



Semi-Markov modeling approach for the reliability analysis of a multiple high altitude platform station system

Ishu Jain^a, Anupam Gautam^a^{*}, Dharmaraja Selvamuthu^b, Min Xie^c

^a Department of Mathematics, Netaji Subhas University of Technology, Dwarka, Delhi 110078, India

^b Department of Mathematics, Indian Institute of Technology Delhi, Hauz Khas, Delhi 110016, India

^c City University of Hong Kong Shenzhen Research Institute, Shenzhen, China

ARTICLE INFO

Keywords:

Multiple high altitude platform station system
Internet of things
 k -out-of- n model
Semi-Markov model
Sensitivity analysis
Mean time to failure
Reliability analysis

ABSTRACT

High Altitude Platform Stations (HAPS) are anticipated as a promising solution for providing wide-area coverage and resilient communication networks by deploying wireless infrastructure in the stratosphere, approximately 20 km above the Earth's surface. With their extensive reach and capability to function as backup systems during disasters, HAPS are expected to play a vital role in enhancing mobile broadband capacity, particularly in future wireless networks. In this paper, the concept of deploying multiple HAPS within a single network is explored to deliver fast, stable, and efficient communication services across a region. To ensure the dependability of such a multiple HAPS system, a comprehensive reliability assessment becomes crucial. This research work proposes to consider a k -out-of- n system for estimating the reliability attributes of multiple HAPS system over 5G New Radio (NR) network. While traditional reliability assessments often utilize k -out-of- n frameworks and Markovian analysis for repairable systems, our investigation proposes an improved methodological approach. To assess the impact of system failures on HAPS availability, a Semi Markov Process (SMP) is adopted for probabilistic evaluation. The SMP framework demonstrates superior analytical capabilities compared to conventional methods. Particularly, in capturing time dependent variations in system failure probabilities when non-exponential sojourn times are considered. Additionally, the effectiveness of the SMP model is evaluated by analyzing steady-state probability, availability, reliability, and mean time to failure.

1. Introduction

A three-layer Vertical Heterogeneous Network (VHetNet) is the concept of a significant advancement in the architecture of Fifth Generation (5G) networks. This innovative framework aligns with the ongoing Non-Terrestrial Networks (NTN) initiatives led by the 3rd Generation Partnership Project (3GPP), as detailed in Technical Report 38.811 [1]. The architecture is composed of three interconnected layers: the terrestrial network, the aerial network, and the space network [2]. Among these, High Altitude Platform Stations (HAPS) play a pivotal role in bridging the aerial layer with the others, making them indispensable for the full realization of the VHetNet vision.

A HAPS is a communication platform positioned in the stratosphere, typically at altitudes of 18–24 km above the ground [3]. Powered primarily by solar energy, it can operate continuously for several months without interruption, providing wide-area coverage with service diameter reaching up to 200 km. Compared to Low Earth Orbit (LEO) satellites, HAPS has significantly lower propagation delays (around 1–2 ms) and minimal doppler shifts, while being more cost-effective

and easier to deploy and maintain [4]. HAPS supports a wide range of applications, including Internet of Things (IoT), mobile broadband in rural areas, public safety, autonomous factories, and disaster relief efforts [5]. Numerous HAPS initiatives, such as HAPS Mobile, Google Loon, and Airbus Zephyr, are under development to deliver 4G LTE and 5G (NR) services to sparsely populated regions, enhancing global wireless connectivity.

A significant challenge for HAPS systems lies in designing and managing a self-organized network. A single HAPS can cover a limited area. Intra-HAPS coordination refers to the mechanisms and strategies used to manage and synchronize the operation of multiple HAPS within the same network [5]. Since multiple HAPS system operate in the same stratospheric environment to collectively provide seamless and efficient services, coordination is crucial to avoid interference, optimize resource usage, and ensure reliable communication. By deploying multiple HAPS system, it is possible to provide seamless connectivity across larger regions, including rural, remote, and underserved areas. Multiple HAPS system can create a fail-safe system. If one platform fails

* Corresponding author.

E-mail address: anupam.gautam@nsut.ac.in (A. Gautam).

or requires maintenance, others can continue to provide uninterrupted service, improving the overall reliability of the network. By coordinating multiple HAPS system, the network can support higher data traffic and more users simultaneously, which is critical for applications like mobile broadband and IoT. Further, multiple HAPS system can dynamically adjust their coverage areas based on user demand, atmospheric conditions, or specific service needs, ensuring optimal resource allocation [6].

1.1. Related work

Many studies have investigated establishing communication links between stratospheric HAPS and terrestrial terminals within the telecommunication industry. A brief overview of the key studies in this field is provided below.

HAPS is presented as a groundbreaking solution for delivering communication services to a wide range of smart IoT devices [7–9]. The books [10,11] illustrate how HAPS nodes can be utilized independently or integrated with other terrestrial network components. Additionally, Wang et al. [12] addresses the particular difficulties brought about by the considerable latency and high mobility of HAPS-LEO routes.

To improve the transmission rate, capacity, and Quality of Service (QoS) in Wireless Sensor Networks (WSN) over a specified coverage area, multiple HAPS system or a constellation of interconnected HAPS are typically used [13]. The collaboration of multiple HAPS system into a network can lead to higher data rates, as each HAPS provides moderate data rates, which adds up to a higher overall rate. As the number of HAPS in the network increases, the workload of each HAPS decreases proportionally. By incorporating more HAPS in a coverage area, each HAPS can allocate greater resources to individual users, thereby meeting higher data rate demands [14,15]. The negative effects of component failures, blockage, or weather conditions such as rain or clouds can be mitigated by utilizing space diversity through redundant communication links [16]. One way to ensure link redundancy is through overlapping HAPS coverage, which means there are multiple assets visible to users. For example, a mobile sensor device can select HAPS with the best line of sight among all visible HAPS, reducing service outages and improving overall dependability [17,18].

Operating a HAPS typically requires a ground-based team of two to four personnel, managing mission planning, flight control, sensor operations, and data evaluation. However, deploying multiple HAPS systems as a coordinated network increases operational complexity and costs. To address these technical and economic challenges, integrating autonomy into HAPS control and coordination is essential. Autonomy reduces reliance on human intervention across various operational levels, enabling HAPS systems to make intelligent, collaborative decisions [19].

1.2. Motivation

The rapid growth of IoT technology in 5G (NR) networks presents several challenges, including those related to connectivity, robustness, energy efficiency, and latency. Sometimes, these IoT devices are situated in regions where terrestrial network coverage is either unavailable or difficult and expensive to establish. In such scenarios HAPS offer an attractive solution by enabling communication networks to collect data from IoT devices in a cost-effective and efficient manner. However, the failure of certain components or individual HAPS units can adversely affect the overall system's functionality. This highlights the need for a comprehensive reliability framework to evaluate and enhance system performance effectively. Since, the coverage area in a single HAPS is limited, multiple HAPS operating together can deliver seamless connectivity across vast areas, provides redundancy for continuous service, and handle higher traffic demands for applications like IoT and mobile broadband. These systems facilitate fast handoffs, minimize latency for mobile devices, and dynamically adjust coverage based on user needs

or environmental conditions. As a result, *multiple HAPS systems* play a critical role in meeting the demands of 5G(NR) and future networks, ensuring reliable and adaptable connectivity.

A multiple HAPS system consists of many HAPS. In general, if a system of n HAPS is considered, then the system is non-functional if k or more of its HAPS fail. The complexity of this system necessitates the implementation of a k -out-of- n analytical framework for reliability assessment. This modeling approach proves particularly effective for both evaluating the dependability of multiple HAPS system and establishing optimal system architectures [20]. The random behavior of component failures and repair time is captured very well by stochastic modeling. It accommodates more flexible and realistic assumptions about state transition times, making it well-suited for modeling the dynamic operations of HAPS systems [21–23].

1.2.1. Comparison with existing work

This section evaluates the performance results of our study in comparison with existing literature. Tables 1 and 2 provide an overview highlighting the differences between our work and prior studies.

Table 1 summarizes existing studies on multiple HAPS systems over 5G (NR). Table 2 shows the relevant foundational studies compared to the existing work for the model-based approach. Work like [3,24–26] explore various aspects of the multiple HAPS system, while [25,26] analyze the reliability of the HAPS. The authors in this article [3] discuss architectural design of multiple HAPS system, while [25] examines network integration of this system. Reliability modeling for HAPS systems with k -out-of- n configurations is investigated in [27]. Studies such as [21,28] use Semi Markov Process (SMP) for steady-state and transient solutions, incorporating different failure and repair distributions, while others such as [29,30] employ Continuous Time Markov Chain (CTMC) and Markov Regenerative Process (MRGP) approaches. Metrics vary between works, with some prioritizing availability and others focusing on transient behavior. Our approach differentiates itself by using SMP with steady-state analysis, integrating availability, reliability, and general repair distributions for a comprehensive evaluation.

However, to the best of authors knowledge, no research paper has been published that determines the reliability of a multiple HAPS system. Therefore, this motivates us to addresses all aspects of repairable multiple HAPS systems, including their associated failures and to analytically evaluate its dependability analysis, specifically in terms of reliability and availability.

