}{v_1}}) + (1 - e^{-\frac{\lambda}{v_2}}) + (1 - e^{-\frac{\lambda}{v_3}}) e^{-\frac{\lambda}{v_2}} + (1 - e^{-\frac{\lambda}{v_4}}) e^{-\lambda(\frac{1}{v_2} + \frac{1}{v_3})} \\ & \left. + e^{\lambda(\frac{1}{v_1} + \frac{1}{\gamma})} (1 - e^{-\frac{\lambda}{\gamma}}) \} + 1 \right] + e^{-\frac{\lambda}{\gamma}} (1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})})^{-1} \frac{\mu}{\lambda + \mu} \frac{1}{\lambda} \\ & \left\{ (1 - e^{-\frac{\lambda}{v_1}}) (1 + e^{\frac{\lambda}{v_1}}) + e^{-\frac{\lambda}{v_1}} (1 + e^{\frac{\lambda}{v_2}}) (1 - e^{-\frac{\lambda}{v_2}}) + e^{-\lambda(\frac{1}{v_2} + \frac{1}{v_1})} (1 + e^{\frac{\lambda}{v_3}}) \right. \\ & \left. (1 - e^{-\frac{\lambda}{v_3}}) + e^{-\lambda(\frac{1}{v_3} + \frac{1}{v_1} + \frac{1}{v_2})} (1 + e^{\frac{\lambda}{v_4}}) (1 - e^{-\frac{\lambda}{v_4}}) + e^{\frac{\lambda}{\gamma}} (1 + e^{\frac{\lambda}{\gamma}}) (1 - e^{-\frac{\lambda}{\gamma}}) \right\} \end{aligned}$$

The steady-state probabilities for the Markov regenerative process model are given as:

$$\begin{aligned} \phi_{A_{i,1}} = \frac{1}{G} & \left[\frac{1}{\lambda + \mu} \left(1 - \frac{\lambda\mu}{(\lambda + \mu)^2} \right)^{-1} \frac{\lambda\mu}{(\lambda + \mu)^2} e^{-\lambda(\frac{1}{\gamma} + \frac{1}{v_1})} \right. \\ & (1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})})^{-1} \{ (1 - e^{-\frac{\lambda}{v_1}})^{i-1} \\ & + e^{-\frac{\lambda}{v_2}} (1 - e^{-\frac{\lambda}{v_2}})^{i-1} + e^{-\frac{\lambda}{v_3}} (1 - e^{-\frac{\lambda}{v_3}})^{i-1} e^{-\frac{\lambda}{v_2}} \\ & \left. + e^{-\frac{\lambda}{v_4}} (1 - e^{-\frac{\lambda}{v_4}})^{i-1} e^{-\lambda(\frac{1}{v_2} + \frac{1}{v_3})} + e^{-\frac{\lambda}{v_1}} (1 - e^{-\frac{\lambda}{\gamma}})^{i-1} \} + 1 \right]; i = 1, 2, \dots \end{aligned} \quad (17)$$

$$\begin{aligned} \phi_{S_{1i,2}} = \frac{1}{G} & \left[\frac{\mu}{\lambda + \mu} e^{-\frac{\lambda}{\gamma}} (1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})})^{-1} \frac{(1 - e^{-\frac{\lambda}{v_1}})}{\lambda} \right. \\ & \left. (1 + (1 - e^{-\frac{\lambda}{v_1}})^i) \right]; i = 0, 1, \dots \end{aligned} \quad (18)$$

$$\begin{aligned} \phi_{S_{2i,3}} = \frac{1}{G} & \left[\frac{\mu}{\lambda + \mu} e^{-\lambda(\frac{1}{v_1} + \frac{1}{\gamma})} (1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})})^{-1} \frac{(1 - e^{-\frac{\lambda}{v_2}})}{\lambda} \right. \\ & \left. (1 + (1 - e^{-\frac{\lambda}{v_2}})^i) \right]; i = 0, 1, \dots \end{aligned} \quad (19)$$

$$\begin{aligned} \phi_{S_{3i,4}} = \frac{1}{G} & \left[\frac{\mu}{\lambda + \mu} e^{-\lambda(\frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})} (1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})})^{-1} \frac{(1 - e^{-\frac{\lambda}{v_3}})}{\lambda} \right. \\ & \left. (1 + (1 - e^{-\frac{\lambda}{v_3}})^i) \right]; i = 0, 1, \dots \end{aligned} \quad (20)$$