1.3. Contribution

This study introduces a k -out-of- n repairable model, where a multiple HAPS system comprises n HAPS and ceases to function if $n - k + 1$ or more HAPS fail. We adopt a SMP to model the system, as it ensures the Markov property holds only at some specific points. The SMP follows the exponential distribution at failure and repair times epoch only. This approach makes the proposed model more realistic.

The Semi-Markov Model (SMM) is a model that is formulated based on the SMP methodology. This SMM is utilized to evaluate steady-state results for various dependability attributes of multiple HAPS system. Closed-form steady state expressions for the availability function of multiple HAPS system are obtained. Further, transient results for reliability and availability are obtained. The practicality of the proposed model is validated through numerical illustrations.

The remaining paper is organized as follows. In Section 2, architecture of multiple high altitude platform station system and its mathematical model description are described. The Steady state analysis and its performance measures are discussed in Sections 3 and 4. Sections 5 and 6 is devoted to numerical methods and its theoretical implications used for the reliability analysis, including analytical results are performed. Concluding remarks and a few future directions are given in Section 7.

Table 1
Comparison of existing works with the current one.

References	Platform type	Ideology	Reliability	Availability	k -out-of- n
[3]	HAPS	Highlights different aspects of HAPS, such as their architectural design, strategies for allocating research resources, approaches to channel modeling and possible applications.	No	Yes	No
[5]	HAPS	Focused on how HAPS provide connectivity in the next communication systems.	No	No	No
[6]	HAPS	Emphasizes the possibilities of HAPS, and explains how they possess capabilities to provide services in urban regions.	No	Yes	No
[24]	HAPS	Discusses the Free Space optics (FSO) communication technology, data transmission methods, and the outcomes of experimental trials conducted for HAPS.	Yes	No	No
[25]	Multiple satellite-aerial networks	Examines the routing layer and designs for integrating multiple networks, including satellite, ground, and aerial connections.	No	Yes	No
[26]	HAPS	Concentrates on addressing matters related to the scope and ability to accommodate data traffic by utilizing HAPS.	No	Yes	No
[27]	UAVs	Investigates mathematical models for systems with shared components, consisting of m subsystems both linear and circular, that have k -out-of- n : F configurations.	Yes	No	Yes
[31]	Satellite	Explains the recent developments in scientific, and satellite communications standards.	No	Yes	No
This paper	Multiple HAPS system	The architecture of multiple HAPS system is proposed and its reliability analysis is performed.	Yes	Yes	Yes

Table 2
Comparisons of the existing works about model-based approaches.

No.	Model	Solution type	Distribution		Evaluation metric		
			Failure	Repair	Availability	Reliability	MTTF
[21]	SMP	Steady-state	Exponential	General	✓	X	✓
[29]	CTMC	Steady-state	General	General	✓	X	✓
[30]	MRGP	Steady-state	Exponential	General	X	X	X
[32]	SMP	Steady-state	Exponential	General	X	X	X
[28]	SMP	Steady-state Transient	General	General	✓	X	✓
[33]	SMP	Steady-state	General	General	✓	X	✓
[34]	MRGP	Steady-state	Exponential	General	X	✓	X
[35]	SMP	Transient	Exponential	General	X	✓	X
Ours	SMP	Steady-state	Exponential	General	✓	✓	✓

2. Architecture of multiple high altitude platform station system

The architecture of a multiple HAPS system, as depicted in Fig. 1, is designed to enable seamless wireless communication between the sky and ground segments. The sky segment comprises several HAPS nodes positioned in the stratosphere at an altitude of approximately 20 km. These platforms, resembling aircraft, function as aerial communication hubs equipped with antennas and transceivers to facilitate data transmission and reception. High-speed inter-platform links (IPL) connect these nodes, enabling efficient data exchange, coordination, and redundancy for stable operations. Each HAPS establishes connections via user links and communicates with ground stations using backhaul links [6].

The ground segment includes gateway antennas that connect the HAPS network to terrestrial infrastructure, routing and aggregating data to end-user devices or external networks such as the internet. Various applications are highlighted in Fig. 1, including personal and professional devices (laptops, smartphones), urban connectivity for dense metropolitan areas, and remote or educational services for underserved regions like schools and rural areas [36]. These interconnected

systems showcase the wide coverage and flexibility offered by the HAPS network, making it adaptable to diverse use cases.

This architecture emphasizes HAPS' capability to provide reliable, large-scale connectivity through inter-HAPS link redundancy and efficient ground network operations. Integration with external networks ensures global data exchange, positioning HAPS as an effective solution for urban, rural, and disaster recovery scenarios.

2.1. Mathematical model description

Consider a multiple HAPS system consisting of n HAPS Units. We assume that for the system to remain operational, at least k out of n HAPS must function. The failure of HAPS occurs due to several reasons, such as engine overheating, rotor failure, communication breakdowns, technical issues, etc. [6]. It is assumed that a failed HAPS can be repaired to maintain the proper functioning of the multiple HAPS system.

In this study, we consider a k -out-of- n repairable multiple HAPS system. This means that there are n repairable HAPS out of which

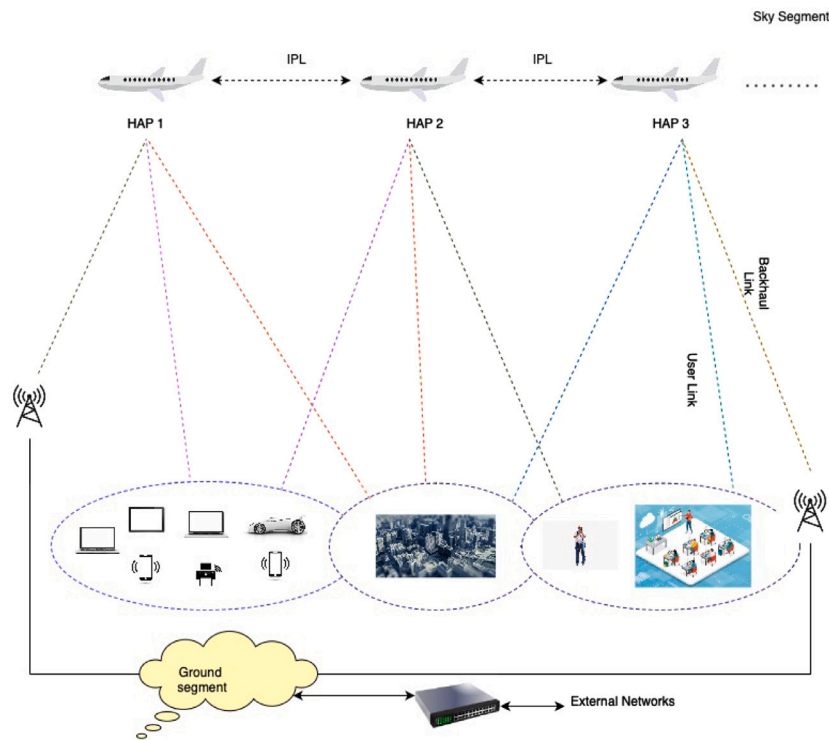


Fig. 1. Multiple HAPS system.

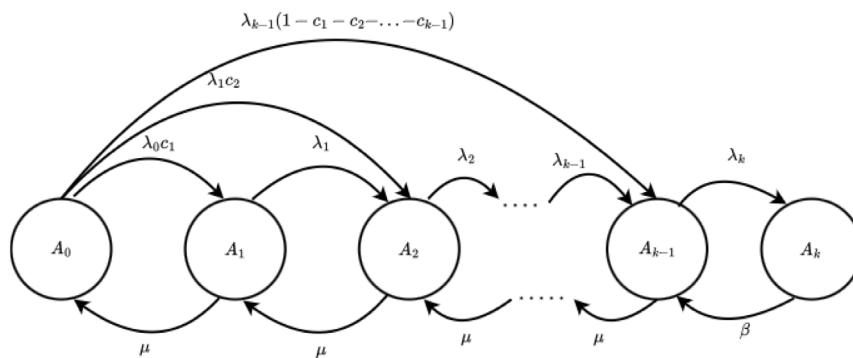


Fig. 2. State transition diagram of k -out-of- n model.

k must be operational. Initially, we assumed that all HAPS are in operational (UP) state. Let us consider a random process $\{Z(t), t \geq 0\}$ in which $Z(t)$ denotes the number of failed HAPS over time t . Therefore, the state space of $\{Z(t), t \geq 0\}$ is given by $S = \{0, 1, \dots, k\}$. Here $j = \overline{0, k}$, denotes the states where j HAPS have failed and remaining $n - j$ HAPS are operational. The failure time of j th HAPS is assumed to follow an exponential distribution with the failure rate $\lambda_j = (n - j)\delta$, where δ denotes the failure rate of individual HAPS. The failures of individual HAPS units are considered to be independent of each other.

Furthermore, it is considered that at any given time t , more than one HAPS can fail only at the starting point. Let c_i , $i = 1, 2, \dots, k-1$, denotes the probability of failure of i HAPS initially such that $\sum_{i=1}^{k-1} c_i = 1$. In addition, it is assumed that these failures are identical and independent of each other. Moreover, these c_i 's are defined as follows:

$$c_i = P\{Z(0^+) = i | Z(0) = 0\}, \quad i = 1, 2, \dots, k-1,$$

In addition, we assumed that there is only one repair device available. While individual components recover automatically, recovery of the system is initiated by the multiple HAPS system coordinator, leading to differences in the associated recovery rates. For $j = 1, k - 1$, the

HAPS repair time is supposed to follow a hypoexponential distribution with parameters μ_1 and μ_2 . In addition, for state k , the repair time of HAPS is assumed to follow a Weibull distribution with parameters β and b . The state transition diagram for this system is depicted in Fig. 2.

2.2. Semi-Markov model

This section presents SMM of the system. In the SMP framework, two random variables operate simultaneously. Let $(\Omega, \mathcal{F}, P)$ be a probability space on which the following two random variables are defined as:

$$X_n : \Omega \rightarrow S, \quad U_n : \Omega \rightarrow \mathbb{R}$$

The variable $X_n = Z(t_n)$, $n \in \mathbb{N}$, denotes the state at the n th transition t_n with state space $S = \{0, 1, \dots, k\}$. Similarly, U_n , where $n \in \mathbb{N}$ represents the time of the n th transition t_n with state space $\mathbb{R}$. It is assumed that the two dimensional stochastic process (X_n, U_n) follows a homogeneous Markov renewal process.

The key distinction between a CTMC and a SMP lies in the distribution of sojourn times. In a Markov framework, this function must follow

an exponential distribution. However, in a SMP, the function can follow general distribution. As a result, the transition intensity is not restricted to a constant rate and may either increase or decrease over time.

The SMP, denoted as $Z = \{Z(t), t \in \mathbb{R}\}$, can now be defined. This process indicates the state occupied at any given waiting time and is expressed as:

$$Z(t) = X_{N(t)}, \quad \text{where } N(t) = \max\{n : U_n \leq t\}.$$

A set of time points, denoted as $\{U_k, k \geq 0\}$, is introduced, where each U_k corresponds to a state in S that satisfies the Markov property at that specific time. It is established that if a process exhibits the same behavior every time it reaches a particular time point, that point is referred to as a regeneration time point. Consequently, all time points in $\{U_k, k \geq 0\}$ serve as regeneration points, forming a renewal sequence that follows the following property:

$$P\{Z(U_n + t) = j, t \geq 0 \mid Z(U_n) = i\} = P\{Z(t) = j, t \geq 0 \mid Z(0) = i\}, \quad i, j \in S.$$

The sequence of states, along with their corresponding time instances, forms an embedded discrete-time Markov chain. Given that certain states exhibit a general distribution for their mean sojourn time and that the time epochs U_k satisfy Markov properties, the resulting stochastic process can be effectively modeled as an SMP. This model is considered as a k -out-of- n system which is depicted in Fig. 2. Therefore, the stochastic process $\{Z(t), t \geq 0\}$ denotes the system's state transitions over the state space $S = \{0, 1, \dots, k\}$.

3. Steady-state analysis

In this section, the steady-state analysis of the SMP for this model determines the availability and reliability of the system. The kernel matrix, $K(t)$, of the SMP, as provided below, fully describes this two-stage method:

$$K(t) = \begin{bmatrix} 0 & K_{0,1}(t) & K_{0,2}(t) & \dots & K_{0,k-2}(t) & K_{0,k-1}(t) & 0 \\ K_{1,0}(t) & 0 & K_{1,2}(t) & \dots & 0 & 0 & 0 \\ 0 & K_{2,1}(t) & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & K_{3,2}(t) & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & K_{k-2,k-1}(t) & 0 \\ 0 & 0 & 0 & \dots & K_{k-1,k-2}(t) & 0 & K_{k-1,k}(t) \\ 0 & 0 & 0 & \dots & 0 & K_{k,k-1}(t) & 0 \end{bmatrix}.$$

The elements of the kernel matrix $K(t)$ can be given as follows:

$$K_{m,n}(t) = P\{Z(U_i) = n, V_i \leq t \mid Z(U_{i-1}) = m\}, \quad (1)$$

where $m, n \in S$ and $V_i = U_i - U_{i-1}$ is the length of the interval.

As established by Pyke [37], the transition probabilities satisfy

$$p_{mn} = \lim_{t \rightarrow \infty} K_{m,n}(t), \quad m, n \in S, \quad t \in \mathbb{R}, \quad (2)$$

where $P = [p_{mn}]$, $m, n \in S$ represents the transition matrix of the embedded Markov chain (EMC) associated with the process.

In this Markov renewal process, we define $K_{m,n}(t)$ as the conditional probability of transitioning from state m to state n within time t . The time-based distribution between states is characterized by $F_{m,n}(t)$, which denotes the cumulative distribution function (CDF) of sojourn times during m to n transitions. This can be formally expressed as

$$F_{m,n}(t) = P\{V_i \leq t \mid Z(U_j) = n, Z(U_{j-1}) = m\}.$$

Furthermore, $f_{m,n}(t)$, $m, n \in S$ represents the probability density function (PDF) of the duration spent in state m before transitioning to state n . The sequential characteristics of state j , $j = \overline{0, k-1}$ specifically the PDF and CDF of time spent before subsequent failure, are defined by the following expressions:

$$f_{0,i}(x) = \begin{cases} c_i \lambda_0 e^{-c_i \lambda_0 x}, & i = \overline{1, k-2} \\ \left(1 - \sum_{i=0}^{k-2} c_i\right) \lambda_0 e^{-\left(1 - \sum_{i=0}^{k-2} c_i\right) \lambda_0 x}, & i = k-1, \end{cases} \quad (3)$$

$$f_{j,j+1}(x) = \lambda_j e^{-\lambda_j x}, \quad j = \overline{1, k-1}$$

$$F_{0,i}(x) = \begin{cases} 1 - e^{-c_i \lambda_0 x}, & i = \overline{1, k-2} \\ 1 - e^{-\left(1 - \sum_{i=0}^{k-2} c_i\right) \lambda_0 x}, & i = k-1, \end{cases} \quad (4)$$

$$F_{j,j+1}(x) = 1 - e^{-\lambda_j x}, \quad j = \overline{1, k-1}.$$

Similarly, the CDF of the time spent in state j , $j = \overline{1, k}$ prior to the completion of the repair of the failed HAPS are expressed as:

$$F_{j,j-1}(x) = 1 - \frac{\mu_2}{\mu_2 - \mu_1} e^{-\mu_1 x} + \frac{\mu_1}{\mu_2 - \mu_1} e^{-\mu_2 x}, \quad \text{where } \mu_1 = \frac{\mu_2}{2}, \quad \mu_1 + \mu_2 = \mu, \quad (5)$$

$$F_{k,k-1}(x) = 1 - e^{-(\beta x)^b}, \quad b > 1.$$

Now, the $\tilde{F}_{i,j}(x)$ is defined as:

$$\tilde{F}_{i,j}(x) := 1 - F_{i,j}(x), \quad i, j \in S.$$

The computation of matrix $K(t)$ elements follows the subsequent analytical procedure.

- For the transition of the system from state 0 to state j for $j = \overline{1, k-2}$, means j components failed, we have

$$\begin{aligned} K_{0,j}(t) &= P\{Z(U_i) = j, V_i \leq t \mid Z(U_{i-1}) = 0\} \quad \forall i, 0 \leq i \leq j \\ &= \frac{P\{(Z(U_i) = j) \cup (Z(U_{i-1}) = 0) \cup (V_i \leq t)\}}{Z(U_{i-1}) = 0}, \\ &= P(V_i \leq t \text{ and } V_i < V_i \quad \forall i \neq j) \\ &= \int_0^t \prod_{i=1, i \neq j}^{k-1} (1 - F_{0,i}(x)) dF_{0,j}(x), \end{aligned} \quad (6)$$

- For the transition of the system from state 0 to state $k-1$, means $k-1$ components failed, we have

$$\begin{aligned} K_{0,k-1}(t) &= P\{Z(U_{k-1}) = k-1, V_{k-1} \leq t \mid Z(U_{k-2}) = 0\}, \\ &= \int_0^t \prod_{i=1, i \neq j}^{k-2} \tilde{F}_{0,i}(x) dF_{0,k-1}(x). \end{aligned} \quad (7)$$

- For the transition of the system from state j to state $j+1$, where $j = \overline{1, k-1}$, means no failed components are repaired and an additional component fails, we have

$$\begin{aligned} K_{j,j+1}(t) &= P\{Z(U_{j+1}) = j+1, V_{j+1} \leq t \mid Z(U_j) = j\}, \\ &= \int_0^t \tilde{F}_{j,j-1}(x) dF_{j,j+1}(x). \end{aligned} \quad (8)$$

- For the transition of the system from state j to state $j-1$, where $j = \overline{1, k-1}$, means a new component does not fail; instead, one of the failed parts is repaired, we have

$$\begin{aligned} K_{j,j-1}(t) &= P\{Z(U_{j-1}) = j-1, V_{j-1} \leq t \mid Z(U_j) = j\}, \\ &= \int_0^t \tilde{F}_{j,j+1}(x) dF_{j,j-1}(x). \end{aligned} \quad (9)$$

- Lastly, when the system moves from state k to state $k-1$, it indicates that one of the failed parts has been repaired, we have

$$\begin{aligned} K_{k,k-1}(t) &= P\{Z(U_{k-1}) = k-1, V_{k-1} \leq t \mid Z(U_k) = k\}, \\ &= \int_0^t dF_{k,k-1}(x). \end{aligned} \quad (10)$$

As $t \rightarrow \infty$, the function $K_{m,n}(t)$ converges to the one-step transition probability p_{mn} of the EMC associated with the given SMP. Additionally, the sum of the elements in each row of $\lim_{t \rightarrow \infty} K(t)$ equals 1.