$$\begin{aligned} \phi_{S_{4i,5}} = \frac{1}{G} & \left[\frac{\mu}{\lambda + \mu} e^{-\lambda(\frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})} (1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})})^{-1} \frac{(1 - e^{-\frac{\lambda}{v_4}})}{\lambda} \right. \\ & \left. (1 + (1 - e^{-\frac{\lambda}{v_4}})^i) \right]; i = 0, 1, \dots \end{aligned} \quad (21)$$

$$\begin{aligned} \phi_{I_{i,0}} = \frac{1}{G} & \left[\frac{\mu}{\lambda + \mu} (1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})})^{-1} \frac{(1 - e^{-\frac{\lambda}{\gamma}})}{\lambda} \right. \\ & \left. (1 + (1 - e^{-\frac{\lambda}{\gamma}})^i) \right]; i = 0, 1, \dots \end{aligned} \quad (22)$$

6 Performance Measures

Performance measures for the proposed models are presented in this section. Firstly, system state probabilities are given, then power saving factor, average power consumption, and throughput are defined using the steady-state probabilities.

6.1 System State Probabilities

System state probabilities for the CTMC and SMP model are given in Sections 3 and 4. Steady-state probabilities for being in an active mode is given by π_A which is defined as:

$$\pi_A = \sum_{i=1}^{\infty} \pi_{A_i}.$$

6.1.1 System State Probabilities of MRGP

Let $\pi_A, \pi_{S_1}, \pi_{S_2}, \pi_{S_3}, \pi_{S_4}, \pi_I$ are the steady-state probabilities for being in an active mode, sleep mode 1, sleep mode 2, sleep mode 3, sleep mode 4 and idle mode, respectively.

$$\begin{aligned} \pi_A = \frac{1}{G} & \left[\frac{1}{\lambda + \mu} \left[\left(1 - \frac{\lambda\mu}{(\lambda + \mu)^2} \right)^{-1} \frac{\lambda\mu}{(\lambda + \mu)^2} e^{-\lambda(\frac{1}{\gamma} + \frac{1}{v_1})} \right. \right. \\ & (1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})})^{-1} \{ (1 - e^{-\frac{\lambda}{v_1}}) e^{\frac{\lambda}{v_1}} \\ & + (1 - e^{-\frac{\lambda}{v_2}}) + (1 - e^{-\frac{\lambda}{v_3}}) e^{-\frac{\lambda}{v_2}} + (1 - e^{-\frac{\lambda}{v_4}}) e^{-\lambda(\frac{1}{v_2} + \frac{1}{v_3})} \\ & \left. \left. + e^{\frac{\lambda}{v_1}} e^{\frac{\lambda}{\gamma}} (1 - e^{-\frac{\lambda}{\gamma}}) \right\} + 1 \right]. \end{aligned} \quad (23)$$

$$\begin{aligned} \pi_{S_1} = \frac{1}{G} & \left[\frac{\mu}{\lambda + \mu} e^{-\frac{\lambda}{\gamma}} (1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})})^{-1} \frac{(1 - e^{-\frac{\lambda}{v_1}})}{\lambda} \right. \\ & \left. (1 + e^{\frac{\lambda}{v_1}}) \right]. \end{aligned} \quad (24)$$

$$\begin{aligned} \pi_{S_2} = \frac{1}{G} & \left[\frac{\mu}{\lambda + \mu} e^{-\lambda(\frac{1}{v_1} + \frac{1}{\gamma})} (1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})})^{-1} \frac{(1 - e^{-\frac{\lambda}{v_2}})}{\lambda} \right. \\ & \left. (1 + e^{\frac{\lambda}{v_2}}) \right]. \end{aligned} \quad (25)$$

$$\begin{aligned} \pi_{S_3} = \frac{1}{G} & \left[\frac{\mu}{\lambda + \mu} e^{-\lambda(\frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})} (1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})})^{-1} \frac{(1 - e^{-\frac{\lambda}{v_3}})}{\lambda} \right. \\ & \left. (1 + e^{\frac{\lambda}{v_3}}) \right]. \end{aligned} \quad (26)$$

$$\begin{aligned} \pi_{S_4} = \frac{1}{G} & \left[\frac{\mu}{\lambda + \mu} e^{-\lambda(\frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})} (1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})})^{-1} \frac{(1 - e^{-\frac{\lambda}{v_4}})}{\lambda} \right. \\ & \left. (1 + e^{\frac{\lambda}{v_4}}) \right]. \end{aligned} \quad (27)$$