The one-step transition probabilities for the EMC are represented by the matrix $X = K(\infty)$ given as follows:

$$P = K(\infty) = \begin{bmatrix} 0 & p_{0,1} & p_{0,2} & \dots & p_{0,k-2} & p_{0,k-1} & 0 \\ p_{1,0} & 0 & p_{1,2} & \dots & 0 & 0 & 0 \\ 0 & p_{2,1} & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & p_{3,2} & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & p_{k-2,k-1} & 0 \\ 0 & 0 & 0 & \dots & p_{k-1,k-2} & 0 & p_{k-1,k} \\ 0 & 0 & 0 & \dots & 0 & p_{k,k-1} & 0 \end{bmatrix}.$$

where each p_{mn} are calculated in the [Appendix](#).

The steady-state probabilities, represented by the vector $\vec{N}$, containing N_k for $k \in S$, are determined as the solution to the following system of equations:

$$\vec{N} = \vec{N}K(\infty), \quad \sum_{k \in S} N_k = 1. \quad (11)$$

The steady-state probabilities for each state in the SMP can be calculated using the following method:

$$\pi_k = \frac{N_k v_k}{\sum_{m \in S} N_m v_m}, \quad k \in S. \quad (12)$$

Here, v_k denotes the expected sojourn time in state $k \in S$,

$$v_0 = \int_0^\infty \left(\prod_{i=1}^{k-1} (1 - F_{0,i}(t)) \right) dt, \quad (13)$$

$$v_1 = \int_0^\infty (1 - F_{1,2}(t))(1 - F_{1,0}(t))dt, \quad (14)$$

$$v_2 = \int_0^\infty (1 - F_{2,3}(t))(1 - F_{2,1}(t))dt, \quad (15)$$

$$v_j = \int_0^\infty (1 - F_{j,j+1}(t))(1 - F_{j,j-1}(t))dt, \quad j = \overline{1, k-1} \quad (16)$$

$$v_k = \int_0^\infty (1 - F_{k,k-1}(t))dt. \quad (17)$$

The system's steady-state probabilities are calculated using Eq. (12) and the mean sojourn time values from Eqs. (14)–(17). The Appendix provides a full description of the computation procedure.

In the next section, the system efficiency will be calculated using the steady-state probabilities in each states.

4. Performance metrics

The main objective of this section is to focus on evaluating the performance metrics, particularly the reliability and availability of multiple HAPS system.

1. **Availability:** The degree to which a system is in a given operable and committable state at the beginning of a task, when the task is at random time is called as availability. "The limit of the availability function as time tends to infinity" is the system's steady-state availability [38]. In the suggested model, the system is functional only in states $0, 1, 2, \dots, k-1$. Therefore, the availability may be computed using Eqs. (24), (25), (26), and (27) as follows:

$$A = \sum_{i=0}^{k-1} \pi_i \quad (18)$$

2. **Reliability:** Reliability is defined as the "likelihood that a system will carry out its intended function over a specified time period under defined conditions" [38]. The most popular metric for assessing reliability is MTTF, which is frequently used in software systems to measure reliability. We preassume that the absorbing state is k in order to calculate the MTTF of the HAPS. As a result, the state space S can be divided into two separate sets, represented by the letters M and N , where:

$$M = \{0, 1, 2, 3, \dots, k-1\}, \quad N = \{k\}.$$

Thus, the transformed single-stage transition probability matrix (TPM), denoted by X , as follows,

$$X = \begin{bmatrix} X_M & X_{MN} \\ X_{NM} & X_N \end{bmatrix}.$$

In this case, the transitions within the transient state set are represented by the rate transition matrix X_M , the absorbing state set N is represented by X_N , and the transitions from the transient state set M to the absorbing state set N and vice versa are given by the matrices X_{MN} and X_{NM} respectively [39].

Let initial distribution of X be represented by $\phi(0)$ and $\phi_M(0)$ be a sub-vector that is limited to the transient states with $j \in M$. The following system of equations can be solved to determine the average amount of time spent in each condition j , or v_j :

$$v_j \cdot X_M = -\phi_M(0), \quad \text{for } j \in M. \quad (19)$$

It is assumed that the initial distribution's sub-vector is:

$$\phi_M(0) = [1, 0, \dots, 0].$$

The following formula can be used to calculate MTTF, which stands for the average time needed for the process to go through all transient states before entering any absorption states:

$$MTTF = \sum_{j \in M} N_j \cdot v_j, \quad (20)$$

In the EMC of the SMP, where N_j represents the mean number of transitions to state j occurring in set M prior to system absorption. The following set of equations can be solved to find the value of N_j :

$$N_j = q_j + \sum_{i \in M} N_i \cdot q_{ij}, \quad \forall j \in M. \quad (21)$$

Here, q_{ij} indicates the (i, j) th element of the matrix X , and q_j indicates the likelihood that the EMC begins in state j . It is presumed that state 0 is the starting state in this study. Consequently, the following represents the initial probability vector q :

$$q = [1, 0, \dots, 0], \quad \text{where } q = [q_j].$$

5. Numerical results

This section examines the viability of the suggested reliability model by using analytical results. To demonstrate the numerical results from Sections 3 and 4, a multiple HAPS system is employed (see Fig. 1). It is assumed that the initial parameters utilized for the numerical demonstration have standard values. Real-time data collection, including statistical data collection and initial model parameter estimation, is associated with multiple HAPS system experimental operation and should be handled as a distinct activity.

Consider a 3-out-of-6 multiple HAPS system in which the system becomes operational as long as at least 3 out of its 6 HAPS are functioning. This implies that multiple HAPS system will cease to operate if 3 or more of its HAPS fail. Consequently, a 3-out-of-6 model is applied to represent this scenario. It is further assumed that all HAPS are identical, meaning that they share the same primary performance metrics, specifically the average operating duration until failure. This time is called mean lifetime and denoted by $\alpha = \delta^{-1}$.

Let $\{0, 1, 2, 3\}$ be the state space of the aforementioned model, and let the states of the model be denoted by the number of failed HAPS in the multiple HAPS system. When 3 out of its 6 HAPS are non-functional and one HAPS is restored over a time period of β then multiple HAPS system is considered to be non-operational. To validate the suggested methodology and the computed aspects discussed in Section 4, we provide moderate evaluations along with the states of steady-state probabilities for the multiple HAPS system. Assume $\alpha = 1$, $c_1 = 0.5$,

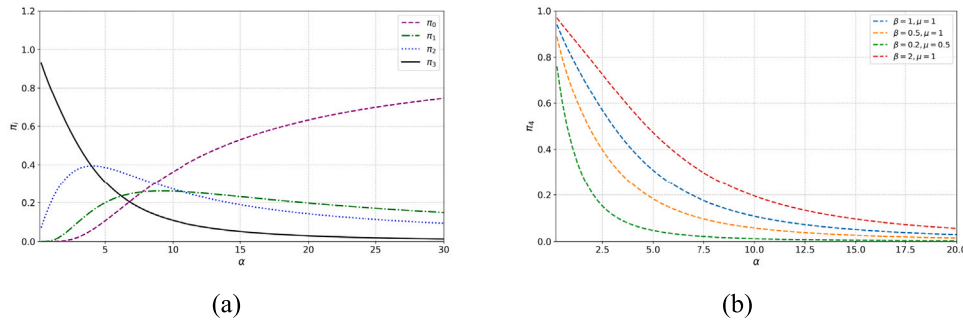


Fig. 3. (a) Steady state probabilities of multiple HAPS system, (b) Failure probability of multiple HAPS system.

$\mu = 1$, $b = 2$ and $\beta = 1$ for this analysis. For each state, the mean sojourn times are as follows:

$$v_0 = \frac{1}{6}, v_1 = \frac{15}{136}, v_2 = \frac{90}{91}, v_3 = 1. \quad (22)$$

By taking into account the failure rate (λ_j) and the repair rate (μ or β) of HAPS, the steady-state probabilities $N_j, j \in [0, 3]$ of the EMC are calculated using Eqs. (29)–(35) as follows:

$$N_0 = 6.19 \times 10^{-5}, N_1 = 9.19 \times 10^{-3}, N_2 = 0.4999, N_3 = 0.4998.$$

Additionally, the steady-state probabilities of the SMP using Eqs. (24)–(28) are determined as the following:

$$\pi_0 = 1.67 \times 10^{-5}, \pi_1 = 2.967 \times 10^{-3}, \pi_2 = 0.1994, \pi_3 = 0.7976.$$

Clearly, the steady-state probabilities satisfy the normalization condition, indicated by $\sum_{j=0}^3 \pi_j = 1$.