$$\begin{aligned} \pi_I = \frac{1}{G} & \left[\frac{\mu}{\lambda + \mu} (1 - e^{-\lambda(\frac{1}{v_4} + \frac{1}{v_3} + \frac{1}{v_2} + \frac{1}{v_1} + \frac{1}{\gamma})})^{-1} \frac{(1 - e^{-\frac{\lambda}{\gamma}})}{\lambda} \right. \\ & \left. (1 + e^{\frac{\lambda}{\gamma}}) \right]. \end{aligned} \quad (28)$$

6.2 Power saving Factor

The power saving factor for the system is defined as the percentage of time the BSs spent in S_1 , S_2 , S_3 and S_4 which is given as

$$P_S = (\pi_{S_1} + \pi_{S_2} + \pi_{S_3} + \pi_{S_4}) \times 100\% \quad (29)$$

where, π_{S_1} , π_{S_2} , π_{S_3} and π_{S_4} are the steady-state probabilities for being the 5G BS are in S_1 , S_2 , S_3 and S_4 mode.

6.3 Average Power Consumption

The average power consumption of 5G BSs is defined as follows:

$$P_{\text{cons}} = PC_A \pi_A + PC_I \pi_I + PC_{S_1} \pi_{S_1} + PC_{S_2} \pi_{S_2} + PC_{S_3} \pi_{S_3} + PC_{S_4} \pi_{S_4} \quad (30)$$

where π_A , π_I and π_{S_i} , $i = 1, 2, 3, 4$ are the steady-state probabilities for the 5G BS to be in active mode, idle mode, and sleep mode. PC_i , $i \in \Omega$ is the power consumption in the state i , and power consumption values for each state are given in Table 4.

6.4 Throughput

Throughput is the rate at which the URs get served. The throughput is the fraction of time the system is available to customers multiplied by the servicing rate, which is same as divided by mean service time. Hence, the throughput of the system is defined as follows (Kalita and Selvamuthu 2023),

$$\text{Throughput} = \pi_A \times \frac{1}{E(S)} \quad (31)$$

where $E(S)$ is the mean service time.

7 Numerical Illustration

The numerical results are discussed in this section for all three models. Depending on the model, we presume some reasonable values for the mean sojourn rates or times for numerical illustration. The graphs are plotted for all the models by varying the parameters, and their behavior is analyzed.

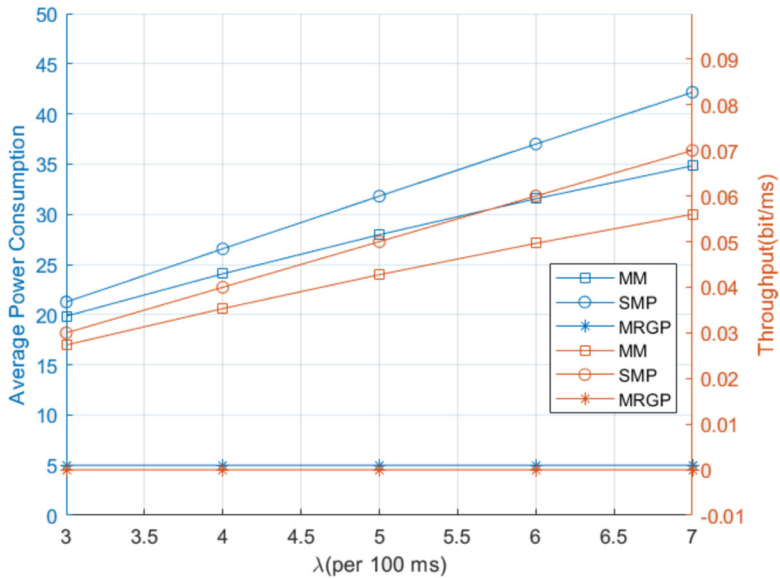
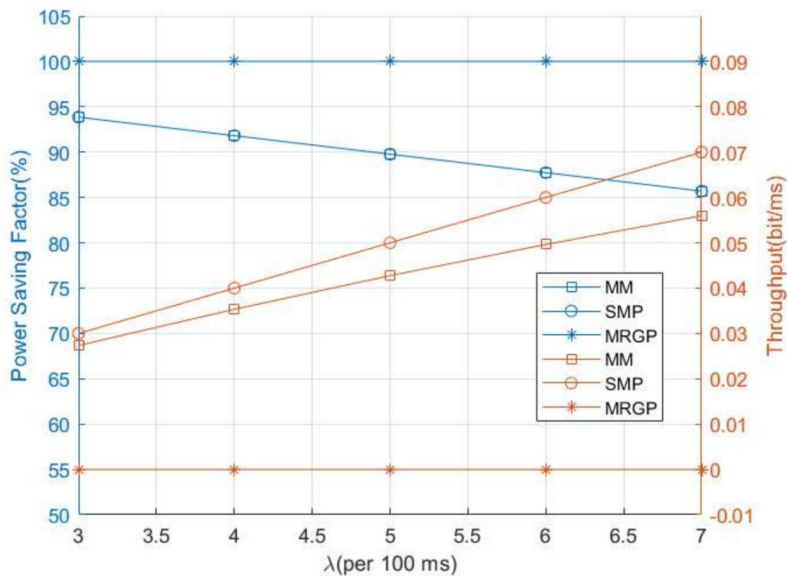
To plot the graphs, the values of parameters are mentioned based on the stability conditions of the proposed models, $\frac{\lambda}{\mu} < 1$, $\frac{\lambda}{\nu_1} < 1$, $\frac{\lambda}{\nu_2} < 1$, $\frac{\lambda}{\nu_3} < 1$ and $\frac{\lambda}{\nu_4} < 1$. Using that the expected sojourn time in mode I needs to be less than expected sojourn time in S_1 mode, i.e., less than expected sojourn time in S_2 mode, i.e. less than expected sojourn time in S_3 mode, i.e., less