In this scenario, the expected number of visits to each state obtained by using Eq. (21) are given as follows:

$$N_0 = 1.1567, N_1 = 1.1934, N_2 = 1.2358. \quad (23)$$

By using Eqs. (20), (22) and (23), we calculate MTTF = 0.7391. For the subsequent plots, we assume that the MTTF of the HAPS changes within the specified interval $\alpha \in [0, 30]$.

5.1. Steady state probabilities

Assume that the failure rate of each HAPS varies over time. Fig. 3(a) illustrates the steady-state probabilities, π_i , of the system states as a function of the mean lifetime of the HAPS, α .

The findings indicate that the probability of failure-free operation, π_0 , increases as the mean lifetime α increases for all HAPS. Conversely, the system probability being in a state of failure, π_3 , declines with increase in α . Unlike π_0 and π_3 , the intermediate probabilities π_1 , and π_2 form non-convex curves. The probability π_1 initially increases for values of $\alpha < 7$ and then decreases to zero as α approaches ∞ . The behavior of other probability curves is similar, with variations occurring at different values of α where their dynamics change. This behavior is expected since the probability of the system being in a functional state generally increases as the mean lifetime grows, aligning with the findings presented in [21].

5.2. Failure probability of HAPS

Along with studying the system's steady-state probabilities, it is also crucial to evaluate its failure probability. This probability is presented in Fig. 3(b), which illustrates the system failure probability for varying repair times, plotted against the mean lifetime of a multiple HAPS system. The figure highlights an inverse relationship between system failure and the mean lifetime of the 3-out-of-6 model. Specifically, the failure probability of the system decreases as α increases for $\beta = 0.2$ and $\mu = 0.5$.

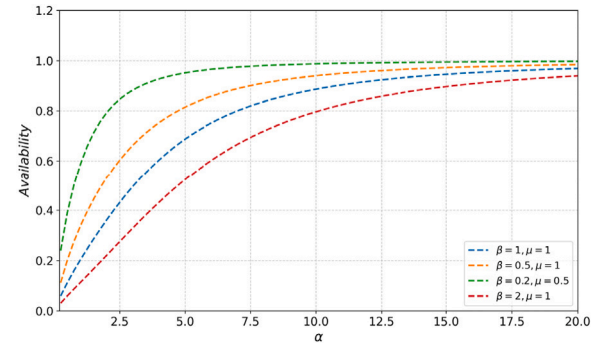


Fig. 4. Availability of multiple HAPS system in case of varying repair times μ and β .

5.3. Availability of a multiple HAPS system

In this section, we analyze the performance metric, availability A in relation to the HAPS mean lifetime α in 3-out-of-6 model, considering various combinations of component repair times μ and system repair times β (refer to Fig. 4).

The graphical results show that as the repair times for both, components μ and the entire system β increase, the system availability of the multiple HAPS system decreases. This dynamics arises because shorter repair time results in greater system availability. Additionally, we observe that the availability increases rapidly as α decreases and approaches to 1.

The most reliable scenario occurs when $\mu = 0.5$ and $\beta = 0.1$ for $\alpha \in [0, 20]$. In this case, at $\alpha = 5$, the availability is nearly 100%. Conversely, for $\mu = 1$ and $\beta = 0.5$, the availability at $\alpha = 5$ is approximately $A \approx 0.8$, reaching 1 only when $\alpha \approx 15$. When $\mu = 1$ and $\beta = 2$, the availability curve lies below all other cases where $\mu \geq \beta$ for $\alpha \in [0, 15]$.

Thus, the lowest availability occurs when the repair time of the multiple HAPS system is greater than the repair time of HAPS, specifically $\mu = 1, \beta = 2$. However, for $\alpha \geq 15$, the availability curves converge and, at $\alpha = 20$, all repair time cases considered for repair time converges to 1, $A \approx 1$.

It is noted that the lifetime of the system is impacted by how long it takes for its components to be repaired, which in turn affects the system's availability. To investigate this, we consider the two scenarios illustrated in Figs. 5(a) and 5(b). In scenario 1, the repair time of the HAPS μ is equal to its mean lifetime ($\mu = \alpha$), which indicates that repairing a non-operational HAPS takes the same amount of time as it did before. In scenario 2, the HAPS repair time is five times shorter than the mean lifetime ($\mu = \frac{\alpha}{5}$).

In both the scenarios, the repair time of the system β varies but remains independent of the components' life and repair times. From the analysis, it becomes clear that the system is more reliable when

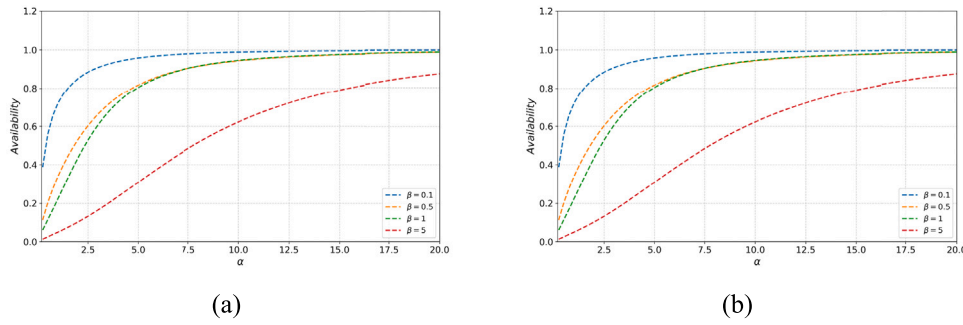


Fig. 5. Availability of multiple HAPS system in case of dependent life and repair times (a) when $\mu = \alpha$, (b) $\mu = \frac{\alpha}{5}$.

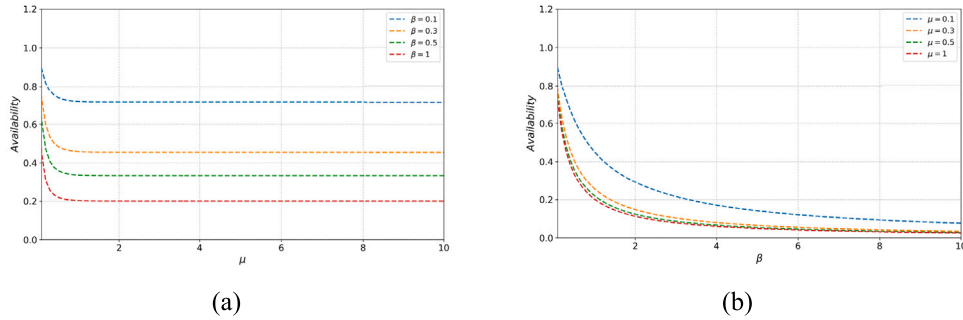


Fig. 6. Availability of multiple HAPS system (a) in case of varying repair times β , (b) in case of varying repair times μ .

the component repair time μ is shorter than its lifetime. This trend is evident in Figs. 5(a) and 5(b).

Additionally, with the increase in system repair time β , the availability decreases. However, the results illustrates that availability, A approaches to 1 with the increase in HAPS mean lifetime α . The convergence of A exhibits a direct relationship with the system's repair parameter β .

Further, assume the component repair time fluctuates within the range $\beta \in [0.1, 10]$, the components mean lifetime is set with $\alpha = 1$, and the parameter β takes different certain values. Fig. 6(a) shows the findings of the experiment.

The results indicate that by keeping α constant and $\mu \rightarrow \infty$, the system repair time β has significant impact on the availability of the 3-out-of-6 model. However, for every value of β , the availability A stabilizes as the component repair time μ grows ($\mu \rightarrow \infty$). This implies that as μ increases, the effect of μ on A decreases, suggesting that A becomes less sensitive to μ .

Next, we analyze the availability of a multiple HAPS system in connection with the repair time of the system. We investigate cases with different values of β and $\mu \in [0.1, 10]$, assuming $\alpha = 1$. The results of this study are shown in Fig. 6(b). As μ rises and $\beta \rightarrow \infty$, the results deviate from the earlier study, demonstrating that availability becomes insensitive to μ with a given α . In particular, availability progressively declines as β increases when $\mu = 0.1$. Nevertheless, availability decreases more quickly at $\mu = 0.3$, and when μ rises further, it terminates to rely on μ , which can be looking at the overlaying curves throughout the whole β range. This indicates that availability becomes insensitive when the mean lifetime approaches to the component repair time.

5.4. Reliability of a multiple HAPS system

This section investigates the reliability analysis of a multiple HAPS system concerning repair time of the system. We analyze cases with different values of β and $\mu \in [0.1, 10]$ assuming $\alpha = 1$. Figs. 7(a) and 7(b) presented the findings of this study.

Unlike the previous analysis, these findings reveal that reliability becomes insensitive to μ when α is fixed, particularly as μ increases and $\beta \rightarrow \infty$. Specifically, for $\mu = 0.1$, reliability decreases gradually as β increases. However, for $\mu = 0.5$, reliability declines more sharply, and beyond a certain point, it no longer depends on μ . It is indicated by the overlapping curves throughout the μ range. This demonstrates that reliability becomes insensitive when the mean lifetime approaches to the component repair time.

The findings reveals that with the fixed mean lifetime, the system reliability is independent of component repair time. In simpler terms, reliability shows insensitivity to μ in cases where recovery of the HAPS increases. Contrary, system recovery time β significantly affects reliability, with shorter β values, leading to enhanced system reliability.

5.5. Mean time to failure

As discussed in Section 4, another key factor influencing system reliability is MTTF. MTTF represents the system expected lifetime until it reaches a state of the failure, which means the repair time of the system β is not considered. For given parameters of the failure-free operational intensity of HAPS δ and their repair time μ , the matrix X_M and its elements q_{ij} for the 3-out-of-6 model are constructed as follows:

$$X_M = \begin{bmatrix} 0 & c_1 & 1 - c_1 \\ e^{-\lambda_1 \mu} & 0 & 1 - e^{-\lambda_1 \mu} \\ 0 & e^{-\lambda_2 \mu} & 0 \end{bmatrix}.$$

Consider $\alpha = 1$ and different values of μ to compute MTTF. Table 3 shows the results of MTTF. It is evident that the repair time μ of HAPS has minimal effect on the average lifespan of the system for smaller values of α . For instance, whether μ is 1 or 0.5, at $\alpha = 0.1$, MTTF stays the same, and at $\alpha = 1$, the difference is insignificant. Moreover, the repair time of the component has a considerable effect on MTTF for $\alpha \geq 5$. This suggests that while the mean lifetime α is stable, MTTF is comparatively insensitive to μ , particularly when component restoration is quick in comparison to α .

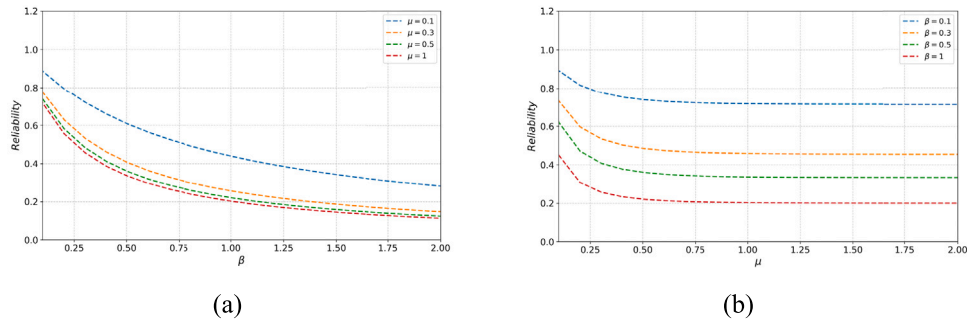


Fig. 7. Reliability of multiple HAPS system (a) in case of varying repair times μ , (b) in case of varying repair times β .

Table 3

MTTF of the multiple HAPS system.

	$\alpha = 0.1$	$\alpha = 1$	$\alpha = 5$	$\alpha = 10$
$\mu = 1$	0.05434	0.062715	5.48869	17.37554
$\mu = 0.5$	0.05434	0.064715	8.48869	74.37554

6. Theoretical implications

The findings demonstrate that increasing the mean lifetime (α) of HAPS components significantly enhances system reliability. In particular, the probability of failure-free operation (π_0) grows with higher values of α , underscoring the importance of improving component reliability to ensure continuous system functionality. In contrast, as α rises, the probability of a complete system failure (π_3) falls, emphasizing the vital role that strong design and efficient maintenance play in preventing catastrophic failures. The intermediate probabilities (π_1, π_2, π_3) exhibit non-linear behaviors, with π_1 rising initially for $\alpha < 7$ before declining to zero as α approaches infinity. This reflects the transitional dynamics between degraded states and full functionality. The shifts in intermediate state probabilities at various α values indicate key reliability thresholds where incremental improvements in α can drive substantial changes in system performance. These insights emphasize the need for a balanced design strategy that maximizes π_0 while minimizing time spent in degraded states. For high-availability applications such as disaster recovery and advanced communication systems, this analysis offers a valuable framework for defining reliability objectives and optimizing HAPS systems to meet demanding operational standards.

This analysis reveals that the failure probability of a multiple HAPS system, as described by the 3-out-of-6 model, is inversely related to the mean lifetime (α) of its components. This relationship underscores the critical role of enhancing the reliability and durability of individual HAPS units to minimize overall system failure. As shown in Fig. 3(b), the failure probability decreases with increasing α for fixed repair rates ($\mu = 0.2$) and failure rate ($\beta = 0.5$), suggesting that systems with higher component lifetimes are more resilient to disruptions. These findings highlight the importance of designing systems with extended operational lifetimes and optimized repair strategies to achieve higher reliability. Furthermore, the results provide valuable insights for system architects in balancing failure rates and repair efficiencies, enabling the development of robust HAPS systems tailored to meet the dependability requirements of critical applications, such as disaster management and next-generation communication networks.

The analysis emphasizes that reducing repair times (μ and β) greatly improves system availability (A), particularly for shorter HAPS mean lifetimes ($\alpha < 15$). Achieving optimal reliability requires minimizing system repair time (β) compared to component repair time (μ). For longer mean lifetimes ($\alpha \geq 15$), availability becomes less dependent on repair times, eventually stabilizing at $A \approx 1$. This underscores the need

to balance efficient repairs and durable components to create reliable and resilient systems. The analysis emphasizes the significant impact of component repair time (μ) and system repair time (β) on system availability (A). When component repair times are shorter than their mean lifetimes ($\mu < \alpha$), system reliability improves substantially. On the other hand, longer system repair times (β) result in decreased availability. As the HAPS mean lifetime (α) increases, the availability (A) approaches 1, with the speed of convergence influenced by β . These insights highlight the necessity of reducing repair times to ensure a reliable and efficient system. The analysis highlights the critical role of component repair time (μ) and system repair time (β) in determining system availability. System reliability improves significantly when component repair times are shorter than their mean lifetimes ($\mu < \alpha$). Conversely, longer system repair times (β) negatively impact availability. As the HAPS mean lifetime (α) increases, availability (A) converges to 1, with the rate of convergence depending on β . These findings underscore the importance of minimizing repair times to achieve a dependable and efficient system.

The analysis demonstrates that while the mean lifetime (α) does not vary, the repair time (μ) of HAPS has little effect on system reliability, especially as $\beta \rightarrow \infty$. For smaller μ values, reliability decreases gradually, while for larger μ , it declines more sharply before stabilizing and becoming independent of μ . This behavior is demonstrated by the overlapping reliability curves across the μ range. The results emphasize that although reliability is insensitive to μ in these scenarios, it is strongly impacted by the system recovery time (β), with shorter β leading to improved reliability.

The results indicate that for mean lifetime values less than 5 ($\alpha < 5$), the impact of component repair time (μ) on the system's MTTF is minimal. For example, changes in μ cause insignificant differences in MTTF at lower α values. However, as α increases ($\alpha \geq 5$), the influence of μ on MTTF becomes more significant. This suggests that when the mean lifetime is consistent, MTTF is largely unaffected by μ , particularly when repair times are relatively short compared to α .

The proposed model provides valuable insights for optimizing the design and operational management of HAPS systems. By enabling reliability predictions across various design configurations and operational scenarios, it supports informed decision making, provided that model parameters can be precisely verified. These challenges emphasize the critical need for accurate estimation of failure parameters (δ) and repair rate parameters μ and β , to ensure model validity. On top of that, putting predictive maintenance into practice in the real world can be challenging, with high initial costs and difficulties in integrating it with existing systems. The assumption of a single repair mechanism further highlights the importance of streamlined maintenance workflows to sustain high system uptime. Ultimately, the model facilitates a structured assessment of key design trade-offs which balances component redundancy, failure tolerance, and reliability targets, thus helping in the selection of system architectures and maintenance protocols for HAPS networks.

7. Conclusion and future directions

This manuscript highlights on assessing the dependability of a multiple HAPS system using redundant HAPS. An analytical model based on a k -out-of- n repairable system is employed for this purpose. While Markov models have traditionally been used to analyze the availability of k -out-of- n repairable systems, this study introduces a comprehensive approach using a SMM. The SMM provides a precise calculation of the failure system probability over time by accounting for the other operational HAPS after some fail. A 3-out-of-6 system is used as a case study for the reliability and availability analysis of the multiple HAPS system. The availability and MTTF for the suggested architecture are computed using closed-form expressions for the steady-state probability of the SMM. The steady-state analysis of the model is demonstrated with numerical examples. Additionally, numerical analysis is conducted to examine how slight variations in the network parameters influence the network's dependability. The analysis identifies the best possible system reliability based on parameters such as the system mean lifetime and repair time.

Our future research will focus on two key areas. We will first examine how dependable HAPS is in recovering from faults or anomalous operating situations using time-dependent analysis. The second one is the interconnectedness of failed HAPS, the correlation between recovery duration and the amount of failed HAPS, and other relevant elements are among the more intricate mathematical models of HAPS failure and recovery that we also hope to investigate.

CRedit authorship contribution statement

Ishu Jain: Writing – original draft, Visualization, Validation, Resources, Methodology, Conceptualization, Writing – review & editing. **Anupam Gautam:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Resources, Project administration, Methodology, Investigation, Funding acquisition. **Dharmaraja Selvamuthu:** Writing – review & editing, Visualization, Validation, Supervision, Software, Resources, Project administration, Funding acquisition, Conceptualization. **Min Xie:** Writing – review & editing, Visualization, Validation, Supervision, Software, Resources.