Table 4 Power consumption in different modes (Aggarwal and Selvamuthu 2023)

State	Active	Idle	S_1	S_2	S_3	S_4
Power Consumption	250 W	109W	52.3W	14.3W	9.51 W	5W

Table 5 Expected sojourn times in different states (Aggarwal and Selvamuthu 2023)

State	Idle	S_1	S_2	S_3	S_4
Expected Sojourn time	$50 \mu s$	$71 \mu s$	$1 ms$	$10 ms$	$1 s$

**Fig. 6** Tradeoff between power consumption and throughput varying values of λ **Fig. 7** Tradeoff between power saving factor and throughput varying values of λ

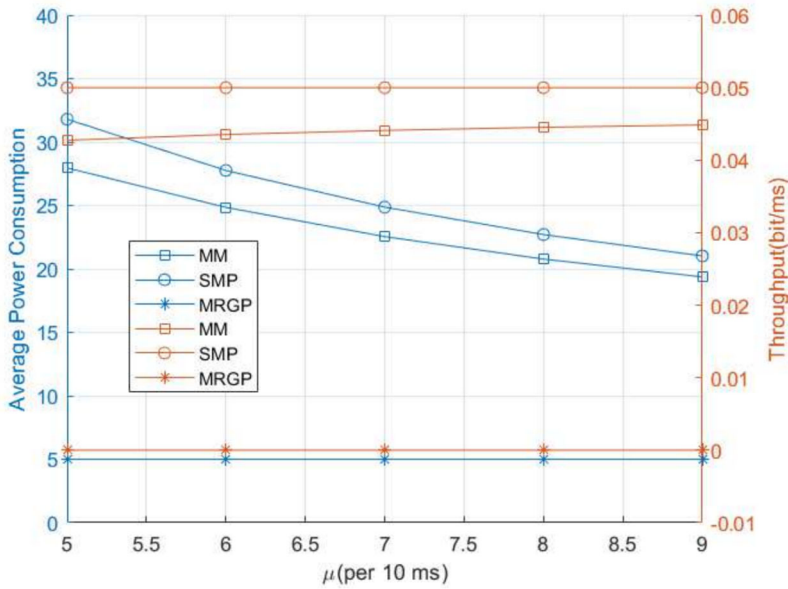


Fig. 8 Tradeoff between power consumption and throughput varying values of μ

than expected sojourn time in S_4 mode, the values of parameters are given in Table 5. The value of λ is 0.05 and μ is 0.5.

To show the results of the Markov model, Semi-Markov model, and Markov regenerative model, *MM*, *SMP*, and *MRGP* are written, respectively.