Consent for publication

The authors affirm that the research article with figures provided are informed consent for publication.

Funding

The reported study has been prepared with the support of DST-RSF research project, with scientific project No. 64800 (DST).

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

Hypoexponential distribution

The CDF of hypo-exponential and Weibull distribution are given as follows:

$$F(x) = 1 - \frac{\mu_2}{\mu_2 - \mu_1} e^{-\mu_1 x} + \frac{\mu_1}{\mu_2 - \mu_1} e^{-\mu_2 x}.$$

$$F_{k,k-1}(x) = 1 - e^{-(\beta x)^b}, b > 1$$

The elements of kernel matrix are computed by using Eq. (6)–(10) as follows.

$$K_{0,j}(t) = \int_0^t \prod_{i=1, i \neq j}^{k-1} (e^{-\lambda_0 c_i x}) d(1 - e^{-\lambda_0 c_j x}) \\ = \lambda_0 c_j \times \left(\sum_{i=1}^{k-1} \lambda_0 c_i \right)^{-1} \times (1 - e^{-(\sum_{i=1}^{k-1} \lambda_0 c_i) t}), t \geq 0 \quad \text{and} \quad \sum_{i=1}^{k-1} c_i = 1.$$

$$K_{0,k-1}(t) = \int_0^t \prod_{i=1, i \neq j}^{k-2} (e^{-\lambda_0 c_j x}) d(1 - e^{-(\lambda_0 \sum_{i=1}^{k-2} c_i) x}) \\ = \lambda_0 \times (1 - \sum_{i=1}^{k-2} c_i) \times \left(\sum_{i=1}^{k-2} \lambda_0 c_i \right)^{-1} \times (1 - e^{-(1 - \sum_{i=1}^{k-2} \lambda_0 c_i) t}), t \geq 0.$$

$$K_{j,j+1}(t) = \int_0^t (1 - F_{j,j-1}(x)) \lambda_j e^{-\lambda_j x} dx \\ = \frac{\lambda_j}{\mu_2 - \mu_1} \left[-\frac{\mu_2}{\mu_1 + \lambda_j} e^{-(\mu_1 + \lambda_j)t} + \frac{\mu_1}{\mu_2 + \lambda_j} e^{-(\mu_2 + \lambda_j)t} \right. \\ \left. + \frac{(\mu_2 - \mu_1)(\mu_1 + \mu_2 + \lambda_j)}{(\mu_1 + \lambda_j)(\mu_2 + \lambda_j)} \right]$$

$$K_{j,j-1}(t) = \int_0^t e^{-\lambda_j x} dF_{j,j-1}(x) \\ = \frac{\mu_1 \mu_2}{\mu_2 - \mu_1} \left[-\frac{1}{\mu_1 + \lambda_j} e^{-(\mu_1 + \lambda_j)t} + \frac{1}{\mu_2 + \lambda_j} e^{-(\mu_2 + \lambda_j)t} \right. \\ \left. + \frac{(\mu_2 - \mu_1)}{(\mu_1 + \lambda_j)(\mu_2 + \lambda_j)} \right]$$

$$K_{k,k-1}(t) = F_{k,k-1}(t) \\ = 1 - e^{-(\beta t)^b}, b > 1$$

Now, the one step transition probabilities of EMC are given as follows:

$$p_{0,j} = \lambda_0 c_j \times \left(\sum_{i=1}^{k-1} \lambda_0 c_i \right)^{-1}, \quad \sum_{i=1}^{k-1} c_i = 1,$$

$$p_{0,k-1} = \lambda_0 \times (1 - \sum_{i=1}^{k-2} c_i) \times \left(\sum_{i=1}^{k-2} \lambda_0 c_i \right)^{-1},$$

$$p_{j,j+1} = \frac{\lambda_j(\mu_1 + \mu_2 + \lambda_j)}{(\mu_1 + \lambda_j)(\mu_2 + \lambda_j)},$$

$$p_{j,j-1} = \frac{\mu_1 \cdot \mu_2}{(\mu_1 + \lambda_j)(\mu_2 + \lambda_j)},$$

$$p_{k,k-1} = 1.$$

On solving the set of Eqs. (12), the steady-state probabilities can be obtained in the following form.

$$\pi_0 = \frac{1}{A'C}, \quad (24)$$

$$\pi_1 = \left[\frac{\lambda_1(\mu_1 + \mu_2 + \lambda_1)}{(\mu_1 + \lambda_1)(\mu_2 + \lambda_1)} \right] C, \quad (25)$$

$$\pi_2 = \frac{1}{\lambda_2 C} \left(e^{\lambda_1 \mu_2} - \frac{\lambda_0 c_1}{A'} \right) \left[\frac{\lambda_2(\mu_1 + \mu_2 + \lambda_2)}{(\mu_1 + \lambda_2)(\mu_2 + \lambda_2)} \right], \quad (26)$$

$$\pi_j = \frac{1}{\lambda_j C} \prod_{i=1}^j \left[\frac{\lambda_i(\mu_1 + \mu_2 + \lambda_i)}{(\mu_1 + \lambda_i)(\mu_2 + \lambda_i)} \right] \left(e^{\lambda_{i-1} \mu_2} - \frac{\lambda_0 c_i}{A'} \right), \quad (27)$$

$$j = \overline{1, k-1},$$

$$\pi_k = \frac{1}{C\beta b} \Gamma\left(\frac{1}{b}\right) \prod_{i=1}^{k-1} \left[\frac{\lambda_i(\mu_1 + \mu_2 + \lambda_i)}{(\mu_1 + \lambda_i)(\mu_2 + \lambda_i)} \right] \left(e^{\lambda_{i-1} \mu_2} - \frac{\lambda_0 c_i}{A'} \right). \quad (28)$$

$$\text{where } C = \frac{1}{A'} + \frac{e^{\lambda_1 \mu} - 1}{\lambda_1} + \sum_{j=2}^{k-1} (e^{\lambda_j \mu} - 1) \prod_{i=1}^{j-1} \left(e^{\lambda_i \mu} - \frac{\lambda_{i-1} c_{i-1}}{A'} \right) \\ + \prod_{i=1}^{k-1} (e^{\lambda_i \mu} - 1),$$

$$\text{and } A' = \sum_{i=1}^{k-1} c_i$$

The steady-state probabilities, represented by the vector $\vec{N}$, containing N_k for $k \in S$, are determined as

$$N_1 = \frac{(\lambda_1 + \mu_1)(\lambda_1 + \mu_2)}{\mu_1 \mu_2} N_0 \quad (29)$$

$$N_2 = \left(\frac{(\lambda_1 + \mu_1)(\lambda_1 + \mu_2)}{\mu_1 \mu_2} - \frac{c_1}{A'} \right) N_0 \quad (30)$$

$$N_3 = \frac{(\lambda_2 + \mu_1)(\lambda_2 + \mu_2)}{\mu_1 \mu_2} \left(\frac{(\lambda_1 + \mu_1)(\lambda_1 + \mu_2)}{\mu_1 \mu_2} - c_2 \right) N_0 \quad (31)$$

$$N_j = \frac{(\lambda_j + \mu_1)(\lambda_j + \mu_2)}{\mu_1 \mu_2} \prod_{i=1}^{j-1} \left(\frac{(\lambda_i + \mu_1)(\lambda_i + \mu_2)}{\mu_1 \mu_2} - \frac{c_{i-1}}{A'} \right) N_0, \quad (32)$$

$$j = 2, k-2, \quad (33)$$

$$N_{k-1} = \frac{(\lambda_{k-1} + \mu_1)(\lambda_{k-1} + \mu_2)}{\mu_1 \mu_2} \prod_{i=1}^{k-2} \left(\frac{(\lambda_{i-1} + \mu_1)(\lambda_{i-1} + \mu_2)}{\mu_1 \mu_2} - \frac{c_{i-1}}{A'} \right) N_0 \quad (34)$$

$$N_k = \prod_{i=1}^{k-1} \left(\frac{(\lambda_i + \mu_1)(\lambda_i + \mu_2)}{\mu_1 \mu_2} - \frac{c_{i-1}}{A'} \right) N_0, \quad (35)$$

As $\sum_{k \in S} N_k = 1$, and by using Eqs. (29)–(35), we have $N_0 = 1/B$, where

$$B = \left[1 + \frac{(\lambda_1 + \mu_1)(\lambda_1 + \mu_2)}{\mu_1 \mu_2} \right] + \sum_{j=2}^{k-1} \left(\frac{(\lambda_j + \mu_1)(\lambda_j + \mu_2)}{\mu_1 \mu_2} - \frac{c_{j-1}}{A'} \right) \frac{(\lambda_{j-1} + \mu_1)(\lambda_{j-1} + \mu_2)}{\mu_1 \mu_2}, \quad (36)$$

Here, v_k denotes the expected sojourn time in state $k \in S$.