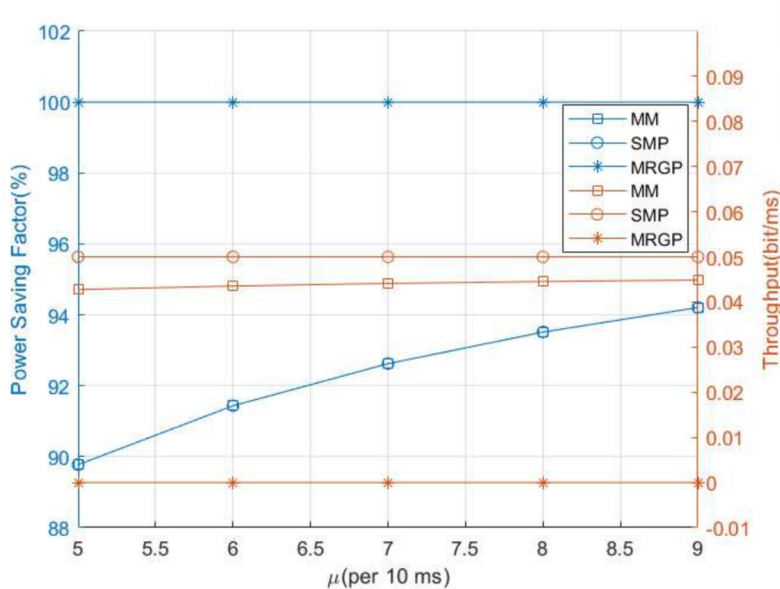
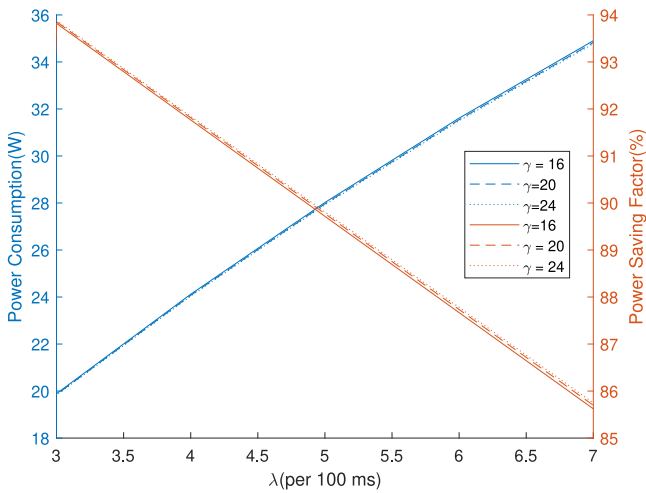


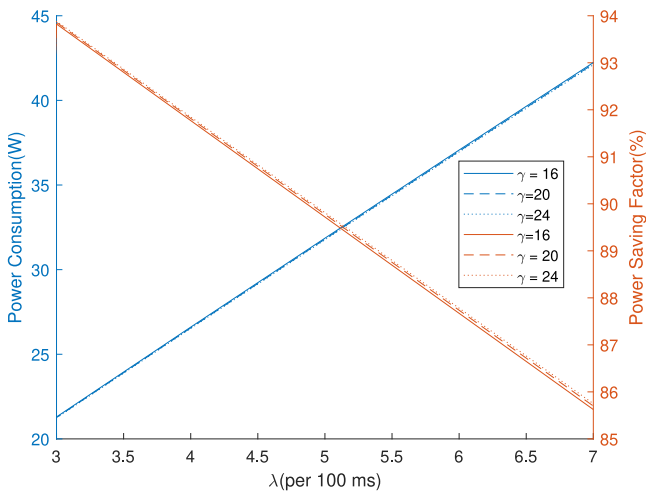
Fig. 9 Tradeoff between power saving factor and throughput varying values of μ

The trade-off between average power consumption and throughput for all the models for varying values of λ is plotted in Fig. 6. It is noticed that the throughput and power consumption, both increase with the increasing value of λ . The power consumption and the throughput are maximum for the *SMP* model, whereas they are minimum for the *MRGP* model.

The trade-off between the power saving factor and throughput for all the models for varying values of λ is plotted in Fig. 7. It is observed that the throughput increases whereas the power saving factor decreases with the increasing value of λ . The power saving factor is maximum for the *MRGP* model and minimum for the *MM*. And, the throughput is maximum for the *SMP* model whereas this is minimum for the *MRGP* model.



(a) Markov model

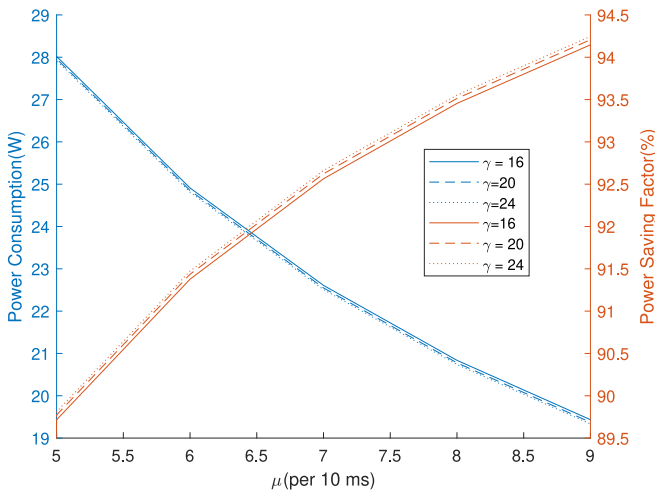


(b) Semi-Markov model

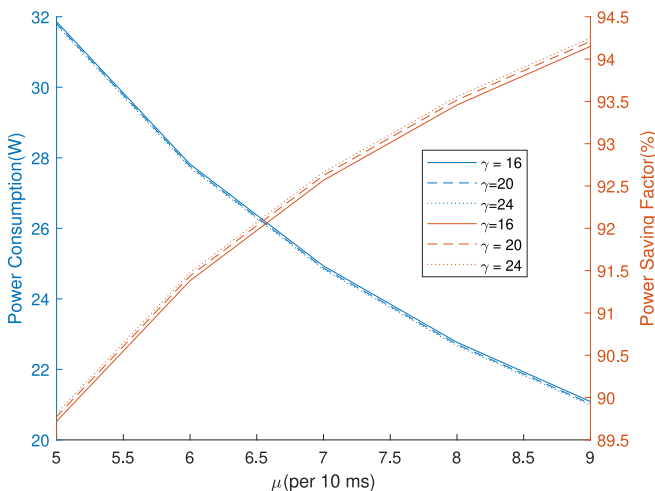
Fig. 10 Tradeoff between power saving factor and power consumption varying values of λ

The tradeoff between power consumption and throughput for all the models for varying values of μ is plotted in Fig. 8. It is observed that the throughput increases whereas power consumption decreases with the increasing value of μ . The power consumption and the throughput are maximum for the *SMP* model, whereas they are minimum for the *MRGP* model.

The tradeoff between the power saving factor and throughput for all the models for varying values of μ is plotted in Fig. 9. It is observed that the throughput, as well as the power saving factor, increases with the increasing value of μ . The power saving factor is maximum for the *MRGP* model and minimum for the *MM* model. And the throughput is maximum for the *SMP* model whereas this is minimum for the *MRGP*.



(a) Markov model



(b) Semi-Markov model

Fig. 11 Tradeoff between power saving factor and power consumption varying values of μ

The tradeoff between the power saving factor and power consumption for varying values of λ and γ is plotted in Fig. 10. It is observed that the power consumption increases and the power saving factor decreases with the increasing value of λ . Power consumption is maximum for a smaller value of γ whereas the power saving factor is maximum for a larger value of γ . The tradeoff between the power saving factor and power consumption for varying values of μ and γ is plotted in Fig. 11. It is observed that the power consumption decreases and the power saving factor increases with the increasing value of μ . Power consumption is maximum for a smaller value of γ whereas the power saving factor is maximum for a larger value of γ .

8 Conclusion and Future Work

In this paper, we investigated three stochastic models to analyze the energy consumption and throughput of a base station, providing valuable insights into their performance under varying assumptions. These models, based on infinite-state continuous-time frameworks, were designed to capture different operational dynamics, allowing us to observe how specific assumptions influence energy efficiency and system throughput. By deriving and leveraging closed-form solutions for steady-state probabilities and expected sojourn times, we quantified key performance metrics, including the energy-saving factor, total energy consumption, and achievable throughput. Our findings highlight the trade-offs between energy efficiency and throughput, offering a comprehensive understanding of how base station performance can be optimized under different operational scenarios. In the future, one can work on the transient analysis for all the models and show the numerical results for the same.

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Data Availability Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

Conflicts of Interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Competing interests The authors declare no competing interests.

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