$$v_0 = \int_0^\infty e^{-(\sum_{i=1}^k \lambda_0 c_i)t} dt, \\ = \left(\sum_{i=1}^k \lambda_0 c_i \right)^{-1}, \quad (37)$$

$$v_1 = \int_0^\infty e^{-\lambda_1 t} \left[\frac{\mu_2}{\mu_2 - \mu_1} e^{-\mu_1 t} - \frac{\mu_1}{\mu_2 - \mu_1} e^{-\mu_2 t} \right] dt, \\ = \frac{\lambda_1(\mu_1 + \mu_2 + \lambda_1)}{(\mu_1 + \lambda_1)(\mu_2 + \lambda_1)}, \quad (38)$$

$$v_2 = \int_0^\infty e^{-\lambda_2 t} \left[\frac{\mu_2}{\mu_2 - \mu_1} e^{-\mu_1 t} - \frac{\mu_1}{\mu_2 - \mu_1} e^{-\mu_2 t} \right] dt, \\ = \frac{\lambda_2(\mu_1 + \mu_2 + \lambda_2)}{(\mu_1 + \lambda_2)(\mu_2 + \lambda_2)}, \quad (39)$$

$$v_j = \int_0^\infty e^{-\lambda_j t} \left[\frac{\mu_2}{\mu_2 - \mu_1} e^{-\mu_1 t} - \frac{\mu_1}{\mu_2 - \mu_1} e^{-\mu_2 t} \right] dt, \\ = \frac{\lambda_j(\mu_1 + \mu_2 + \lambda_j)}{(\mu_1 + \lambda_j)(\mu_2 + \lambda_j)}, \quad j = 1, k-1 \quad (40)$$

$$v_k = \frac{1}{\beta b} \Gamma \left(\frac{1}{b} \right) \quad (41)$$

Data availability

No data was used for the research described in the article.

References

- [1] 3GPP. Study on New Radio (NR) to support non-terrestrial networks V15.2.0 (rel. 15). 2019, [Accessed 06 July 2019].
- [2] Cao X, Yang P, Alzenad M, Xi X, Wu D, Yanikomeroglu H. Airborne communication networks: A survey. *IEEE J Sel Areas Commun* 2018;36(9):1907–26.
- [3] Karapantazis S, Pavlidou F. Broadband communications via high-altitude platforms: A survey. *IEEE Commun Surv Tutor* 2005;7(1):2–31.
- [4] Hsieh F, Jardel F, Visotsky E, Vook F, Ghosh A, Picha B. UAV-based multi-cell HAPS communication: System design and performance evaluation. In: *GLOBECOM 2020-2020 IEEE global communications conference*. IEEE; 2020, p. 1–6.
- [5] Mohammed A, Mehmood A, Pavlidou F-N, Mohorcic M. The role of high-altitude platforms (HAPs) in the global wireless connectivity. *Proc IEEE* 2011;99(11):1939–53.
- [6] Kurt GK, Khoshkholgh MG, Alfattani S, Ibrahim A, Darwish TS, Alam MS, Yanikomeroglu H, Yongacoglu A. A vision and framework for the high altitude platform station (HAPS) networks of the future. *IEEE Commun Surv Tutor* 2021;23(2):729–79.
- [7] Wang W-Q, Jiang D. Integrated wireless sensor systems via near-space and satellite platforms: A review. *IEEE Sens J* 2014;14(11):3903–14.
- [8] Abbasi O, Yanikomeroglu H, Kaddoum G. Hemispherical antenna array architecture for high-altitude platform stations (HAPS) for uniform capacity provision. *IEEE Trans Wirel Commun* 2024.
- [9] Khanna A, Kaur S. Internet of things (IoT), applications and challenges: a comprehensive review. *Wirel Pers Commun* 2020;114:1687–762.
- [10] Aragón-Zavala AA, Cuevas-Ruiz JL, Delgado-Penín JA. High-altitude platforms for wireless communications. John Wiley & Sons; 2008.
- [11] Grace D, Mohorcic M. Broadband communications via high altitude platforms. John Wiley & Sons; 2011.
- [12] Wang J, Gong S, Xiao J, Guo P, Wang J, Xie W, Li X. A lightweight channel prediction network for UAV-LEO satellite communications. *IEEE Wirel Commun Lett* 2024.
- [13] Shah AS, Karabulut MA, Rabie K. Multiple access schemes for 6G enabled NTN-assisted IoT technologies: recent developments, prospects and challenges. *IEEE Internet Things Mag* 2024;7(1):48–54.
- [14] Shaw GB. The generalized information network analysis methodology for distributed satellite systems (Ph.D. thesis), Massachusetts Institute of Technology; 1999.
- [15] Yang Y, Guo X, Hai X, Feng Q, Sun B, Wang Z. Modeling and vulnerability analysis of UAV swarm based on two-layer multi-edge complex network. *Reliab Eng Syst Saf* 2024;110779.
- [16] Vázquez-Castro M, Belay-Zelek D, Curieses-Guerrero A. Availability of systems based on satellites with spatial diversity and HAPS. *Electron Lett* 2002;38(6):1.
- [17] Grace D, Thornton J, Chen G, White GP, Tozer TC. Improving the system capacity of broadband services using multiple high-altitude platforms. *IEEE Trans Wirel Commun* 2005;4(2):700–9.
- [18] Gautam A, Jain I. Stochastic fluid queue modelling and its analysis for power saving in high altitude platform station system: an analytical approach. *OPSEARCH* 2025;1–22.
- [19] Ryu K, Kim W. Energy efficient deployment of aerial base stations for mobile users in multi-hop UAV networks. *Ad Hoc Networks* 2024;157:103463.
- [20] Wang C, Wang X, Xing L, Guan Q, Yang C, Yu M. Efficient reliability approximation for large k-out-of-n cold standby systems with position-dependent component lifetime distributions. *Reliab Eng Syst Saf* 2023;240:109548.
- [21] Choudhary N, Khaitan nee Gupta V. High altitude aeronautical platform for VoIP: Dependability analysis. *Wirel Pers Commun* 2022;125(3):2277–303.
- [22] Wang C, Xing L, Yu J, Guan Q, Yang C, Yu M. Phase reduction for efficient reliability analysis of dynamic k-out-of-n phased mission systems. *Reliab Eng Syst Saf* 2023;237:109349.
- [23] Zhang J, Fouladirad M, Limnios N. Sensitivity analysis of an imperfect maintenance policy for Proton-exchange membrane fuel cell based on geometric a semi-Markov model. *Reliab Eng Syst Saf* 2025;255:110674.
- [24] Sharma M, Chadha D, Chandra V. High-altitude platform for free-space optical communication: Performance evaluation and reliability analysis. *J Opt Commun Netw* 2016;8(8):600–9.
- [25] Liu J, Shi Y, Fadlullah ZM, Kato N. Space-air-ground integrated network: A survey. *IEEE Commun Surv Tutor* 2018;20(4):2714–41.
- [26] Arum SC, Grace D, Mitchell PD. A review of wireless communication using high-altitude platforms for extended coverage and capacity. *Comput Commun* 2020;157:232–56.
- [27] Yin J, Cui L, Sun Y, Balakrishnan N. Reliability modelling for linear and circular k-out-of-n: F systems with shared components. *Reliab Eng Syst Saf* 2022;219:108172.
- [28] Bai J, Chang X, Machida F, Trivedi KS. Understanding container-based services under software aging: Dependability and performance views. *IEEE Trans Sustain Comput* 2024.
- [29] Lalropuia K, Khaitan V. Availability and reliability analysis of cloud computing under economic denial of sustainability (EDoS) attack: a semi-Markov approach. *Clust Comput* 2021;24:2177–91.
- [30] Raj R. Stochastic modeling of enhancing energy efficiency for HAP in HAP-LEO systems using Markov regenerative process. *Comput Netw* 2024;242:110253.
- [31] Kodheli O, Lagunas E, Maturro N, Sharma SK, Shankar B, Montoya JFM, Duncan JCM, Spano D, Chatzinotas S, Kisseleff S, et al. Satellite communications in the new space era: A survey and future challenges. *IEEE Commun Surv Tutor* 2020;23(1):70–109.
- [32] Raj R. Stochastic modelling of multi-layer HAP-LEO systems in 6G for energy saving: An analytical approach. *Comput Commun* 2023;210:22–34.

- [33] Bai J, Li Y, Chang X, Machida F, Trivedi KS. Understanding NFV-enabled vehicle platooning application: A dependability view. *IEEE Trans Cloud Comput* 2023;11(4):3367–80.
- [34] Bai J, Chang X, Ning G, Zhang Z, Trivedi KS. Service availability analysis in a virtualized system: A markov regenerative model approach. *IEEE Trans Cloud Comput* 2020;10(3):2118–30.
- [35] D'Amico G, Raj R, Selvamuthu D. Reliability analysis of a K-out-of-N system with random rate of repair. *Probab Engrg Inform Sci* 2025;1–30.
- [36] Vittori C. Analysis of high altitude platform stations for network reliability improvement (Ph.D. thesis), Politecnico di Torino; 2024.
- [37] Pyke R. Markov renewal processes with finitely many states. *Ann Math Stat* 1961;1243–59.
- [38] Trivedi KS. Probability and statistics with reliability, queuing, and computer science applications. John Wiley & Sons; 2001, p. 1–313.
- [39] Trivedi KS, Bobbio A. Reliability and availability engineering: modeling, analysis, and applications. Cambridge University Press; 2017.